SWISS COTTAGE SECONDARY SCHOOL

SECONDARY THREE EXPRESS
FIRST SEMESTRAL EXAMINATION

ADDITIONAL MATHEMATICS 4038/1
PAPER 1

Thursday 10 May 2007 2 hours

Additional materials:
Answer Paper (8)
Graph Paper (1)

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number on all the work you hand in.
Write your answers and working on separate writing paper provided and not on question paper.
Write in dark blue or black pen on both sides of paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

At the end of the examination, fasten all work securely together in two bundles with the Summary Page as the Cover Page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B.$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin A + \sin B = 2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\sin A - \sin B = 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

$$\cos A + \cos B = 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\cos A - \cos B = -2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc\cos A.$$

$$\Delta = \frac{1}{2}bc\sin A.$$

1 The line $x + y = 4$ intersects the curve $y(2x+1) = 10$ at A and B. Find the coordinates of A and B. [5]

2 Evaluate $\frac{1-\sqrt{7}}{3+\sqrt{7}} + 2\sqrt{7}$. [4]

3 **(a)** Find the range of values of x for which $x(x-2) \leq 15$. [3]

(b) Find the range of values of k for which $x^2 + 6x > 4 - k$ for all values of x. [2]

4 Given that $\frac{4^x}{2^7} = 16^{\frac{1}{x}}$, find the possible values of x. [4]

5 **(a)** Find the remainder when $x^3 + 2x^2 - 4x - 8$ is divided by $x^2 - 9$. [2]

(b) Find the value of a and of b given that

$$x^3 + 2x^2 - 4x - 8 \equiv (x-3)(x+1)Q(x) + ax + b,$$

where $Q(x)$ is a polynomial.

[4]

6 Given that $y = (x-1)^2 + 3$,

(i) state the minimum value of y and the corresponding value of x, [1]

(ii) sketch the graph of $y = (x-1)^2 + 3$ for $0 \leq x \leq 3$. [3]

Hence, state the range of values of p for which the roots of the equation $(x-1)^2 = p - 3$ are real. [1]

7 **(a)** Solve the equation $e^x - 12e^{-x} = 4$. [5]

(b) Using the substitution $u = 3^x$, find the value of x such that $\frac{6}{9^x} + \frac{5}{3^x} = 1$. [5]

8 Given that the roots of the equation $2x^2 - x - 2 = 0$ are α and β,

(i) show that $\alpha^2 + \beta^2 = 2\frac{1}{4}$, [4]

(ii) form an equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [4]

9 (i) On the same diagram, sketch the graphs of $y = |3x + 5|$ and $y = 5 - x$. [3]

(ii) Solve the simultaneous equations $y = |3x + 5|$ and $y = 5 - x$. [3]

(iii) Hence, find the solution set for which $5 > |3x + 5| + x$. [2]

10 (a) Given that $\sqrt{p}\left(\sqrt{98} + 8\sqrt{2}\right) = 30\sqrt{q}$, express p in terms of q. [4]

(b) Express $\dfrac{2x + 1}{x^2 - 12x + 27}$ in partial fractions. [4]

11 (i) Solve the equation $2x^3 - 11x^2 = x - 30$. [4]

(ii) Hence, solve the equation $16x^3 - 44x^2 = 2x - 30$. [2]

12 Solve the equations

(i) $\sqrt{4 + \dfrac{3}{x}} = \dfrac{1}{\sqrt{x}} + 2$, [5]

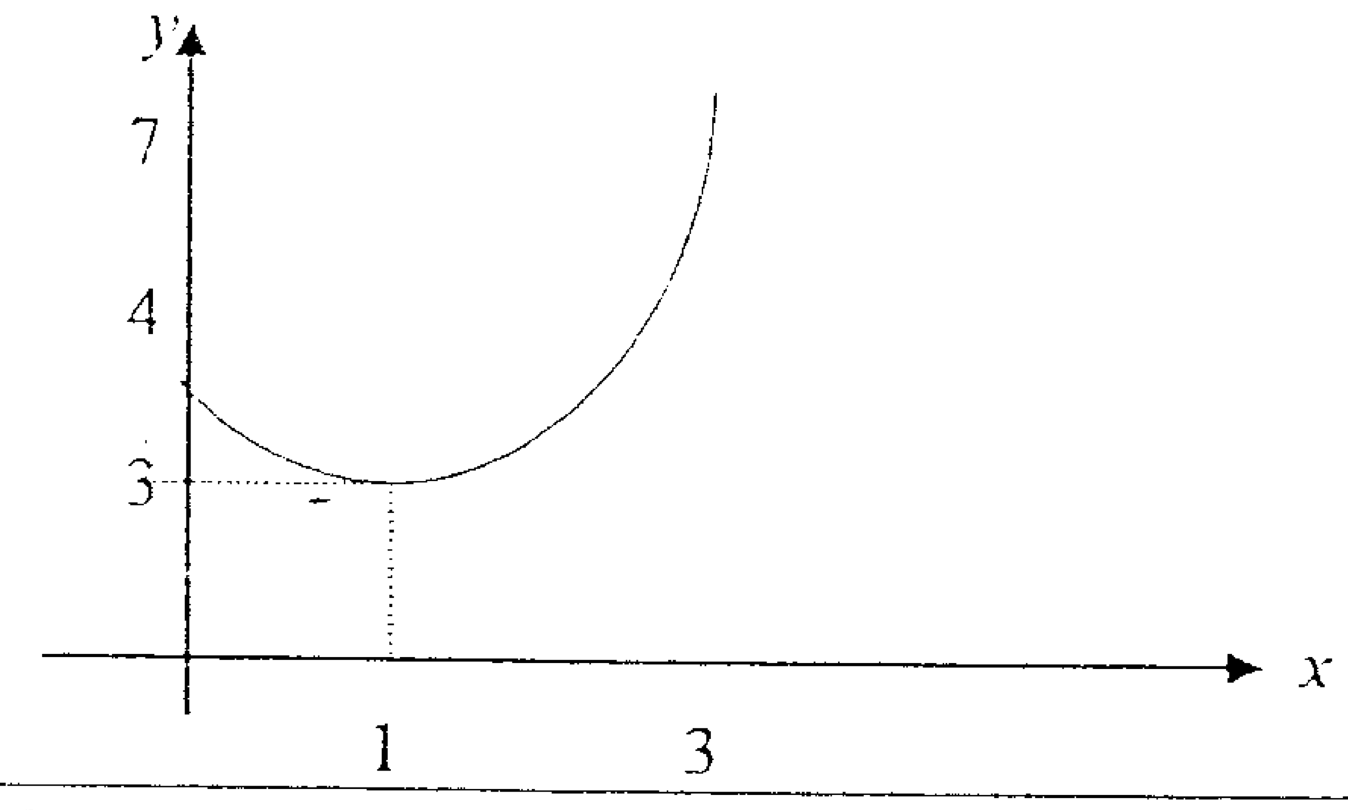
(ii) $\log_{27}(3x + 42) - \log_4 16 = \log_8 \dfrac{1}{4}$. [6]

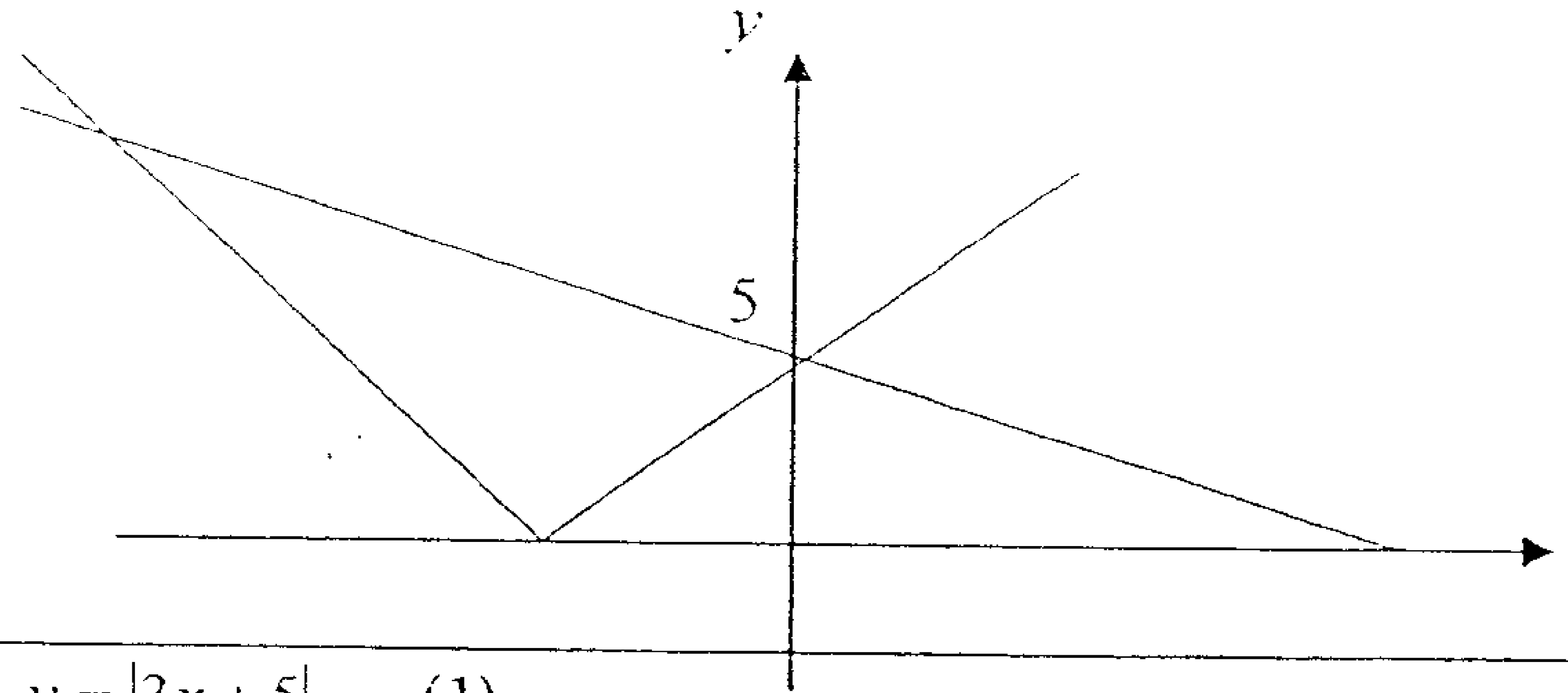
THE END

Paper (80 marks)

Qn.	Solution		Mark Allocation
1	$x+y=4$ $x=4-y \quad \cdots(1)$ $y(2x+1)=10 \quad \cdots(2)$ Subst. (1) into (2), $y(2(4-y)+1)=10$ $y(8-2y+1)=10$ $9y-2y^2=10$ $2y^2-9y+10=0$ $(2y-5)(y-2)=0$ $y=2\frac{1}{2} \quad or \quad 2$ When $y=2\frac{1}{2}$, $x=1\frac{1}{2}$ When $y=2$, $x=2$ $A\left(1\frac{1}{2}, 2\frac{1}{2}\right)$, $B(2, 2)$	**OR** $x+y=4$ $y=4-x \quad \cdots(1)$ $y(2x+1)=10 \quad \cdots(2)$ Subst. (1) into (2), $(4-x)(2x+1)=10$ $8x+4-2x^2-x=10$ $2x^2-7x+6=0$ $(2x-3)(x-2)=0$ $x=1\frac{1}{2} \quad or \quad 2$ When $x=1\frac{1}{2}$, $y=2\frac{1}{2}$ When $x=2$, $y=2$ $A\left(1\frac{1}{2}, 2\frac{1}{2}\right)$, $B(2, 2)$	M1 (substitution) M1 (factorise) M1 A2
2	$\frac{1-\sqrt{7}}{3+\sqrt{7}}+2\sqrt{7}$ $=\frac{1-\sqrt{7}}{3+\sqrt{7}}\times\frac{3-\sqrt{7}}{3-\sqrt{7}}+2\sqrt{7}$ $=\frac{3-4\sqrt{7}+7}{9-7}+2\sqrt{7}$ **OR** $=5-2\sqrt{7}+2\sqrt{7}$ $=5$	$\frac{1-\sqrt{7}}{3+\sqrt{7}}+2\sqrt{7}$ $=\frac{1-\sqrt{7}+2\sqrt{7}\left(3+\sqrt{7}\right)}{3+\sqrt{7}}$ $=\frac{1-\sqrt{7}+6\sqrt{7}7}{9-7}+2\sqrt{7}$ $=5-2\sqrt{7}+2\sqrt{7}$ $=5$	M1 (conjugate) M1 (expansion) M1 (simplify) A1
3a	$x(x-2)\le 15$ $x^2-2x-15\le 0$ $(x-5)(x+3)\le 0$ $-3\le x\le 5$		M1 (factorise) M1 (graph or number line) A1
3b	$x^2+6x+k-4>0$ Then, $b^2-4ac<0$ $36-4(k-4)<0$ $9<k-4$ $k>13$		M1 (no real roots) A1

Qn. #	Solution	Mark Allocation
4	$\frac{4^x}{2^7} = 16^{\frac{1}{x}}$ $2^{2x-7} = \left(2^4\right)^{\frac{1}{x}}$ Comparing indices on both sides, $-7 + 2x = \frac{4}{x}$ $2x^2 - 7x - 4 = 0$ $(2x+1)(x-4) = 0$ $x = -\frac{1}{2}$ or 4	M1 (change to base 2) M1 M1 A1
5a	$x^2 - 9 \overline{) x^3 + 2x^2 - 4x - 8}$ quotient $x + 2$ $(-)\ x^3 - 9x$ $2x^2 + 5x - 8$ $(-)\ 2x^2 - 18$ $5x + 10$ Remainder $= 5x + 10$	M1 A1
5b	$x^3 + 2x^2 - 4x - 8 = (x-3)(x+1)Q(x) + ax + b$ When $x = 3$, $3^3 + 2(3)^2 - 4(3) - 8 = 3a + b$ $25 = 3a + b$ $b = 25 - 3a \cdots (1)$ When $x = -1$, $(-1)^3 + 2(-1)^2 - 4(-1) - 8 = -a + b$ $-3 = -a + b$ $b = a - 3 \cdots (2)$ $(1) = (2)$, $a - 3 = 25 - 3a$ $4a = 28$ $a = 7$ and $b = 7 - 3 = 4$	M1 M1 A1 A1

Qn. #	Solution	Mark Allocation
6i	Minimum value of $y = 3$, Corresponding value of $x = 1$	B1
6ii		M1 – turning pt. (1, 3) M1 – symmetry & smoothness of curve, 1 more pt. M1 – y-intercept
	$(x-1)^2 + 3 = p$ Using graph, for the roots of equation to be real, $p \geq 3$ **OR** using discriminant, $b^2 - 4ac \geq 0$ $(-2)^2 - 4(4-p) \geq 0$ $4 \geq 4(4-p)$ $1 \geq 4 - p$ $p \geq 3$	B1
7a	$e^x - 12e^{-x} = 4$ $e^{2x} - 4e^x - 12 = 0$ $(e^x - 6)(e^x + 2) = 0$ $e^x = 6 \quad or \quad e^x = -2\ (NA)$ $x = \ln 6 \approx 1.79$	M1 M1 A1 (reject -ve) M1/ A1
7b	$\frac{6}{9^x} + \frac{5}{3^x} = 1$ $\frac{6}{(3^x)^2} + \frac{5}{3^x} = 1$ Given $y = 3^x$, $\frac{6}{y^2} + \frac{5}{y} = 1$ $6 + 5y = y^2$ $y^2 - 5y - 6 = 0$	M1 (simplify before using substitution mtd)

Qn. #	Solution	Mark Allocation
	$(y-6)(y+1)=0$ $y=6 \quad or \quad y=-1\,(NA)$ $3^x=6$ $x=\dfrac{\lg 6}{\lg 3}\approx 1.63$	M1 A1 (NA value) M1 A1
8i	$2x^2-x-2=0$ $x^2-\dfrac{1}{2}x+(-1)=0$ $\alpha+\beta=\dfrac{1}{2},\ \alpha\beta=-1$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=2\dfrac{1}{4}$	M2 – sum & prod of roots M1, A1
8ii	$\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=\dfrac{\alpha^2+\beta^2}{\alpha\beta}=-2\dfrac{1}{4}$ $\dfrac{\alpha}{\beta}\left(\dfrac{\beta}{\alpha}\right)=1$ Equation: $x^2+\dfrac{9}{4}x+1=0$ $4x^2+9x+4=0$	B1 B1 M1 A1
9i		M1 – use the graph of $y=\|3x+5\|$ A1 – modulus function B1 – straight line graph
9ii	$y=\|3x+5\|\ \ldots(1)$ $y=5-x\ \ldots(2)$ (1) = (2), $\|3x+5\|=5-x$ $3x+5=5-x \quad or \quad 3x+5=x-5$ $4x=0 \quad\quad 2x=-10$ $x=0 \quad\quad x=-5$ $y=5 \quad\quad y=10$	 M1 A2 (each pair)

Qn. #	Solution	Mark Allocation
9iii	$5 > \|3x+5\| + x$ $5 - x > \|3x+5\|$ Comparing y values of both graphs, $x : -5 < x < 0$	M1 A1
10a	$\sqrt{p}\left(\sqrt{98}+8\sqrt{2}\right)=30\sqrt{q}$ $\sqrt{p}\left(7\sqrt{2}+8\sqrt{2}\right)=30\sqrt{q}$ $\sqrt{p}\left(15\sqrt{2}\right)=30\sqrt{q}$ $15\sqrt{2p}=30\sqrt{q}$ $\sqrt{2p}=2\sqrt{q}$ $2p=4q$ $p=2q$	M1 – simplify surds M1 - $\sqrt{a}\sqrt{b}=\sqrt{ab}$ M1 – sq both sides A1
10b	$x^2-12x+27=(x-9)(x-3)$ $\frac{2x+1}{x^2-12x+27}=\frac{A}{x-9}+\frac{B}{x-3}$ $2x+1=A(x-3)+B(x-9)$ When $x=3$, $6+1=B(3-9)$ $B=-\frac{7}{6}$ When $x=9$, $18+1=A(9-3)$ $A=\frac{19}{6}$ $\frac{2x+1}{x^2-12x+27}=\frac{19}{6(x-9)}-\frac{7}{6(x-3)}$	B1 M1 M1 A1
11i	$2x^3-11x^2-x+30=(x-a)Q(x)+R$ Try $x-2$, $2x^3-11x^2-x+30=(x-2)Q(x)+R$ When $x=2$, $R=2(8)-11(4)-2+30=0$ Therefore, $2x^3-11x^2-x+30=(x-2)\left(2x^2+bx-15\right)$ Compare coeff. of x^2, $-11=b-4$ $b=-7$	M1 (obtain 1 factor) M1 (use identities or long division)

Qn.	Solution	Mark Allocation
	$2x^3-11x^2-x+30=0$ $(x-2)(2x^2-7x-15)=0$ $(x-2)(2x+3)(x-5)=0$ $x=2,-\frac{3}{2},5$	M1 – factorise completely A1
11ii	$16x^3-44x^2=2x-30$ $2(8x^3)-11(4x^2)-2x+30=0$ $2(2x)^3-11(2x)^2-2x+30=0$ $2x=2,-\frac{3}{2},5$ $x=1,-\frac{3}{4},\frac{5}{2}$	M1 (suitable substitution) A1
12i	$\sqrt{4+\frac{3}{x}}=\frac{1}{\sqrt{x}}+2$ $4+\frac{3}{x}=\frac{1}{x}+4+\frac{4}{\sqrt{x}}$ $\frac{2}{x}=\frac{4}{\sqrt{x}}$ $\sqrt{x}=2x$ $x=4x^2$ $x(4x-1)=0$ $x=0\ (na)\ or\quad x=\frac{1}{4}$	M1 (square both sides) M1 - expansion M1(square both sides) A1 (NA), A1
12ii	$\log_{27}(3x+42)-\log_4 16=\log_8\frac{1}{4}$ $\log_{27}(3x+42)=\log_4 4^2+\frac{\lg 2^{-2}}{\lg 2^3}$ $\log_{27}(3x+42)=2-\frac{2}{3}$ $3x+42=27^{\frac{4}{3}}=(3^3)^{\frac{4}{3}}$ $3x=81-42$ $x=13$ **or** $Let\ \log_8\frac{1}{4}=x$ $\frac{1}{4}=8^x$ $2^{-2}=2^{3x}$ $x=-\frac{2}{3}$	M1 - $\log_4 16=2$ M1, A1 – change for $\log_8\frac{1}{4}=-\frac{2}{3}$ M1 (log to exp) M1 – $27^{\frac{4}{3}}=(3^3)^{\frac{4}{3}}$ A1