National
Qualifications
SPECIMEN ONLY

SQ30/H/02

Mathematics
Paper 2

Date — Not applicable

Duration — 1 hour and 30 minutes

Total marks — 70

Attempt ALL questions.

You may use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not you may lose all the marks for this paper.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

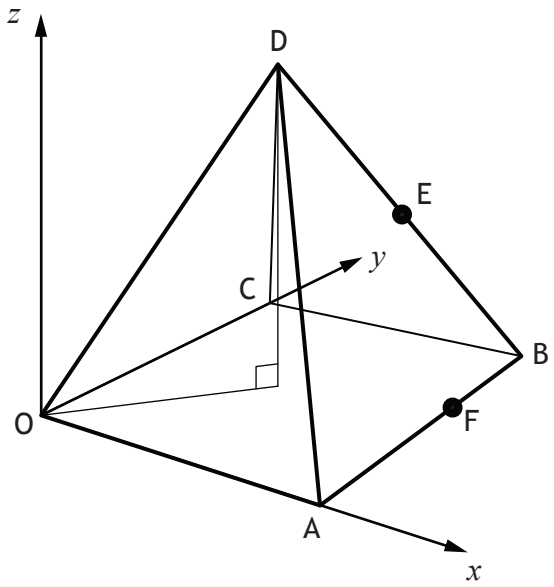
Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Attempt ALL questions **MARKS**

Total marks — 70

1.



A square based right pyramid is shown in the diagram.

Square OABC has a side length of 60 units with edges OA and OC lying on the x-axis and y-axis respectively.

The coordinates of D are (30, 30, 80).

E is the midpoint of BD and F divides AB in the ratio 2:1.

(a) Find the coordinates of E and F. **2**

(b) Calculate $\overrightarrow{ED}.\overrightarrow{EF}$. **2**

(c) Hence, or otherwise, calculate the size of angle DEF. **4**

2. A wildlife reserve has introduced conservation measures to build up the population of an endangered mammal. Initially the reserve population of the mammal was 2000. By the end of the first year there were 2500 and by the end of the second year there were 2980.

It is believed that the population can be modelled by the recurrence relation:

$u_{n+1} = au_n + b$,

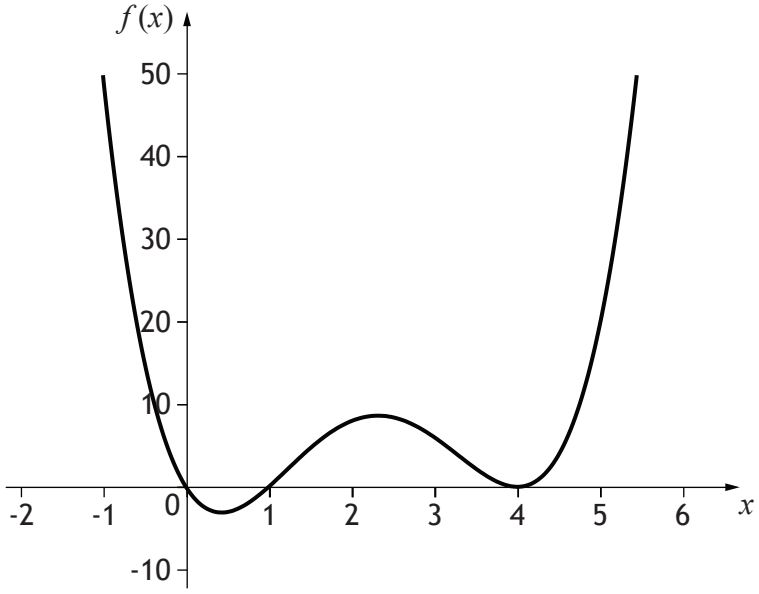
where a and b are constants and n is the number of years since the reserve was set up.

(a) Use the information above to find the values of a and b. **4**

(b) Conservation measures will end if the population stabilises at over 13 000. Will this happen? Justify your answer. **3**

MARKS

3. The diagram shows the graph of $f(x) = x(x-p)(x-q)^2$.



(a) Determine the values of p and q. **1**

(b) Find the equation of the tangent to the curve when $x = 1$. **4**

4. (a) Express $y = \log_4 2x$ in the form $y = \log_4 x + k$, clearly stating the value of k. **2**

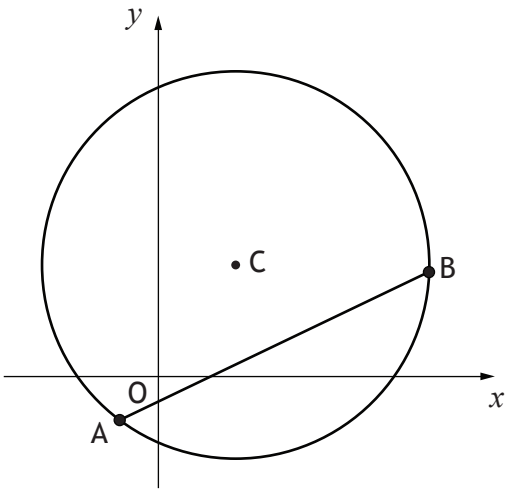
(b) Hence, or otherwise, describe the relationship between the graphs of $y = \log_4 2x$ and $y = \log_4 x$. **1**

(c) Determine the coordinates of the point where the graph of $y = \log_4 2x$ intersects the x-axis. **2**

(d) Sketch and annotate the graph of $y = f^{-1}(x)$, where $f(x) = \log_4 2x$. **3**

MARKS

5.



Points A(−1, −1) and B(7, 3) lie on the circumference of a circle with centre C, as shown in the diagram.

(a) Find the equation of the perpendicular bisector of AB. **4**

CB is parallel to the x-axis.

(b) Find the equation of the circle, passing through A and B, with centre C. **4**

6. The points A(0, 9, 7), B(5, −1, 2), C(4, 1, 3) and D(x, −2, 2) are such that AB is perpendicular to CD.

Determine the value of x. **5**

7. Given that $P(t) = 30e^{t-2}$ decide whether each of the statements below is true or false. Justify your answers.

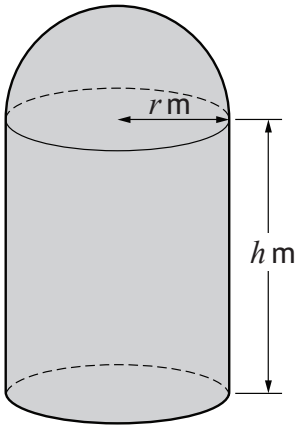
Statement A $P(0) = 30$.

Statement B When $P(t) = 15$, the only possible value of t is 1·3 to one decimal place. **6**

MARKS

8. A design for a new grain container is in the shape of a cylinder with a hemispherical roof and a flat circular base. The radius of the cylinder is r metres, and the height is h metres.

The volume of the **cylindrical** part of the container needs to be 100 cubic metres.



(a) Given that the curved surface area of a hemisphere of radius r is $2\pi r^2$ show that the surface area of metal needed to build the grain container is given by:

$$A = \frac{200}{r} + 3\pi r^2 \text{ square metres}$$

3

(b) Determine the value of r which minimises the amount of metal needed to build the container.

6

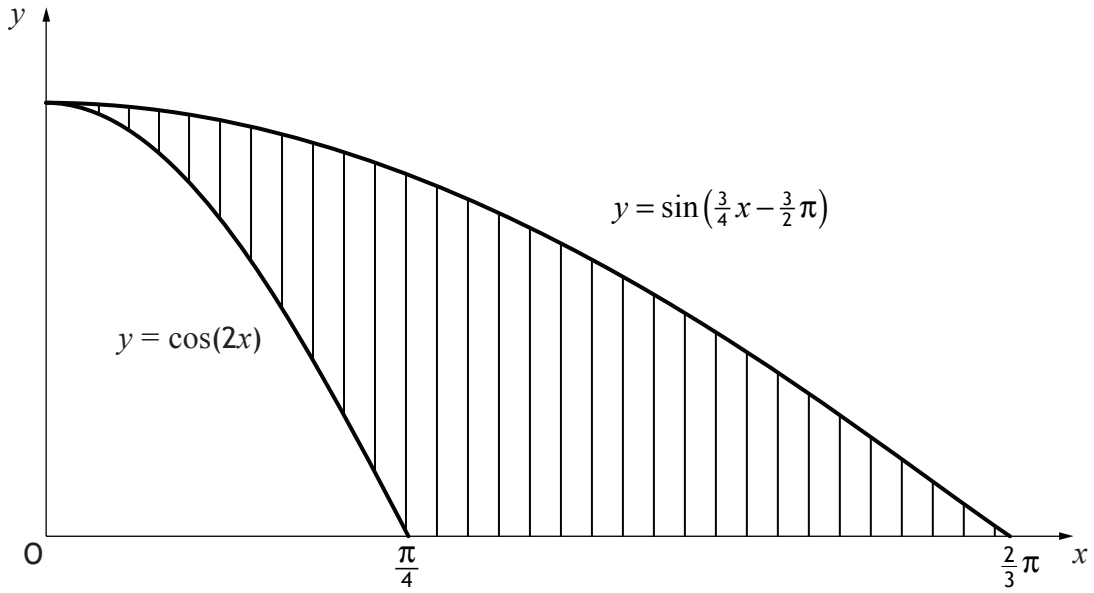
MARKS

9. A sea-life visitor attraction has a new logo in the shape of a shark fin.

The outline of the logo can be represented by parts of

- the x axis
- the curve with equation $y = \cos(2x)$
- the curve with equation $y = \sin\left(\frac{3}{4}x - \frac{3}{2}\pi\right)$

as shown in the diagram.

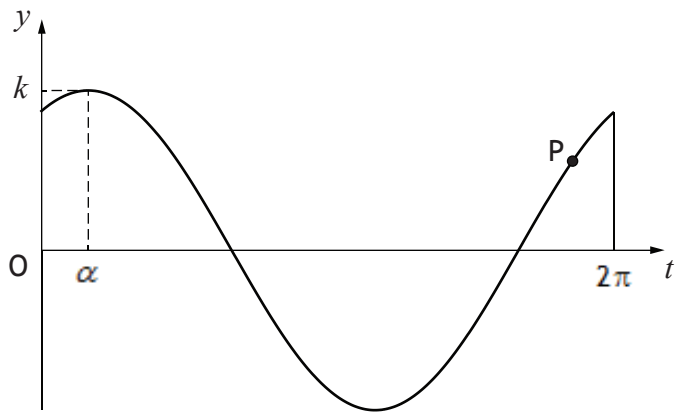


Calculate the shaded area. **6**

MARKS

10. Two sound sources produce the waves $y = \sin t$ and $y = \sqrt{3}\cos t$.

An investigation into the addition of these two waves produces the graph shown, with equation $y = k\cos(t - \alpha)$ for $0 \leq t \leq 2\pi$.



(a) Calculate the values of k and α. **4**

The point P has a y-coordinate of 1·2.

(c) Hence calculate the value of the t-coordinate of point P. **4**

[END OF SPECIMEN QUESTION PAPER]

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Mathematics
Paper 2

Marking Instructions

These Marking Instructions have been provided to show how SQA would mark this Specimen Question Paper.

SQA ©

General Marking Principles for Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

(a) Marks for each candidate response must always be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) Credit must be assigned in accordance with the specific assessment guidelines.

(d) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(e) Working subsequent to an error must be followed through, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is easier, candidates lose the opportunity to gain credit.

(f) Where transcription errors occur, candidates would normally lose the opportunity to gain a processing mark.

(g) Scored out or erased working which has not been replaced should be marked where still legible. However, if the scored out or erased working has been replaced, only the work which has not been scored out should be judged.

(h) Unless specifically mentioned in the specific assessment guidelines, do not penalise:

- Working subsequent to a correct answer
- Correct working in the wrong part of a question
- Legitimate variations in solutions
- Repeated error within a question

Definitions of Mathematics-specific command words used in this Specimen Question Paper.

Determine: find a numerical value or values from the information given.

Expand: multiply out an algebraic expression by making use of the distributive law or a compound trigonometric expression by making use of one of the addition formulae for $\sin(A \pm B)$ or $\cos(A \pm B)$.

Show that: use mathematics to prove something, eg that a statement or given value is correct — all steps, including the required conclusion, must be shown.

Express: use given information to rewrite an expression in a specified form.

Hence: use the previous answer to proceed.

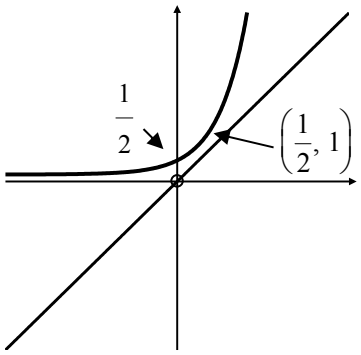
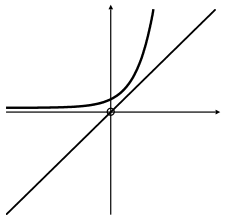
Hence, or otherwise: use the previous answer to proceed; however, another method may alternatively be used.

Justify: show good reason(s) for the conclusion(s) reached.

Specific Marking Instructions for each question

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
1	(a)		•[1] find coordinates of E •[2] find coordinates of F	2	•[1] $E(45,\ 45,\ 40)$ •[2] $F(60,\ 40,\ 0)$
	(b)		Ans: -1750 •[3] find $\overrightarrow{ED}$ and $\overrightarrow{EF}$ •[4] correct calculation of scalar product	2	•[3] $\overrightarrow{ED}=\begin{pmatrix}-15\\-15\\40\end{pmatrix}$, $\overrightarrow{EF}=\begin{pmatrix}15\\-5\\-40\end{pmatrix}$ •[4] $\overrightarrow{ED}.\overrightarrow{EF}=-225+75-1600=-1750$
	(c)		Ans: 154° •[5] know how to find angle DEF using formula •[6] find $\left\|\overrightarrow{ED}\right\|$ •[7] find $\left\|\overrightarrow{EF}\right\|$ •[8] calculates angle DEF	4	•[5] $\cos DEF=\dfrac{\overrightarrow{ED}.\overrightarrow{EF}}{\left\|\overrightarrow{ED}\right\|\left\|\overrightarrow{EF}\right\|}$ or equivalent •[6] $\left\|\overrightarrow{ED}\right\|=\sqrt{2050}$ •[7] $\left\|\overrightarrow{EF}\right\|=\sqrt{1850}$ •[8] $\cos DEF=\dfrac{-1750}{\sqrt{2050}\sqrt{1850}}$ $DEF=153\cdot977...=154°$
2	(a)		Ans: $a=0\cdot96$, $b=580$ •[1] set up one equation •[2] set up second equation •[3] solve for one variable •[4] solve for second variable	4	•[1] $2500=2000a+b$ •[2] $2980=2500a+b$ •[3] $480=500a$ or $12\,500=10\,000a+5b$ $a=\dfrac{480}{500}$ $11920=10\,000a+4b$ $a=0\cdot96$ $580=b$ •[4] $b=2500-2000(0\cdot96)$ $b=2500-1920$ $b=580$ or $2000a=2500-580$ $a=\dfrac{1920}{2000}$ $a=0\cdot96$

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
	(b)		Ans: Yes. Stabilises at 14 500 •5 knows how to find the limit •6 calculate limit •7 conclusion	3	•5 $u_{n+1} = 0\cdot 96u_n + 580, \quad -1 < a < 1$ $L = \frac{b}{1-a}$ $L = \frac{580}{1-0\cdot 96}$ •6 $L = 14\,500$ •7 yes, conservation measures will end, since the predicted population stabilises at 14 500 and 14 500 > 13 000
3	(a)		Ans: $p = 1, q = 4$ •1 state values of p and q	1	•1 $p = 1, q = 4$
	(b)		Ans: $y = 9(x-1)$ •2 expand brackets •3 differentiate •4 calculate gradient of tangent •5 substitutes gradient and (1,0) into equation of line	4	•2 $f(x) = x^4 - 9x^3 + 24x^2 - 16x$ •3 $f'(x) = 4x^3 - 27x^2 + 48x - 16$ •4 $f'(1) = 4 - 27 + 48 - 16 = 9$ •5 $y = 9(x-1)$

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
4	(a)		Ans: $y = \log_4 x + \frac{1}{2}$ •[1] using law of logarithms •[2] evaluating $\log_4 2$	2	•[1] $\log_4 2x = \log_4 2 + \log_4 x$ •[2] $\log_4 2 = \frac{1}{2}$
	(b)		Ans: Graph of $y = \log_4 x$ moved up by $\frac{1}{2}$ or graph of $y = \log_4 x$ compressed horizontally by a factor of 2. •[3] valid description of relationship	1	•[3] valid description – see answer
	(c)		Ans: $x = \frac{1}{2}$ •[4] setting $y = 0$ •[5] solving for x	2	•[4] $\log_4 2x = 0$ •[5] $x = \frac{1}{2}$
	(d)		Ans:  •[6] reflecting $y = \log_4 2x$ in the line $y = x$ •[7] correct shape •[8] annotating (2 points) **(or other valid method)**	3	•[6] reflect in $y = x$ •[7]  •[8] $\left(0, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, 1\right)$

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
5	(a)		Ans: $y-1=-2(x-3)$	4	
			•[1] calculate midpoint of AB		•[1] (3, 1)
			•[2] calculate gradient of AB		•[2] $\frac{1}{2}$
			•[3] state gradient of perpendicular bisector		•[3] -2
			•[4] substitute into equation of line		•[4] $y-1=-2(x-3)$
	(b)		Ans: $(x-2)^2+(y-3)^2=25$	4	
			•[5] knowing and using $y=3$		•[5] $3=-2x+7$
			•[6] solving for x		•[6] $x=2$
			•[7] identifying the radius		•[7] $r=5$
			•[8] obtain circle equation		•[8] $(x-2)^2+(y-3)^2=25$
6			Ans: $x=-3$	5	
			•[1] use perpendicular property		•[1] If $\overrightarrow{CD}$ is perpendicular to $\overrightarrow{AB}$ then $\overrightarrow{CD}.\overrightarrow{AB}=0$
			•[2] find $\overrightarrow{CD}$		•[2] $\begin{pmatrix} x-4 \\ -3 \\ -1 \end{pmatrix}$
			•[3] find $\overrightarrow{AB}$		•[3] $\begin{pmatrix} 5 \\ -10 \\ -5 \end{pmatrix}$
			•[4] correct substitution into scalar product formula		•[4] $5(x-4)+(-10)(-3)+(-5)(-1)=0$
			•[5] calculates value of x		•[5] $x=-3$

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
7			Ans: A False and B True •[1] valid reason for statement A •[2] selecting true or false for statement A with valid reason •[3] setting $P(t)=15$ •[4] taking log to base e •[5] completing valid reason •[6] selecting true or false for statement B with valid reason	6	•[1] $P(0)=30e^{-2}=4\cdot06$ •[2] false, since $P(0)\neq 30$ (do not award without valid reason) •[3] $15=30e^{t-2}$ •[4] $\ln e^{t-2}=\ln 0\cdot5$ •[5] $t-2=\ln 0\cdot5$ $t=\ln 0\cdot5+2$ (1·3) •[6] true, since $t=1\cdot3$ to one decimal place and there is only one solution (do not award without valid reason)
Notes			Substituting $t=1\cdot3$ into $P(t)=30e^{t-2}$ is not sufficient to show that statement B is true, since it does not prove that $t=1\cdot3$ is the <u>only</u> solution.		
8	(a)		•[1] know to equate volume to 100 •[2] obtain an expression for h •[3] complete area evaluation	3	•[1] $V=\pi r^2h=100$ •[2] $h=\dfrac{100}{\pi r^2}$ •[3] $A=\pi r^2+2\pi r^2+2\pi r\times\dfrac{100}{\pi r^2}$
	(b)		Ans: $r=2\cdot20$ m •[4] know to and start to differentiate •[5] complete differentiation •[6] set derivative to zero •[7] obtain r •[8] justify nature of stationary point •[9] interpret result	6	•[4] $A'(r)=6\pi r\ldots$ •[5] $A'(r)=6\pi r-\dfrac{200}{r^2}$ •[6] $6\pi r-\dfrac{200}{r^2}=0$ •[7] $r=2\cdot20$ metres •[8] $A''(r)=6\pi+\dfrac{400}{r^3}\Rightarrow A''(2\cdot1974\ldots)=56\cdot5\ldots$ •[9] minimum (when $r=2\cdot20$ m)
Notes			Candidates may use a nature table at •[8] to justify a minimum turning point when $r=2\cdot1974\ldots$		

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
9			Ans: $\frac{5}{6}$ •[1] knowing to use integration •[2] using correct limits •[3] integrating correctly •[4] integrating correctly •[5] substituting limits correctly •[6] evaluating correctly	6	•[1] $\int \sin(\frac{3}{4}x-\frac{3}{2}\pi)dx-\int \cos(2x)dx$ •[2] $\int_0^{\frac{2}{3}\pi} \sin(\frac{3}{4}x-\frac{3}{2}\pi)dx-\int_0^{\frac{\pi}{4}} \cos(2x)dx$ •[3] $\left[-\frac{4}{3}\cos(\frac{3}{4}x-\frac{3}{2}\pi)\right]$……. •[4] $-[\frac{1}{2}\sin(2x)]$ •[5] See * below •[6] $(\frac{4}{3}-0)-(\frac{1}{2}-0)=\frac{5}{6}$
* $([-\frac{4}{3}\cos(\frac{3}{4}\times\frac{2}{3}\pi-\frac{3}{2}\pi)]-[-\frac{4}{3}\cos(0-\frac{3}{2}\pi)])-([\frac{1}{2}\sin(2\times\frac{1}{4}\pi)]-[\frac{1}{2}\sin(2\times0)])$					
10	(a)		Ans: $k=2, \alpha=\frac{\pi}{6}$ or equivalent •[1] knows to set wave function equal to addition of individual waves •[2] knows to expand •[3] knows to compare coefficients •[4] interpret comparison	4	•[1] $\sin t+\sqrt{3}\cos t=k\cos(t-\alpha)$ or equivalent •[2] $k\cos\alpha\cos t+k\sin\alpha\sin t$ or equivalent •[3] $k\sin\alpha=1,\quad k\cos\alpha=\sqrt{3}$ or equivalent •[4] $k=2,\quad \alpha=\frac{\pi}{6}$ or equivalent
			Ans: 5·9 •[5] equates wave function with y-coordinate of P •[6] rearranges correctly •[7] solve equation for $t-\frac{\pi}{6}$ •[8] find t-coordinate of P by interpreting diagram	4	•[5] $2\cos\left(t-\frac{\pi}{6}\right)=1\cdot2$ or equivalent •[6] $\cos\left(t-\frac{\pi}{6}\right)=0\cdot6$ or equivalent •[7] $t-\frac{\pi}{6}=0\cdot927\ldots$ & $5\cdot355\ldots$ $1\cdot45\ldots$ & •[8] $5\cdot879\ldots$

[END OF SPECIMEN MARKING INSTRUCTIONS]