EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'
Thursday 17th March 2016

Organised by the United Kingdom Mathematics Trust and the
Association Kangourou Sans Frontières

This competition is being taken by 6 million students in over 50 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.

2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.

3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.

4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.

5. **Use B or HB pencil only**. For each question, mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.

6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.

7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.

8. The questions on this paper challenge you **to think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office,
School of Mathematics Satellite, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. Which of the following numbers is the closest to the value of $\dfrac{17 \times 0.3 \times 20.16}{999}$?

    A 0.01   B 0.1   C 1   D 10   E 100

2. Four of the following points are vertices of the same square. Which point is not a vertex of this square?

    A (−1, 3)   B (0, −4)   C (−2, −1)   D (1, 1)   E (3, −2)

3. When the positive integer x is divided by 6, the remainder is 3. What is the remainder when $3x$ is divided by 6?

    A 4   B 3   C 2   D 1   E 0

4. How many weeks are equivalent to 2016 hours?

    A 6   B 8   C 10   D 12   E 16

5. Little Lucas invented his own way to write down negative numbers before he learned the usual way with the minus sign in front. Counting backwards, he would write: 3, 2, 1, 0, 00, 000, 0000, …. What is the result of 000 + 0000 in his notation?

    A 1   B 00000   C 000000   D 0000000   E 00000000

6. Marie changed her dice by replacing 1, 3, and 5 with −1, −3 and −5 respectively. She left the even numbers unchanged. If she throws two such dice, which of the following totals cannot be achieved?

    A 3   B 4   C 5   D 7   E 8

7. Angelo wrote down the word TEAM. He then swapped two adjacent letters around and wrote down the new order of the letters. He proceeded in this way until he obtained the word MATE. What is the least number of swaps that Angelo could have used?

    A 3   B 4   C 5   D 6   E 7

8. Sven wrote five different one-digit positive integers on a blackboard. He discovered that none of the sums of two different numbers on the board equalled 10. Which of the following numbers did Sven definitely write on the blackboard?

    A 1   B 2   C 3   D 4   E 5

9. Four numbers a, b, c, d are such that $a + 5 = b^2 − 1 = c^2 + 3 = d − 4$. Which of them is the largest?

    A a B b C c D d E more information required

10. A square is split into nine identical squares, each with sides of length one unit. Circles are inscribed in two of these squares, as shown. What is the shortest distance between the two circles?
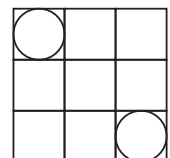
    A $2\sqrt{2} - 1$ B $\sqrt{2} + 1$ C $2\sqrt{2}$ D 2   E 3

11. A tennis tournament was played on a knock-out basis. The following list is of all but one of the last seven matches (the quarter-finals, the semi-finals and the final), although not correctly ordered: Bella beat Ann; Celine beat Donna; Gina beat Holly; Gina beat Celine; Celine beat Bella; and Emma beat Farah. Which result is missing?

A Gina beat Bella   B Celine beat Ann   C Emma beat Celine
D Bella beat Holly   E Gina beat Emma

12. The large triangle shown has sides of length 5 units. What percentage of the area of the triangle is shaded?

A 80%   B 85%   C 88%   D 90%
E impossible to determine
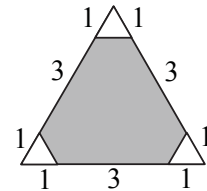
13. Sepideh is making a magic multiplication square using the numbers 1, 2, 4, 5, 10, 20, 25, 50 and 100. The products of the numbers in each row, in each column and in the two diagonals should all be the same. In the figure you can see how she has started. Which number should Sepideh place in the cell with the question mark?

A 2   B 4   C 5   D 10   E 25

14. Eight unmarked envelopes contain the numbers: 1, 2, 4, 8, 16, 32, 64, 128. Eve chooses a few envelopes randomly. Alie takes the rest. Both sum up their numbers. Eve's sum is 31 more than Alie's. How many envelopes did Eve take?
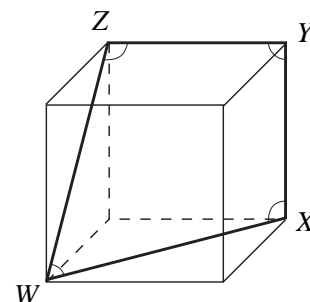
A 2   B 3   C 4   D 5   E 6

15. Peter wants to colour the cells of a 3×3 square in such a way that each of the rows, each of the columns and both diagonals have cells of three different colours. What is the least number of colours Peter could use?

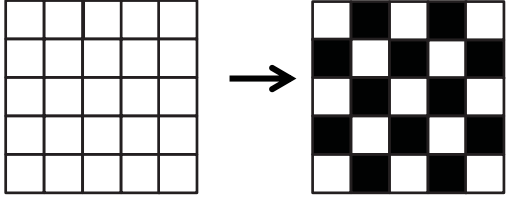A 3   B 4   C 5   D 6   E 7
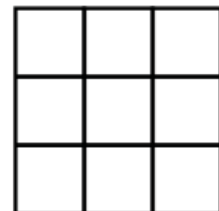
16. The picture shows a cube with four marked angles, $\angle WXY$, $\angle XYZ$, $\angle YZW$ and $\angle ZWX$. What is the sum of these angles?

A 315°   B 330°   C 345°   D 360°   E 375°

17. There are 2016 kangaroos in a zoo. Each of them is either grey or pink, and at least one of them is grey and at least one is pink. For every kangaroo, we calculate this fraction: the number of kangaroos of the other colour divided by the number of kangaroos of the same colour as this kangaroo (including himself). Find the sum of all the 2016 fractions calculated.

A 2016   B 1344   C 1008   D 672   E more information required

18. What is the largest possible remainder that is obtained when a two-digit number is divided by the sum of its digits?

    A 13   B 14   C 15   D 16   E 17

19. A 5 × 5 square is divided into 25 cells. Initially all its cells are white, as shown. Neighbouring cells are those that share a common edge. On each move two neighbouring cells have their colours changed to the opposite colour (white cells become black and black ones become white).
    What is the minimum number of moves required in order to obtain the chess-like colouring shown on the right?

    A 11   B 12   C 13   D 14   E 15

20. It takes 4 hours for a motorboat to travel downstream from X to Y. To return upstream from Y to X it takes the motorboat 6 hours. How many hours would it take a wooden log to be carried from X to Y by the current, assuming it is unhindered by any obstacles? [Assume that the current flows at a constant rate, and that the motorboat moves at a constant speed relative to the water.]

    A 5   B 10   C 12   D 20   E 24

21. In the Kangaroo republic each month consists of 40 days, numbered 1 to 40. Any day whose number is divisible by 6 is a holiday, and any day whose number is a prime is a holiday. How many times in a month does a single working day occur between two holidays?

    A 1   B 2   C 3   D 4   E 5

22. Jakob wrote down four consecutive positive integers. He then calculated the four possible totals made by taking three of the integers at a time. None of these totals was a prime. What is the smallest integer Jakob could have written?

    A 12   B 10   C 7   D 6   E 3

23. Two sportsmen (Ben and Filip) and two sportswomen (Eva and Andrea) − a speed skater, a skier, a hockey player and a snowboarder − had dinner at a square table, with one person on each edge of the square. The skier sat at Andrea's left hand. The speed skater sat opposite Ben. Eva and Filip sat next to each other. A woman sat at the hockey player's left hand. Which sport did Eva do?

    A speed skating   B skiing   C hockey
              D snowboarding   E more information required

24. Dates can be written in the form DD.MM.YYYY. For example, today's date is 17.03.2016. A date is called 'surprising' if all 8 digits in its written form are different. In what month will the next surprising date occur?

    A March   B June   C July   D August   E December

25. At a conference, the 2016 participants were registered from P1 to P2016. Each participant from P1 to P2015 shook hands with exactly the same number of participants as the number on their registration form. How many hands did the 2016th participant shake?

    A 1   B 504   C 672   D 1008   E 2015