X202/701

NATIONAL QUALIFICATIONS 2008

THURSDAY, 29 MAY
1.00 PM – 4.00 PM

APPLIED MATHEMATICS
ADVANCED HIGHER
Statistics

Read carefully

1. Calculators may be used in this paper.
2. Candidates should answer all questions.

   Section A assesses the Units Statistics 1 and 2
   Section B assesses the Unit Mathematics for Applied Mathematics
3. **Full credit will be given only where the solution contains appropriate working.**
4. A booklet of Statistical Formulae and Tables is supplied for all candidates.

Section A (Statistics 1 and 2)

Marks

Answer all the questions.

A1. One in ten thousand members of a bird population is thought to have an avian influenza virus and a test used to screen birds for this virus detects it in 99·9% of infected birds. The test has a false alarm rate of 0·2% ie the probability that the test incorrectly indicates the presence of the virus in an uninfected bird is 0·2%. Calculate the probability that a bird from this population testing positive is, in fact, uninfected. **4**

A2. Random Digit Dialling (RDD) may be used to select samples of households for inclusion in surveys. On an island, all land-line telephones have 10-digit numbers of the form 012345****. By generating random numbers in the range 0000 to 9999 and appending them to the string 012345, a consumer research organisation created a set of potential contact numbers for a sample of households on the island.

(*a*) Give two possible reasons why this procedure would be highly unlikely to yield a random sample of households on the island. **2**

(*b*) Explain briefly how you would set about taking a simple random sample of households on the island. **2**

A3. Current acoustic standards in the UK, for separating floors between flats used for residential purposes, require that maximum impact sound transmission should be 62dB. Tests performed on random samples of a type of floor yielded maximum impact sound transmission results that could be adequately modelled by the $N(53{\cdot}4, 4{\cdot}7^2)$ distribution. Calculate:

(*a*) the proportion of floors that would fail to meet the standard; **3**

(*b*) the value to which mean maximum impact sound transmission for this type of floor would have to be reduced in order to ensure that at most 1% of floors would fail to meet the standard. (Assume that the variability remains unchanged.) **3**

Marks

A4. The evaluation of the nutritional status of hospital patients requires both height and weight measurements. In the case of patients who are too unwell to stand up, height is often estimated from demi-span ie the distance from the middle of the sternum to the tip of the middle finger. In order to investigate the relationship between height and demi-span, a student dietician took a random sample of 50 male students and measured height (y) and demi-span (x) for each, in metres. He calculated the following summary statistics from the data:

$\bar{x} = 0{\cdot}8916 \qquad \bar{y} = 1{\cdot}7268$

$S_{xx} = 0{\cdot}144672 \qquad S_{xy} = 0{\cdot}170456 \qquad S_{yy} = 0{\cdot}221088$

(*a*) Obtain the equation of the least squares linear regression of y on x and demonstrate that there is very strong evidence that the population slope parameter differs from zero. Interpret this evidence. **6**

(*b*) Explain why prediction intervals are more likely to be of interest in this context than confidence intervals. **1**

A5. In a study of the therapeutic benefits of horse-riding for disabled children, a researcher obtained Rosenberg self-esteem scores for each of ten children suffering from cerebral palsy, both before and after participation in a therapeutic riding programme. The Rosenberg score is such that the higher the value then the greater is the subject's self-esteem. The results are tabulated below.

Subject	1	2	3	4	5	6	7	8	9	10
Before	2·8	3·2	2·8	3·3	3·1	3·0	3·5	3·2	3·0	3·1
After	3·6	3·7	3·8	3·5	3·4	3·4	3·3	3·4	3·0	3·5

Stating any assumption required, perform a hypothesis test in order to determine whether or not the data provide evidence that horse-riding leads to increased self-esteem. **7**

A6. The progeny that result from cross-breeding two types of plant can be any one of three genotypes A, B or C. A genetic model predicts that the frequencies of occurrence of the genotypes A, B and C are in the ratios 1:2:1. The observed frequencies obtained from an experiment involving 100 cross-bred plants were as follows.

Genotype	A	B	C
Frequency	17	56	27

Perform a chi-squared goodness-of-fit test in order to assess the applicability of the genetic model in this case. **5**

[Turn over

A7. Sentence-length in the known works of a medieval author has mean 24·9 words with standard deviation 4·6 words. A random sample of 30 sentences from a medieval work of unknown authorship has mean length 23·3 words. Stating any assumption required, obtain a 95% confidence interval for the mean sentence length for the unknown author and comment on the authorship of the manuscript sampled. **6**

A8. An emergency services team based in a city may be called to assist at road traffic accidents and at fires. The number of requests per day for assistance at road traffic accidents, R_t, has the Poisson distribution with mean 5·5 and the number of requests per day for assistance at fires, R_f, has the Poisson distribution with mean 3·5. The total number of requests per day is denoted by T and the variables R_t and R_f may be assumed to be independent.

(*a*) Write down the ordered pairs (R_t, R_f) corresponding to $T = 2$ and hence use the laws of probability to calculate $P(T = 2)$. **5**

(*b*) Write down the mean and variance of T. **2**

(*c*) Given that T also has a Poisson distribution, confirm your answer to part (*a*). **1**

(*d*) The manager of the emergency services team wishes to know the value of the smallest integer, k, such that $P(T > k) < 0{\cdot}01$. Determine the value of k. **2**

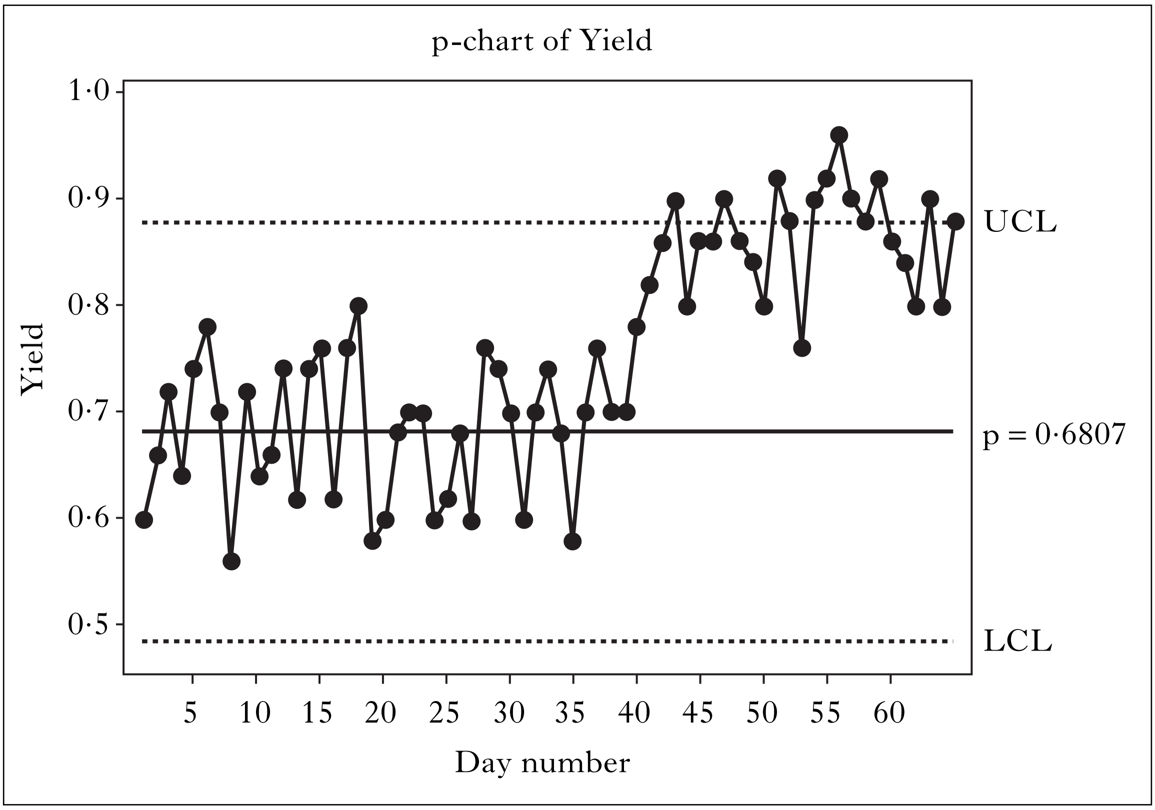
A9. Market researchers wished to compare the effectiveness of television adverts for two new, similar products, A and B. The researchers believed that the advert for product B was more effective in terms of likelihood to purchase than the advert for product A. Independent groups of ten potential purchasers were randomly selected. Each group viewed the advert for one of the products and then rated the product on a scale from 1 to 10, with 1 representing "certain not to purchase" and 10 representing "certain to purchase". The ratings are tabulated below.

Product A	1	7	3	6	8	3	5	5	8	5
Product B	6	5	9	4	8	8	8	7	5	8

(*a*) Display the data appropriately and comment. **3**

(*b*) Perform a suitable hypothesis test in order to investigate whether the data provide any evidence that the advert for product B is superior to that for product A. **7**

A10. In a manufacturing operation, daily samples of items were checked for nonconformities, the sample size being constant. The proportion of items free from nonconformities is referred to as Yield. The p-chart of Yield for 60 days production is shown below with 3-sigma limits. Process changes were made after 40 days.



(*a*) Given that the sample size is 50, calculate the upper and lower control limits. **3**

(*b*) Explain how the chart provides evidence that the process changes have had a beneficial effect on Yield. **1**

It was decided to chart the data from day 41 onwards in a new p-chart with limits based on the data for days 41 to 60 inclusive. The mean Yield for days 41 to 60 inclusive was 0·8648.

(*c*) Show that this would result in an impossible upper limit. **2**

(*d*) By solving an inequality, determine the smallest sample size that would lead to a viable upper limit for monitoring the modified process. **3**

[*END OF SECTION A*]

[Turn over for Section B on *Pages six* and *seven*

Section B (Mathematics for Applied Mathematics)

Answer all the questions.

Marks

B1. Given that A, B, C and D are square matrices where:

$$A = \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 6 \\ 0 & -3 \end{pmatrix} \quad C = \begin{pmatrix} x & 2 \\ 0 & y \end{pmatrix} \quad D = \begin{pmatrix} 2 & 7 \\ 12 & -1 \end{pmatrix}$$

(*a*) Find AB. **1**

(*b*) Express $4C + D$ as a single matrix. **2**

(*c*) Given that $AB = 4C + D$, find the values of x and y. **2**

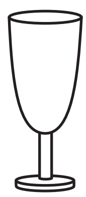
B2. Given that $y = e^{2x}\cos x$, find $\dfrac{dy}{dx}$. **3**

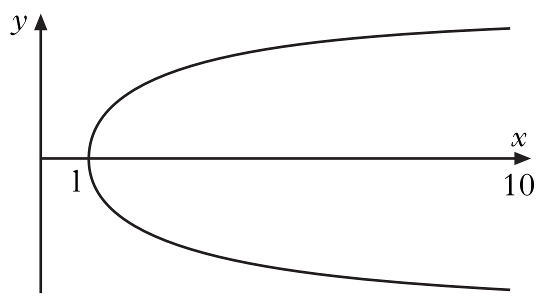
B3. Express $y = \dfrac{4x-3}{x(x^2+3)}$, $x \neq 0$, in partial fractions. **4**

B4. (*a*) Use integration by parts to show that $\int \ln x \, dx = x\ln x - x + c$. **2**

(*b*) A goblet consists of a bowl and a short stem. The diagram below shows the bowl section of the goblet (on its side).

The equation of the upper half of the curve is $y = 2\sqrt{\ln x}$ for $1 \le x \le 10$.





Given that the stem has length 1 and the overall height is 10, what is the capacity of the bowl? **4**

Marks

B5. (*a*) Use the standard formulas for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n}(6r^2 - r) = \tfrac{1}{2}\,n(n+1)(4n+1).$$ **3**

(*b*) Hence evaluate $\sum_{r=5}^{10}(6r^2 - r)$. **2**

B6. Newton's law of cooling states that a body loses heat at a rate which is proportional to the difference in temperature between itself and its surroundings. So, in a room with constant temperature 22 °C, the temperature T °C of a body after a time t minutes satisfies

$$\frac{dT}{dt} = k(T - 22)$$

where k is a negative constant.

(*a*) Hence show that T can be expressed in the form $T = Ae^{kt} + 22$ for some arbitrary constant A. **4**

(*b*) In a restaurant, where the temperature remains constant at 22 °C, a freshly baked roll, with temperature 82 °C, is placed on a cooling tray. After 5 minutes, the temperature of the roll has fallen by 20 degrees.

Calculate the values of A and k.

Write down an expression for the temperature of the roll after t minutes.

Supposing the roll remains uneaten after a further 5 minutes, what will its temperature be? **5**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

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