FST1 2009

OCR

TCC April 4

1. Find all prime numbers p such that

$$\frac{2^{p-1}-1}{p}$$

is a perfect square.

2. Each point of the plane is painted one of three colours. Show that there exists a triangle in the plane such that the following three conditions are satisfied:

   (a) The three vertices have the same colour.

   (b) The radius of the circumcircle of the triangle is 2009.

   (c) One angle of the triangle is either two or three times larger than one of the other two angles of the triangle.

3. Let the quadrilateral $ABCD$ be inscribed in a circle with centre O. Suppose that $\angle B$ and $\angle C$ are both obtuse. Let the lines AB and CD meet at E. Let P are R be the feet of the perpendiculars dropped from E to the lines BC and AD respectively. Let Q be the intersection of the lines EP and AD, and S be the intersection of ER and BC. Let K be the midpoint of the line segment QS. Prove that E, K and O are collinear.