# First Selection Test: Paper 1

Trinity College, Cambridge

$16^{\text{th}}$ April 2011

1. Let $ABC$ be an acute triangle with $D$, $E$, $F$ the feet of the altitudes lying on $BC$, $CA$, $AB$ respectively. One of the intersection points of the line $EF$ and the circumcircle is $P$. The lines $BP$ and $DF$ meet at point $Q$. Prove that $AP = AQ$.

2. There is a conference attended by 512 mathematicians, who have been assigned to share 256 twin rooms. Each is interested in some subset of the following nine subjects: algebraic topology, Banach spaces, combinatorics, differential manifolds, Euclidean geometry, fluid dynamics, group theory, harmonic analysis and inequalities. Every mathematician has a distinct set of interests (so, in particular, one is interested in nothing, and one in everything).

   Show that, at the conference dinner, they can be sat in one big circle such that everyone is sat next to his roommate, and such that, if two people are sat next to one another who are not roommates, then they have sets of interests which are identical except for one subject.

3. Let $x_1, \ldots, x_{100}$ be nonnegative real numbers such that $x_i + x_{i+1} + x_{i+2} \leq 1$ for all $i$ (we interpret subscripts modulo 100 so that $x_{101} = x_1$, and so on).

   Find the maximal possible value of the sum

$$S = \sum_{i=1}^{100} x_i x_{i+2}.$$

*Each question is worth seven marks.*
*Time: 4 hours, 30 minutes.*