FST2 2009

OCR

TCC April 6

1. Triangle ABC has a right-angle at C, and the point M on AB is strictly between A and B. Let S, S_1 and S_2 denote the circumcentres of $\triangle ABC$, $\triangle AMC$ and $\triangle MBC$ respectively.

   (a) Show that the points M, C, S, S_1 and S_2 lie on a circle.

   (b) For which position of M does this circle have the least radius?

2. The sequence $a_1, a_2, a_3, \ldots$ is defined by $a_1 = 3, a_2 = 11$ and

$$a_n = 4a_{n-1} - a_{n-2}$$

   for $n \geq 3$. Prove that each term of the sequence is of the form $a^2 + 2b^2$ for some integers a and b.

3. Find all pairs of primes (p, q) such that $p^p + q^q + 1$ is divisible by pq.