FINAL SELECTION TEST

SUNDAY 2 APRIL 1995

1. Given any collection $\mathcal{K}$ of 1995 heaps of stones, each containing some positive number of stones ≤ 1995, you have to remove all the stones using a sequence of moves of the following kind:

on each move you choose certain heaps, and remove the same number of stones from each of these heaps, leaving the remaining heaps unchanged.

For each initial collection $\mathcal{K}$, let k denote the minimum number of moves needed. Find the maximum possible value of k (as $\mathcal{K}$ varies over all possible arrangements).

2. A positive integer is a *3d-number* if it is divisible by the sum of its decimal digits. Prove that every positive integer is a divisor of some 3d-number.

3. *ABC* is a triangle. Its incircle touches the sides *BC*, *CA*, *AB* at *X*, *Y*, *Z* respectively. The points *A'*, *B'*, *C'* are the feet of the perpendiculars from *X* to *YZ*, from *Y* to *ZX*, and from *Z* to *XY* respectively.

   (i) Prove that *AA'*, *BB'*, *CC'* are concurrent.

   (ii) Prove that $\text{area}\{\Delta ABC)\} \geq 16 \cdot \text{area}\{\Delta(A'B'C')\}$.

Time allowed: 4 1/2 hours.

Marks: 7 marks per question