Mark Scheme (Results)

Summer 2007

GCE

O Level Pure Mathematics (7362_02)

A PEARSON COMPANY

Pure Mathematics 7362

Paper 2

1	$\dfrac{dy}{dx}=\dfrac{-2\sin 2x\left(x^2+3\right)-2x\cos 2x}{\left(x^2+3\right)^2}$	M1A1A1 (3)
2	$\dfrac{da}{dt}=15 \quad a=\pi r^2 \quad \dfrac{da}{dr}=2\pi r, \quad r=\sqrt{\tfrac{50}{\pi}},$ $\dfrac{dr}{dt}=\dfrac{dr}{da}\times\dfrac{da}{dt},=\dfrac{15}{2\pi r}=\dfrac{15}{2\pi\times\sqrt{\frac{50}{\pi}}}=0.598$	M1A1,B1 M1A1,A1 (6)
3	(a) $MN^2=5.6^2+8.2^2-2\times5.6\times8.2\cos57, \quad MN=6.97\text{cm}$ (b) $\dfrac{\sin N}{5.6}=\dfrac{\sin57}{MN}, \quad N=42.4°$	M1A1,A1 M1A1,A1 (6)
4	(a) $x^2+x-12=3x+3$ $x^2-2x-15=0 \qquad (x-5)(x+3)=0$ $x=5 \ \ y=18; \quad x=-3 \ \ y=-6$ Points are $(-3,-6)$ and $(5,18)$ (b) $x^2-2x-15 \geq 0 \quad x \leq -3, \ x \geq 5$	M1 M1A1 A1;A1 M1A1⊦ (7)
5	(a) $\log_4 2=p \quad 4^p=2 \quad p=\tfrac{1}{2}$ (b) $\log_2 3=\tfrac{\log_4 3}{\log_4 2},=\tfrac{\log_4 3}{\frac{1}{2}} \quad k=2$ (c) $5x\log_4 x-2\log_4 x-10x\times2\log_4 3+4\times2\log_4 3$ $=\log_4 x^{5x}-\log_4 x^2-\log_4 3^{20x}+\log_4 3^8$ $=\log_4\dfrac{x^{5x}\times3^8}{x^2\times3^{20x}}, \quad =\log_4\dfrac{x^{5x-2}}{3^{20x-8}}$ (d) $\log_4\dfrac{x^{5x-2}}{3^{20x-8}}=0 \quad \dfrac{x^{5x-2}}{3^{20x-8}}=1, \quad \left(\dfrac{x}{3^4}\right)^{5x-2}=1$ $x=3^4=81$ or $5x-2=0 \quad x=\tfrac{2}{5}$	B1 M1,A1 M1 M1 M1,A1 M1, M1 A1A1 (11)
6	(a) $3x^2h=450$ $h=\tfrac{150}{x^2}$ $A=2\left(3x^2+xh+3xh\right)$ $A=6x^2+8xh=6x^2+8x\times\tfrac{150}{x^2}=6x^2+\tfrac{1200}{x}$ (b) $A=6x^2+1200x^{-1} \qquad \tfrac{dA}{dx}=12x-1200x^{-2}$ $\tfrac{dA}{dx}=0 \quad 12x^3=1200 \quad x^3=100 \quad x=4.641... \quad x=4.64$ $\tfrac{d^2A}{dx^2}=12+2400x^{-3}>0$ when $x>0 \quad \therefore$ minimum. (c) $A_{\min}=6\times4.641...^2+\tfrac{1200}{4.641}=387.7...=388$	B1 M1 M1A1 M1 M1A1 M1A1 Ö

		M1A1 (11)
7	(a) (i) $\overrightarrow{PB} = \mathbf{b} - \frac{2}{5}\mathbf{a}$ (ii) $\overrightarrow{AD} = \frac{5}{2}\mathbf{b} - \mathbf{a}$ (iii) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ (iv) $\overrightarrow{PQ} = \frac{3}{5}\mathbf{a} + \frac{5}{7}(\mathbf{b} - \mathbf{a}) = \frac{5}{7}\mathbf{b} - \frac{4}{35}\mathbf{a}$ (v) $\overrightarrow{PD} = \frac{5}{2}\mathbf{b} - \frac{2}{5}\mathbf{a} \quad \left(= \frac{5}{2}\left(\mathbf{b} - \frac{4}{25}\mathbf{a}\right)\right)$ (b) (i) $\overrightarrow{PB} = \mathbf{b} - \frac{2}{5}\mathbf{a} = \frac{2}{5}\left(\frac{5}{2}\mathbf{b} - \mathbf{a}\right) = \frac{2}{5}\overrightarrow{AD}$ $\therefore$ PB parallel to AD (ii) $\overrightarrow{PQ} = \frac{5}{7}\left(\mathbf{b} - \frac{4}{25}\mathbf{a}\right) = \frac{5}{7} \times \frac{2}{5}\ \overrightarrow{PD}$ $\therefore P, Q, D$ collinear	M1A1 B1 B1 M1A1 M1A1 M1A1 M1A1(12)
8	(a) $\frac{a}{1-r} = 243$ $\frac{a(1-r^4)}{1-r} = 240$ $1 - r^4 = \frac{240}{243}$ $\quad 243 - 243r^4 = 240 \quad r^4 = \frac{3}{243} \quad r = \pm\frac{1}{3}$ (b) $r = \frac{1}{3} \quad \frac{a}{2/3} = 243 \quad a = 243 \times \frac{2}{3} = 162$ $r = -\frac{1}{3} \quad \frac{a}{4/3} = 243 \quad a = 243 \times \frac{4}{3} = 324$ (c) $r = -\frac{1}{3} \quad ar^7 = -324 \times \frac{1}{3^7} = -\frac{324}{2187} = -\frac{4}{27}$ (d) $S_8 = \frac{324\left(1 - \left(-\frac{1}{3}\right)^8\right)}{1 + \frac{1}{3}}$, $=242.96$	M1 A1 M1A1A1 M1A1 M1A1 M1A1 M1A1 (13)
9	(a) multiply out brackets for (i) and (ii) (b) $\alpha + \beta = -\frac{7}{2} \quad \alpha\beta = 2$ $(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2 = (\alpha + \beta)^2 - 4\alpha\beta = \frac{49}{4} - 8 = 4\frac{1}{4}$ (c) $\alpha^3 + \beta^3 = (\alpha + \beta)\left((\alpha + \beta)^2 - 3\alpha\beta\right) = \left(-\frac{7}{2}\right)\left(\frac{49}{4} - 6\right) = -\frac{175}{8}$ (d) $\alpha^3 - \beta^3 = (\alpha - \beta)\left((\alpha + \beta)^2 - \alpha\beta\right) = \frac{\text{Ö}17}{2}\left(\frac{49}{4} - 2\right) = \frac{41}{8}\text{Ö}17$ (e) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = -\frac{175}{8} \times \frac{1}{2} = -\frac{175}{16}$ $\frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta = 2$ $x^2 + \frac{175}{16}x + 2 = 0, \quad 16x^2 + 175x + 32 = 0$	M1A1A1 B1 M1A1 M1A1 M1A1 M1 B1 M1,A1 (14)

10	(a) $\sin 2\theta = 2\sin\theta\cos\theta$	B1
	(b) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 1-\sin^2\theta - \sin^2\theta = 1-2\sin^2\theta$	M1
	$\sin^2\theta = \frac{1}{2}\left(1-\cos 2\theta\right)$	A1
	(c) $\sin^2\left(A+B\right) - \sin^2\left(A-B\right)$	
	$=\left(\sin A\cos B + \cos A\sin B\right)^2 - \left(\sin A\cos B - \cos A\sin B\right)^2$	M1
	$=\left(\sin^2 A\cos^2 B + 2\sin A\cos A\sin B\cos B + \cos^2 A\sin^2 B\right)$	M1
	$-\left(\sin^2 A\cos^2 B - 2\sin A\cos A\sin B\cos B + \cos^2 A\sin^2 B\right)$	A1
	$= 4\sin A\cos A\sin B\cos B = \sin 2A\sin 2B$	M1A1
	(d) (i) Use $A = 2\theta,\ \ B=\theta$ $\quad \sin^2 3\theta - \sin^2\theta = \sin 4\theta\sin 2\theta$	M1A1
	(ii) $\sin^2 3\theta - \sin^2\theta = \frac{1}{2}\left(1-\cos 6\theta\right) - \frac{1}{2}\left(1-\cos 2\theta\right)$	M1
	$= \frac{1}{2}\left(\cos 2\theta - \cos 6\theta\right)$	A1
	(e) $\int_0^{\frac{\pi}{3}}\left(6\sin 4\theta\sin\theta + 2\right)\mathrm{d}\theta = \int_0^{\frac{\pi}{3}} 6\left(\sin^2 3\theta - \sin^2\theta\right) + 2\mathrm{d}\theta$	
	$= \int_0^{\frac{\pi}{3}}\left(3\cos 2\theta - 3\cos 6\theta + 2\right)\mathrm{d}\theta$	M1
	$=\left[\frac{3}{2}\sin 2\theta - \frac{3}{6}\sin 6\theta + 2\theta\right]_0^{\frac{\pi}{3}}$	M1A1
	$= \frac{3}{2}\sin\frac{2\pi}{3} - \frac{1}{2}\sin 2\pi + \frac{2\pi}{3} - 0 \ = \frac{3}{2}\times\frac{\sqrt{3}}{2} + \frac{2\pi}{3} = \frac{3\sqrt{3}}{4} + \frac{2\pi}{3}$	M1A1 (17)