BRITISH MATHEMATICAL OLYMPIAD COMMITTEE

FINAL SELECTION TEST

Sunday 10th April 1994

Time allowed : $4\frac{1}{2}$ hours

1. Prove that if a, b, c are positive real numbers, then

$$a^5 + b^5 + c^5 \geq 5abc(b^2 - ac)$$

and find when equality occurs.

2. Let S be the circumcircle of an isosceles triangle ABC with $AB = AC$. The point P lies on the arc BC of S on the opposite side of BC to A. Let X be the point on AP such that $AX = AC$, and Y be the point on BP such that $BY = BC$.
   (i) Prove that C, Y, P, X lie on a circle.
   (ii) Prove that YX, the tangent at B to the circumcircle S and the line through C perpendicular to AB are concurrent.

3. Dwarves, as everyone knows, have a sweet tooth. This Easter the ten younger members of the Took family were each allowed to choose three different sorts of Easter Egg. To add to the fun (and to minimise jealousy), each pair of children had to choose at least one sort of Egg in common. If the most popular sort of Egg was chosen by n children, what is the smallest possible value of n?