EXAMINATIONS OF THE ROYAL STATISTICAL SOCIETY

ORDINARY CERTIFICATE IN STATISTICS, 2008

Paper II

Time Allowed: Three Hours

Candidates may attempt ***all*** *the questions.*

The number of marks allotted to each question or part-question is shown in brackets.

The total for the whole paper is 100.

A pass may be obtained by scoring at least 50 marks.

Graph paper and Official tables are provided.

Candidates may use calculators in accordance with the regulations published in the Society's "Guide to Examinations" (document Ex1).

This examination paper consists of 7 pages, **each printed on one side only**.
This front cover is page 1.
Question 1 starts on page 2.

There are 8 questions altogether in the paper.

1. Nathan is training for a combined running and cycling race. He runs at 15 kilometres per hour and cycles at 30 kilometres per hour.

How far will he run in (i) 45 minutes, (ii) 1 hour 20 minutes? (2)

How long will it take him to cycle (iii) 20 kilometres, (iv) 50 kilometres? (2)

What is his average speed when he runs for 30 minutes and cycles for 2 hours? (2)

What will be his average speed for a race consisting of a 10 kilometre run and a 40 kilometre cycle ride? (2)

2. A class of 30 students took tests in Mathematics and English. The results were graded A, B or C and are shown in the table.

Student	Mathematics	English	Student	Mathematics	English
1	C	A	16	A	C
2	A	B	17	C	A
3	B	A	18	C	C
4	B	A	19	C	B
5	C	A	20	C	B
6	C	A	21	B	A
7	C	A	22	A	C
8	A	A	23	B	B
9	A	C	24	A	B
10	C	B	25	B	B
11	C	A	26	C	B
12	C	C	27	B	C
13	B	A	28	C	B
14	C	C	29	A	A
15	A	B	30	C	B

Construct a contingency table showing the results, with rows the Mathematics grades and columns the English grades. Include the marginal totals in your table. (4)

What is the modal grade in Mathematics? What is the modal grade in English? (1)

Find the probability that a randomly selected student obtains As in both subjects. (Give your answer as a fraction.) (1)

Given that Alice has a grade A in Mathematics, what is the probability that she also has a grade A in English? (2)

Given that David has a grade A in English, what is the probability that he also has a grade A in Mathematics? (2)

3. The 2006 Stern report, "The Economics of Climate Change", contained the following data, showing the relative contributions to greenhouse gas emissions for different activities in the year 2000.

Activity	*Contribution*
Industry	30%
Commercial buildings	5%
Residential buildings	10%
Transport	14%
Non-energy	35%
Miscellaneous	6%

Draw a bar chart of the data.

(5)

If you were to draw a pie chart of the data, what angle in degrees would be used to represent (i) Non-energy, (ii) Commercial buildings?

(2)

If by 2025 each sector reduces its emissions by 10%, what would be the overall percentage reduction in emissions and what angles in degrees would be used (in a pie chart) to represent Non-energy and Commercial buildings respectively?

(2)

If by 2025 Non-energy and Industry emissions were reduced by 20% and each of the other activities reduced their emissions by 10%, what would be the overall reduction in emissions and what angles in degrees would be used to represent Non-energy and Commercial buildings respectively?

(4)

4. A book contains 90 Sudoku puzzles at four levels of difficulty: Easy, Mild, Difficult and Fiendish. I have recorded the times, in minutes, it took me to solve all the puzzles and I have done some analysis, as follows.

Easy:	Number of puzzles 4 Times taken 11, 9, 8, 8.
Mild:	Number of puzzles 16 Times taken 14, 13, 14, 12, 11, 12, 13, 12, 10, 11, 12, 10, 11, 12, 10, 11.
Difficult:	Number of puzzles 45 Mean time taken = 18.4 minutes Standard deviation 2.3 minutes
Fiendish:	Number of puzzles 25 Mean time taken = 25.3 minutes Standard deviation 3.4 minutes

Showing full details of your working, calculate the means and the standard deviations of the times taken to solve (i) the Easy puzzles, (ii) the Mild puzzles.

(7)

Calculate the coefficients of variation of the times taken in each of the four categories. Display all your results (mean, standard deviation, coefficient of variation) in a summary table, and comment briefly on what the table shows.

(6)

5. In my drawer I have 5 identical red socks, 3 identical blue socks and 2 identical green socks. I draw socks out of the drawer in turn and without replacement, colour unseen.

Using probability trees or otherwise, evaluate the following probabilities. Show your arguments clearly.

(i) I draw two socks out of the drawer. What is the probability that they are a pair (i.e. both the same colour)?

(4)

(ii) Suppose the first sock drawn is a red one. If I draw two more socks, what is the probability that I have a pair among the three socks?

(4)

(iii) Suppose the first two socks are one red and one blue. If I draw two more socks, what is the probability that I have two pairs?

(4)

6. One day in early November a newspaper weather column gives the following maximum and minimum temperatures, in degrees Celsius, for 12 cities in the Northern Hemisphere.

City	*Maximum °C*	*Minimum °C*
Athens	19	11
Beijing	14	2
Berlin	10	4
Cairo	24	14
Istanbul	15	9
Kuala Lumpur	32	24
Madrid	20	6
Moscow	0	–7
Mumbai	35	19
New York	16	10
Oslo	3	1
Rome	20	8

Denoting the values in the "Maximum" column by x, and in the "Minimum" column by y, you are given that

$$\Sigma x^2 = 4772 \qquad \Sigma y^2 = 1605 \qquad \Sigma xy = 2624$$

Calculate $\bar{x}$, $\bar{y}$, $\Sigma(x-\bar{x})^2$, $\Sigma(y-\bar{y})^2$, $\Sigma(x-\bar{x})(y-\bar{y})$; in each case give your answer correct to four decimal places.

(5)

Evaluate the product-moment correlation coefficient r, correct to two decimal places, and comment on its sign and magnitude.

(3)

You wish to fit a regression line of the form $y = a + bx$. Find the least squares estimates, $\hat{a}$ and $\hat{b}$, of a and b, each correct to two decimal places.

(2)

The formula for converting from the Celsius to the Fahrenheit scale is

Degrees Fahrenheit = 1.8 × degrees Celsius + 32.

Without going back to the original data, obtain the values of

(i) $\bar{x}$, $\bar{y}$, $\Sigma(x-\bar{x})^2$, $\Sigma(y-\bar{y})^2$ and $\Sigma(x-\bar{x})(y-\bar{y})$ in the Fahrenheit scale,

(ii) r, $\hat{a}$ and $\hat{b}$, all correct to two decimal places, when the data are converted to the Fahrenheit scale.

(5)

7. Explain the meaning of the following terms relating to a time series.

(i) Trend.

(ii) Seasonal variation.

(iii) Multiplicative model.

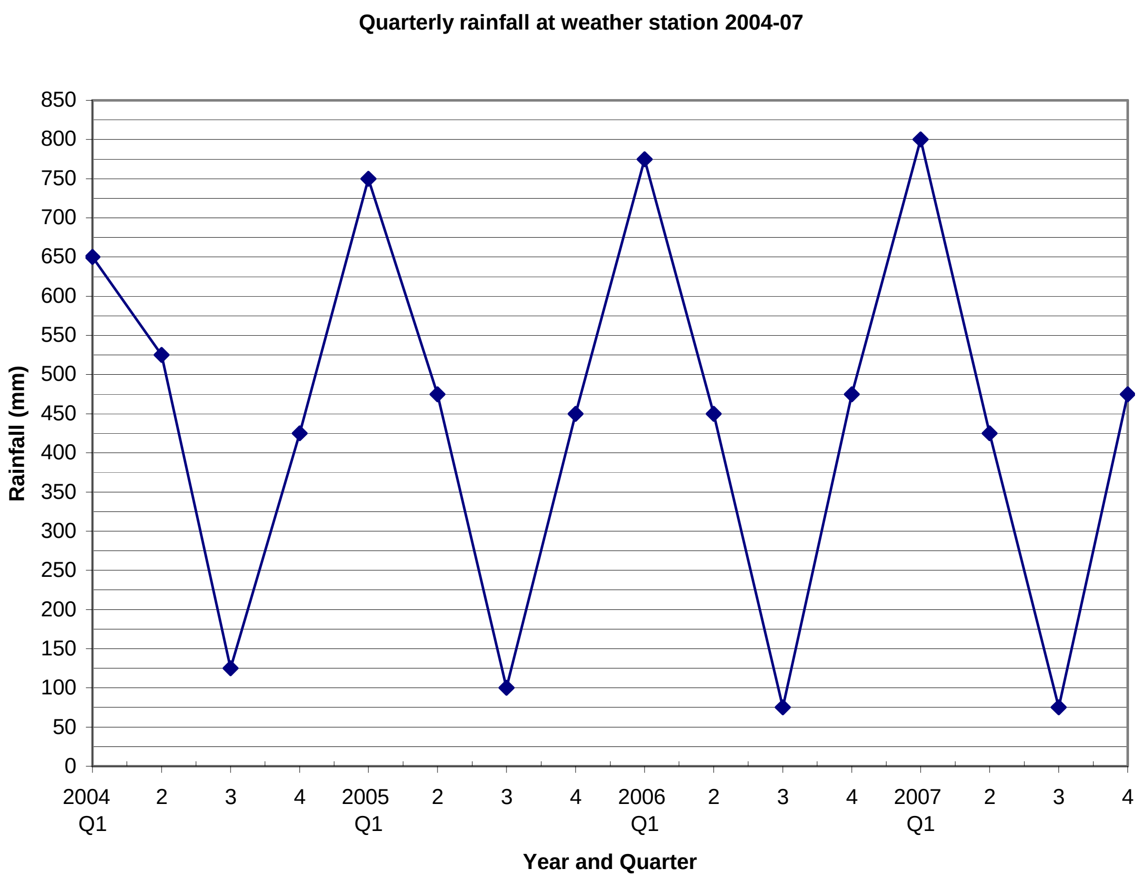
(3)

The quarterly rainfall at a weather station over the period 2004–2007 is as shown in the chart below.

Comment on what the chart shows. (2)

It is desired to perform a time series analysis on these data using a multiplicative model. Construct a suitable table that shows the actual quarterly rainfall data, the centred four-quarterly moving averages, and the detrended data. (Other columns may be included in the table to help with the calculations.) (10)

Comment on the trend and the detrended data. (2)



Turn over

8. In 2006 the percentage monthly changes in a company's energy costs were as shown below.

Month	Jan	Feb	Mar	Apr	May	Jun
Percentage change from previous month	n.a.*	1.5	1.3	0.9	0.4	–0.5

* n.a. means "not available"

Draw a time chart to show the data.

(4)

Convert these figures to a series of chain-based index numbers of energy costs for February to June 2006.

(2)

Convert the chain-based index numbers to a fixed base with January 2006 = 100. Work to one decimal place throughout.

(3)

Using your chart and the index numbers, make three comments about these energy costs over the period shown.

(3)