

# FST 2 2010

Trinity College, Cambridge

12<sup>th</sup> April 2010

4. Find all solutions to

$$p(p+1) + q(q+1) = n(n+1)$$

where  $p$  and  $q$  are prime numbers and  $n$  is a positive integer.

5. Let  $S$  be a set of 1953 points in the plane. Every two points of  $S$  are at least distance 1 apart. Prove that  $S$  contains a subset  $T$  of 217 points, every two at least distance  $\sqrt{3}$  apart.
6. The monic polynomial

$$P(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$$

of degree  $n > 1$  has  $n$  distinct negative real roots. Prove that

$$a_1P(1) > 2n^2a_0.$$

*Each question is worth seven marks.  
Time: 4 hours, 30 minutes.*