

X202/701

NATIONAL
QUALIFICATIONS
2010

TUESDAY, 25 MAY
1.00 PM – 4.00 PM

APPLIED
MATHEMATICS
ADVANCED HIGHER
Statistics

Read carefully

1. Calculators may be used in this paper.
2. Candidates should answer all questions.

Section A assesses the Units Statistics 1 and 2

Section B assesses the Unit Mathematics for Applied Mathematics

3. **Full credit will be given only where the solution contains appropriate working.**
4. A booklet of Statistical Formulae and Tables is supplied for all candidates.



Section A (Statistics 1 and 2)

Marks

Answer all the questions.

- A1.** A medical researcher, investigating body temperature ($^{\circ}\text{C}$) in a population of healthy males, determined that the random variable could be modelled adequately by a $N(36.2, 0.16)$ distribution.

- (a) Determine the reference range (ie the range of body temperature, symmetrical about the mean, that includes 95% of the population).

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In order to make comparisons with the results of an earlier study the researcher wished to convert the mean and standard deviation to degrees Fahrenheit.

- (b) Given that t degrees Celsius is equivalent to $(1.8t + 32)$ degrees Fahrenheit, calculate the required values.

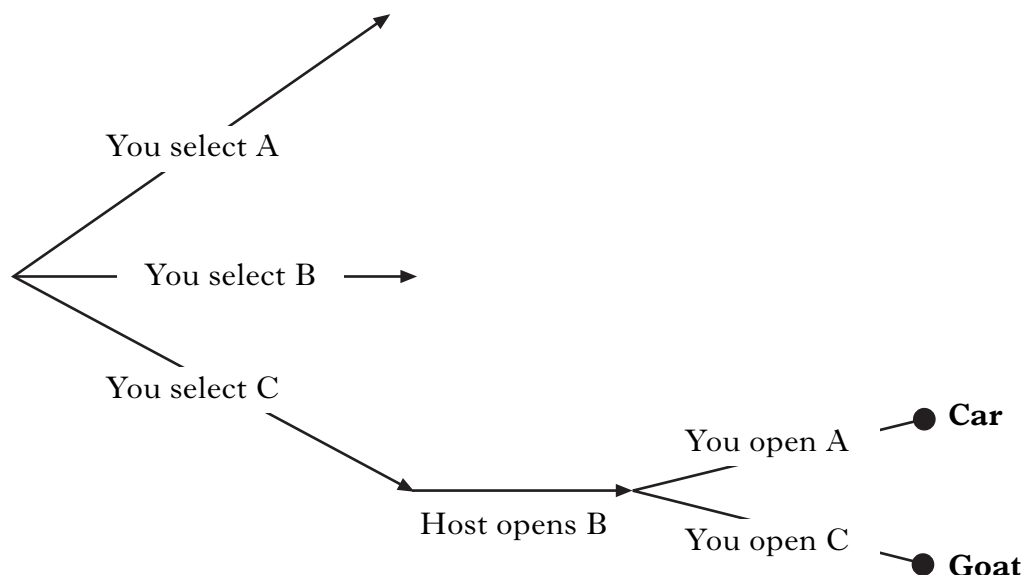
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- A2.** Imagine that you are taking part in a television game show during which you are given the choice of opening one of three doors A, B and C. Behind one of the doors is a car and behind each of the other two is a goat. Behind the door that you ultimately open will be your prize.

You are asked to select, **but not open**, one of the three doors. The host then opens one of the two remaining doors to reveal a goat. You are then given the options to either open the door that you selected initially or to switch and open the other closed door. The game show host knows where the car is located.

- (a) Given that both you and the host will make random selections when appropriate and that the car is behind door A, copy and complete the tree diagram to indicate the eight possible sequences of events and the probabilities of the corresponding eight outcomes.

4



- (b) Determine the conditional probability of winning the car, given that you switch and comment on the strategy that a player who wants to win a car should adopt.

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- A3.** At the end of 2006, the newspaper *USA Today* reported the results of a poll of predictions for 2007 made by a sample of adult citizens. It stated:

“The telephone poll of 1000 adults was conducted Dec 12-14 by *Ipsos*, an international polling firm. The margin of sampling error was plus or minus three percentage points.”

- (a) State why such sampling could potentially lead to biased results. 1

A 95% confidence interval for a population proportion, based on a random sample of size n , is given approximately by:

$$\hat{p} \pm 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}},$$

which may be considered as Estimate $\pm$ Margin of Sampling Error.

- (b) Show that the maximum value of $\hat{p}(1-\hat{p})$ is 0.25 and hence deduce that the maximum margin of sampling error for a random sample of 1000 is just over three percentage points. 5

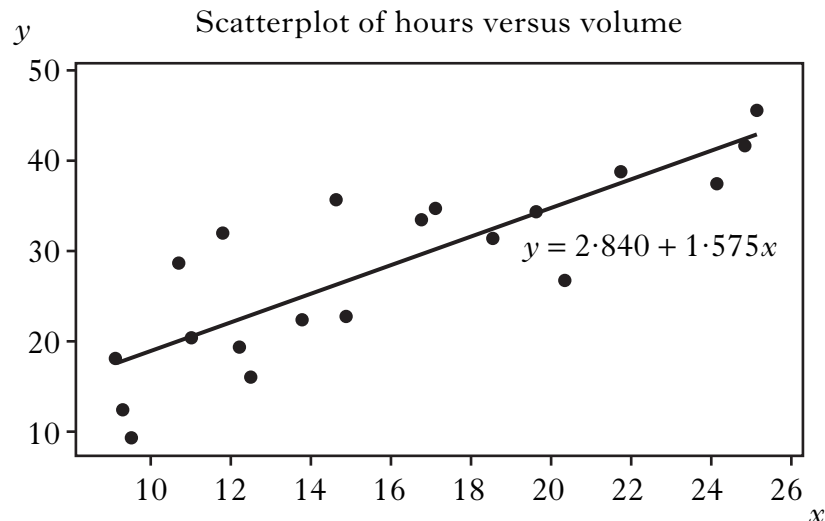
- A4.** Needlestick injuries are skin punctures caused by hypodermic needles and are of major concern in health care because of the associated risk of infection by blood-borne viruses. In an investigation of a random sample of 400 incidents, in each of which a nurse suffered a needlestick injury, it was ascertained that 57 of the nurses were left-handed and the remainder were right-handed. Given that 10% of the nurse population is left-handed and the remainder right-handed, carry out a chi-squared test and suggest a subsequent action that might be usefully pursued. 7

- A5.** A scuba diver and her buddy intend making a series of dives to a depth of 15 metres. At that depth their gas cylinders provide breathing times of X minutes and Y minutes respectively, where $X \sim N(52.0, 1.5^2)$ and $Y \sim N(55.0, 2.0^2)$. Stating any assumption required, calculate the probability that her buddy will require to switch to his reserve gas supply before the diver switches to her reserve. 6

[Turn over

- A6.** A removal company is seeking to improve its estimation process for contracts and collected data on 20 randomly chosen removals. The random variable X was the total volume (m^3) of the packages and articles and the random variable Y the number of hours of labour involved in loading and unloading.

The scatterplot of y versus x with the fitted least squares regression line of y on x is shown below. The mean values of x and y for the data set were 15.915 and 27.910 respectively.



Initial analysis showed that the slope parameter differed significantly from zero and that

$$S_{xx} = 546.385, \quad S_{xy} = 860.707 \quad \text{and} \quad S_{yy} = 1943.76.$$

Calculate a 95% confidence interval for the mean number of hours required for loading and unloading during removals which involve volumes of 20 m^3 and state any assumption required.

7

- A7.** For patients experiencing transient ischaemic attacks, the mean waiting time between referral by a GP and being seen by a consultant in a neurovascular clinic was 41 days. After improvements to shorten waiting times, a random sample of 10 patients had the following number of days to wait:

35 24 27 43 46 25 42 41 32 37

- (a) Stating an assumption required, show that there is evidence of a reduction in mean waiting time by performing a hypothesis test.

6

It was ultimately concluded that the initiative had yielded a process for which the mean waiting time was 36 days with standard deviation 9 days. The clinic management team decided to monitor waiting times. They did this by plotting the mean waiting time for a random sample of 25 of the patients each month on a control chart.

- (b) Calculate 3-sigma control limits for the chart.
- (c) Give a reason why it is not necessary for waiting time to be normally distributed for valid use of the control chart and why not all “out of control points” need be a cause for concern.

2

2

- A8.** During the development of a new adhesive, a research team tested the tensile strength (N/m^2) of a random sample of five bonds made with Formulation A and ten with Formulation B. The results were:

Formulation A	245	191	226	219	233					
Formulation B	283	258	261	236	239	242	250	260	232	257

- (a) Display the data and comment. 3
- (b) Perform a non-parametric hypothesis test to determine whether or not the data provide evidence that Formulation B makes stronger bonds than Formulation A. 7
- A9.** A budget airline operates a fleet of aircraft that each have seats for 296 passengers. On a particular route the demand for tickets is always well in excess of 300 seats so the airline has a policy of selling 300 seats for each flight on the route. If more than 296 passengers actually check in for a flight, allocation of the 296 available boarding passes is done on a first come, first served basis. Any passenger who is unable to board is given £200 in compensation and allocated a seat on a flight with another airline.
- Let the random variable F be the number of passengers who fail to turn up and assume that they do so independently with probability 0.002.
- (a) State the probability distribution of F with its parameters. 1
- (b) By using a suitable approximation for the distribution of F , show that $P(F \geq 4)$ is $1 - 1.816e^{-0.6}$. 5
- (c) By approximating four more probabilities, find the expected compensation paid out per flight. 3
- (d) Explain why the events of individual passengers failing to turn up for flights will, in reality, not be independent. 1

[END OF SECTION A]

[Turn over for Section B on Pages six and seven]

Section B (Mathematics for Applied Mathematics)*Marks***Answer all the questions.****B1.** Differentiate the following, simplifying your answers as appropriate.

(a) $f(x) = e^{2x} \tan x, \frac{-\pi}{2} < x < \frac{\pi}{2}.$ **3**

(b) $g(x) = \frac{\cos 2x}{x^3}.$ **4**

B2. Find the term in a^6 in the binomial expansion of $\left(\frac{1}{a} + 3a\right)^{10}.$ **4**

B3. Express $\frac{3x}{(x+1)^2}$ in partial fractions. **3**

Hence obtain $\int \frac{3x}{(x+1)^2} dx$ **2**

B4. An industrial process is modelled by the differential equation

$$\frac{dy}{dt} = \frac{9te^{3t}}{y},$$

where $y > 0$ and $t \geq 0$.

Given that $y = 2$ when $t = 0$, find y explicitly in terms of t . **7**

- B5.** (a) Find the value(s) of m for which the matrix

$$A = \begin{pmatrix} m & 1 & 1 \\ 0 & m & -2 \\ 1 & 0 & 1 \end{pmatrix}$$

is singular.

3

- (b) The matrix $B = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & -3 \end{pmatrix}$. Use elementary row operations to obtain B^{-1} . **4**

Hence, or otherwise, solve the system of equations

$$x + y - z = 3$$

$$y + z = -2$$

$$x - 3z = 7.$$

2

[END OF SECTION B]

[END OF QUESTION PAPER]

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