

Mark Scheme (Results)

Summer 2008

GCE O Level

AO Level Pure Mathematics (7362) Paper 1

Pure Mathematics 7362

Paper 1

Q.	Scheme	Marks
1	$\cos \theta = \frac{4^2 + 5^2 - 6^2}{2 \times 4 \times 5} = \frac{16 + 25 - 36}{40} \left(= \frac{1}{8} \right)$ $\theta = 82.8^\circ$	M1 M1A1 A1 (4)
2	$5 - x = x^2 - 8x + 11$ $x^2 - 7x + 6 = 0$ $(x - 6)(x - 1) = 0$ $x = 6 \quad y = -1$ $x = 1 \quad y = 4$	M1 A1 M1 A1 A1 (5)
3	$\frac{r}{h} = \tan 30 \quad h = \frac{r}{\tan 30}$ $V = \frac{1}{3} \pi r^2 \times \frac{r}{\tan 30}$ $\frac{dV}{dr} = \frac{\pi r^2}{\tan 30}$ $\frac{dV}{dt} = -5 \quad \frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV}$ $= -5 \times \frac{\tan 30}{\pi r^2} = -5 \times \frac{\tan 30}{\pi (h \tan 30)^2}$ $h = 10 \quad \frac{dr}{dt} = \frac{-5}{100\pi \tan 30} = -0.0275... = -0.028 \text{ cm/s}$ Rate of decrease = 0.028 cm/s	B1 M1 M1 B1 M1 A1 A1 (7)
4	(a) $\frac{dy}{dx} = 10xe^{2x} + 2(5x^2 - 2)e^{2x}$ (b) $\frac{dy}{dx} = \frac{3x^2(x - x^2) - (x^3 + 2)(1 - 2x)}{(x - x^2)^2}$ $= \frac{2x^3 - x^4 + 4x - 2}{(x - x^2)^2}$	M1A1A1 (3) M1 A2,1,0 A1 (4)

5	(i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 5\mathbf{i} + 10\mathbf{j}$ (ii) $\overrightarrow{OC} = \overrightarrow{OA} + \frac{2}{5}\overrightarrow{AB} = 4\mathbf{i} + 5\mathbf{j} + 2\mathbf{i} + 4\mathbf{j} = 6\mathbf{i} + 9\mathbf{j}$ (b) unit vector $= \frac{\overrightarrow{AB}}{ \overrightarrow{AB} } = \frac{5\mathbf{i} + 10\mathbf{j}}{\sqrt{125}} = \frac{\mathbf{i} + 2\mathbf{j}}{\sqrt{5}}$ (c) $\overrightarrow{DA} = -\lambda\mathbf{i} + 4\mathbf{i} + 5\mathbf{j} = (4 - \lambda)\mathbf{i} + 5\mathbf{j}$ Parallel to $5\mathbf{i} + 10\mathbf{j} \quad \therefore 5 = 2(4 - \lambda)$ $\lambda = \frac{3}{2}$	B1 M1A1 (3) M1A1 (2) M1 M1 A1 (3)
6	(a) $p^5 = 243, \quad p = 3$ (b) $3q + 4 = 4^3$ $3q + 4 = 64 \quad 3q = 60 \quad q = 20$ (c) $f(x) = 2x \log_x 3 - 10 \log_x 3 - x + 5$ $= 2 \log_x 3 (x - 5) - (x - 5)$ $= (x - 5)(2 \log_x 3 - 1) \quad a = 2, \quad b = 1$ (d) $\log_x 3 = \frac{1}{2}$ $x^{\frac{1}{2}} = 3 \quad x = 9$ or $x = 5$	M1,A1 (2) M1A1 (2) M1 M1A1 (3) M1A1 B1 (3)
7	(a) $A = 2(10x^2 + 5xh + 2xh)$ (b) $(V =) 10x^2h = 500$ $A = 20x^2 + 14x \times \frac{50}{x^2}$ $A = 20x^2 + \frac{700}{x}$ (c) $\frac{dA}{dx} = 40x - \frac{700}{x^2}$ $\frac{dA}{dx} = 0 \quad x^3 = \frac{700}{40} \quad (x = 2.596...)$ $A_{\min} = 20 \times 2.596^2 + \frac{700}{2.596} = 404.4... = 404 \text{ cm}^3$ (d) $\frac{d^2A}{dx^2} = 40 + \frac{700 \times 2}{x^3}$ $x > 0 \Rightarrow \frac{d^2A}{dx^2} > 0 \quad \therefore \min A \text{ at } x = 2.596...$	B1 (1) B1 M1 A1 (3) M1 M1A1 M1A1 (5) M1 A1ft (2)

8	(a) $(15x+6)(x+4) = (6x-3)^2$ $15x^2 + 66x + 24 = 36x^2 - 36x + 9$ $21x^2 - 102x - 15 = 0 \quad 7x^2 - 34x - 5 = 0$ $(7x+1)(x-5) = 0 \quad x = -\frac{1}{7} \quad x = 5$ (b) $x = 5 \quad r = \frac{6x-3}{15x+6} = \frac{27}{81} = \frac{1}{3}$ $x = -\frac{1}{7} \quad r = \frac{-\frac{6}{7}-3}{-\frac{15}{7}+6} = \frac{-27/7}{27/7} = -1$ (c) $r = \frac{1}{3} \quad a = 81 \quad S_\infty = \frac{a}{1-r} = \frac{81}{2/3} = \frac{243}{2} = 121\frac{1}{2}$ (d) $S_n = \frac{81\left(1 - \left(\frac{1}{3}\right)^n\right)}{\frac{2}{3}} = \frac{243}{2}\left(1 - \left(\frac{1}{3}\right)^n\right)$ $\% \text{ error} = \frac{(-)\frac{243}{2} \times \left(\frac{1}{3}\right)^n}{\frac{243}{2}} \times 100\% = (-)100\left(\frac{1}{3}\right)^n \%$	M1 M1 A1A1 (4) M1A1ft A1ft (3) B1M1A1 (3) M1 M1A1ft (on r) (3)
9	(a) $\cos 2A \equiv \cos^2 A - \sin^2 A = \cos^2 A - (1 - \cos^2 A) = 2\cos^2 A - 1$ (b) $\sin 2A = 2\sin A \cos A$ (c) $\cos 3A = \cos(2A + A) = \cos 2A \cos A - \sin 2A \sin A$ $= (2\cos^2 A - 1)\cos A - 2\sin^2 A \cos A$ $= 2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A$ $= 4\cos^3 A - 3\cos A$ (d) $\cos 3x = 0.6$ $3x = 53.13^\circ, \quad 306.86^\circ, \quad 413.13^\circ$ $x = 17.7^\circ, \quad 102.3^\circ, \quad 137.7^\circ$ (e) $\frac{1}{4} \int_0^{\frac{\pi}{3}} (\cos 3\theta + 3\cos \theta) d\theta, \quad = \frac{1}{4} \left[\frac{1}{3} \sin 3\theta + 3\sin \theta \right]_0^{\frac{\pi}{3}}$ $= \frac{1}{4} \left[\frac{1}{3} \sin \pi + 3\sin \frac{\pi}{3} - 0 \right] = \frac{3}{4} \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{8} \quad a = 3, \quad b = 8, \quad c = 3$	M1A1 (2) B1 (1) M1 M1 M1 A1 (4) M1 M1 A3,2,1,0 (5) M1,M1A1 M1A1 (5)

10	(a) $(-2, 0)$ on curve: $-8 + 4p - 2q + 6 = 0$ $4p - 2q = 2 \quad 2p - q = 1$ $(2, -4)$ on curve: $8 + 4p + 2q + 6 = -4$ $2p + q = -9$ Add: $4p = -8$ $p = -2 \quad q = -5$ (b) $f(x) = (x+2)(x^2 - 4x + 3) = (x+2)(x-3)(x-1)$ D is $(1, 0)$ E is $(3, 0)$ (c) $y = x^3 - 2x^2 - 5x + 6 \quad \frac{dy}{dx} = 3x^2 - 4x - 5$ $x = 2 \quad \frac{dy}{dx} = 12 - 8 - 5 = -1$ grad normal $= 1$ eqn. normal: $y + 4 = x - 2 \quad (y = x - 6)$ (d) $\int_1^2 (x^3 - 2x^2 - 5x + 6) dx = \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{5x^2}{2} + 6x \right]_1^2$ $= \left(4 - \frac{16}{3} - 10 + 12 \right) - \left(\frac{1}{4} - \frac{2}{3} - \frac{5}{2} + 6 \right) = -2\frac{5}{12}$ Normal cuts x-axis at $x = 6$ Area $\Delta = \frac{1}{2} \times 4 \times 4 = 8$ Total area $= 8 + 2\frac{5}{12} = 10\frac{5}{12}$ units²	M1 A1 M1 A1A1 (5) M1 A1A1 (3) M1 A1ft B1ft B1 (4) M1 M1A1 M1A1 B1ft (6)
----	--	--