Training Weekend 1989

GEOMETRY TEST

Time allowed : 1½ hours

1. A, B are two points on a sphere whose centre is 0 and whose radius is r. AB subtends an angle 2θ at 0. Two planes are drawn through AB, each plane inclined at an angle α to the plane AOB. C_1 and C_2 are the centres of the circles in which these planes cut the sphere. Find the length C_1C_2.

2. ABC is a triangle; D, E, F are the feet of the perpendiculars from A, B, C respectively to the opposite sides; D', E', F' are the midpoints of EF, FD, DE respectively.
    The line E'F' meets CA at Q and AB at R';
            F'D' meets AB at R and BC at P';
            D'E' meets BC at P and CA at Q'.
    Prove that
    (i) $QR' = RP' = PQ'$;
    (ii) P, P', Q, Q', R, R' lie on a circle.

3. ABCD is a quadrilateral inscribed in a circle. BD meets AC at X and (produced) passes through the point of intersection T of the tangents at A and C. Prove that the perpendicular distances of X from the sides AB, BC, CD, DA of the quadrilateral are proportional to the lengths of those sides.