

Class	Index Number	Name
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RIVER VALLEY HIGH SCHOOL
SECONDARY FOUR PRELIMINARY EXAMINATION

MATHEMATICS

4017/1

Paper 1

13 September 2006

2 hours

Candidates answer on the Question Paper.

Additional Materials: Geometrical instruments

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen in the spaces provided on the Question Paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question, it must be shown in the space below that question.
Omission of essential working will result in loss of marks.
The total marks for this paper is 80.

**NEITHER ELECTRONIC CALCULATORS NOR MATHEMATICAL TABLES MAY
BE USED IN THIS PAPER.**

This document consists of 16 printed pages including this page.

NEITHER ELECTRONIC CALCULATORS NOR MATHEMATICAL TABLES
MAY BE USED IN THIS PAPER

- 1 (a) Find the fraction which is exactly halfway between $\frac{2}{9}$ and $\frac{5}{9}$.
(b) Subtract 239 millilitres from 4.6 litres. Give your answer in litres.

Answer (a) _____ [1]

(b) _____ litres [1]

- 2 Express 0.088

- (a) as a percentage,
(b) as a fraction in its simplest form.

Answer (a) _____ % [1]

(b) _____ [1]

- 3 Evaluate

(a) $13\frac{2}{3} - 4\frac{3}{5}$,

(b) $9.6 \div 0.04$.

Answer (a) _____ [1]

(b) _____ [1]

- 4 (a) Write down the cube root of $2\frac{10}{27}$.

(b) State which of the following numbers are irrational.

$$\sqrt{8} \times \sqrt{2}, \frac{22}{7}, \pi, 3\sqrt{4}, 2\sqrt{3}.$$

Answer (a) _____ [1]

(b) _____ [1]

- 5 At noon on a particular day, the temperature at the bottom of a mountain was 14°C and the temperature at the top of the mountain was -9°C .

(a) Calculate the difference between these temperatures.

(b) The height of the mountain is 4600 m. Given that the temperature changed uniformly with height, calculate the height above the bottom of the mountain at which the temperature was 0°C .

Answer (a) _____ $^{\circ}\text{C}$ [1]

(b) _____ m [1]

- 6 (a) Find the lowest common multiple of 4, 9 and 12

(b) Three buses leave the bus interchange at 6 a.m. The first bus leaves the bus interchange at 9 minute intervals, the second bus at 12 minute intervals and the third bus at 4 minute intervals. At what time will the three buses next leave the terminal together?

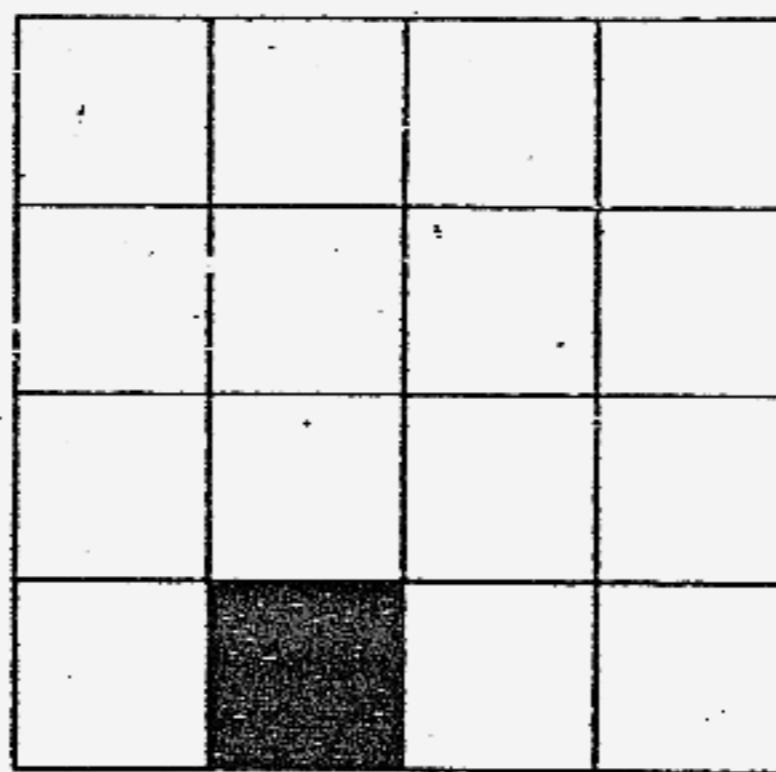
Answer (a) _____ [1]

(b) _____ [1]

- For Examiner's Use
- 7 (a) Shade three more squares so that the completed square grid has rotational symmetry of order 4.

For Examiner's Use

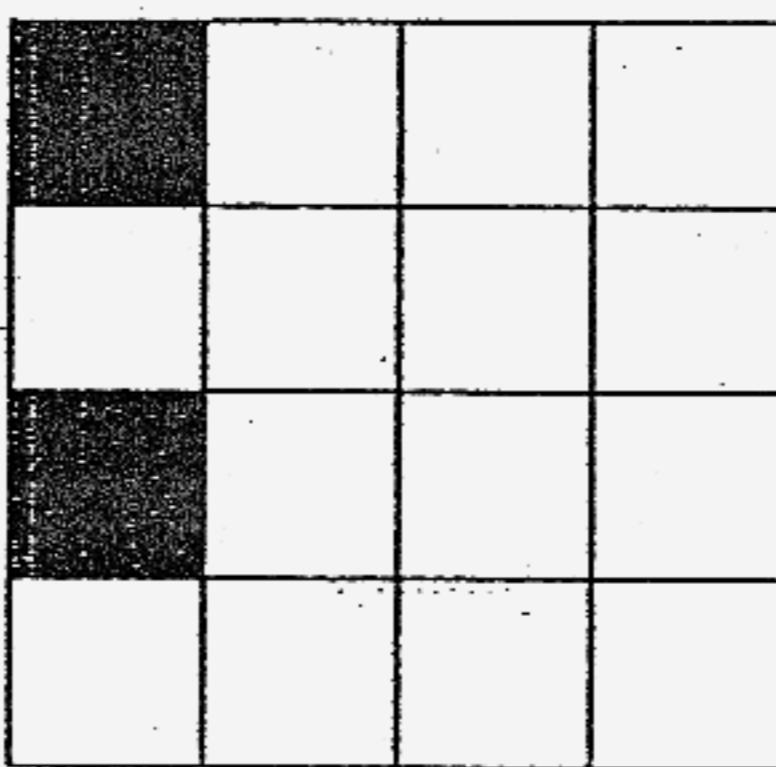
Answer



[1]

- (b) Shade one more square so that the completed square grid has only one line of symmetry.

Answer



[1]

- 8 (a) Rearrange the following numbers in order of size, starting from the smallest.

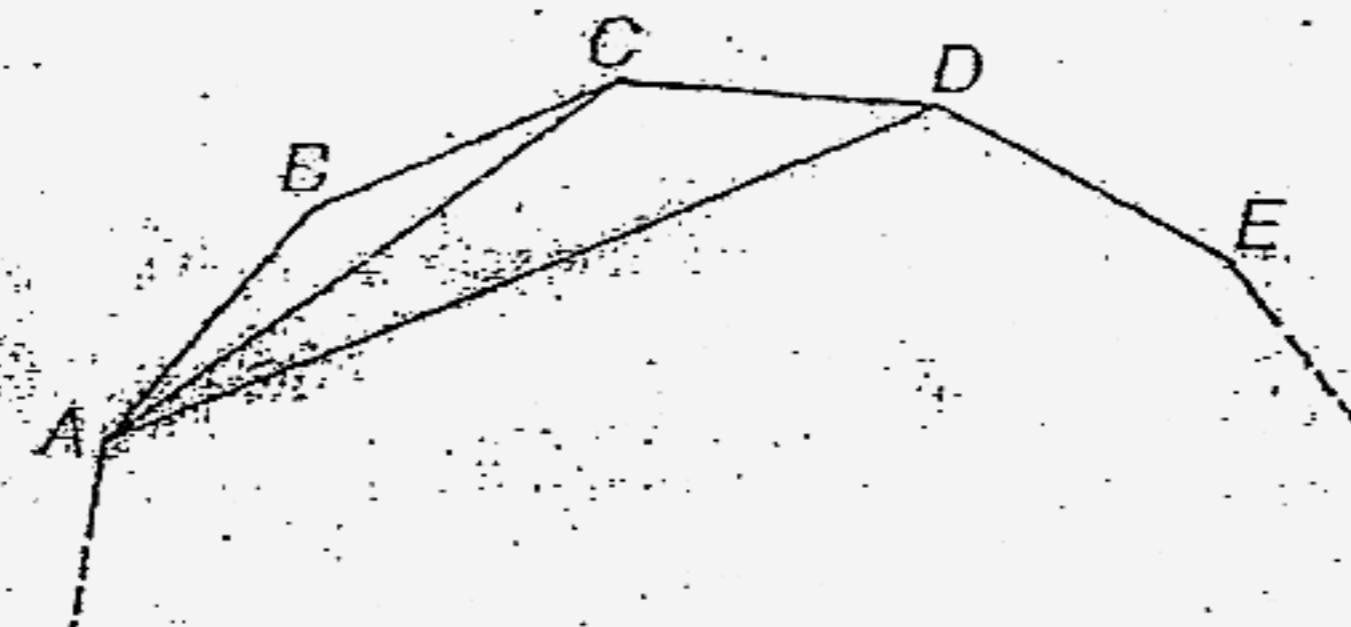
$$-0.19, 0, 0.4, -1.4, -0.3$$

- (b) The thickness of a sheet of paper is 6×10^{-4} cm. Find the thickness of eight sheets of paper, giving your answer in standard form.

Answer (a) _____ [1]

(b) _____ cm [1]

- 9 The diagram shows part of a regular polygon with n sides. Each interior angle of a regular polygon is 162° . Find



- (a) The value of n ,
 (b) $\angle ACD$.

Answer (a) _____ [2]

(b) _____ [1]

- 10 y is inversely proportional to x^2 . It is known that $y = 16$ for a particular value of x .

(a) Find the value of y when the value of x is doubled.

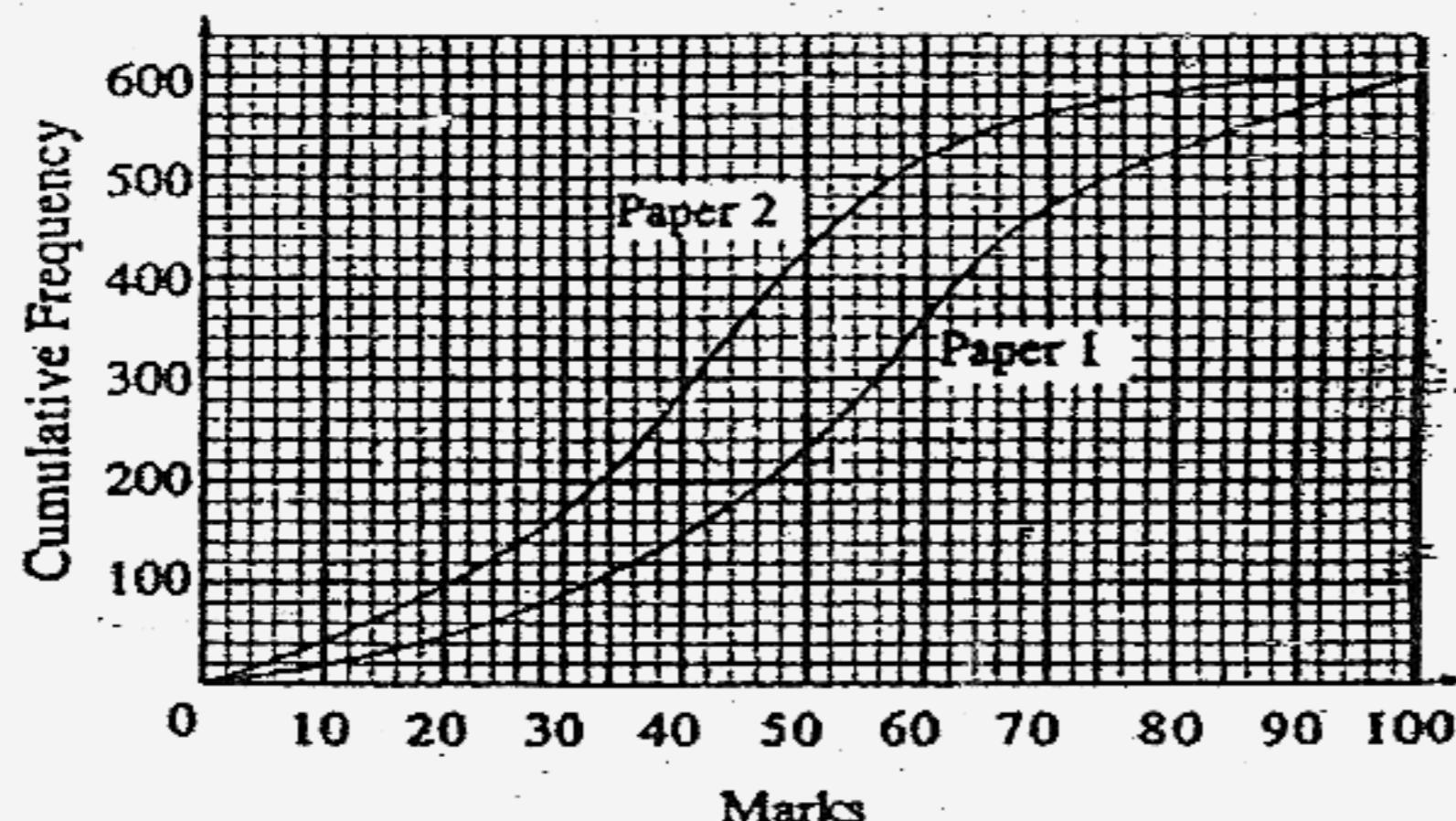
(b) Given further that when $x = 6$, $y = 1\frac{7}{9}$, express y in terms of x .

Answer (a) _____ [2]

(b) _____ [2]

- 11 Six hundred candidates took a Mathematics examination which consisted of 2 papers. Each paper was marked out of 100.

The diagram shows, on the same axes, the cumulative frequency curves for Paper 1 and Paper 2.



- (a) Estimate the median mark of the students taking Paper 1.
(b) Estimate the percentage of the students taking Paper 2 who gained more than 78 marks.
(c) State, with a reason which you think was the easier paper.

Answer (a)

[1]

(b) % [1]

(c) Paper _____ is easier because _____

[1]

12

(a) Solve the following equation $9^x = \frac{1}{729}$.

(b) Evaluate

(i) $25^{\frac{3}{2}}$.

(ii) $121^{\frac{1}{2}} \times (11^{-3})^0$

Answer (a) _____ [1]

(b) (i) _____ [1]

(ii) _____ [1]

13 Factorise fully $a^2 - ac - 9b^2 + 3bc$.

Answer: _____ [3]

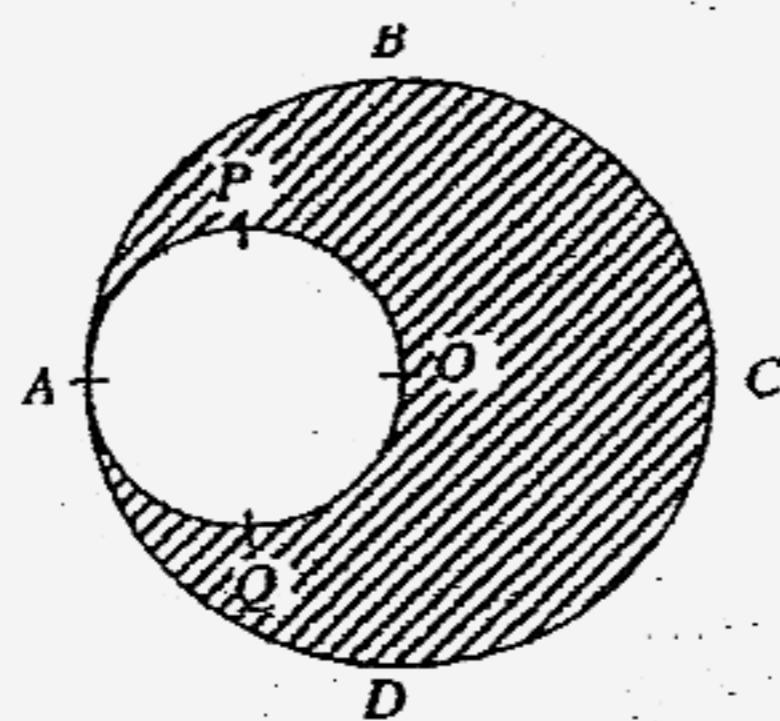
- 14 A model of the RV spirit statue is made to a scale of 1 to 300. The statue has a volume of 2700 cubic metres. Calculate the volume of the model in cubic metres. Give your answer in standard form.

Answer _____ m^3 [3]

- 15 A, B, C and D lie on a circle, centre O , of radius $4x$ cm. AO is a diameter of the circle through A, P, O and Q .

(a) A point is chosen at random, inside the larger circle. Find in its simplest fractional form, the probability that this point is in the shaded area.

(b) Find in its simplest form, the ratio of the circumference of the larger circle : sum of circumferences of the 2 circles.



Answer (a) _____ [2]

(b) _____ [2]

- 16 Find the smallest integer x such that $\frac{3x}{2} - \frac{x-4}{3} > 5$.

Answer _____ [2]

- 17 Look at this pattern

$$1 - \frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

$$\frac{1}{4} - \frac{1}{5} = \frac{1}{20}$$

$$\dots \dots \dots$$

(a) Write down

(i) the 10th line of the pattern,

(ii) the n^{th} line of the pattern.

(b) Use the pattern to find the sum of

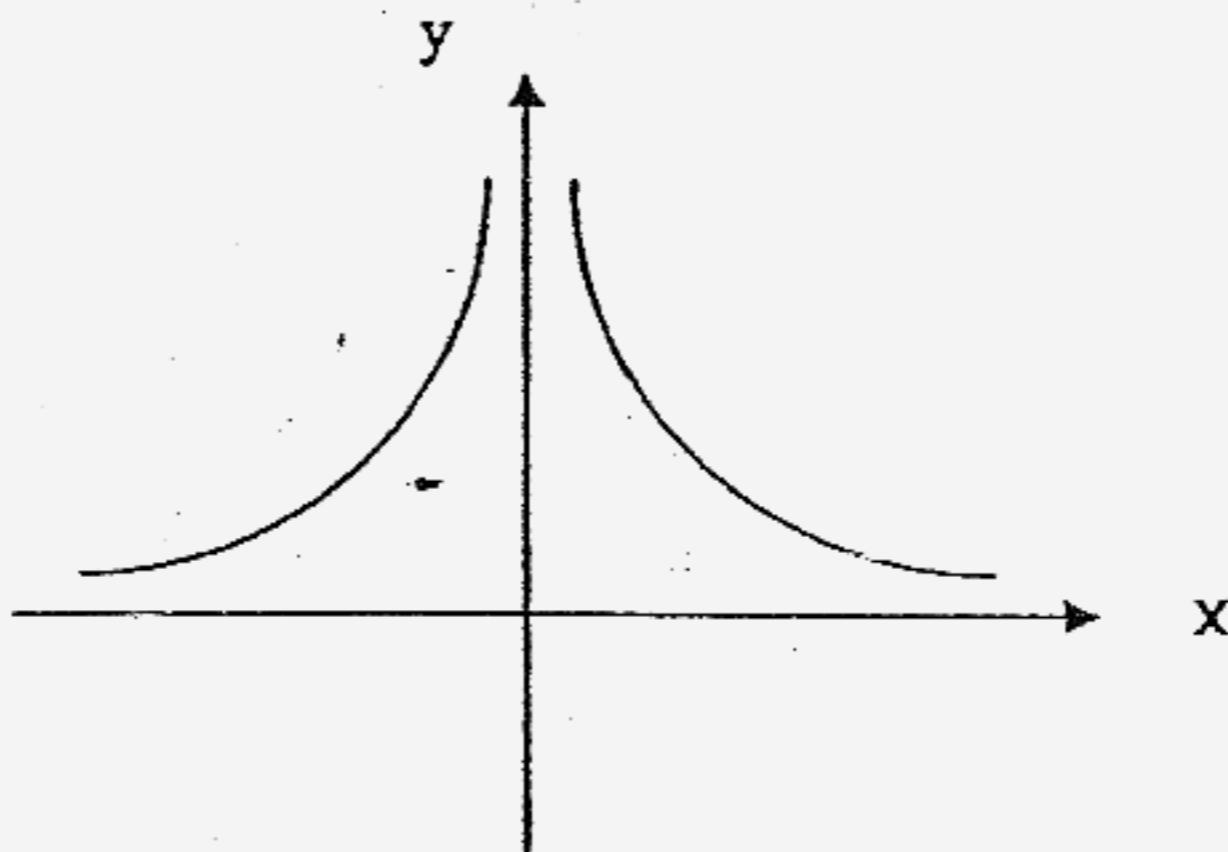
$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{9900}$$

Answer (a) (i) _____ [1]

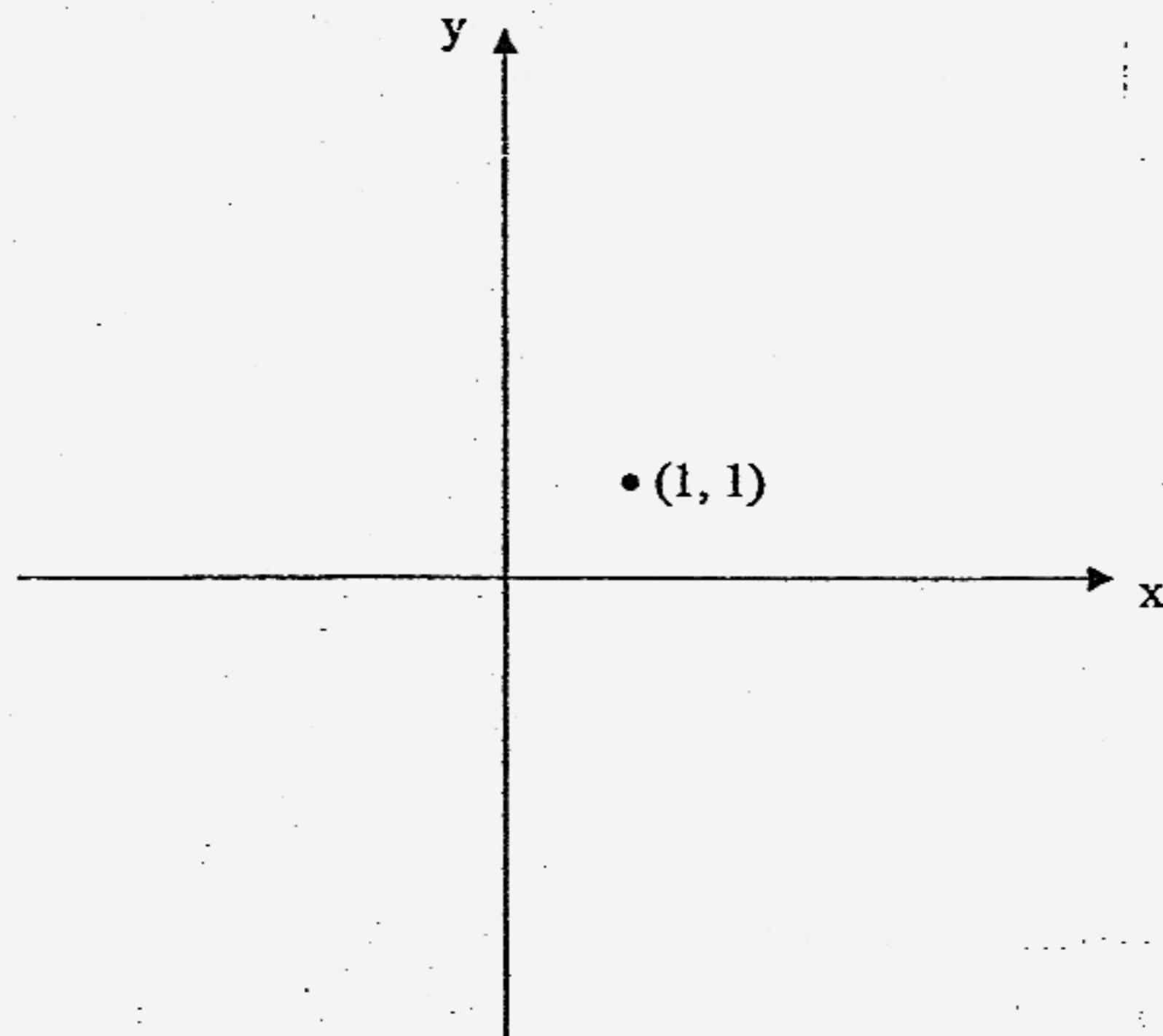
(ii) _____ [1]

(b) _____ [2]

- 18 (a) The sketch below represents the graph of $y = x^n$.
Write down a possible value of n .



- (b) (i) On the following diagram, sketch the graph of $y = x^3 - 1$. [1]
The point $(1, 1)$ is marked on the diagram.



(ii) Determine the straight line which must be added to the graph to solve the equation $x^3 + x - 3 = 0$.

(iii) Hence, determine the number of real value(s) of x which satisfy the above equation.

Answer (a) _____ [1]

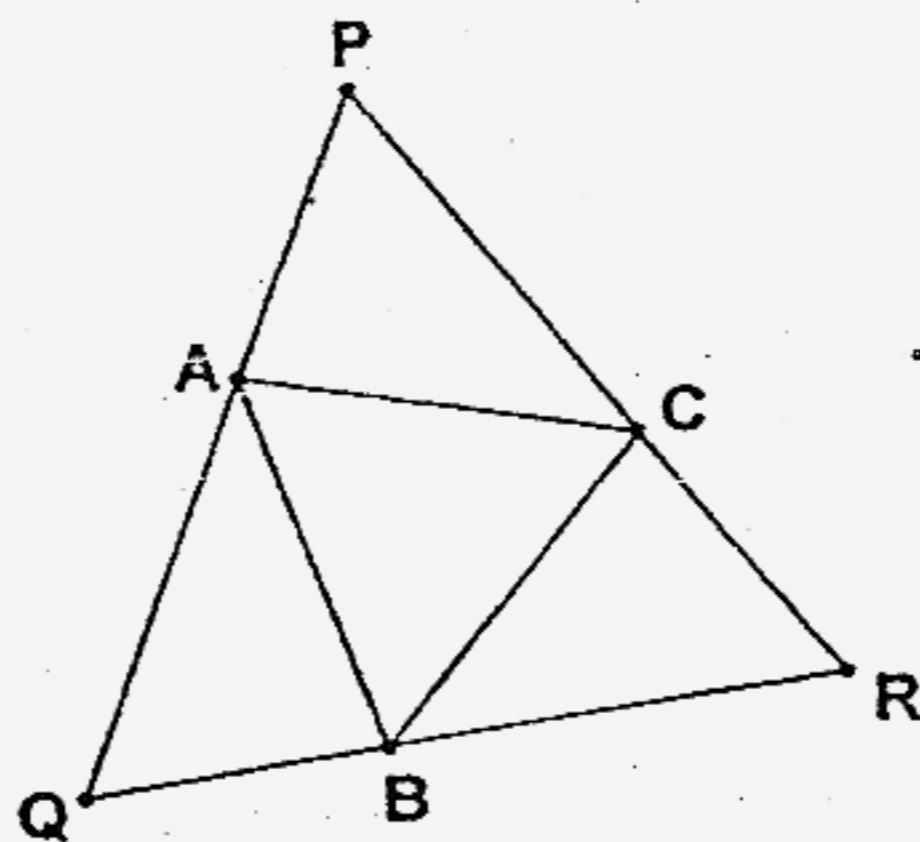
(b) (ii) _____ [1]

(iii) _____ [1]

- 19 (a) (i) Factorise $5x^2 + 8x + 3$.
(ii) Hence find all the factors of 583.
(b) Given that $8.52 \times 10^{n+1} + 3.65 \times 10^{n-1} - 7.62 \times 10^n = 77945$, calculate the value of n .

Answer (a) (i) _____ [1]
(ii) _____ [1]
(b) _____ [2]

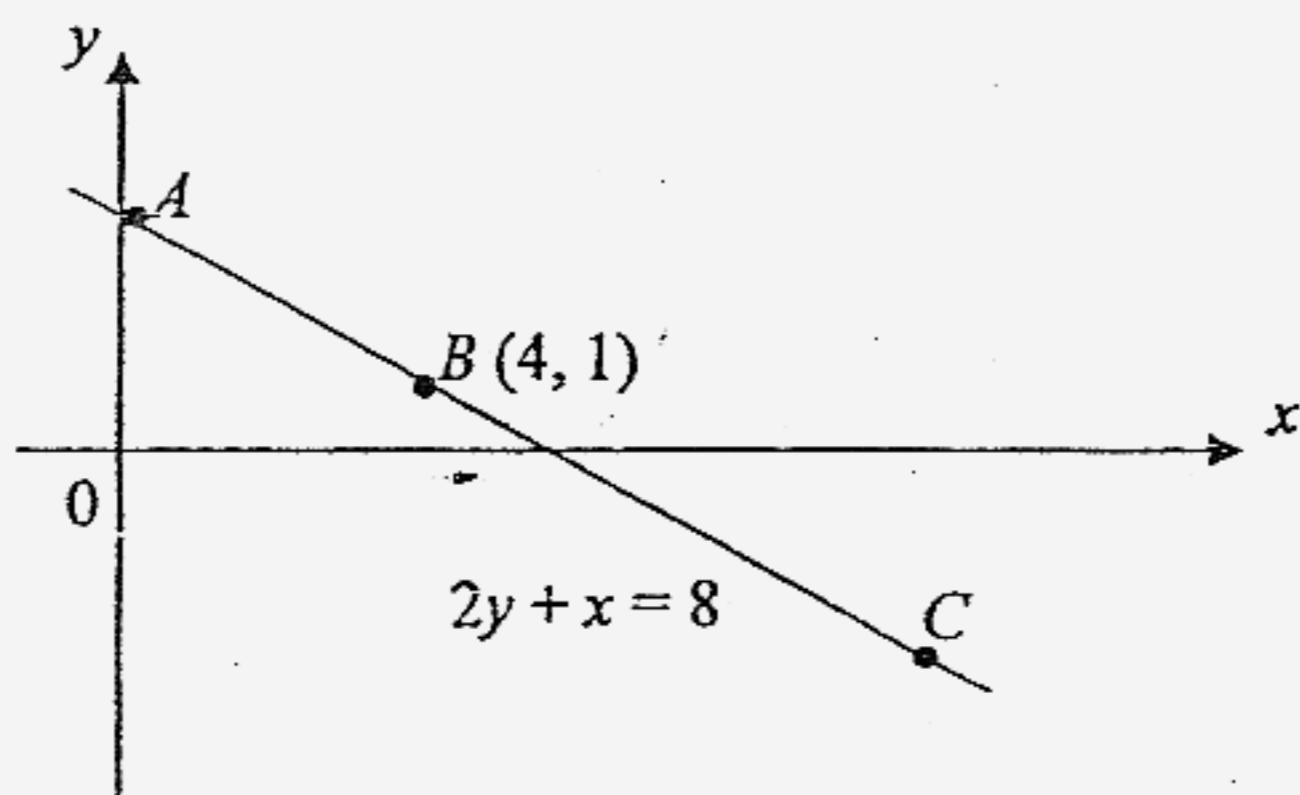
- 20 In the diagram, triangle PQR is an equilateral triangle of side 16 cm. A , B and C are points on PQ , QR and RP respectively such that $PA = QB = RC = 4$ cm.



(a) Prove that triangles APC and BQA are congruent. [2]

(b) Show that triangle ABC is an equilateral triangle. [2]

21

For
Examine
Use

In the diagram above, A , B and C are three points on the line $2y + x = 8$ such that $AB : BC = 1 : 2$. Find

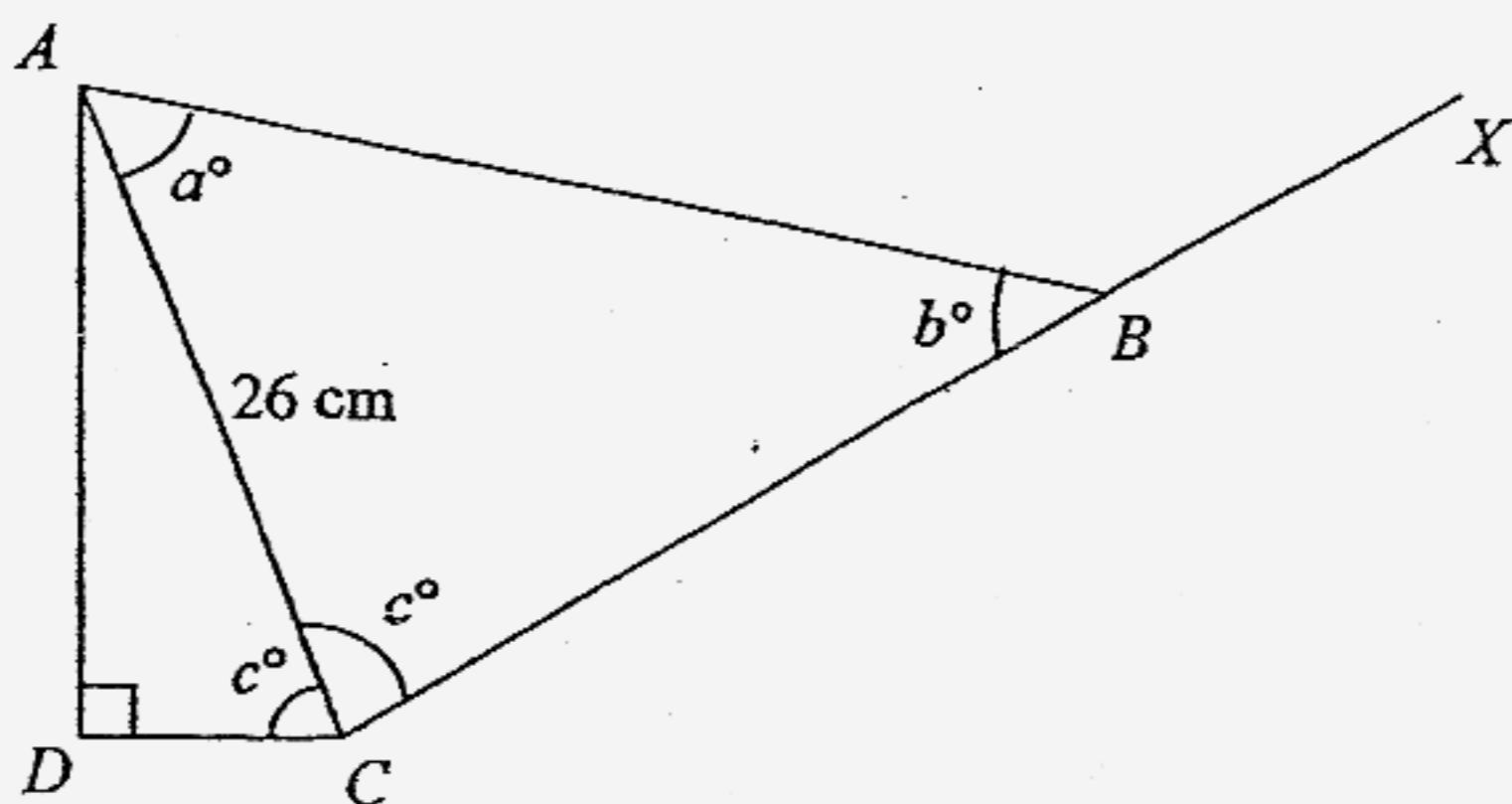
- the coordinates of the point A ,
- the coordinates of the point C ,
- the equation of the line which passes through point B and is perpendicular to the line $2y + x = 8$.

Answer (a) _____ [1]

(b) _____ [2]

(c) _____ [2]

22



In the diagram, $AC = 26 \text{ cm}$, $\angle ADC = 90^\circ$, $\angle DCA = \angle ACB$ and CBX is a straight line.

Using as much information given in the table below as is necessary,

	\sin	\cos	\tan
α°	$\frac{56}{65}$	$\frac{33}{65}$	$\frac{56}{33}$
b°	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{4}{3}$
c°	$\frac{12}{13}$	$\frac{5}{13}$	$\frac{12}{5}$

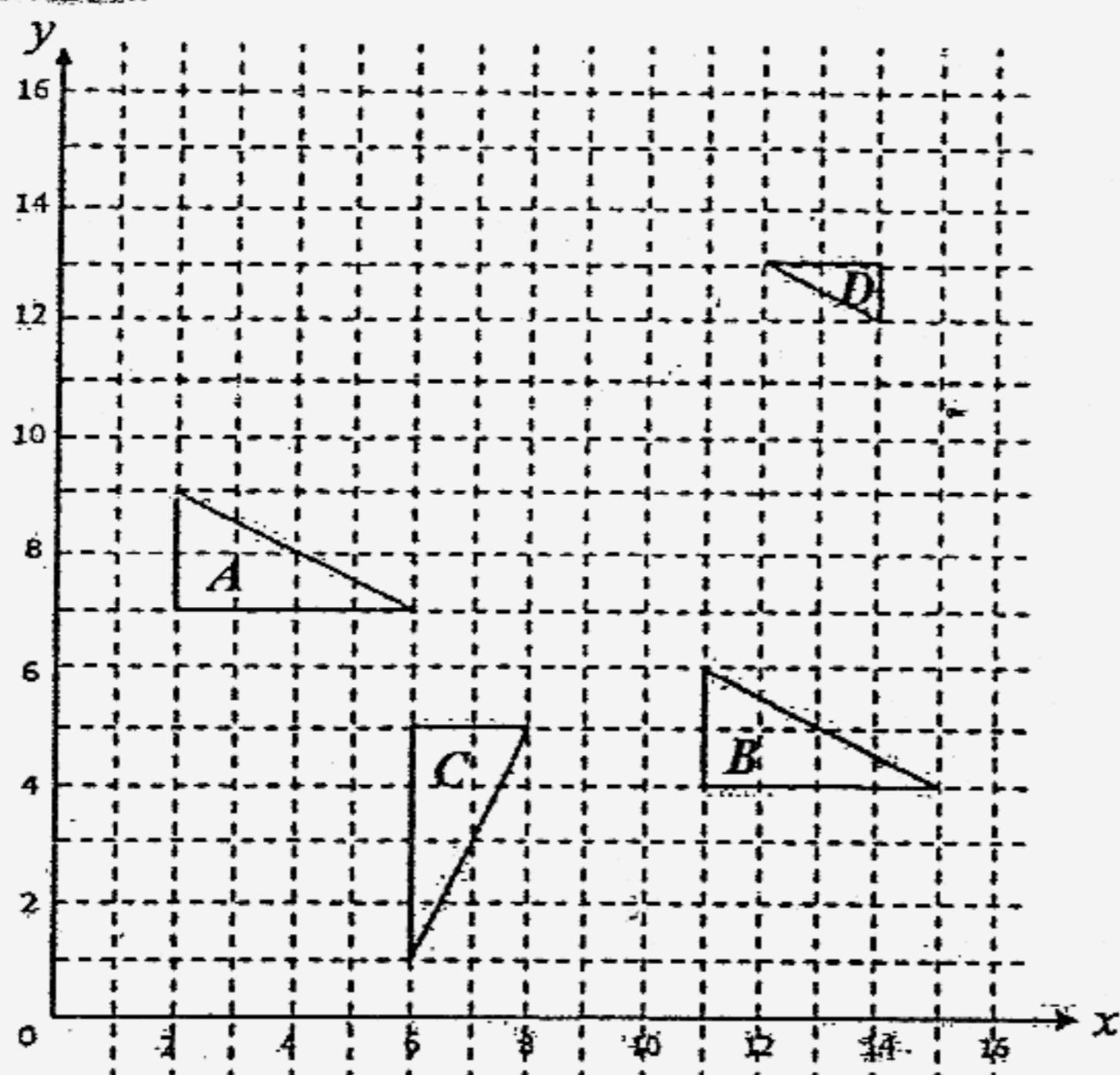
- (a) calculate the length of DC ,
- (b) calculate the length of CB ,
- (c) write down the value of $\cos \angle ABX$.

Answer (a) _____ [1]

(b) _____ [2]

(c) _____ [2]

23



- (a) A translation maps triangle A onto triangle B .
Write down the column vector representing this translation.
- (b) A rotation maps triangle A onto triangle C .
Write down the coordinates of the centre of this rotation and the angle of rotation.
- (c) An enlargement maps triangle A onto triangle D .
Write down the coordinates of the centre of this enlargement and its scale factor.

Answer (a) _____ [1]

(b) _____

[2]

(c) _____

[2]

- 24 *A, B and C are the 3 edges of a triangular plot of land such that A is 9 km due west of B, and C is above AB where BC is 10 km and AC is 11 km.*

Using the scale of 1 cm to represent 1 km, construct this triangular plot of land using only ruler and compasses. Label the 3 edges clearly. [2]

A farmer intends to reserve a region S within the triangular land ABC for paddy plantation. The paddies in S are

- I nearer to *B* than *A*,
- II less than 8 km from *A*,
- III nearer to *BC* than *BA*,
- IV 2.5 km or further away from *AB*.

Using ruler and compasses only, use conditions I, II, III and IV to construct appropriate loci. Hence, shade the region S. [5]

The farmer also intends to set up 2 scarecrows in the region S to protect the plantation. By taking appropriate measurement, suggest the furthest distance the 2 scarecrows from one another.



Answer Furthest distance = _____ km [1]

RVHS S4 PRELIM EXAM 2006 E MATH P2

Date _____

No. _____

1 (i) Price of each ball = \$ $\left(\frac{450}{x}\right)$ *

(ii) Price of each ball after reduction = \$ $\left(\frac{450}{x} - 3\right)$
 $= \$\left(\frac{450-3x}{x}\right)$ *

(iii) $(x+5)\left(\frac{450-3x}{x}\right) = x\left(\frac{450}{x}\right)$

$$\Rightarrow (x+5)\left(\frac{450-3x}{x}\right) = 450$$

$$\Rightarrow (x+5)(450-3x) = 450x$$

$$\Rightarrow (x+5)(150-x) = 150x$$

$$\Rightarrow -x^2 + 145x + 750 = 150x$$

$$\therefore x^2 + 5x - 750 = 0 \quad (\text{shown})$$

(iv) $x^2 + 5x - 750 = 0$

$$(x-25)(x+30) = 0$$

$$x = 25 \quad \text{or} \quad x = -30 \quad (\text{le})$$

(v) Price of each ball = \$ $\left(\frac{450}{25}\right)$

$$= \$18 \quad *$$

Total price of 25 balls = $25 \times (18-3)$

after reduction = \$375 *

Date _____

No. _____

2(a) Let $A(a, -8)$

$$\Rightarrow 5(-8) - 12(a) = 80$$

$$\Rightarrow 12a = -120$$

$$a = -10$$

$$\therefore A(-10, -8) \#$$

Let $B(0, b)$

$$\Rightarrow 5(b) - 12(0) = 80$$

$$\Rightarrow 5b = 80$$

$$b = 16$$

$$\therefore B(0, 16) \#$$

(b) Gradient of $AB = \frac{16 - (-8)}{0 - (-10)}$

$$= \frac{12}{5}$$

Let $y = mx + c$

$$\text{At } C(0, -8): -8 = \frac{12}{5}(0) + c$$

$$c = -8$$

$$\therefore \text{eqn of line: } y = \frac{12}{5}x - 8$$

$$5y - 12x = -40 \#$$

(c) Length of $AB = \sqrt{(-10-0)^2 + (-8-16)^2}$

$$= \sqrt{676}$$

$$= 26 \text{ units} \#$$

(d) $\tan \angle DAB = -\tan \angle CAB$

$$= -\frac{BC}{AC}$$

$$= -\frac{24}{10}$$

$$= -\frac{12}{5} \#$$

Date _____

No. _____

$$3(a) \text{ (i) No. of girls} = \frac{3}{5} \times \frac{80}{100} \times 50 \\ = 24 \quad *$$

$$\text{(ii) No. of boys} = \frac{2}{5} \times \frac{80}{100} \times 50 \\ = 16$$

$$\text{Ratio of boys to girls} = 16 : 24 \\ = 2 : 3 \quad *$$

$$(b) \text{ (i) Math mark} = \frac{8}{21} \times 231 \\ = 88 \quad *$$

$$\text{(ii) Physics mark} = \frac{\frac{7}{21} \times 231}{110} \times 100\% \\ = \frac{77}{110} \times 100\% \\ = 70\% \quad *$$

$$\text{(iii) Ratio of English mark to Japanese mark} = \frac{6}{21} \times 231 : 54 \\ = 66 : 54 \\ = 11 : 9 \quad *$$

$$\text{(iv) \% increase} = \frac{68 - 54}{54} \times 100\% \\ = \frac{14}{54} \times 100\% \\ = 25\frac{25}{27}\% \text{ or } 25.9\% \text{ (3 s.f.)} \quad *$$

Date _____

No. _____

4(a) (i) $\triangle ZXA$ is similar to $\triangle ZDC$ #

$$(ii) \frac{AZ}{CZ} = \frac{XZ}{DZ}$$

$$AZ = \frac{14}{10} \times 9$$

$$\therefore AZ = 12.6 \text{ cm} #$$

$$(iii) \hat{\triangle} ZDC = \hat{\triangle} CBA \text{ (opp } \hat{x}s \text{ of } ||\text{gram)}$$

$$\hat{\triangle} ZCD = \hat{\triangle} CAB \text{ (alt } \hat{x}s)$$

$$\hat{\triangle} ZDC = \hat{\triangle} CBA \text{ (alt } \hat{x}s)$$

\therefore by AAA ppty, $\triangle ZDC$ and $\triangle CBA$ are similar.

(b) (i) $\triangle WBC$ is similar to $\triangle WYZ$.

$$(ii) \frac{WZ}{WC} = \frac{YZ}{BC}$$

$$\frac{WZ}{WZ+9} = \frac{8}{24}$$

$$\frac{WZ}{WZ+9} = \frac{1}{3}$$

$$3WZ = WZ + 9$$

$$2WZ = 9$$

$$WZ = 4.5 \text{ cm}$$

$$\text{Hence } AW = AZ - WZ$$

$$= 12.6 - 4.5$$

$$= 8.1 \text{ cm} #$$

Date _____

No. _____

5 (a) $\angle COD = 360^\circ - 90^\circ - 90^\circ - 52^\circ$ (tangent \perp radius, \angle sum of quadrilateral)
 $= 128^\circ$ # ✓

(b) $\angle CAD = \frac{1}{2} \angle COD$ (\angle at ctr = $2 \times$ \angle at O°)
 $= \frac{1}{2}(128^\circ)$
 $= 64^\circ$ # ✓

(c) $\angle DCT = \frac{1}{2}(180^\circ - \angle DTC)$ (tangents from ext pt have equal length $\Rightarrow \triangle DCT$ isosceles)
 $= \frac{1}{2}(180^\circ - 52^\circ)$
 $= 64^\circ$ #

OR $\angle DCT = \angle DAC$ (alt seg thm)
 $= 64^\circ$ ✓

(d) $\angle BCG = 180^\circ - \angle DCT - \angle DCB$ (sum of \angle s on str. line)
 $= 180^\circ - 64^\circ - 90^\circ$ (right \angle in semicircle)
 $= 26^\circ$ # ✓

(e) $\angle DEA = \angle ABE + \angle BAE$ (ext \angle of \triangle)
 $= 48^\circ + \angle BAD - \angle CAD$ (right \angle in semicircle, complementary \angle s)
 $= 48^\circ + (90^\circ - 64^\circ)$
 $= 74^\circ$ # ✓ OR $= 48^\circ + \angle BCG$ (alt seg thm)
 $= 48^\circ + 26^\circ$
 $= 74^\circ$

6(a) (i) $P(\text{two nos. of diff value})$

$$= 1 - P(\text{both of same value})$$

$$= 1 - [3 \times (\frac{1}{6} \times \frac{1}{5})]$$

$$= \frac{9}{10} \quad \# \quad \checkmark$$

(ii) $P(\text{product is even no.})$

$$= P(\text{Box A even no., Box B odd no.})$$

$$= \frac{3}{6} \times \frac{5}{5}$$

$$= \frac{1}{2}$$

(b) (i)

+	3	4	5	6	7	8	
3	6	7	8	9	10	11	
5	8	9	10	11	12	13	
7	10	11	12	13	14	15	
9	12	13	14	15	16	17	
11	14	15	16	17	18	19	

(ii) (a) $P(\text{sum is odd})$

$$= \frac{15}{30}$$

$$= \frac{1}{2} \quad \# \quad \checkmark$$

(b) $P(\text{sum is prime})$

$$= \frac{10}{30}$$

$$= \frac{1}{3} \quad \# \quad \checkmark$$

(c) $P(\text{sum is } \geq 13)$

$$= \frac{15}{30}$$

$$= \frac{1}{2} \quad \# \quad \checkmark$$

Date _____

No. _____

$$7(a) \frac{OJ}{OL} = \frac{SN}{PL} \quad (\triangle OSN \text{ similar to } \triangle OPL)$$

$$\Rightarrow \frac{4+LN}{4} = \frac{10}{4}$$

$$4+LN = \frac{10}{4} \times 4$$

$$\therefore LN = 6 \text{ cm} \quad *$$

$$(b) \text{ Height of cylindrical base} = TU + MN$$

$$= 6 + \frac{6}{2} \quad (\text{M is midpt of LN})$$

$$= 9 \text{ cm} \quad *$$

(c) Curved surface area of the lampshade

$$= \text{CSA of cone OSR} - \text{CSA of cone OPQ}$$

$$= \pi(10)\sqrt{10^2+10^2} - \pi(4)\sqrt{4^2+4^2}$$

$$= \pi(10\sqrt{200} - 4\sqrt{32})$$

$$= 373.202\dots$$

$$= 373 \text{ cm}^2 \quad (3 \text{ s.f.}) \quad *$$

Area of circular ring

$$= \pi(10)^2 - \pi(4)^2$$

$$= \pi(100 - 16)$$

$$= 263.89\dots$$

$$= 264 \text{ cm}^2 \quad (3 \text{ s.f.}) \quad *$$

$$8(a) (i) \angle ABC = 151^\circ - (246^\circ - 180^\circ) \quad (\text{adj } \times s, \text{ alt } \times s)$$

$$= 151^\circ - 66^\circ$$

$$= 85^\circ$$

$$(ii) AC = \sqrt{90^2 + 160^2 - 2(90)(160)\cos 85^\circ} \quad (\text{cos rule})$$

$$= 176.606\dots$$

$$= 177 \text{ m (3 s.f.)} \#$$

$$(iii) \frac{\sin \angle ACD}{210} = \frac{\sin 50^\circ}{AC} \quad (\text{sin rule})$$

$$\angle ACD = \sin^{-1} \left[\frac{\sin 50^\circ}{AC} \times 210 \right]$$

$$= 65.6287\dots$$

$$= 65.6^\circ \text{ (1 d.p.)} \#$$

$$(iv) \text{Area of quadrilateral } ABCD = \frac{1}{2}(90)(160)\sin 85^\circ + \frac{1}{2}(210)(AC)\sin \hat{C}$$

$$= \frac{1}{2}(90)(160)\sin 85^\circ + \frac{1}{2}(210)(AC)\sin(180^\circ - 50^\circ - \hat{A})$$

$$= 24063.8818$$

$$= 24000 \text{ m}^2 \text{ (3 s.f.)}$$

(b) (i) Let T be the top of the coconut tree.

$$\tan \hat{CAT} = \frac{CT}{AC}$$

$$\Rightarrow CT = AC \times \tan 8^\circ$$

$$\therefore \text{height of the tree} = 24.820\dots$$

$$= 24.8 \text{ m (3 s.f.)} \#$$

(ii) Let M be the monkey's position on the tree.

$$\tan \hat{MBC} = \frac{\frac{1}{3}CT}{BC}$$

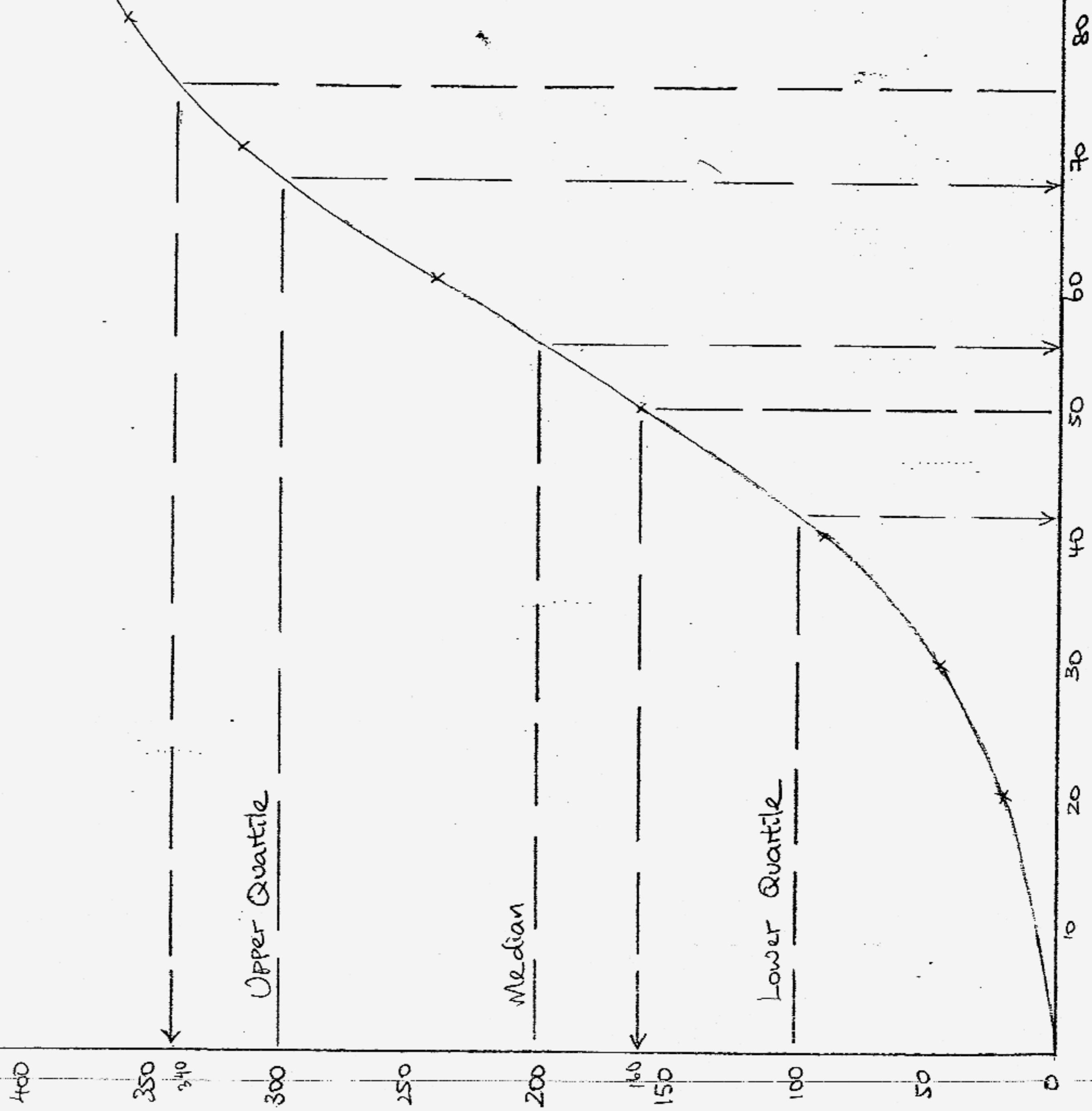
$$\hat{MBC} = \tan^{-1} \left(\frac{\frac{1}{3}CT}{160} \right)$$

$$= 2.960\dots$$

$$\therefore \times \text{ of elevation} = 3.0^\circ \text{ (1 d.p.)}$$

↑ No. of students

(a)



Q1 #9

(b) (i) Median position = $\frac{1}{2} \times 400$
= 200th

Median = 55 marks # ✓

(ii) Upper quartile = $\frac{3}{4} \times 400$
= 300th

= 67.5 marks

Lower quartile = $\frac{1}{4} \times 400$
= 100th

= 41.5 marks

Interquartile range

U.Q. - L.Q.

= 67.5 - 41.5

= 26 marks # ✓

(c) P (Fail, Distinction)
= $\left(\frac{160}{400} \times \frac{160}{399}\right) + \left(\frac{160}{400} \times \frac{160}{399}\right)$

= $\frac{16}{399}$ #

(d) (i) P = 160 - 90 = 70 # ✓
q = 315 - 240 = 75 # ✓

(ii) Height = $\frac{40}{20} \times 2 = 4 \text{ cm } \#$

$$10(a) (i) a = 999982000081 \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Same number of } '9's \text{ and } '0's$$

$$b = 99999820000081 \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

$$c = 55 \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

$$\text{Pattern: } \begin{aligned} 1+9 &= 10 \\ 2+8 &= 10 \\ 3+7 &= 10 \\ 4+6 &= 10 \end{aligned}$$

$$d = 64 \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

$$(ii) \text{ Sum of } n\text{th row} = 9(n-1) + 8 + 2 + 8 + 1 \\ = 9n - 9 + 19 \\ = 9n + 10 \quad \#$$

$$(b) (i) u = 2 + 6 + 12 + 20 + 30 = 70 \quad \#$$

$$v = 5 \quad \#$$

$$w = (5+1)(5+2) = 42 \quad \#$$

$$x = 2 + 6 + 12 + 20 + 30 + 42 = 112 \quad \#$$

$$y = 6 \quad ,$$

$$z = (6+1)(6+2) = 56 \quad \#$$

$$(ii) \text{ Sum of series} = \frac{1}{3}(N \times M)$$

$$= \frac{1}{3}[n \times (n+1)(n+2)]$$

$$= \frac{n}{3}(n+1)(n+2)$$

$$(iii) \text{ Let } 2345 = (N+1)(N+2)$$

Since the result for N is not a whole no.

$$\Rightarrow 2345 = N^2 + 3N + 2$$

$\Rightarrow 2345$ can NOT appear in column M

$$\Rightarrow N^2 + 3N - 2343 = 0$$

$$N = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-2343)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{9381}}{2}$$

$$= -49.9 \quad \text{or} \quad 46.9 \quad (\text{3 s.f.})$$

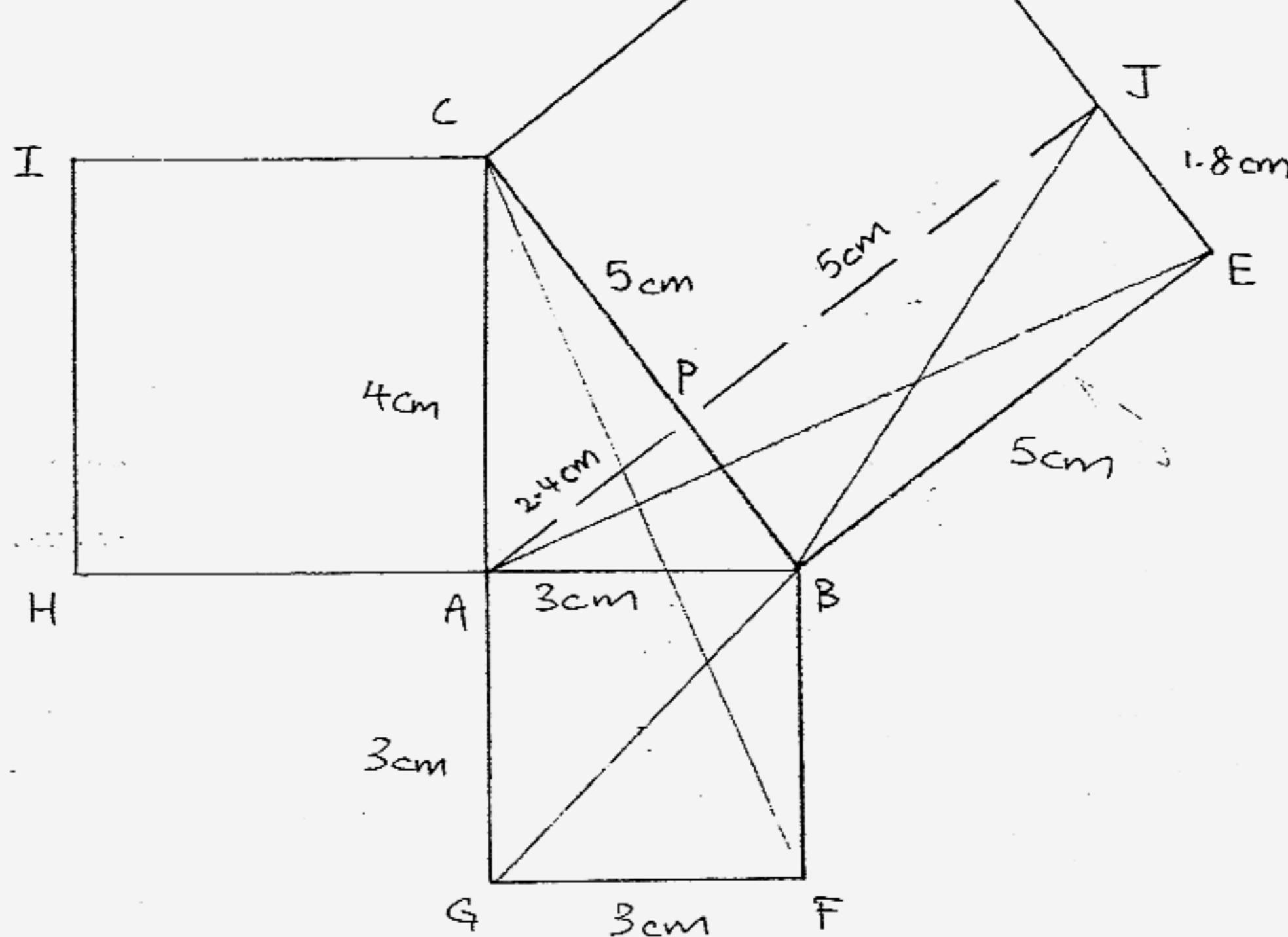
(a) Net area under curve = ...
 $\therefore \text{Area } LRN = B$

Area semicircle $MLN = \text{Area } P$
 $= \text{Area } Q + \text{Area } R$

Shaded Area = Area $Q + \text{Area } R - \text{Area } A$
 $- \text{Area } B$

Area $T = \text{Area semicircle } MLN - \text{Area } A$
 $- \text{Area } B$
 $= \text{Area } Q + \text{Area } R - \text{Area } A - \text{Area } B$
 $= \text{Shaded Area } \# \text{ (proven)}$

(a), (b)



(c) Area $\triangle ABF = \text{Area } \triangle BAE = \text{Area } \triangle BJE = \frac{3 \times 3}{2} = 4.5 \text{ cm}^2$

Area $\triangle ABC = \frac{4 \times 3}{2} = 6 \text{ cm}^2 \Rightarrow \frac{1}{2} \times 5 \times \text{ht} = 6 \text{ cm}^2 \Rightarrow \text{ht} = 2.4 \text{ cm}$

$AJ = 2.4 + 5 = 7.4 \text{ cm}$

$JE = \frac{4.5 \times 2}{5} = 1.8 \text{ cm}$
 $\Rightarrow \text{shear factor} = \frac{7.4}{7.4 + 1.8} = \frac{37}{91} \approx 0.41$

(d) Area $\triangle ABG = \text{Area } \triangle ABCF \text{ (shear)}$

$= \text{Area } \triangle BEA \text{ (Rotation)}$

$= \text{Area } \triangle BJE \text{ (shear)}$

(e) $\triangle IHC$ is mapped onto $\triangle IBC$ by

a shear with IC as invariant line and shear factor $\frac{7}{4}$;

$\triangle IBC$ is mapped onto $\triangle CDA$ by

a 90° anticlockwise rotation with centre C.

Let L be the intersection of Q and P

(g) Area $P = \frac{1}{2}\pi\left(\frac{MN}{2}\right)^2$

$= \frac{1}{2}\pi\left[\frac{(ML)^2 + (NL)^2}{2^2}\right]$

$= \frac{1}{2}\pi\left(\frac{ML}{2}\right)^2 + \frac{1}{2}\pi\left(\frac{NL}{2}\right)^2$

$= \text{Area } Q + \text{Area } R$

$$12(a) (i) \tan \alpha^\circ = \frac{4}{3}$$

$$\alpha^\circ = \tan^{-1} \left(\frac{4}{3} \right)$$

$$= 53.130 \dots$$

$$\alpha = 53.1^\circ \#$$

$$(ii) \text{ Let } 4k = 100 \quad (\because \alpha > 45^\circ \Rightarrow \text{vert dist} = RQ)$$

$$\Rightarrow k = 25 \#$$

$$(iii) 25 \left(\begin{matrix} 3 \\ 4 \end{matrix} \right) = \left(\begin{matrix} 75 \\ 100 \end{matrix} \right)$$

\therefore coord of the ball are $(75, 100)$ #

$$(iv) \text{ New vector} = \left(\begin{matrix} 4 \\ -3 \end{matrix} \right) \#$$

$$(v) \text{ Let } 4x = 25$$

$$\Rightarrow x = \frac{25}{4}$$

$$\left(\begin{matrix} 75 \\ 100 \end{matrix} \right) + \frac{25}{4} \left(\begin{matrix} 4 \\ -3 \end{matrix} \right) = \left(\begin{matrix} 75 \\ 100 \end{matrix} \right) + \left(\begin{matrix} 25 \\ -18.75 \end{matrix} \right) = \left(\begin{matrix} 100 \\ 81.25 \end{matrix} \right)$$

\therefore coord of ball are $(100, 81.25)$ #

$$(b) (vi) \vec{AC} = \vec{AB} + \vec{BC} \#$$

$$(viii) \vec{AM} = \frac{2}{3} \vec{AB}$$

$$\Rightarrow \vec{OM} - \vec{OA} = \frac{2}{3} (\vec{OB} - \vec{OA})$$

$$(vii) \vec{AC} = \vec{AB} + \vec{BC}$$

$$\vec{OM} = \frac{2}{3} (\vec{OB} - \vec{OA}) + 3\vec{P}$$

$$= (\vec{OB} - \vec{OA}) + (\vec{OC} - \vec{OB})$$

$$= 2\vec{q} - 2\vec{p} + 3\vec{p}$$

$$= 3\vec{q} - 3\vec{p} + 4(3\vec{q}) - 3\vec{q}$$

$$\therefore \vec{OM} = 2\vec{q} + \vec{p} \#$$

$$= 12\vec{q} - 3\vec{p} \text{ (shown)} \#$$

$$\vec{AN} = \frac{1}{3} \vec{AC}$$

$$\Rightarrow \vec{ON} - \vec{OA} = \frac{1}{3} (\vec{OC} - \vec{OA})$$

$$\vec{ON} = \frac{1}{3} [4(3\vec{q}) - 3\vec{p}] + 3\vec{p}$$

$$= 4\vec{q} - \vec{p} + 3\vec{p}$$

$$(vii) \text{ Since } \vec{ON} = 4\vec{q} + 2\vec{p} \\ = 2(2\vec{q} + \vec{p})$$

$$\therefore \vec{ON} = 4\vec{q} + 2\vec{p} \#$$

\Rightarrow The points O, M and N
are collinear