AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

Level 3 Technical level: Engineering
MATHEMATICS FOR ENGINEERS

Unit J/506/5953

SPECIMEN QUESTION PAPER

Time allowed: 1 hour 45 minutes

Materials
For this paper you must have:
- pens
- pencils
- simple drawing instruments
- scientific calculator (non-programmable).

Instructions
- Answer all questions on the paper.
- Answer to 3 significant figures unless otherwise instructed.
- Include units in all answers, where required, as marks are given for units in some questions.

Information
- There are 80 marks available on this paper.

Advice
Do not spend too long on one question. Read each question thoroughly before starting your answer. Show all working in the spaces provided.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
TOTAL	

Formula Sheet

Area of a circle $A = \pi r^2$ or $A = \frac{\pi D^2}{4}$	Density $\rho = \frac{m}{V}$
Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$ $b^2 = a^2 + c^2 - 2ac\cos B$ $c^2 = a^2 + b^2 - 2ab\cos C$
Angular measure $360^\circ \equiv 2\pi$ radians	Newton's second law $F = ma$
Trigonometry $\sin = \frac{opp}{hyp}$, $\cos = \frac{adj}{hyp}$ and $\tan = \frac{opp}{adj}$	Quadratic equation $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $ax^2 + bx + c = 0$
Mean value $\bar{x} = \frac{\sum x}{n}$	Standard deviation $\sigma = \sqrt{\left\{\frac{\sum (x - \bar{x})^2}{n}\right\}}$
Cartesian to polar conversion $r = \sqrt{x^2 + y^2}$ $\tan\theta = \frac{y}{x}$	Polar to Cartesian conversion $x = r\cos\theta$ $y = r\sin\theta$
Straight line graph $y = mx + c$	Energy $Potential\ energy = mgh$ and $Kinetic\ energy = \frac{mv^2}{2}$
The gravitation constant: $g = 9.81\ m\ s^{-2}$	

Standard derivatives

$f(x)$	$\frac{dy}{dx}$
ax^n	anx^{n-1}
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$
$\ln ax$	$\frac{1}{x}$
e^{ax}	ae^{ax}

Standard integrals

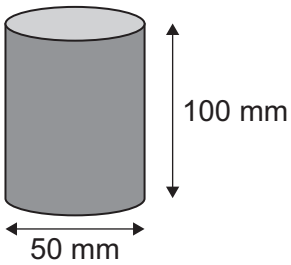
$f(x)$	$\int f(x)\,dx$
ax^n	$\frac{ax^{n+1}}{n+1} + c$ if $n \neq -1$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Section A

Answer **all** questions in this section.

0 1 An engineering company needs to source some solid steel billets 50 mm diameter and 100 mm long – as shown in **Figure 1**.

Figure 1



0 1 . 1 Determine the volume of the material in each billet.

[4 marks]

0 1 . 2 If the density of the steel is 7908 kg m^{-3}, what is the mass of each billet?

[3 marks]

0 1 . 3 Determine the weight, in Newton's, of each billet.

[2 marks]

0 1 . 4 If the cost of the steel used is £4.79 per kg, calculate the cost of the steel for 75 billets to the nearest £.

[2 marks]

11

0 2 An electrical engineer is taking current measurements from two circuits in an electronic control panel and gets the following results:

$I_1 + 6I_2 = 12$ Amps and $3I_1 - 4I_2 = 5$ Amps

Determine the values for I_1 and I_2 giving the correct units in your answer.

[7 marks]

7

0 3 Simplify the following algebraic fraction into its simplest partial fractions and write down the final partial fractions:

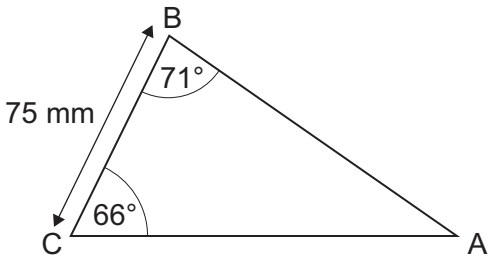
$$\frac{5x - 4}{(x - 2)\ (x + 1)}$$

[6 marks]

6

0 4 A strengthening member for a commercial jet liner is manufactured from aluminium alloy and is shown in **Figure 2**.

Figure 2



Determine the length of the side AC to the nearest mm.

[5 marks]

5

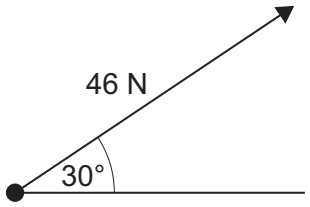
0 5 . 1 A motor turns through an angle of 240°.

Convert this angle into radians.
Your answer must be shown in fraction multiples of π.

[3 marks]

0 5 . 2 **Figure 3** shows a force vector acting on a riveted joint in an engineering mechanism.

Figure 3



Determine the component forces of the above system, give your answers to 3 decimal places.

[3 marks]

6

0 6 A set of 20 rivets has been manufactured. The lengths of the rivets were measured (mm) and the results are shown in **Table 1**:

Table 1

25	23	22	24	26	25	26	24	23	22
24	25	23	25	24	25	22	23	24	25

0 6 . 1 Determine the mean length of the sample.

[3 marks]

0 6 . 2 Determine the modal length of the sample.

[1 mark]

0 6 . 3 Determine the standard deviation of the sample.

[4 marks]

8

0 7 The crane in **Figure 4** lifts the beam with linear velocity function (where v = velocity m s^{-1} and t = time seconds):

$$v = 10\ sin\left(\frac{t}{2}\right)$$

Figure 4

By using the process of integration, determine how far the beam will have been lifted between 0 and 6 seconds of operation. Show all working.

[7 marks]

7

Section B

Answer **all** questions in this section.

0 8 The position of a 5 tonne earth moving machine in **Figure 5** is given by the function:
(where s = position (m) and t = time (seconds))

$$s = 3t^2 + 2t + 3$$

Figure 5

0 8 . 1 By using the process of differentiation, determine a function for the earth mover's velocity.

[3 marks]

0 8 . 2 Determine the velocity of the earth mover at $t = 0.5$ seconds and include the units in your answer.

[2 marks]

0 8 . 3 Calculate the kinetic energy of the earth mover at this velocity.

[3 marks]

0 8 . 4 Determine the acceleration of the earth mover.

[2 marks]

10

0 9 An engineering company performed stiffness tests on a selection of three engineering materials. The load against extension results are provided in **Table 2**.

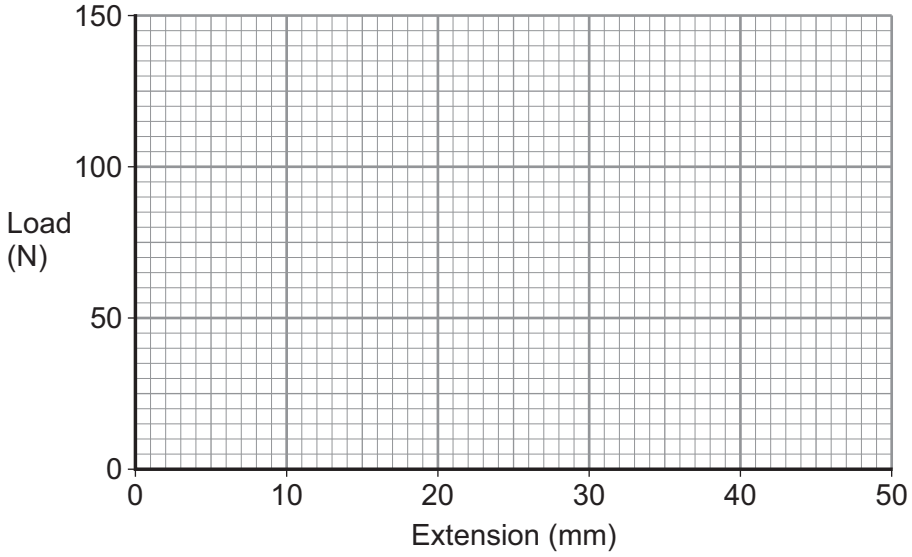
Table 2

Material	Load (N)	Ext'n (mm)	Load (N)	Ext'n (mm)	Load (N)	Ext'n (mm)	Load (N)	Ext'n (mm)	Load (N)	Ext'n (mm)
Steel A	25	10	50	20	75	30	100	40	125	50
Steel B	25	7.1	50	14.3	75	21.4	100	28.6	125	35.7
Steel C	25	5.5	50	11.1	75	16.7	100	22.2	125	27.8

0 9 . 1 Plot the load against extension graph for each of the steels on **graph 1**.

[6 marks]

Graph 1



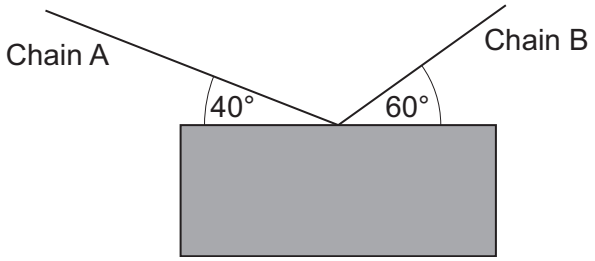
0 9 . 2 From your graph, determine the stiffness values for each of the steel samples using the correct units in your answer.

[4 marks]

10

1 0 A $8\ \text{kN}$ container is suspended above the ground by two chains, A and B, as shown in **Figure 6**:

Figure 6



Determine the tension, in Newtons, in the chains A and B.

[10 marks]

10

END OF QUESTIONS

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Level 3
Technical Level Engineering

Mathematics for Engineers

Specimen Mark scheme

Unit Number: J/506/5953
SPECIMEN-SAM 2

Version: 0.1

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

MARKING METHODS

In fairness to candidates, all examiners **must** use the same marking methods. The following advice may seem obvious, but all examiners **must** follow it as closely as possible.

1 If you have any doubt about how to allocate marks to an answer, consult your Team Leader.
2 Refer constantly to the mark scheme and standardising scripts throughout the marking period.
3 Use the full range of marks. Don't hesitate to give full marks when the answer merits them.
4 The key to good and fair marking is **consistency**.

INTRODUCTION

The information provided for each question is intended to be a guide to the kind of answers anticipated and is neither exhaustive nor prescriptive. **All appropriate responses should be given credit**.

Where literary or linguistic terms appear in the Mark Scheme, they do so generally for the sake of brevity. Knowledge of such terms, other than those given in the specification, is **not** required. However, when determining the level of response for a particular answer, examiners should take into account any instances where the candidate uses these terms effectively to aid the clarity and precision of the argument.

DESCRIPTIONS OF LEVELS OF RESPONSE

The following procedure must be adopted in marking by levels of response:
□ read the answer as a whole
□ work up through the descriptors to find the one which best fits
□ where there is more than one mark available in a level, determine the mark from the mark range judging whether the answer is nearer to the level above or to the one below.

Since answers will rarely match a descriptor in all respects, examiners must allow good performance in some aspects to compensate for shortcomings in other respects. Consequently, the level is determined by the 'best fit' rather than requiring every element of the descriptor to be matched. Examiners should aim to use the full range of levels and marks, taking into account the standard that can reasonably be expected of candidates.

01 An engineering company needs to source some solid steel billets 50mm diameter and 100mm long – as shown in figure 1.

01.1 Determine the volume of the material in each billet.

[4 marks]

$\text{Volume} = \frac{\pi D^2}{4} \times \text{L} = \frac{\pi \times 0.05^2}{4} \times 0.1 = 196 \times 10^{-6} m^3$ or equivalent.

1 mark for the correct formula.
1 mark for the correct values in the calculation.
1 mark for the correct answer.
1 mark for the correct units in the calculation.

01.2 If the density of the steel is 7908kg m^{-3}, what is the mass of each billet?

[3 marks]

$\rho = \frac{m}{v}$ then $m = \rho v \ \therefore m = 7908 \ \times 196 \times 10^{-6} = 1.55kg.$

1 mark for the correct formula.
1 mark for the correct transposition.
1 mark for the correct answer in kilogrammes.

Allow follow through if the answer to part 1.1 is incorrect.

01.3 Determine the weight, in Newton's, of each billet.

[2 marks]

$F(\text{or weight}) = mg \ \therefore F = 1.55 \times 9.81 = 15.2N.$

1 mark for the correct formula.
1 mark for the correct answer.

Allow follow through if the answer to parts 1.1 and / or 1.2 are incorrect.

01.4 If the cost of the steel used is £4.79 per kg, calculate the cost of the steel for 75 billets to the nearest £.

[2 marks]

$\text{Cost} = 75 \times 1.55 \times £4.79 = £557.$

1 mark for the correct method – any order will suffice.
1 mark for the correct value to the nearest £.

Total for question 1 is 11 marks.
Allow follow through if the answers to part 1.1, 1.2 and / or 1.3 are incorrect.

02 An electrical engineer is taking current measurements from two circuits in an electronic control panel and gets the following results:

$$I_1 + 6I_2 = 12\ Amps \text{ and } 3I_1 - 4I_2 = 5\ Amps$$

Determine the values for I_1 and I_2 giving the correct units in your answer.

[7 marks]

$I_1 + 6I_2 = 12 \text{ Amps} - \text{equation } 1$
$3I_1 - 4I_2 = 5 \text{ Amps} - \text{equation } 2$

From equation 1:

$I_1 = 12 - 6I_2 - \text{equation } 3.$

Substitute equation 3 into equation 2:

$3(12 - 6I_2) - 4I_2 = 5$

$36 - 18I_2 - 4I_2 = 5$

$\therefore I_2 = \frac{31}{22} \text{ Amps.}$

Now substitute the value for I_2 into equation 2:

$3I_1 - 4\left(\frac{31}{22}\right) = 5 \text{ and } 3I_1 = \frac{117}{11}$

$\therefore I_1 = \frac{39}{11} \text{ Amps.}$

1 mark for suitable identification of the initial equations.
1 mark for suitable rewriting / transposing of one equation in terms of one of the variables.
1 mark for substitution into an equation.
1 mark for correct solutions.
1 mark for substituting that value into the outstanding equation.
1 mark for correct transposition.
1 mark for correct evaluation / answer.

7 marks in total for Question 2

03 Simplify the following algebraic fraction into is simplest partial fractions and write down the final partial fractions:

$$\frac{5x-4}{(x-2)(x+1)}$$

[6 marks]

From $\frac{5x-4}{(x-2)(x+1)} \equiv \frac{A}{x-2} + \frac{B}{x+1}$ we can rewrite the original equation as:

$5x - 4 \equiv A(x+1) + B(x-2)$

Let $x = -1$:

$5(-1) - 4 = 0A + B(-1-2) \therefore B = \frac{-9}{-3} = 3.$

Let $x = 2$:

$5(2) - 4 = A(2+1) + 0B \therefore A = \frac{6}{3} = 2.$

Finally we have:

$\frac{5x-4}{(x-2)(x+1)} \equiv \frac{2}{x-2} + \frac{3}{x+1}$ as required.

1 mark for rewriting the original equation in terms of two constants (A and B in this answer but allow any suitable constants and allow them to be evaluated in any order).
1 mark for cancelling the denominators correctly.
1 mark for substituting a suitable value for *x*.
1 mark for the correct evaluation of one of the constants.
1 mark for substituting another suitable value for *x*.
1 mark for the correct evaluation of the other constant

6 marks in total for Question 3.

04 A strengthening member for a commercial jet liner is manufactured from aluminium alloy and is shown in figure 2.

Determine the length of the side AC to the nearest mm.

[5 marks]

Using the sine rule of the type:

$\frac{a}{\sin A} = \frac{b}{\sin B}$ and transposing we have:

$b = \frac{a \sin B}{\sin A}$. By inserting the correct values, we have:

$b = \frac{75 \times \sin 71^0}{\sin 43^0} = 104\ mm$. To the nearest mm.

1 mark for the correct form of the sine rule using the correct symbols.
1 mark for the correct transposition to determine *b*.
1 mark for the correct values used in the equation.
1 mark for the correct evaluation.
1 mark for showing the answer to the nearest mm.

5 marks for Question 4

05.1 A motor turns through an angle of 240^0.

Convert this angle into radians. Your answer must be shown in fraction multiples of $\boldsymbol{\pi}$.

[3 marks]

Any of the following solutions:

$\theta = \frac{240^0}{360^0} \times 2\pi \equiv \frac{240^0}{180^0} \times \pi = \frac{4\pi}{3}$ radians.

1 mark for either correct method.
1 mark for the correct evaluation.
1 mark for the correct fraction multiple of Pi.

3 marks for Question 5.1.

05.2 **Figure 3** shows a force vector acting on a riveted joint in an engineering mechanism. Determine the component forces of the above system; give your answers to 3 decimal places.

[3 marks]

By converting the force vector to Cartesian co-ordinates:

Horizontally, we have: $46 \times \cos 30^0 = 39.837N$ and
Vertically, we have: $46 \times \sin 30^0 = 23.000N$.

1 mark for the correct horizontal component.
1 mark for the correct vertical component.
1 mark for both answers to 3 decimal places.

3 marks for Question 5.2.

6 marks in total for Question 5.

06 A set of 20 rivets has been manufactured. The lengths of the rivets was measured (mm) and the results are shown in **Table 1**:

25	23	22	24	26	25	26	24	23	22
24	25	23	25	24	25	22	23	24	25

Table 1

06.1 Determine the mean length of the sample.

[3 marks]

The mean length can be determined by:

$\bar{L} = \frac{\sum L}{n} = \frac{25+23+22+24+26+25+26+24+23+22+24+25+23+25+24+25+22+23+24+25}{20}$ Therefore, we have:

$\bar{L} = \frac{480}{20} = 24\ mm.$

1 mark for using the correct method.
1 mark for using the correct values.
1 mark for the correct answer.

3 marks for Question 6.1.

06.2 Determine the modal length of the sample.

[1 mark]

25 is the most commonly occurring value; therefore, it is the **modal value** of this data set.

1 mark for correctly identifying the modal value as 25 mm.

1 mark for Question 6.2.

06.3 Determine the standard deviation of the sample.

[4 marks]

The standard deviation of the sample can be found by using:

$\sigma = \sqrt{\left\{\frac{\sum(L-\bar{L})^2}{n}\right\}}$ Therefore, we have:

$\sum(L-\bar{L})^2$: $(25-24)^2 + (23-24)^2 + (22-24)^2 + (24-24)^2 + (26-24)^2 + (25-24)^2 + (26-24)^2 + (24-24)^2 + (23-24)^2 + (22-24)^2 + (24-24)^2 + (25-24)^2 + (23-24)^2 + (25-24)^2 + (24-24)^2 + (25-24)^2 + (22-24)^2 + (23-24)^2 + (24-24)^2 + (25-24)^2 = 30$

$\frac{1+1+4+4+1+4+1+4+1+1+1+1+4+1+1}{20} = \frac{30}{20}$. Finally we have:

$\sigma = \sqrt{\frac{30}{20}} = 1.22\ mm.$

Continued Overleaf.

1 mark for correct method of squaring.
1 mark for the correct result.
1 mark for correct values in the formula.
1 mark for the correct answer.

4 marks in total for Question 6.3

Allow follow-through from part 6.1.

8 marks in total for Question 6.

07 The crane in **figure 4** lifts the beam with linear velocity function (where *v* = velocity and *t* = time):

$$v = 10\ sin\left(\frac{t}{2}\right)$$

By using the process of integration, determine how far the beam will have been lifted between 0 and 6 seconds of operation. Show all working.

[7 marks]

By forming the definite integral, we have:

$\int_0^6 10\sin\left(\frac{t}{2}\right).dt = \left[-20\cos\left(\frac{t}{2}\right)\right]_0^6$ Then we can evaluate:

$-20\cos\left(\frac{6}{2}\right) - -20\cos\left(\frac{0}{2}\right)$ which returns:

$19.8 - -20 \equiv 39.8\ m.$

1 mark for the $-20\ cos$.
1 mark for the substitution $t = 6$.
1 mark for the substitution $t = 0$.
1 mark for the double negative becoming a "+".
1 mark for the 19.8.
1 mark for the 20.
1 mark for the units of "m" in the answer.

7 marks in total for Question 7.

08 The position of a 5 tonne earth moving machine in **figure 5** is given by the function: (where *s* = position and *t* = time)

$$s = 3t^2 + 2t + 3$$

08.1 By using the process of differentiation, determine a function for the earth mover's velocity.

[3 marks]

Using differentiation we get:

$\frac{ds}{dt} = 6t + 2$. Therefore; the velocity function is:

$v = 6t + 2$.

1 mark for the derivative "*6t*".
1 mark for the derivative "2".
1 mark for the correct notation.

3 marks total for Question 8.1

08.2 Determine the velocity of the earth mover at $t = 0.5$ seconds.

[2 marks]

The velocity of the earth mover at $t = 0.5\ s$:

$v = 6t + 2 = 6(0.5) + 2 = 5\ ms^{-1}$.

1 mark for the correct evaluation.
1 mark for the correct units.

Allow follow-through where necessary.

2 marks total for Question 8.2

08.3 Calculate the kinetic energy of the earth mover at this velocity.

[3 marks]

The kinetic energy of the earth mover can be calculated by:

$KE = \frac{1}{2}mv^2 = \frac{5000 \times 5^2}{2} = 125kJ$.

1 mark for the correct formula.
1 mark for the correct answer.
1 mark for the correct units.

3 marks in total for Question 8.3

08.4	Determine the acceleration of the earth mover.	**[2 marks]**

$\frac{dv}{dt} = 6$ Therefore, the acceleration of the earth mover is $6\,ms^{-2}$.

1 mark for the derivative.
1 mark for the units.

2 marks in total for Question 8.4.

10 marks in total for Question 8.

09 An engineering company performed stiffness tests on a selection of three engineering materials, the load against extension results are provided in **table 2**.

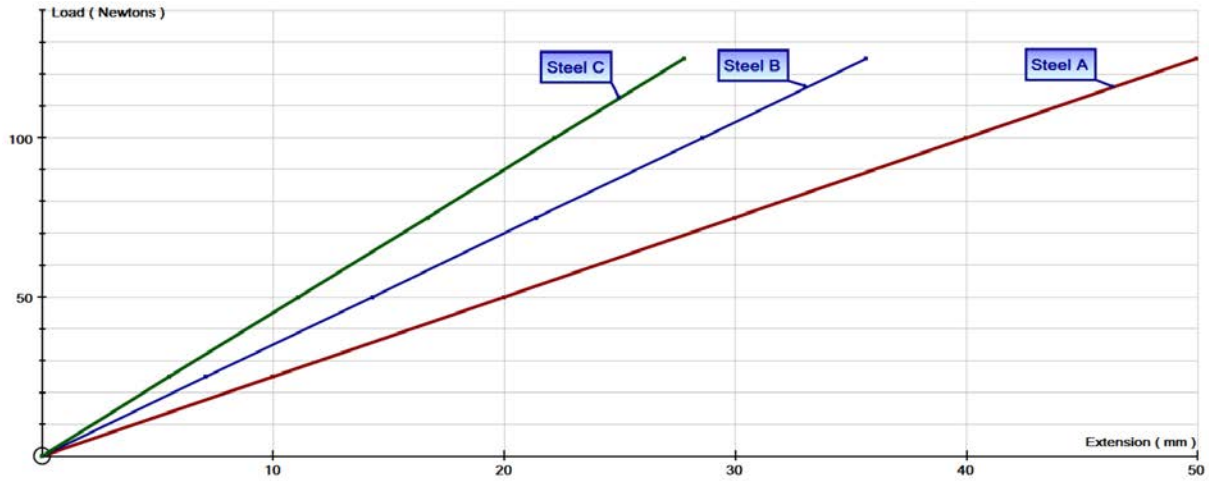
Table 2

Material	Load (N)	Ext'n (mm)	Load (N)	Ext'n (mm)	Load (N)	Ext'n (mm)	Load (N)	Ext'n (mm)	Load (N)	Ext'n (mm)
Steel A	25	10	50	20	75	30	100	40	125	50
Steel B	25	7.1	50	14.3	75	21.4	100	28.6	125	35.7
Steel C	25	5.5	50	11.1	75	16.7	100	22.2	125	27.8

09.1 Plot the load against extension graph for each of the steels on **graph 1.**

[6 marks]

A graph similar to that below should be produced.



2 marks for each material sample producing a straight line.

6 marks in total for Question 9.1

09.2 From your graph, determine the stiffness values for each of the steel samples using the correct units in your answer.

[4 marks]

Using the following method:

$\frac{dL}{dExt} = \frac{125}{50\times10^{-3}} = 2500Nm^{-1}$. Steel A.

$\frac{dL}{dExt} = \frac{125}{35.7\times10^{-3}} = 3500Nm^{-1}$. Steel B.

$\frac{dL}{dExt} = \frac{125}{27.8\times10^{-3}} = 4500Nm^{-1}$. Steel C.

1 mark for each correct answer.
1 mark for the correct units.
4 marks in total for Question 9.2
10 marks in total for Question 9.

10 An **8 kN** compressor in suspended, above the ground, by two chains, A and B, as shown in figure 6:

Determine the tension in the chains A and B.

[10 marks]

Resolving horizontally:

$B\cos 60^0 - A\cos 40^0 = 0$

$\therefore B\cos 60^0 = A\cos 40^0$ equation 1.

Resolving vertically:

$8kN - B\sin 60^0 - A\sin 40^0 = 0$

$\therefore 8kN = B\sin 60^0 + A\sin 40^0$ equation 2.

From equation 1 we have:

$B = \frac{A\cos 40^0}{\cos 60^0} = 1.53\ A$ equation 3.

Substitute from equation 3 into equation 2:

$8000 = 1.53\ A\sin 60^0 + A\sin 40^0 = 1.97\ A$

$\therefore A = 4062\ N$

Substitute A into equation 1:

$B\cos 60^0 = 4062\cos 40^0 =$ therefore, $B = \frac{3111}{\cos 60^0}$

Finally:

$B = 6220\ N$.

1 mark for the horizontal resolving.
1 mark for simplifying to equation 1.
1 mark for resolving vertically – equation 2.
1 mark for rewriting equation 1 in terms of one of the unknowns.
1 mark for evaluating one unknown against the other unknown – equation 3.
1 mark for the substitution of equation 3 into equation 2.
1 mark for correct value of A.
1 mark for substituting the A-value into equation 1.
1 mark for evaluating the above.
1 mark for the correct solution for B.

10 marks in total for Question 10.

Assessment outcomes coverage

Assessment Outcomes	Marks and % of marks available in section A	Marks and % of marks available in section B	Total Marks
AO1:	0 marks 0%	30 marks 100%	30 marks
AO2:	11 Marks 22%	0 marks 0%	11 marks
AO3:	13 Marks 26%	0 marks 0%	13 marks
AO4:	11 Marks 22%	0 marks 0%	11 marks
AO5:	8 Marks 16%	0 Marks 0%	8 marks
AO6:	7 Marks 14%	0 Marks 0%	7 marks
Total Marks	50	30	80

Question	AO1	AO2	AO3	AO4	AO5	AO6
1		11				
2			7			
3			6			
4				5		
5				6		
6					8	
7						7
8	10					
9	10					
10	10					
Totals	**30**	**11**	**13**	**11**	**8**	**7**