

# MATHEMATICS

Paper 4024/11

Paper 11

## Key Messages

All working should be clearly shown. It is important that candidates should take care when transcribing answers from their working to the answer line as errors at this stage may lead to the loss of credit.

Accuracy with basic number work is essential. Candidates are strongly advised to check their working carefully and to consider whether or not their answers are sensible.

## General Comments

The paper was accessible to almost all candidates with the full range of credit scored. Generally the work was neatly presented.

There was no evidence that candidates had difficulty in completing the paper in the allotted time. Where questions were left unanswered, this appeared to be due to a problem of understanding rather than a lack of time.

In **Questions 2(a), 2(b), 4(a), 9(a), 10(a), 10(b), 11(a), 12(a), 12(b), 12(c), 20(a)(i), 20(a)(ii) and 25(a)**, candidates scored well.

The questions which candidates had most difficulty with were **8(a), 8(b), 16(b), 19(b)(i), 19(b)(ii), 21(c), and 22(c)**.

## Comments on Specific Questions

### Question 1

(a) Generally the concept of perimeter was understood. Its application required care. With all the sides in place, accurate totals were not always given. A common omission was the short side of 3 cm, leading to the answer 97 cm. The diagram in the question showed the lengths of 6 of the 8 sides, leading some candidates to the answer 77 cm. There was some inappropriate use of the formula  $2(L+W)$  which meant that some sides were duplicated.

(b) The underlying concept of area was generally understood but was not always applied accurately.

Answer: (a) 100 (b) 475

### Question 2

(a) Some candidates found the use of decimal points in this question challenging, but generally this was answered accurately, with the working showing understanding of equivalent forms.

(b) This was usually correctly converted into a fractional form. Not all those with the correct  $\frac{350}{7}$  reached 50.

Answer: (a) 0.06 (b) 50

### Question 3

- (a) This question was well answered. The relationship between the units was well known. As usual, care in lining up the appropriate digits was necessary for a successful subtraction. 3556 was a frequent wrong answer.
- (b) The conversion of units here caused problems for some candidates. For example, both 100 and 1000 were seen in attempts to convert to  $\text{cm}^2$ .

Answer: (a) 3.556 (b) 12000

### Question 4

- (a) Most candidates inserted the correct symbol.
- (b) This proved to be a straightforward question which was well answered by the majority.

Answer: (a) < (b) 0.07

### Question 5

In this question, the underlying structure of a right-angled triangle and Pythagoras was usually spotted. The application of Pythagoras was often successful. Some candidates gave 8 as their final answer. Where the structure of the question had not been appreciated, answers such as  $6 + 6$  were seen for PQ, and sometimes formulas involving  $\pi$  were attempted.

Answer: 16

### Question 6

This question was well understood, with appropriate working carefully set out. A common incorrect answer was  $\frac{8-5}{20} = \frac{3}{20}$ . Candidates could be reminded that rather more than  $1 - (\frac{2}{5} + \frac{1}{4})$  is required for the answer.

Answer:  $\frac{7}{20}$

### Question 7

Many candidates clearly understood that some form of the fraction  $\frac{2.7}{4.5}$  was required, but they were not always able to adjust the units appropriately. A common incorrect answer was 1:270 000.

Answer: 1 : 60 000

### Question 8

- (a) Some correct answers were written down without any extraneous working. Others rewrote the statement  $\sin x = 0.53$  without any idea of how to proceed. Some speculative calculations using 0.53 and multiples of 32 were seen.
- (b) There were very few correct answers to this part. Candidates tried to use the given right-angled triangle, reaching answers such as  $\frac{12}{13}$ , but other ratios were also seen. The evaluation of  $1 - \frac{12}{13}$  was sometimes seen.

Answer: (a) 148 (b)  $-\frac{12}{13}$

**Question 9**

- (a) This question was well answered.
- (b) As well as purely arithmetical solutions, some candidates constructed the relevant algebraic equation and solved it. A common misconception was to calculate 10% of \$81. Candidates could be encouraged to think through problems of this type using an algebraic approach.

Answer: (a) 18 (b) 90

**Question 10**

- (a) There were many correct answers to this question with the  $-(-3)$  well negotiated.
- (b) Again, some very confident work presented. Both of the expected approaches were seen in equal measure, and both were well handled.

Answer: (a) 55 (b)  $\frac{ma - b}{m}$

**Question 11**

- (a) Most candidates answered this part correctly.
- (b) This question was well answered.
- (c) Slightly more likely to attract other answers, such as Rhombus or Parallelogram.

Answer: (a) square (b) trapezium (c) kite

**Question 12**

- (a) The general principles underlying this part were well understood with a high rate of success.
- (b) Many correct answers here. A division by 7 was not always accurate.
- (c) Candidates generally seemed to know what they were looking for and most earned credit. A few candidates introduced digits which were not given in the question.

Answer: (a) 619 (b) 196 (c) 169, 196 or 961

**Question 13**

- (a) Some candidates saw immediately that they could go for the area of the trapezium in order to form an equation in terms of  $v$ . A few used 1375 in this formula instead of  $v$ .

Others built up the area part by part and were reasonably successful in reaching an accurate value for  $v$ .

A common misconception was the use of proportionality to calculate  $v$ . Others used the formula Distance  $\div$  Time, leading to  $1375 \div 70 = 19.64$ .

- (b) This part was generally well understood.

Answer: (a) 25 (b) 1.25

**Question 14**

- (a) The answer was usually correct. A common wrong answer was  $58^\circ$ , presumably from  $90^\circ - 32^\circ$ .
- (b) This part was not answered particularly well, with an angle of  $32^\circ$  frequently given.
- (c) Again, this caused problems for many candidates.

Answer: (a)  $32^\circ$  (b)  $26^\circ$  (c)  $58^\circ$

**Question 15**

- (a) (i) A good number of candidates drew the correct angle bisector, but some were led into constructing the perpendicular bisector of AD.
  - (ii) The correct arc was generally seen.
- (b) There were some well drawn accurate diagrams achieving full credit.

**Question 16**

- (a) The idea of proportionality was well understood here, and accurately applied. Those candidates who did not gain credit usually gave 99 as their answer from  $66 \div 6 = 11$  followed by  $11 \times 9$ .
- (b) Changing the appropriate ratio for volume was not always successful in many cases and, as a result, there were very few correct answers. Some candidates were still using 3:2.

Answer: (a) 44 (b) 5400

**Question 17**

- (a) Whilst many candidates can negotiate this type of problem successfully, it is clear that other candidates would benefit from developing their knowledge of this area. The numbers and powers tend to be seen as separate entities that can be combined independently, so solutions such as  $8.4 \times 10^5$  were given.
- (b) Candidates had more success here because with multiplication and division, the numbers appearing as part of  $p$  and  $q$  can be written in as given and dealt with independently of the powers of 10.

Answers: (a)  $6.24 \times 10^3$  (b)  $8 \times 10^{-2}$

**Question 18**

- (a) This question was well answered.
- (b) Most candidates understood how to interpret the graph to obtain the median. The main problem encountered was the reading of the horizontal scale, where 5 small squares to 10 units required care.
- (c) The method for finding the interquartile range was not understood so well as the median. Again, candidates are advised to take care when reading scales.

Answers: (a) 30 (b) 66 (c) 30

### Question 19

- (a) The idea of sector area was generally understood, but the given area was not always seen to be the difference of two. The formula for this was occasionally confused with the formula for arc length. Some candidates were unsure what value to take for the radius as there were a number of radii to choose from. Many candidates were clear what giving your answer in terms of  $\pi$  meant. More than  $\frac{16}{9}\pi - \pi$  was required for the final answer.
- (b)(i) Many candidates seemed to have difficulty in relating what was required here to the information given, and the area  $\pi 5^2$  of the whole metal cover. One misconception was to give 12 times the answer to the previous part. There were few correct answers and the question was omitted frequently.
- (ii) The question was not answered by many candidates and there were very few correct answers.

Answers: (a)  $\frac{7\pi}{9}$  (b)(i)  $6\frac{2}{3}\pi$  (ii)  $\frac{11}{15}$

### Question 20

- (a)(i) This question was usually answered accurately, with the occasional 0 or 5 for  $5^0$ .
- (ii) Generally well answered. However, some candidates often halved 36 and gave 18 as their answer.
- (iii) This part was answered reasonably well, but some candidates found the presence of a fractional index challenging.
- (b) Most candidates could not deal with this directly, and many could not manipulate the equation into a form that they could understand.

Answer: (a)(i) 26 (ii) 6 (iii) 16 (b) -2

### Question 21

- (a) The majority of candidates answered this question quite well, but some left the formula as  $R = kp^3$ . There was some confusion between direct and inverse proportion.
- (b) Candidates understood the substitutions required and most scored full credit.
- (c) This part was rarely correct. Most candidates opted for Diagram 4.

Answer: (a)  $3p^3$  (b) 4 (c) 2

### Question 22

- (a) The correct translation was given by a fair number of candidates. Occasionally it was triangle  $B$  that was translated.
- (b) This part of the question proved demanding for many and there were few correctly placed triangles drawn. Sometimes, a partially correct rotation was given.
- (c) Clearly the transformation was correctly analysed by some candidates, and the matrix for it well remembered. There were a few candidates who tried to work the matrix out by forming equations using corresponding points, but generally, candidates found it challenging to approach this question in a systematic way.

Answer: (a) Vertices  $(-2,4)$ ,  $(1,4)$  and  $(-1,5)$  (b) Vertices  $(1,0)$ ,  $(1,-3)$  and  $(2,-1)$  (c)  $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

**Question 23**

- (a) (i) Many candidates calculated the gradient correctly. Some responses highlighted the importance of mental arithmetic in a non-calculator paper, where errors such as simplifying  $\frac{4}{6}$  to 1.5 were seen.
- (ii) The principles here were fairly well understood with a reasonable number of correct answers.
- (iii) Many gave the answer as  $y = 3x + c$  which did not answer the question as set.
- (b) Usually one correct inequality stated. Few candidates gave both.

Answer: (a)(i)  $\frac{4}{6}$  (ii) e.g.  $y = \frac{4}{6}x + 3$  (iii)  $y = 3x + 2$  (b)  $y \geq 2$  and  $y \leq \frac{4}{6}x + 2$

**Question 24**

- (a) (i) Many correct answers.
- (ii) A good number of candidates calculated the correct matrix. In some cases, more care was needed in evaluating the determinant as numerical errors were evident.
- (b) A good number of complete lists of factors were seen. Some candidates still omit 1 from such lists.
- (c) Some correct answers, but some candidates seemed unfamiliar with the notation required.

Answer: (a)(i)  $\begin{pmatrix} 6 & 9 \\ 1 & 3 \end{pmatrix}$  (ii)  $\frac{1}{5} \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$  (b) 1, 2, 3, 4, 6, 8, 12 (c)  $M' \cap N$

**Question 25**

- (a) Many achieved the correct answer.
- (b) The correct factors were usually given.
- (c) Most candidates understood the principle of combining fractions, although some found working to a successful conclusion challenging. Some candidates put themselves at a disadvantage from the start by choosing a more complicated denominator than necessary.
- (d) A successful start was often made here with a correct transition to multiplication. Correct ideas of cancellation were usually seen. However, some candidates attempted to cancel too much at once. Candidates could be encouraged perhaps to use more steps, cancelling a bit at a time. As before in other questions, an answer such as  $\frac{a}{2} \times \frac{b}{2}$  was not considered to be quite finished.

Answer: (a)  $5xy(2x + 3y)$  (b)  $(5a - b)(5a + b)$  (c)  $\frac{1-2x}{(x+1)^2}$  (d)  $\frac{ab}{6}$

# MATHEMATICS

---

Paper 4024/12

Paper 12

## **Key messages**

All working should be clearly shown and answers accurately transcribed to the appropriate answer space.

Accuracy in basic number work is essential.

Care must be taken to form letters and digits clearly and unambiguously.

## **General comments**

There were many well-presented scripts of a good standard.

Candidates of all abilities continue to do well in questions involving standard algebraic procedures, such as factorisation and the manipulation of algebraic fractions.

When solving equations, candidates could be encouraged to check that their solution(s) are valid.

Candidates also continue to do well in questions testing the basic procedures in the manipulation of fractions and the four rules of arithmetic.

Where such calculations occur at intermediate stages of more extended problems, it is important to maintain the degree of accuracy that will lead to a correct final answer. Many candidates lose credit by rounding prematurely in the early stages of more extended problems.

Most candidates show all their working by using effectively the working space provided on the paper.

It should not be necessary to attach extra sheets of working. In some cases this year, large numbers of extra sheets were included, many of which were completely blank.

It is also important that the working is clearly and neatly set out.

Particular care could be given to the formation of letters and digits that are clear and unambiguous.

To improve clarity, candidates could be discouraged from working initially in pencil and then inking over.

### Question 1

Candidates clearly understood that they should work in fractions. Decimals were seen only very rarely. There were very few instances where rules were confused and the fractions were combined incorrectly.

- (a) Always attempted and often accurate, with most candidates finding the correct common denominator. After  $\frac{6}{35}$  was reached in the working, the answer  $5\frac{1}{6}$  was sometimes seen. Some  $\frac{16}{35}$ 's were also seen.
- (b) Generally well answered.

Answer: (a)  $\frac{6}{35}$  (b)  $\frac{15}{16}$

### Question 2

Most candidates interpreted the tabulated data successfully.

- (a) Usually understood and well answered. A common wrong answer was  $\frac{8}{15}$ .
- (b) Again, usually understood and well answered. There were some unusual equivalent formulations of the ratio given, such as  $\frac{11}{23} : \frac{12}{23}$ .

Answer: (a)  $\frac{8}{23}$  (b) 11 : 12

### Question 3

- (a) Many candidates realised that the numbers given had to be assessed in terms of the units stated and were able to complete the list correctly. When the order was incorrect, it seemed that candidates had considered only the units, or only the numbers.
- (b) The conversion of units in the context of area was a problem for many candidates. There were many incorrect answers. Common errors were multiplication by 10 or 1000. Divisions by powers of 10 were also seen.

Answer: (a) 5 cm, 500 mm, 500 m, 50 km (b) 4160 mm<sup>2</sup>

### Question 4

- (a) Candidates who compared the given equation with  $y = mx + c$  often drew the correct inference. Some answers were given as  $-\frac{1}{3}x$ . Some candidates gave the complete equation  $y = -\frac{1}{3}x + \frac{2}{3}$  as their answer. A common wrong answer was  $-1$  given by candidates who had not adjusted the given equation from  $3y$  to  $y$ .
- (b) Candidates were generally confident with the substitution required here.

Answer: (a)  $-\frac{1}{3}$  (b)  $-1$



### Question 5

- (a) A lot of correct answers were seen. Some candidates did not seem to understand what was being asked, giving such as a list of 4 regions for their answer, or even further inequalities in  $x$  and  $y$ .
- (b) Region  $A$  was a common wrong answer for this part of the question. As in part (a), sometimes a list of 3 regions was given, perhaps one for each inequality.

Answer: (a) F (b) E

### Question 6

- (a) The correct reflection was frequently seen. The idea of reflection was understood but sometimes Triangle  $B$  was translated from its correct position when lines such as  $x = -1$ ,  $y = 1$  or  $y = -1$  had been used.
- (b) Successful solutions usually showed appropriate working on the grid. In this part of the question, there was more room for error, such as rotating anticlockwise instead of clockwise. A common incorrect response was to use the centre of rotation as one of the vertices of Triangle  $C$ . In some cases, Triangle  $C$  could be almost anywhere, in any quadrant, in any orientation, and even any shape. This part of the question was sometimes omitted.

Answer: (a) Triangle  $B$ , vertices (4,3), (6,3), (6,2) (b) Triangle  $C$ , vertices (-1,4), (-1,6), (-2,6)

### Question 7

- (a) Generally well done. Sometimes the minus sign was omitted. Other Incorrect readings seen were  $-1.7$ ,  $-1.25$  and  $0.7$ .
- (b) Generally understood. Some candidates used the first and last values given in the table, arriving at the answer  $2.7$ . The answer  $0.6$  was also seen. Some candidates gave an answer of  $4.5$ , using the highest and lowest values printed on the scale in the diagram.
- (c) Again, generally well done. Here again the minus sign was sometimes omitted. Incorrect answers seen from time to time were  $-2.3$  and  $+2.3$ .

Answer: (a)  $-1.3$  (b)  $3.2$  (c)  $-1.5$

### Question 8

In parts (a) and (b), the instruction concerning two digit numbers was sometimes missed by candidates.

- (a) Usually understood. The variety of answers seen included 4,  $4^3$ , 8, 16 and 27.
- (b) Again, usually understood. Common incorrect answers were 39 and 26.
- (c)  $\sqrt{2}$  and  $\sqrt{3}$  were common correct answers. Candidates using  $\pi$  did not always get into the range required in the question. There were a number of solutions of the type  $\sqrt{1.5}$ . A common incorrect answer was 1.5. Extended and recurring decimals were also common incorrect answers.

Answer: (a) 64 (b) 13 (c) Any irrational number  $n$  such that  $1 < n < 2$

### Question 9

- (a) Generally well done. It was important here not to add extra zeros after the digits 41.
- (b) Quite well done. There were solutions such as  $\sqrt{130}$  and  $\sqrt{132}$ ,  $\sqrt{121}$  and  $\sqrt{141}$  as well as 10 and 12 where the integers are not consecutive.

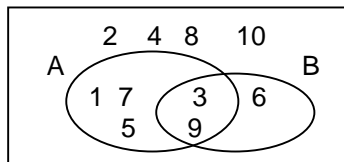
- (c) Care was needed to leave the index 2 outside the final bracket. Some candidates added too many brackets.

Answer: (a) 0.0041 (b)  $11 (< \sqrt{131} <) 12$  (c)  $(3 \times 2 + 1)^2$

#### Question 10

- (a) Generally well done. A common error was to omit the elements 2, 4, 8 and 10. Some candidates included numbers more than once in the diagram. A number of candidates left their Venn diagrams blank.
- (b) Many answered this part correctly even without the Venn diagram. Some listed the correct elements of  $A \cup B$  but did not realise that the number of them was required. Some answers were given as  $n(6)$ .
- (c) There were some correct interpretations of their Venn diagrams. Some candidates made a fresh start, working independently of their Venn diagram. A common incorrect answer was to list the elements of  $A \cap B$ .

Answer: (a)



(b) 6 (c) 1, 5, 7

#### Question 11

- (a) Mostly correct. Some answers were left as 2.
- (b) Most candidates realised that  $12^2$  was involved. Some had difficulty getting the correct ratio into the required form. For example, the long division  $36 \div 25$  was often poorly handled. A common error was to give an answer based on the ratio 6:5.

Answer: (a) 12 (b) 1.44 : 1

#### Question 12

- (a) Often well done. There was a very good response to this question. Many candidates did the full construction accurately, showing all the relevant arcs. There were a number of candidates who bisected one of the angles.
- (b) A lot of correct answers. The circle centre C was not always accurate. Some incorrect areas were shaded.

#### Question 13

- (a) Generally very well answered. As usual, negative signs need extra care.
- (b) The working in this question showed that many candidates knew how to construct the inverse matrix. Having reached the form shown in the answer below, many made errors in attempting to write it in a different form. The negative sign arising in the calculation of the determinant needed careful handling. Some candidates multiplied by their determinant instead of dividing. Candidates could be encouraged to check their answer.

Answer: (a)  $\begin{pmatrix} 4 & -1 \\ 1 & -1 \end{pmatrix}$  (b)  $\frac{1}{6} \begin{pmatrix} 0 & -3 \\ 2 & 2 \end{pmatrix}$

#### Question 14

- (a) Candidates generally seemed to understand that this problem involved the product of \$7.60 and a time difference. One difficulty was the evaluation of the correct time difference. Many different times were seen for this, ranging from 3 hour 15 minutes to 12 hours 15 minutes. Deciding on the length of the working day was a challenge for many candidates. The next difficulty was to express the time difference in a form that could be used in calculating the required product. Candidates reaching 8 hours 15 minutes often used 8.15 as the multiplier rather than 8.25. Again, candidates could be encouraged to check that the answer they have obtained is a realistic one.
- (b) This part of the question was generally well done. The answer 65 was sometimes seen. A typical cancelling error was  $525 \div 5 = 55$ .

Answer: (a) 62.70 (b) 35

#### Question 15

- (a) There was a good response to this question, with many correct answers. Many candidates still think that the answer required in this type of question is  $P = kQ^2$ . Sometimes the final answer used a proportional sign rather than an equals sign. A common error was to find that  $k = 4$ . There were incorrect interpretations of the type of proportionality required here.
- (b) This part of the question was usually completed satisfactorily.  $\sqrt{100}$  rather than  $-10$  was sometimes given as the second answer.

Answer:  $\frac{1}{4}Q^2$  (b) 10 or  $-10$

#### Question 16

- (a) Usually 16 was seen in the working, but not everyone managed to reach  $\frac{1}{16}$ .  $-16$  was frequently given as the final answer.
- (b) Probably the more successful strategy here was to go for cancelling first, followed by the square root. Candidates opting for the square root first tended to have more difficulty in sorting out the cancellation. In fact, in some cases, the expression was simply left with each individual term square rooted, although it was common to see only the numerator adjusted in this way and 4.5 was frequently seen as the square root of 9. Clearly, some candidates lost track of the individual adjustments they were making. Cancelling and rewriting powers all in the same expression made it difficult for candidates to decipher their own work. There was some misconception apparent with some candidates changing to the power 2 and turning the original expression upside down.

Answer: (a)  $\frac{1}{16}$  (b)  $\frac{3y^2}{x}$

#### Question 17

- (a) Candidates showed a good knowledge of the formula and strategy required to solve this problem. Common errors included the use of  $2\pi r^2$  and  $2\pi r$  and there were some candidates who tried to use radian forms of the formula. Many candidates appreciated the form in which the answer was required.
- (b) Correct answers in part (a) were invariably correctly evaluated. After substituting  $\pi = 3$ , candidates were expected to work exactly, so that premature rounding such as  $1.875 = 1.9$  did not lead to an acceptable answer. A common misconception led to the answer 4.8.

Answer: (a)  $\frac{5\pi}{8}$  (b) 3

**Question 18**

- (a) This was well answered by candidates. However, some candidates seemed to think that  $48 \times 10^6$  was standard form.
- (b) There was a good response here also. There were some subtractions in the wrong order, achieving positive answers of the type  $7.35 \times 10^k$ . Some candidates attempted division here.
- (c) A lot of success in this question. Again, some candidates did not give their answer in standard form.

Answer: (a)  $4.8 \times 10^7$  (b)  $9.3 \times 10^6$  (c)  $5.1 \times 10^8$

**Question 19**

- (a) (i) This was well answered by candidates. Where mode was not understood, the usual incorrect answers were 6, 4 or 2.5.
- (ii) Generally understood. In constructing the mean,  $0 \times 4$  needed care. The final division,  $42 \div 20$ , was not always correct. When not properly understood, the common errors were division by 15 or 6.
- (b) Again, generally understood. Some candidates arrived at answers such as 3.4 and 340.

Answer: (a)(i) 1 (ii) 2.1 (b) 34

**Question 20**

- (a) A good number of successful solutions. Candidates who spotted the common denominator 4 seemed to be more successful than those who opted for 8. Often the correct processes to remove fractions and brackets were seen, but the resulting work contained errors where signs were not adjusted when numbers were collected on the right hand side of the equation.  $7x - 2 = 12$  followed by  $7x = 10$  was particularly common. Some candidates who reached  $\frac{7x-2}{4} = 3$  went on to  $7x = 5 \times 4$ . When solving equations of this type, candidates could be encouraged to test if their solution actually works.
- (b) The basics of handling algebraic fractions were well understood by many candidates. Again, care is needed when collecting terms together. Some candidates showed a linear denominator, and some candidates ignored the denominator completely.

Answer: (a) 2 (b)  $\frac{7x+3}{(x+4)(x-1)}$

**Question 21**

- (a) This was generally accurate. Occasionally misunderstood, with frequency densities or mid-class values used.
- (b) The values in the table were usually successfully converted into points on the graph. Some candidates were not aware of the basic shape of a cumulative frequency curve.
- (c) The graph was correctly interpreted by many candidates. Subtraction from 80 was missed by some. In a minority of cases, candidates read across from their cumulative frequency of 45.

Answer: (a) 4 16 30 52 70 80

### Question 22

- (a) Usually attempted, with a good number of correct lines drawn. Misinterpretations of the situation included the starting point, where a common error was to begin Kiran's graph at (13 10, 0), the direction in which Kiran was travelling, and the inclusion of a stationary part in the middle of her journey.
- (b) The idea of reading off at the intersection of the graphs was often understood. The given scale needed careful interpretation. Many candidates who had non-intersecting graphs also attempted an answer, usually 6.
- (c) This part was answered quite well. The use of data from the graph given in the question proved a difficult challenge for many candidates. Frequent incorrect answers were 0.3 km/h from  $6 \text{ km} \div 20 \text{ minutes}$ , and 20 km/h from  $6 \div 0.3$ .
- (d) The implications of this part of the question were not appreciated by the majority of candidates. There were many incorrect graphs and many omissions of this part of the question. Only a minority of candidates realised that constant speed gave a horizontal line on a speed/time graph.

Answer: (c) 18

### Question 23

- (a) There were a good number of solutions that stated a correct set of three facts, giving reasons for the choice. Most candidates who attempted this question gained credit for relevant statements. These were often included in lists that contained angles that could only be implied to be equal once congruency had been established. Some solutions recognised that  $AB$  and  $AD$  were tangents without saying that they were equal in length.
- (b) This was well answered. Rhombus and parallelogram were the common incorrect answers.
- (c) A good number of candidates scored full credit. Some who worked their way around to  $\widehat{BAO} = 22^\circ$  forgot to double it. There was some confusion as to which angle was  $136^\circ$ . Some thought it to be the reflex angle  $\widehat{BOD}$ .

Answer: (b) Kite or Cyclic Quadrilateral (c) 44

### Question 24

- (a) This was well answered by candidates. For the answer, some candidates gave the expression they started from. Common errors were  $-5t + 3t = -8t$  and  $-5 \times 3 = -8$ .
- (b) This part was well answered. The difference of two squares was recognised and usually correctly factorised.
- (c) Candidates needed to group the terms carefully and to take care with negative signs in order to complete the factorisation successfully. Many were able to do this. Most candidates achieved at least half credit for dealing correctly with a common factor. Quite a number of candidates started by writing down the given expression with all signs changed.
- (d)(i) Most candidates would benefit from more practice in converting quadratic expressions into the form required here.
- (ii) Only a minority of candidates achieved the correct answer. Some candidates missed the word "hence" in the question and started again, applying the formula for a quadratic equation. They were unable to give the solution in the form required.

Answer: (a)  $t^2 - 2t - 15$  (b)  $(8x - 3y)(8x + 3y)$  (c)  $(3a + 2)(2b - a)$  (d)(i)  $(x - 3)^2 - 6$  (ii)  $3 \pm \sqrt{6}$

# MATHEMATICS

Paper 4024/21

Paper 21

## General Comments

The paper proved accessible to all candidates and provided them with opportunities and challenges to demonstrate their ability at all levels. Generally, candidates provided working to support their answers and on the whole it was quite neatly presented. Only a small percentage of candidates answered more than the required 4 questions from **Section B** and it appeared that it was a limit of knowledge rather than time that prevented some candidates from attempting 4 questions.

The majority of candidates knew how to apply the quadratic equation formula to solve an equation and this topic seems to have been well taught and understood. Calculators were set on degrees and very few instances of them being set on Rads or Grads by mistake, were seen. Candidates need to be aware that calculations should be done to 4 figure accuracy when the answer is to be given to 3 significant figures and that any premature approximation of values used in the working can result in the loss of accuracy marks. Candidates should also ensure that they have read the question thoroughly and that they have given the answer to the required accuracy, for example, **Question 12b(iii)**. Nearly all candidates used a pair of compasses when required for the construction in the scale diagram question.

## Comments on Specific Questions

### **Section A**

#### **Question 1**

- (a) Many fully correct answers were seen showing that candidates had a good understanding of the process needed to solve the equation. Even those not scoring full credit, nearly always removed the brackets successfully. The errors occurred when candidates did not change the signs in transposing like terms.
- (b) There were many candidates who scored full credit on this part and the general idea of solving simultaneous equations seems to have been well understood. Some candidates did not multiply all of the terms in the first equation by 2 when choosing to eliminate 'x' first of all, or similarly did not multiply all terms by 3 if choosing to eliminate 'y' first. Again, when eliminating the  $x$  terms by subtracting, some candidates did not remember that when subtracting a negative term it becomes a positive term and did not change  $-3y$  to  $+3y$ . Candidates who chose to eliminate 'y' first of all were usually more successful as this problem was avoided.
- (c)(i) A good number of correct answers to this part were seen. A common error was to omit  $-1$  from the answer.
- (ii) Many candidates were aware that they could employ the same rules to solving this inequality as they would to solving an ordinary linear equation and most correctly transferred the 2 but some did not change the sign from 2 to  $-2$ . Those who successfully reached  $-3y < 6$  went on to give their answer as either  $y = 2$  or  $y < -2$ . A smaller number remembered that dividing by a negative number reverses the inequality sign and correctly gave  $y > -2$  as the final answer. A few candidates reached  $y > -2$  in their working but then wrote either  $y = -2$  or just  $-2$  as the final answer.

Answer: (a)  $x = 3$  (b)  $x = 4, y = -1$  (c)(i)  $-1, 0, 1$  (ii)  $y > -2$

## Question 2

- (a) The majority of candidates tried to use the formula  $180 \times (n - 2)$  for calculating the total sum of the interior angles and did not realise that they were already given the interior angle. Only a small percentage of candidates knew to find the exterior angle by doing  $180^\circ - 165^\circ = 15^\circ$  first and that the sum of the exterior angles  $360^\circ \div 15^\circ$  would give the number of sides of the regular polygon.
- (b)(i)(a) Quite well done. Many candidates realised that angle  $BCA$  and angle  $DEA$  were corresponding angles and correctly found angle  $BCG$  from angles on a straight line add up to  $180^\circ$ .
- (b) This was not so well done. A good number of candidates knew that angles  $ADE$  and  $ABC$  were corresponding angles, but in trying to find angle  $ADE$  from  $180 - (180 - p + q)$ , errors occurred when removing the brackets and trying to simplify the expression.
- (ii)(a) Corresponding sides being in the same ratio was not well understood by the majority of candidates with only a small percentage of correct answers seen. Candidates need to take more care in selecting which sides correspond with which in the two given triangles, e.g.  $AE/DE = AC/BC$ .
- (b) Selection of the correct corresponding sides proved difficult for many candidates. They did not know that  $(AD + 2.1) / AC = AD / 7$  were the required pairs of corresponding sides and so further progress towards obtaining the correct answer was not possible.

Answer: (a) 24 (b)(i)(a)  $180 - q$  (b)  $p - q$  (ii)(a) 8cm (b) 4.9cm

## Question 3

- (a)(i) The 2 correct values were found by nearly all candidates.
- (ii) The correct answer was often seen as most candidates realised that to get the number of small triangles they had to multiply the parallelogram number by 2.
- (b)(i) Again, nearly all candidates were able to obtain the correct 2 values, showing a good understanding of square numbers.
- (ii) This also was done well by a large majority of candidates.
- (iii) Many candidates knew to take the square root of 324 and the correct answer of 18 was often seen.
- (c)(i) Not so many candidates were able to see that by combining the diagrams of 3(a) and 3(b) together then they would get the diagrams of 3(c). Thus, there were fewer correct answers of  $2t + t^2$  seen for this part, compared with the other parts.
- (ii) This part was answered correctly by the majority of candidates, even by some who could not find the correct answer to the previous part.

Answer: (a)(i) 10,12 (ii) 2m (b)(i) 25,36 (ii)  $n^2$  (iii) 18 (c)(i)  $t^2 + 2t$  (ii) 675



#### Question 4

- (a) (i) This was not answered well by a large proportion of candidates. Many had difficulty in getting the correct denominators for the fractions of both the second and third chocolates. It was common to see  $10/12$  and  $1/12$  being used for the second chocolate and  $9/11$  and  $1/11$  for the third. Other candidates attempted some of the branches but left others blank, while others used  $9/11$  and  $2/11$  again to fill in the lower branches for the second chocolate and repeated all of the top branch probabilities when filling in the lower branches for the third chocolate.
- (ii)(a) Fairly well answered, with those who knew to multiply the 3 correct probabilities also managing to give the correct answer in its lowest terms.
- (b) This was not as well answered. Many candidates only used one combination of probabilities and not the 3 possible routes on the tree diagram. So it was common to see  $10/12 \times 9/11 \times 2/10$  being used and the answer of  $3/22$  on many scripts. Those who did correctly use all 3 routes were able to give the correct answer in its lowest terms.
- (b)(i) The mode was well understood by majority of candidates, as most were able to give the correct answer here.
- (ii) Not as well answered. There seemed to be a common misconception here that the median was 3, presumably because the bar representing 3 letters was the middle bar out of the 7 bars. Candidates need to remember that the median is the middle value in an ordered set of values, which in this case contained 31 values, so the middle value was  $31/2 = 15.5$ th value which lies in the bar representing 2 letters.

Answer: (a)(i) 10/11, 1/11, 9/10, 1/10, 10/10, 0/10 (ii)(a) 6/11 (b) 9/22 (b)(i) 1 (ii) 2

#### Question 5

- (a) (i) Many correct answers were seen here, with the majority of candidates performing the correct multiplication of  $300 \times €0.72$ .
- (ii) Again, many correct answers were seen, but not quite so well answered as part (i). The common error here was for candidates to invert the conversion fraction and evaluate  $\$75 / €51$  instead, leading to the commonly seen incorrect answer of €1.47.
- (b)(i) Not many candidates were able to arrive at a fully correct answer here. The fact that there was more than one approach that could be used involving 3 calculations, proved confusing for a good number of candidates, who often mixed the different approaches together. Some candidates found 75% of £2.00 and 35% of £2.00 but then multiplied both by 60 and not 25 and 18 respectively when trying to work out the individual profits made. Others using this approach correctly obtained \$6.80 for the remaining 17, but added it on rather than subtracting it, forgetting that it would be a loss not a profit. Those that chose to find the total sales often could only get 1 or 2 of the sales figures correct out of \$87.50, \$48.60 and \$27.20. However, they did know to subtract the cost price of \$120 from the total of their 3 sales figures.
- (ii) This part was done well. Those who had the correct answer to the previous part had no difficulty in obtaining the percentage profit. However, candidates with incorrect answers to (i) knew how to calculate a percentage profit or loss which earned them the follow through mark.

Answer: (a)(i) €216 (ii) €0.68 (b)(i) Profit, \$43.30 (ii) 36 to 36.1%



### Question 6

- (a) (i) Many candidates chose to use the correct trigonometric ratio of  $\tan BAE$  for their calculation and obtained the correct answer. Some candidates used Pythagoras' Theorem first of all and correctly calculated the hypotenuse  $AB$  and then employed either the  $\cos$  or  $\sin$  ratio successfully. Some candidates lost the accuracy in their calculations by either truncating or rounding prematurely. Candidates need to remember to work with 4 figure accuracy when the answer is to be given to 3 significant figures.
- (ii) The majority of candidates employed the area of a trapezium formula in working out the required area and it was good to see so many correctly obtaining the length  $ED$  first of all from use of  $\tan 55^\circ$ . Many then went on to obtain the correct final answer for the area. However, some candidates forgot to add the 7 cm to  $ED$  to get  $AD$  and just had 9 cm + 12.6 cm for the sum of the parallel sides. A common misconception by a number of candidates was to think that  $ED$  was also 9 cm. A small number of candidates having correctly evaluated  $ED$ , chose to find the area of the 3 triangles with mixed success.
- (b) This was not a well answered question with only a handful of correct responses seen. It was good to see that many knew that angle  $QPR$  was also  $41^\circ$  from alternate angles, but then it was too common to be given the answer of  $27^\circ$  from  $180^\circ - 153^\circ$ . It appears that very many candidates forgot that 'measured to the nearest degree' means that the given angles had upper and lower bounds. One or two candidates gave angle  $QPR$  as  $41.4$  and angle  $PQR$  as  $112.4^\circ$  leading to an answer of  $26.2^\circ$ . A few others who realised that  $0.5^\circ$  was involved subtracted this from  $112^\circ$  and  $41^\circ$  instead of adding it on, so reaching a final answer of  $28^\circ$ .

Answer: (a)(i)  $68.7^\circ$  (ii) 257 to 257.5 (b)  $26^\circ$

### Question 7

- (a) A good question for the majority of candidates with  $s = d / t$  being well known and used to give many correct answers. A small number of candidates however, employed the wrong method by dividing 19.94 by 2 to get 9.97 and then subtracted this from 9.98 which gave the same figures as the answer.
- (b) (i) This proved a difficult question for very many candidates and many made no response at all. The ablest candidates gave the required equation and could then simplify it and equate it to zero. A small number of candidates could give the correct expressions  $120 / x$  and  $120 / (x+3)$  for the times of the cars, but then either could go no further or set up the equation with these expressions incorrectly. The common error by candidates was to equate the difference between these expressions to 6 minutes and not convert to 6/60 or 0.1 hours. Sometimes candidates made the error of subtracting the expressions in the reverse order. The more able candidates showed a good level of algebraic skills in the manipulation and simplification of the correct equation.
- (ii) This was a good part question for many candidates. The majority of candidates more often than not were able to score full credit here, showing that the solving of a quadratic equation using the formula method is a well understood topic. There were relatively few examples of candidates not giving their answers to the required one decimal place. However some candidates did make the error of not extending their division line far enough and divided the discriminant by 2 instead of the whole numerator. Others omitted the minus sign in  $-3600$  when substituting into the formula and thus ended up with a negative value for the discriminant. Only a handful of candidates used the completion of the square method to solve the equation and usually were successful.
- (iii) Many candidates knew the method here and divided 120 km by their positive value of  $x$  from (ii), but not many converted the answer into minutes by multiplying by 60. Other candidates thought that they had to add 6 minutes onto the 58.5 before dividing into the 120.

Answer: (a) 0.01 (b)(i) Correct equation shown (ii) 58.5 or  $-61.5$  (iii) 123

## Section B

### Question 8

- (a) (i) Many candidates who attempted this question were able to correctly substitute  $x = -2$  and arrive at the correct answer.
- (ii) There were not so many candidates who could employ the method fully and correctly to reach the right answer to this part. Some candidates started off correctly by letting  $y = (4x - 3) / 2$  and reached  $2y = 4x - 3$  but then transposed  $-3$  incorrectly or obtained  $x = (2y + 3) / 4$  and gave this as their answer overlooking to replace  $y$  with  $x$ .
- (iii) There were slightly more candidates able to arrive at the correct answer to this part than in the previous part. Many candidates substituted  $2g$  for  $x$  into the function, but then the common error was to omit to equate to  $g$ , which led to the equation  $2 = 8g - 3$  and giving the answer  $g = 5/8$ . Others who did equate to  $g$ , sometimes multiplied both terms by 4 in the numerator of the fraction giving  $8g - 12 = 2g$  leading to the answer of  $g = 2$ .
- (b) (i) Nearly all candidates knew that enlargement was the transformation and scored at least half credit. Some however went on to say that the triangle was then reflected, which by mentioning a second transformation did not earn credit. Not many candidates got the scale factor correct, with 4 or  $-4$  being the common error. Even fewer candidates gave the centre as  $A$ . A fully correct answer was only rarely seen.
- (ii) On the whole this part was well attempted. Many candidates showed that they understood the modulus of a vector and correctly employed Pythagoras' Theorem to reach  $\sqrt{5}$  or an acceptable equivalent decimal answer.
- (iii) This was not answered well with only a small percentage of candidates using the correct vectors and reaching the fully correct answer. Few candidates chose the correct path to follow, i.e. vector  $CB$  followed by vector  $BD$ . However, they did not show that vector  $CB = -\frac{1}{3}$  vector  $ED$  and that vector  $BD = 4 \times$  vector  $BA$ .
- (iv) This was not well answered by the majority of candidates, with only a handful of correct answers seen. Those candidates who did not evaluate vector  $BD$  in part (iii) could not obtain vector  $DF$  here, for use in the route vector  $EF =$  vector  $ED +$  vector  $DF$ . Likewise, many candidates did not realise that vector  $AF =$  vector  $BA$  which prevented progress on the other possible route of vector  $EF =$  vector  $EA +$  vector  $AF$ . Some candidates who correctly worked out the value of vector  $FD$ , overlooked that vector  $DF$  would be the negative value of it.

Answer: (a)(i)  $-5.5$  (ii)  $\frac{2x+3}{4}$  (iii)  $0.5$  (b)(i) Enlargement, scale factor  $-3$ , centre  $A$  (ii)  $\sqrt{5}$  or  $2.24$

$$(iii) \text{ vector } CD = \begin{pmatrix} 0 \\ -7 \end{pmatrix} \quad (iv) \text{ vector } EF = \begin{pmatrix} 10 \\ 1 \end{pmatrix}$$

### Question 9

- (a) (i) A good question for many candidates with the correct answer often seen. However, some candidates thought that the volume of a cylinder was  $\frac{1}{3} \pi r^2 h$  or even  $2\pi r h$  or  $2\pi r^2 h$ .
- (ii) Very few correct answers were given here. Commonly seen were answers of 10, 1000, 10 000 or 100 000. A few candidates misunderstood the question and gave their answer as 60 seconds, from the 0.9 litres per minute.
- (b) Not very many fully correct answers were given. Some candidates reached  $x^2 = 6.25$  and omitted to take the square root to obtain the value of ' $x$ '. A good percentage of candidates did obtain the method mark for knowing how to calculate the triangular area of cross-section. However, a number of candidates wrongly used  $1/3$  instead of  $1/2$  in their calculation.

- (c)(i) Factorising was well understood by the majority of candidates and there were many correct answers here.
- (ii) Again a very well answered part question with many candidates scoring credit.
- (iii) Getting the correct answer here depended on whether or not candidates knew the correct area of cross-section rule from part (b). Those that did, knew to use the positive value of  $y = 1.5$  and went on to reach the correct answer. Some did score the follow through mark available here.
- (iv) There was a mixed response by candidates to this part. The more able candidates often obtained the fully correct answer. Others knew that they had to use twice their answer to part (iii), but then omitted one of the rectangular faces from their calculation, usually the base  $8 \times 7.5$ .
- (d) Not many correct answers were seen in this part. Those getting the correct answer often worked out the volume of shape III first of all as  $540 \text{ cm}^3$  and then cancelled  $540/1500$  to its lowest terms, rather than use the ratio  $1.5^2 / 2.5^2$  to obtain  $225/625$  and then cancel down.

Answer: (a)(i) 29.8 to 29.85 (ii) 100 (b) 2.5 (c)(i)  $(2y - 3)(2y + 11)$  (ii) 1.5 or  $-5.5$  (iii) 67.5 (iv) 495  
(d)  $\frac{9}{25}$

#### Question 10

- (a)(i) Many correctly found the two correct values, but some gave  $y = -0.66$  when  $x = -3$ . Others lost credit for not giving the values to at least 2 figure accuracy, by giving 0.6 for both values.
- (ii) The plotting of the points was usually very accurate and the attempts at drawing the curves were usually good. A small number of candidates lost credit by using ruled line segments in their drawings instead of drawing a smooth freehand curve. The common error here was to join the tops of the curves from  $(-1,6)$  to  $(1,6)$  with either a straight line or a curve.
- (iii) Most candidates read off values in the given range or earned the follow through mark for readings from their graph. A small number omitted the negative sign in the lower value reading of  $x$ .
- (iv) Most candidates knew that the gradient was calculated from rise / run and attempted calculations involving 'difference in  $y$ ' / 'difference in  $x$ ' with mixed success. The figures used in the calculations sometimes bore no relation to the line attempt at the drawing of the tangent at  $x = 1.5$ . There were some very good attempts at drawing the tangent, whilst others were less successful. Some candidates thought that as long as the line included  $x = 1.5$ , then it did not matter if the line they drew cut across the curve. Candidates need to remember that a tangent should only touch the curve at the given point.
- (v) This was not well done by the majority of candidates who either left this blank or drew the line  $y = 2$  instead and not the required line. The most able candidates drew the correct line and then read off the correct value of  $x$ .
- (b)(i) The ablest candidates appeared to know how to proceed here and correctly substituted  $x = 2$  and  $y = 45$  into the given equation to obtain the value of ' $a$ '. Some answered by giving  $\pm 3$  as the answer for  $a$  and not just the positive value. Most then correctly gave  $b = 405$ .
- (ii) Slightly better answered with more candidates knowing to substitute  $x = 0$  into the equation and getting  $y = 5$  and thus the correct coordinates.
- (iii) Those candidates able to answer the previous parts usually obtained the correct answer here too. Also there were a few who earned the follow through mark available.

Answer: (a)(i) 0.66, 0.66 (iii)  $-1.7$  to  $-1.8$  and  $1.7$  to  $1.8$  (iv)  $-2.5$  to  $5$  (v)  $-1.3$  to  $-1.4$   
(b)(i)  $a = 3$ ,  $b = 405$  (ii)  $(0, 5)$  (iii) 20

### Question 11

- (a) (i) Most candidates who chose to answer this question were able to give an answer in the allowed range.
- (ii) Nearly all candidates used a pair of compasses for this construction and the majority of arcs were clearly shown. The accuracy shown in the constructions was also very good with  $C$  usually labelled. Most candidates obtained full credit for this part.
- (iii) There was a mixed response to this part. Some candidates knew what to do and gave acceptable answers. Others had obviously measured angle  $BAC$  and gave  $34^\circ$  for the bearing, whilst others left it blank.
- (iv) This was not well answered with only a handful of fully correct quadrilaterals seen. Many candidates did not attempt this part. The majority of candidates who did make an attempt tried to position point  $D$  on a line drawn from  $A$  on a bearing of  $034^\circ$  and appeared not to understand what was meant by ' $AC$  is the line of symmetry of quadrilateral  $ABCD$ '.
- (b) (i) There were a fair number of candidates who were able to use the Cosine rule correctly and obtained full credit for this part. A number also reached angle  $QPR = 110^\circ$  but then did not remember to add the  $54^\circ$  to get the bearing from  $P$ . Some evaluated the Cosine rule as far as  $27500 / 80000$  but with the negative sign missing and this led to their angle  $QPR = 69.9^\circ$ , but they were still able to pick up the method mark if they did remember to add the  $54^\circ$ . A few candidates took the long way around approach and used the Cosine rule to evaluate angle  $PQR$  and the Sine rule to get angle  $PRQ$  and finally obtained angle  $QPR$  from the angle sum of a triangle. A number of other candidates thought that angle  $QPR$  was a right angle, while some candidates made no attempt.
- (ii) There was a mixed response to this part. Those candidates who answered part (i) correctly invariably reached the correct answer here by the correct use of  $\frac{1}{2} ab \sin C$  for the area of the triangle. Errors arose from candidates using  $\sin 164^\circ$  instead of  $\sin 110^\circ$  or 340 instead of 250 in the calculation. Others omitted the  $\sin C$  and just did  $\frac{1}{2} \times 160 \times 250$ . In a few instances some candidates chose to use Hero's formula and more often than not were successful in this. Again, some candidates made no response here.

Answer: (a)(i) 510 to 520m (ii)  $C$  positioned 7 cm from  $A$  and 6 cm from  $B$  with construction arcs shown  
(iii)  $146^\circ$  (iv)  $D$  positioned 10.3 cm from  $A$  and angle  $DAC = 34^\circ$  (b)(i) 164 to  $164.11^\circ$   
(ii) 18780 to 18800

### Question 12

- (a) (i) On the whole this part was answered well and many candidates showed a good understanding of the method involved and usually obtained the correct answer. Some candidates, who multiplied the frequencies by either the lower end of the class intervals or the upper end, and not the mid-points of the intervals, earned the method mark but lost the accuracy mark for the final correct answer. Candidates must remember that if they use the class width  $\times$  frequency then this will not earn the method mark. The majority of candidates knew to divide  $\sum fx$  by 140 and most earned this method mark. Some candidates answered  $140 / 7$  and did not gain credit.
- (ii) Very few candidates knew that they had to use the frequency densities when drawing this histogram because of the different class widths. The majority who attempted this part drew a histogram just using the frequencies.
- (iii) Very few correct answers were seen for this part. By far the most common error was to do the calculation  $(15 + 14 + 20) = 49 / 140$  leading to  $7 / 20$  as the answer. Candidates did not realise that they should have evaluated  $(\frac{3}{4} \times 20) = 15$  for the class width 10 to 14. The ablest candidates knew this and usually went on to obtain the correct answer.

- (b)** The whole of this part was usually well answered by the majority of candidates.
- (i)** Credit was often obtained in this part. Candidates clearly knew the relationship between  $360^\circ$  and 60 students and used this to progress to the correct answer.
- (ii)** Another part where credit was often obtained. Slightly more candidates favoured evaluating  $126^\circ / 360^\circ$  as a percentage directly, rather than first converting  $126^\circ$  to 21 students and then finding this as a percentage of 60.
- (iii)** There were a good number of fully correct responses seen here as well, but some candidates lost credit by not rounding to the nearest degree. Candidates need to check that they have read the question carefully to avoid losing credit. There were a few common errors made as well. Firstly, some candidates forgot to add the 4 new students to the 60, so did the calculation  $\frac{17}{60} \times 360$  giving 102 as the answer. Others did  $\frac{2}{64} \times 360 = 11.25 = 11$ .

Some candidates made the incorrect assumption that 60 students were represented by  $90^\circ$  and this led to 64 students being equivalent to the sector with an angle of  $96^\circ$ . This gained no credit.

*Answer:* **(a)(i)** 14.8 **(iii)** 11/35 **(b)(i)** 9 **(ii)** 35% **(iii)**  $96^\circ$

# MATHEMATICS

---

Paper 4024/22

Paper 22

## GENERAL COMMENTS

The paper was generally of a similar standard to previous years with most questions accessible to the majority of candidates but with a number of useful discriminatory parts.

Candidates seemed to have adequate time to complete the paper and sometimes did all 12 questions.

The graph and construction questions were generally well presented although there was the usual concern regarding the large indistinct plotting of the points in the graph question.

Relatively few candidates realised that they needed to use upper bounds in **Question 5(a)** and more than usual lost credit through premature approximation.

Many candidates also lost credit through incorrect or careless use of standard formulae such as the quadratic formula, the area of a trapezium and the areas and volumes used in **Question 12**.

## COMMENTS ON INDIVIDUAL QUESTIONS

### Question 1

Accuracy was a problem for many candidates. Answers were rounded from one part to the next or even within the same part producing final answers outside the acceptable boundaries.

- (a)(i) The majority of candidates found this part straightforward and scored full credit. Most performed  $25\,200 \div 72$  and then  $350 \times 2.06$ . Several performed the first operation only and stopped at a solution of 350. A few found the number of rupees in a dollar, or vice versa, and then used this value.
- (ii) There were virtually no problems here, most evaluating  $551 \div 380$  successfully.
- (b)(i) Again there was a high success rate, most candidates computing the simple interest and then adding it on to \$8000. A significant number left the answer as \$272 or subtracted the interest amount, arriving at \$7728.
- (ii) The majority of candidates scored credit here by calculating 3.5% of their (b)(i) and adding it on to their previous answer. A few again subtracted, whereas some added it on to \$8000.
- (iii) A quite large number used compound instead of simple interest in this part. Simone's \$8000 then became \$8569.80 and the response of "Simone had \$8.28 more" was fairly commonplace. Some candidates offered two solutions in the response area.

Answer: (a)(i) \$721 (ii) \$1.45 (b)(i) \$8272 (ii) \$8560 – 8562 (iii) Lydia by \$1.52



## Question 2

The later parts proved to be difficult for many candidates. Finding expressions in terms of algebraic notation was not well understood as exemplified by the workings on many scripts.

- (a) Full credit was usually scored in this part. Almost everyone got the  $T$  value right but just occasionally the  $D$  and/or  $R$  value was wrong.
- (b) The vast majority of candidates scored here for  $n^2$  or  $n \times n$  or  $(n)^2$ . On occasions, candidates misinterpreted the question and responded for  $n$  not  $T$ , giving  $n = \sqrt{T}$  as their answer.
- (c) This was a well attempted part, many giving the correct response of 32. Some candidates failed to square root and simply divided by 2. Although 1024 was regularly seen, some candidates, while effecting the right substitutions, then made errors rearranging their equation to make  $T$  the subject.
- (d) Only very able candidates gave a correct expression, often with terms not fully collected. A common contribution was  $\frac{1}{2}R(R+1)$ , simply substituting  $R$  for  $n$  in the given expression. Some candidates did not relate this question part back to the given table in part (a). It was not unusual to find expressions involving  $D$ ,  $T$ ,  $n$  and  $R$ .
- (e) Many simply evaluated  $\frac{1}{2} \times 15 \times 16$ . Some candidates started (e) from scratch and, despite an incorrect expression or no response in part (d), obtained the correct solution of 360. On a few occasions, candidates did extend the table to obtain a solution.
- (f) Very few scored in this part, many expressions presented being in terms of  $D$ ,  $T$ ,  $n$  and  $R$ . There was no response on a lot of scripts.

Answer: (a) 25, 21, 45 (b)  $n^2$  (c) 32 (d)  $\frac{3}{2}n(n+1)$  (e) 360 (f)  $\frac{1}{2}(n+1)(n+2)$

## Question 3

- (a) Candidates performed well on this question showing a good understanding of the topic. Typical incorrect responses were  $x = 4$  and  $x = 11$ .
- (b)(i) A similar degree of competence as shown in part (a) was illustrated here in manipulating this inequality. Having solved correctly, a significant number of candidates put 4.25 or  $17/4$  on the answer line. A number had the inequality reversed or replaced with  $y = -$ . There were a few candidates getting  $11/4$  as an answer.
- (ii) This part was not well answered, with many candidates including 4.25 with the 3 and 4. It was often difficult to see in some cases where the list of values had come from because they did not resemble what was being asked in the question.
- (c) There was a high success rate in this question, candidates being well briefed in the technique of solving simultaneous equations. While a high majority scored full credit, some only found one value correctly, usually the  $x$  value by equalising the coefficients of  $y$  values. As to the method of solution, equating of coefficients was by far the most popular.

Answer: (a)  $-4$  (b)(i)  $y \leq 4.25$  (ii) 3, 4 (c)  $x = 1.5$ ,  $y = -3$

## Question 4

- (a) This proved to be a relatively difficult part. Most incorrect answers did not involve using the area of the trapezium and so generally answers scored 0 or 2. There were some who tried to use the formula for a trapezium but used the wrong values. The most common wrong answer was 12 from the incorrect assumption that  $ACDF$  was a square. Another incorrect approach resulted in an answer of 11.5 from  $\frac{1}{2} \times 6 \times (x + x) = 69$ . Other answers came from calculations using 6 and 16 in various ways ( $16 - 6 = 10$ , etc.).

- (b)(i) This was answered better than part (a), most candidates identifying the area of triangle  $DEF$  as 34.5. The subsequent work using the area formula for a triangle almost always resulted in an answer of 11.5 or the correct answer of 5.75.
- (ii) About half of those who got part (i) correct, gained full credit in this part. Most of the others got partial credit for 5.75:10.25. A lot of those who got (i) wrong also benefitted for stating the ratio "their 5.75:16 – their 5.75". A common error was in attempting to give the ratio in the form  $1:n$ . In the case of those who started with 5.75:10.25, this often became 1:2. Some lost credit because they misunderstood the question regarding integers and they rounded their answer to (i) to the nearest whole number before writing their ratio thus often finishing with an answer of 3:5.

Answer: (a) 7 (b)(i) 5.75 (ii) 23:41

### Question 5

- (a) This was not well answered as most candidates simply worked out  $4 \times 180 + 5 \times 15 = 795$  and concluded that it was not possible. It was very rare to see 180.5 and 15.5, candidates not appreciating the need to use the upper bounds for the widths. A small number used lower bounds and a few did not compare their answer with the given width of 8 metres.
- (b)(i) About a quarter of candidates answered this correctly. The majority of the remaining candidates worked out 30% of \$35.10 and subtracted it from \$35.10 giving them \$24.57. Only on a small minority of scripts was evidence of  $130\% = \$35.10$ .
- (ii) This was attempted very well by most candidates although a lot of them did not do the correct final stage. The majority got to the value \$832.26 from the correct first 2 steps but then often left this as their answer. Others subtracted \$378.3 from it. There were instances of people who added \$50.7 and \$35.1 and then worked out 220% of their answer. More commonly, some candidates interpreted the fence as being just the panels, their next stage of working being to find 220% of \$202.80. Although most did find the 220% correctly, there were some who divided by 220 and then multiplied by 100.

Answer: (a) No and 799.5 cm (b)(i) \$27 (ii) \$1210 to \$1211

### Question 6

Many marks were lost in this question when candidates assumed that  $PS$  and  $RQ$  were parallel.

- (a) This part was generally done well. Common wrong answers were 30 {from  $(360 - 210 - 90) \div 2$ }, 60 {from  $360 - 210 - 90$ } and 65 {thinking it was alternate to the given  $65^\circ$  angle}. A few candidates found angle  $SPQ$  by trigonometry and, assuming  $PS$  to be parallel to  $RQ$ , gave this as their answer for angle  $PQR$ .
- (b) The majority of candidates seemed to know which distance they were trying to calculate, and a lot gained credit for sin their  $35 = x/500$ . Those who got (a) correct usually got this part correct too. Quite a few used the sine rule when they did not need to but this still led to the right answer. Less direct methods were also seen, a significant number finding  $PS$ . Some candidates assumed that  $PS$  was the same length as  $QR$  and others that triangle  $PQR$  was right angled at  $P$ .
- (c) This was fairly well answered. A lot of candidates were able to work out angle  $SPQ$  correctly, although some then did not know exactly which bit was the bearing, so some subtracted 33.8 from 360, etc. The most common error in this part was to just halve 65.

Answer: (a)  $35^\circ$  (b) 287 (c)  $031^\circ$

### Question 7

- (a)(i) A lot of candidates successfully completed the histogram. There were some who had no attempt at this part. Those that were wrong had a variety of heights, 2.8 being the most common, the others seemingly not arising from any specific misconceptions.



- (ii) This was done quite well, generally with both values correct. If not, it was common to see  $q = 42$  correct but  $p$  incorrect, frequently  $p = 24$ . There were some who answered using decimals ( $p = 2.4$ ,  $q = 4.2$ ) from just reading the scale at the side ignoring the fact it was a histogram, but this was relatively rare.
- (iii) Candidates would benefit from improving their knowledge of this area, as the process of linear interpolation was rarely applied. Many candidates answered with a figure rather than a probability. Most correct answers were in fraction form although decimals were seen. A common error was to get  $86/200$ , by using the whole frequency from the 90 – 100 group.
- (b)(i) Almost all candidates identified the group unambiguously. A very small number wrote 85 (the frequency rather than the group) but this was rare.
- (ii) This standard type of question was answered well. The most common error was to multiply all the frequencies by 20 (the class width) but then most candidates divided by 200. Very few used end points instead of mid points. A small group added the frequencies and divided by 4.

Answer: (a)(ii)  $p=48$ ,  $q=42$  (iii)  $\frac{57}{200}$  (b)(i)  $40 < y \leq 60$  (ii) 39.9

### Question 8

- (a) This part was found fairly straightforward and answered correctly by most candidates. Incorrect responses were as expected, particularly off by a factor of 10, for example, 1500 instead of 150.
- (b) Sometimes  $C$  was positioned east of  $A$  but on most scripts it was either east of  $B$  or equidistant from  $A$  and  $B$ . The distance  $AC$  was rarely inaccurate.
- (c) Most assumed that measuring and then multiplying by the scale was equivalent to calculating. There were very few who used Pythagoras' Theorem.
- (d) In this part, candidates usually obtained  $1800/x$  or  $1500/(x+1)$  for the time taken for either boat to get to  $A$ . A good number however did not form the equation correctly, the most commonly occurring error being in the failure to change 1 minute into seconds. This question was answered rather better than similar questions in previous years.
- (e) Most of the stronger candidates showed sound ability in solving the quadratic equation, but many others made errors in this question. Substitution in the formula was often inaccurate with the discriminant not always evaluated to 136 and  $-4$  appearing instead of  $+4$  on many occasions; the latter resulting in solutions with the signs reversed. Candidates regularly did not give their answers to 2 decimal places correctly.
- (f) A significant number did see to divide their positive solution in part (e) into 1800. Some confused the boats, while a few, having effected the right division, added 60 onto their result.

Answer: (a) 150 (c) 995 (e) 7.83,  $-3.83$  (f) 229 to 230

### Question 9

A significant number of candidates omitted this question altogether. A number omitted part (a) but then attempted part (b).

- (a)(i) A large number did see how to use the two given vectors and confusion with signs sometimes led to negative components. It was also common to see the vectors added giving  $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$ .
- (ii) Many candidates showed that they knew how to apply Pythagoras correctly to find the length of a vector. Most of these used the appropriate vector for  $AC$  and correctly computed its length. Some found the length of the vector  $BD$ .

**(iii)(a)** Attempts at this part very frequently had the centre of enlargement missing or occasionally written as the origin instead of point  $B$ . Candidates usually recognised the transformation to be an enlargement. Only very occasionally were two transformations mentioned.

**(iii)(b)** Very few candidates appeared to see that the vector  $SR$  was  $\frac{3}{2}$  times the vector  $BD$ . It was more common to see it as  $\frac{3}{2}$  times the vector  $AC$ . There were not many correct outcomes in this part.

**(b)(i)** Almost all candidates who attempted this were successful.

**(ii)** This was again very well done but the evaluation of  $\frac{3(7)+2}{5}$  was quite often seen.

**(iii)** Most candidates answered this using the change of subject technique: there were few flow diagram approaches. Many were successful, just a small minority of candidates not changing  $y$  back to  $x$  in the final response.

Answer: **(a)(i)**  $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$  **(ii)** 6.71 **(iii)(a)** Enlargement, centre  $B$ , scale factor 3 **(iii)(b)**  $\begin{pmatrix} 7.5 \\ 3 \end{pmatrix}$  **(b)(i)**  $-2$   
**(ii)** 11 **(iii)**  $\frac{5x-2}{3}$

#### Question 10

A number of candidates omitted the early parts of the question and started with the factorisation.

**(a)(i)** This part was well answered, most realising the need for a denominator of 24. Occasionally the numerators were reversed but for many of those who attempted it, full credit was earned.

**(ii)(a)** A majority of candidates extracted the correct probabilities from the tree diagram and multiplied them. Just occasionally addition was seen or a 25 was 'cancelled'.

**(ii)(b)** This part proved to be particularly difficult. The vast majority equated their previous part to  $\frac{1}{p}$  but on most scripts there was little further progress.

**(iii)** There were attempts at factorising but these were often wrong with the negative values being given. The use of the quadratic equation formula sometimes gave the correct solutions but often produced errors.

**(iv)** This part was very often omitted and only rarely was a correct answer seen. Those who made an attempt often left  $n$  in the response, the common answer being  $\frac{n^2-n}{600}$ .

**(b)(i)** Some candidates did not see that the total number of children was 4 times the number in the red quadrant. Nevertheless there were a lot of correct solutions.

**(ii)** Many gave as their answer the number of children who preferred green. That is 25. Those who gave a fraction often did not reduce it to its lowest terms.

**(iii)** This was usually correct, candidates appearing to appreciate that the difference amounted to  $30^\circ$ , which equated to 25 children, the answer which many had given in part **(ii)**.

Answer: **(a)(i)**  $\frac{n}{24}, \frac{24-n}{24}$  **(ii)(a)**  $\frac{n(25-n)}{600}$  **(ii)(b)** 4 **(iii)** 15,10 **(iv)**  $\frac{3}{20}$  **(b)(i)** 300 **(ii)**  $\frac{1}{12}$  **(iii)** 25

### Question 11

- (a) (i) Many candidates correctly found  $-8.5$  although  $5$  was sometimes seen.
- (ii) Most candidates used the scale stated in the question, although occasionally that used on the  $x$ -axis was half of that required. There were some non-uniform scales used. However, elegantly drawn smooth curves on the correct axes were strongly in evidence. Generally points were plotted accurately but a not unusual error was to plot  $(-1.5, 8.5)$  despite a correct table entry of  $(-1.5, -8.5)$ .
- (iii) In cases when a tangent was drawn it was usually to an acceptable degree of accuracy. A quite large number of candidates gave the value  $5$  without any evidence of a tangent, probably stating the  $y$ -coordinate at  $x = 1.5$ .
- (iv) A few very able candidates rearranged the cubic equation  $2x^3 - 3x^2 + 4 = 0$  to  $2x^3 - 3x^2 + 5 = 1$  and so established that the solution of the former was the intersection of the curve with the line  $y = 1$ . They almost always arrived at the correct solution. A significant number used the line  $y = -1$  while others attempted to draw a curve with  $y$  values  $1$  less than the curve drawn in answer to part (ii).
- (b) (i) This was very well answered, most seeing that  $p$  was the product of  $x$  and  $y$  and arriving at the response  $1.2$ . There was slightly less success in finding  $q$  but many did correctly compute  $1.2 \div 2.4$ .
- (ii) The majority knew how to find the gradient of a line joining two points and used their value of  $q$  appropriately to arrive at  $-0.8$ .

Answer: (a)(i)  $-8.5$  (iii)  $2.5$  to  $6.5$  (iv)  $-0.85$  to  $-0.95$  (b)(i)  $p = 1.2$ ,  $q = 0.5$  (ii)  $-0.8$

### Question 12

This question was omitted by many candidates, possibly because they had attempted the questions in the order that they were offered rather than making a deliberate choice. Those who did attempt the question generally did well.

- (a) In this part many candidates had difficulty with the mixed units although most wrote the volume of the cylinder as  $\pi r^2 \times 46$ . A few did not give their answer to the nearest centimetre.
- (b) (i) Most used the appropriate area formula for the triangle and reached the value  $18$ . It was not too unusual to see the area evaluated from  $\frac{1}{2} \times 4 \times 11$ .
- (ii) Many, but by no means all, saw the need to multiply their answer to (i) by  $20$ . Some candidates started again, not always using the formula for the volume of a prism.  $\frac{1}{3}$ ,  $\frac{1}{2}$  and  $\pi$  were all seen in attempts at this formula.
- (iii) Many candidates attempted to use the cosine rule, but many had difficulty in doing so, some using sine  $125$ , others making a sign error. Also, the error of calculating  $(4^2 + 11^2 - 2 \times 4 \times 11) \cos 125$  was sometimes seen. A significant number of candidates who rounded their answers to intermediate steps in their calculations produced final answers outside the acceptable range.
- (iv) This part proved to be difficult for candidates. Often, the prism was perceived to have four rather than five surfaces. In some cases, the calculation suggested a multi-faceted shape in contradiction to the given diagram. The two faces which were most commonly left out were those with dimensions  $4 \times 20$  and  $11 \times 20$ .

Answer: (a)  $22$  (b)(i)  $18.0$  (ii)  $360$  to  $360.5$  (iii)  $13.7$  (iv)  $610$