FST 1 2010

Trinity College, Cambridge

10^{th} April 2010

1. Find all polynomials $P(x)$ with real coefficients which have the property that if a is a real number and $P(a)$ is an integer, then a is an integer.

2. Let $ABCD$ be a trapezium with AB parallel to DC and $|AB| > |CD|$. Let E and F be points on the segments AB and DC respectively, such that $AE{:}\ EB = DF{:}\ FC$. Let K and L be points on the segment EF such that

$$\angle BKA = \angle BCD \qquad \text{and} \qquad \angle DLC = \angle ABC.$$

   Show that K, L, B and C are concyclic.

3. Does there exist a positive integer n satisfying the following condition?

   For each rational number r there exist an integer b and nonzero integers $a_1, a_2, \ldots, a_n$ such that

$$r = b + \frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}.$$

Each question is worth seven marks.
Time: 4 hours, 30 minutes.