① $V = 10{,}000e^{-pt}$

i) when $t = 0$

$V = 10{,}000(1)$

Ans $= \$10{,}000$ ← must have \$.

ii) when $t = 12$, $V = 4000$

$4000 = 10000e^{-12p}$

$0.4 = e^{-12p}$

$\ln 0.4 = \ln e^{-12p}$

$-0.91629 = -12p$

$p = 0.076357$

$\therefore V = 10{,}000e^{-0.076357t}$

when $t = 18$

$V = 10{,}000e^{(-0.076357)(18)}$

$= 2529.847$

Ans $\approx \$2529.85$

iii) when $V = 1000$

$1000 = 10{,}000e^{-0.076357t}$

$\ln 0.1 = \ln e^{-0.076357t}$

$-2.30258 = -0.076357t$

$t = 30.155$

≈ 30 (nearest month)

Ans : The age of the motorcycle is 30mth.

2) $2x^2 - 4x + 3 = 0$

$$x^2 - 2x + \frac{3}{2} = 0$$

$\alpha + \beta = 2$

$\alpha\beta = \frac{3}{2}$

$(\alpha+\beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$(2)^2 = \alpha^2 + \beta^2 + 2(\frac{3}{2})$

$\alpha^2 + \beta^2 = 1$

Sum of new roots

$= (\alpha^2 + 2) + (\beta^2 + 2)$

$= \alpha^2 + \beta^2 + 4$

$= 1 + 4$

$= 5$

Pdt of new roots

$= (\alpha^2 + 2)(\beta^2 + 2)$

$= \alpha^2\beta^2 + 2\alpha^2 + 2\beta^2 + 4$

$= (\frac{3}{2})^2 + 2(\alpha^2 + \beta^2) + 4$

$= \frac{9}{4} + 2(1) + 4$

$= \frac{33}{4}$.

$\therefore x^2 - 5x + \frac{33}{4} = 0$

Ans $4x^2 - 20x + 33 = 0$

3i) L.H.S

$= \tan A + \cot A$

$= \dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}$

$= \dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}$

$= \dfrac{1}{\frac{\sin 2A}{2}}$ *

$\sin 2A = 2\sin A\cos A$

$\sin A\cos A = \dfrac{\sin 2A}{2}$ *

$= \dfrac{2}{\sin 2A}$

$= 2\operatorname{cosec} 2A$

$=$ R.H.S (proven).

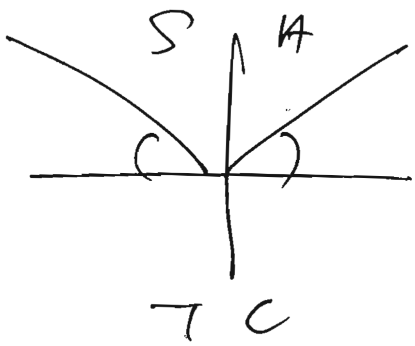
$\tan A + \cot A = 3$

$2\operatorname{cosec} 2A = 3$

$\operatorname{cosec} 2A = \dfrac{3}{2}$

$\sin 2A = \dfrac{2}{3}$

Reference $\angle = 41.81^\circ$



$0^\circ < A < 360^\circ$

$0^\circ < 2A < 720^\circ$

$2A = 41.81^\circ, 138.19^\circ, 401.81^\circ, 498.19^\circ$

Ans $A = 20.9^\circ, 69.1^\circ, 200.9^\circ$ or 249.1°

4i) $2+\log_3(3x-7) = \log_3(2x-3)$

$\log_3(2x-3) - \log_3(3x-7) = 2$

$\log_3\left(\frac{2x-3}{3x-7}\right) = 2$

$\frac{2x-3}{3x-7} = 3^2$

$2x-3 = 9(3x-7)$

$2x-3 = 27x-63$

$60 = 25x$

$x = 2.4$

Ans

ii) $3\log_5 y - \log_y 5 = 2$

$3\log_5 y - \frac{\log_5 5}{\log_5 y} = 2$

$3\log_5 y - \frac{1}{\log_5 y} = 2$

Let $\log_5 y = a$

$3a - \frac{1}{a} = 2$

$3a^2 - 2a - 1 = 0$

$(3a+1)(a-1) = 0$

$a = -\frac{1}{3}$ or $a = 1$

$\log_5 y = -\frac{1}{3}$ or $\log_5 y = 1$

$y = 5^{-\frac{1}{3}}$ or $y = 5^1$

Ans $y = 0.585$ or 5 #

5) i) $f(x) = 2(x^2-3x+1)(x+1)(x-2)$

$= 2(x^2-3x+1)(x^2-x-2)$

$= 2(x^4 - x^3 - 2x^2 - 3x^3 + 3x^2 + 6x + x^2 - x - 2)$

$= 2(x^4 - 4x^3 + 2x^2 + 5x - 2)$

Test ~~Ans~~ $f(x) = 2x^4 - 8x^3 + 4x^2 + 10x - 4$

whether

$f(-1)=0$

and

$f(2)=0$.

Can't be bothered

ii) $f(x) = 0$

~~$(x-1)(x+1)$~~

$2(x+1)(x-2)(x^2-3x+1) = 0$

$x = -1$, $x = 2$ or $x^2 - 3x + 1 = 0$

$x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$

$= \dfrac{3 \pm \sqrt{5}}{2}$

$= \dfrac{3+\sqrt{5}}{2}$ or $\dfrac{3-\sqrt{5}}{2}$

Ans 4 real roots.

5iii) $f(\tfrac{1}{2}) = 2(\tfrac{1}{2})^4 - 8(\tfrac{1}{2})^3 + 4(\tfrac{1}{2})^2 + 10(\tfrac{1}{2}) - 4$

$= \frac{1}{8} - 1 + 1 + 5 - 4$

Ans $= \frac{9}{8}$ # $\Rightarrow 1\frac{1}{8}$

6 i) $AE = CE$ (tangents to circles)

$EO = EO$ (common line)

$AO = CO$ (Radii of circle

$\therefore \triangle AEO$ and $\triangle CEO$ are congruent. (SSS)

State the reason.

ii) $DB^2 = AD^2 + AB^2$

??? waste of time

$E\hat{A}O = D\hat{A}B$ (common $\angle$)

$A\hat{O}E = \frac{1}{2} \cancel{A\hat{O}E}\ A\hat{O}C$ (OAEC is a kite as A and C are tangents) ①

$D\hat{B}A = \frac{1}{2} A\hat{O}C$ ($\angle$ at centre $= 2\angle$ at ~~circ~~ ce) ②

$\therefore A\hat{O}E = D\hat{B}A$ (from ① & ②)

$\therefore A\hat{E}O = A\hat{D}B$ ($\angle$ sum of $\triangle$s)

$\therefore \triangle AEO$ and ADB are similar

Since $\frac{OA}{AB} = \frac{1}{2}$ (similar

$\therefore \frac{AE}{AD} = \frac{1}{2}$

$AE = \frac{1}{2} AD$

$\therefore$ E is the midpoint of AD. (proven)

7) $f(x) = 4\cos 2x - 2$

Ans i) 4

Ans ii) 180°

iii) when $\cos 2x = -1$ $\qquad 0^\circ \leq 2x \leq 360^\circ$

$2x = 180^\circ$

$x = 90^\circ$

$y = 4(-1) - 2$

$= -6$

must put °

Ans co-ordinate of the min pt is $(90^\circ, -6)$

iv) when $y = 0$

$4\cos 2x - 2 = 0$

$\cos 2x = \frac{1}{2}$

$\Rightarrow$ Basic $\angle = 60^\circ$

$2x = 60^\circ, 300^\circ$

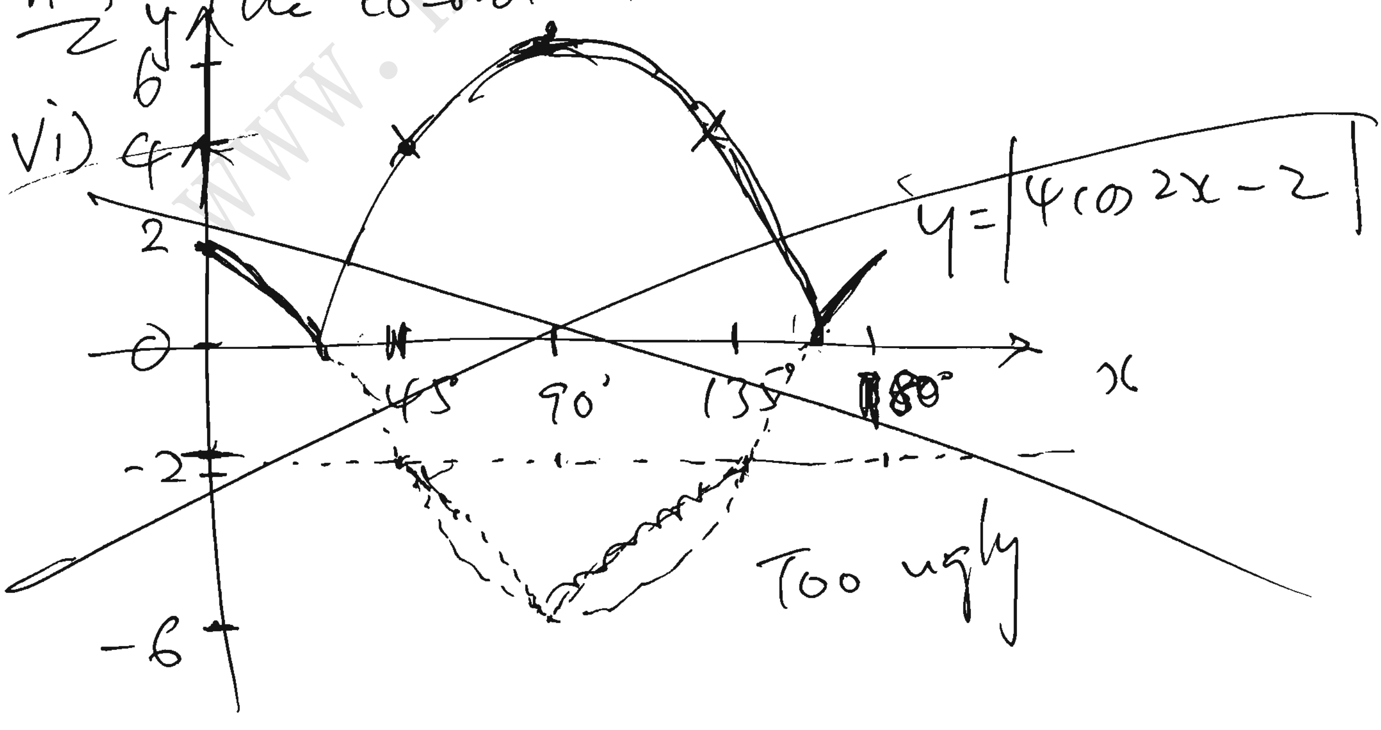
$x = 30^\circ$ or 150°

S A

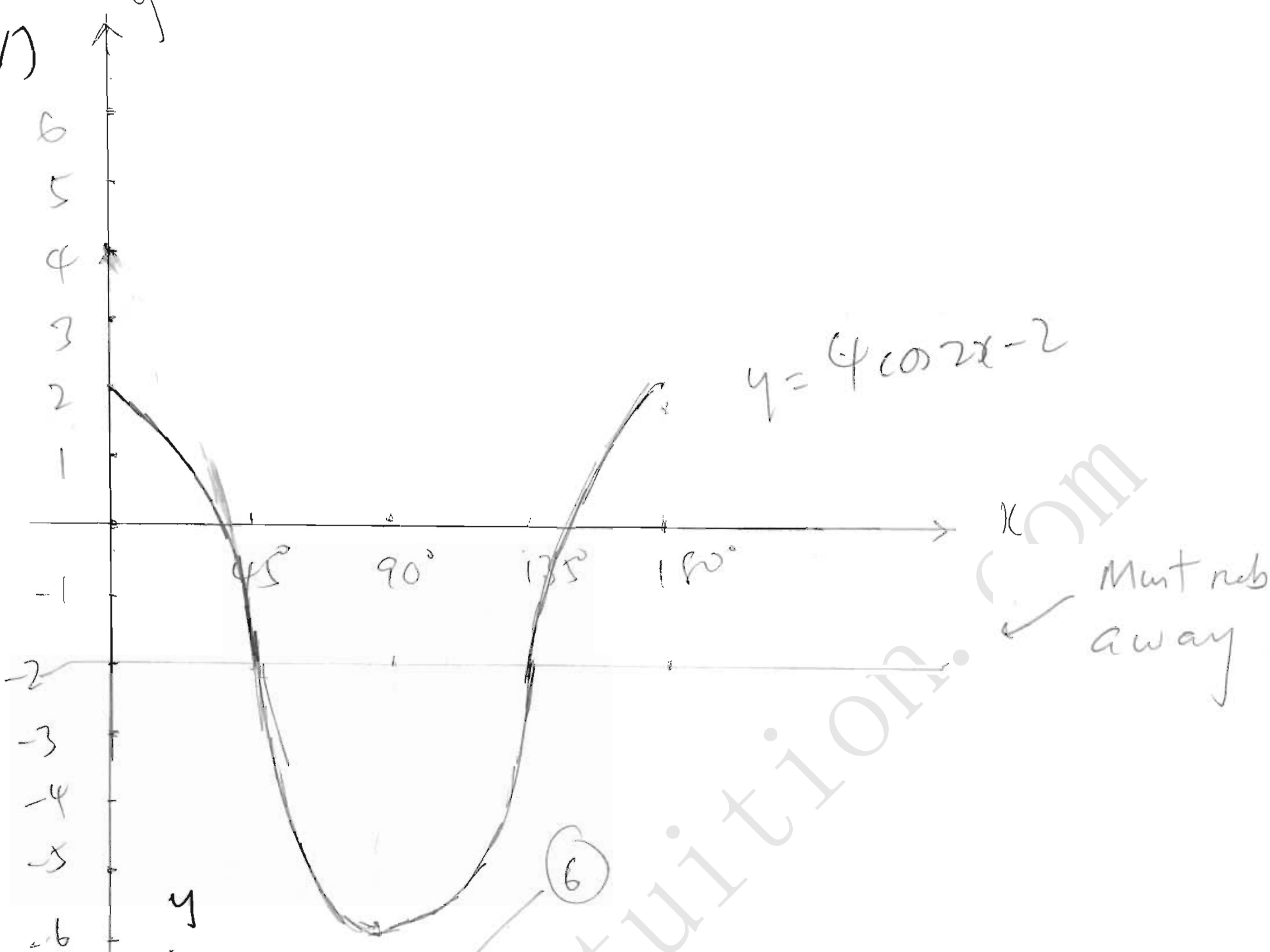
T C

Ans The co-ordinates are $(30^\circ, 0)$ and $(150^\circ, 0)$

vi)

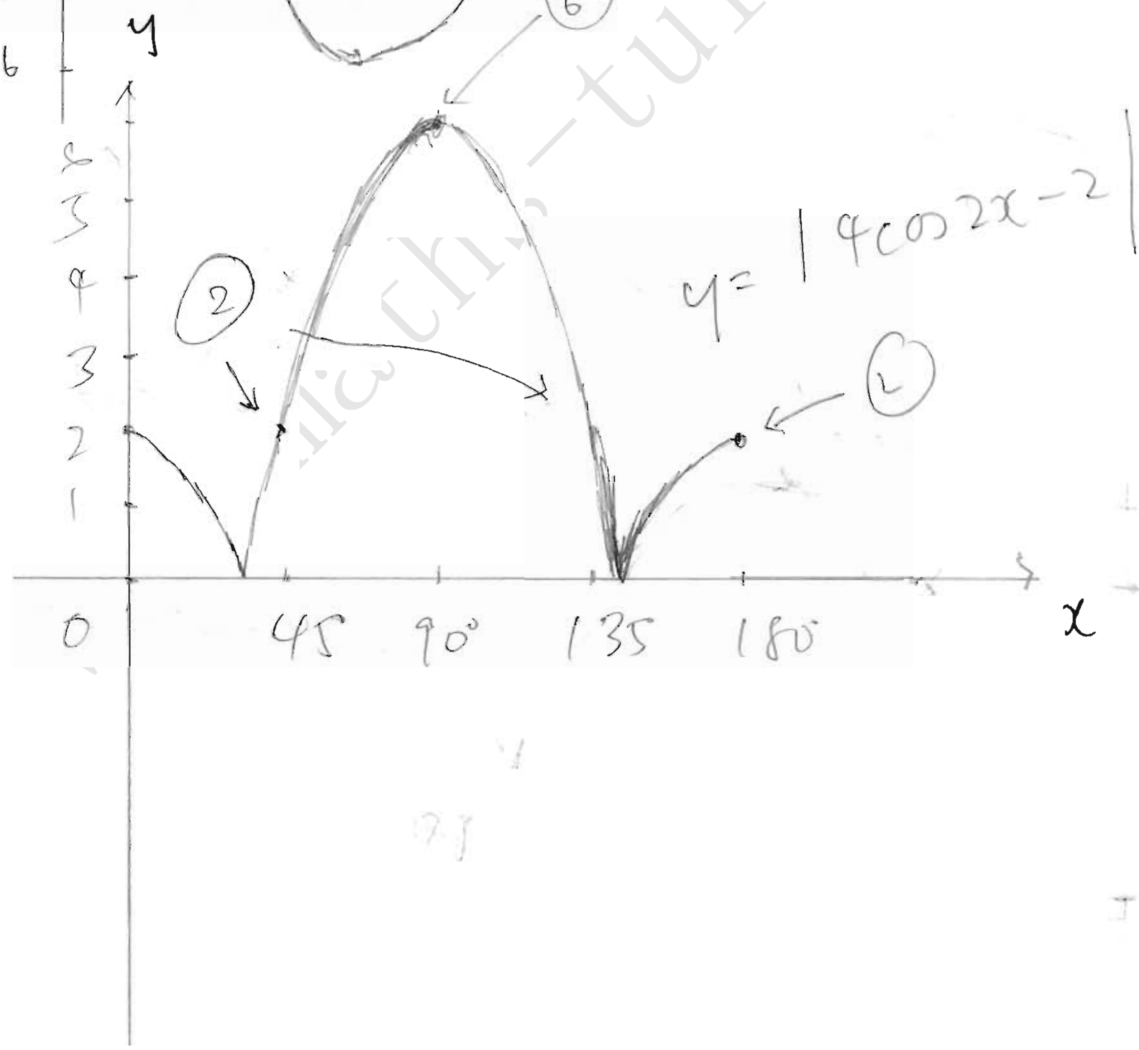


7(v)



y
6
5
4
3
2
1
-1
-2
-3
-4
-5
-6
45°
90°
135°
180°
x
$y = 4\cos 2x - 2$
Must rub away

vi)



y
6
5
4
3
2
1
0
45
90°
135
180
x
$y = |4\cos 2x - 2|$
6
2
2

8i) $y = x^3 - ax + b$

$\frac{dy}{dx} = 3x^2 - a$

At $(2,0)$, $\frac{dy}{dx} = 0$

$0 = 3(2)^2 - a$

$a = 12$

$y = x^3 - 12x + b$

At $(2,0)$

$0 = 8 - 24 + b$

$b = 16$

Ans : $a = 12$, $b = 16$

$y = x^3 - 12x + 16$

$\frac{dy}{dx} = 3x^2 - 12$

When $\frac{dy}{dx} = 0$

$3x^2 - 12 = 0$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x = -2$ or $x = 2$

$\therefore x = -2$

$y = (-2)^3 - 12(-2) + 16$

$= -8 + 24 + 16$

$y = 32$ (Proof of max not asked for)

$\therefore$ co-ordinates of max pt are $(-2, 32)$.

8(iii) Area of shaded region

$$= \int_0^2 (x^3 - 12x + 16)\,dx$$

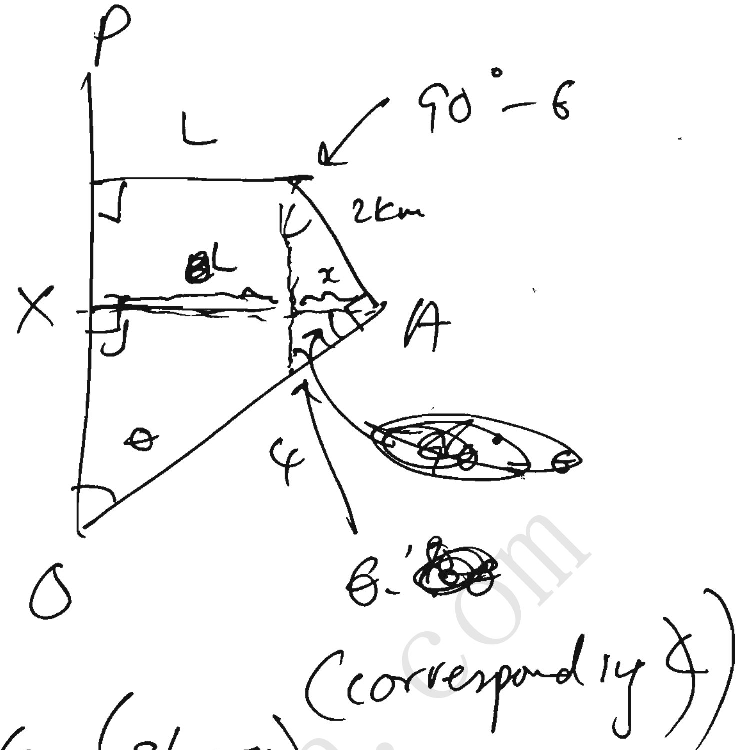
$$= \left[\frac{x^4}{4} - 6x^2 + 16x\right]_0^2$$

$$= \left[\frac{2^4}{4} - 6(2)^2 + 16(2)\right] - (0 - 0 + 0)$$

$$= 4 - 24 + 32$$

Ans $= 12$ unit^2

9 i)



$$\frac{L+x}{4} = \sin\theta$$

$$L = 4\sin\theta - x$$

$$\sin(90^\circ-\theta) = \frac{x}{2}$$

$$2\cos\theta = x$$

$$\therefore L = 4\sin\theta - 2\cos\theta \text{ (shown)}$$

ii) $4\sin\theta - 2\cos\theta = R\sin(\theta-\alpha)$

$$\tan\alpha = \frac{2}{4}$$

$$\alpha = 26.565^\circ$$

$$R = \sqrt{4^2+2^2}$$

$$= \sqrt{20}$$

$$\therefore L = \sqrt{20}\sin(\theta - 26.\cancel{5}^\circ)$$

cannot use 26.6°

iii) $L = 3$

$$\sqrt{20}\sin(\theta - 26.565^\circ) = 3$$

$$\sin(\theta - 26.565^\circ) = \frac{3}{\sqrt{20}}$$

$$\theta - 26.565^\circ = 42.130^\circ$$

$$\theta = 68.695^\circ$$

$$\theta \approx 68.7^\circ$$

Ans

(10) $\frac{dy}{dx} = \frac{6}{(2x-1)^2}$

At $P(2,9)$

$\frac{dy}{dx} = \frac{6}{3^2}$

$= \frac{2}{3}$

Gradient of normal $= -\frac{3}{2}$

Eqn of normal $\Rightarrow y = mx + c$

$y = -\frac{3}{2}x + c$

At $(2,9)$

$9 = -\frac{3}{2}(2) + c$

$c = 12$

$\therefore y = -\frac{3}{2}x + 12$

when $x = 0$, $y = 12$ $Q(0,12)$

when $y = 0$ $0 = -\frac{3}{2}x + 12$

$\frac{3}{2}x = 12$

$x = 8$ $R(8,0)$

Midpt of QR $= \left(\frac{0+8}{2}, \frac{12+0}{2}\right)$

$= (4,6)$

Ans

(ii) $\frac{dy}{dx} = 6(2x-1)^{-2}$

$y = \frac{6(2x-1)^{-1}}{(-1)(2)} + c$

$= \frac{-3}{2x-1} + c$

$= \frac{3}{1-2x} + c$

CANNOT LEAVE as $\frac{-3}{2x-1}$

At $(2,9)$

$9 = \frac{3}{-3} + c$

$c = 10$

$\therefore y = \frac{3}{1-2x} + 10$

10(iii) $\frac{dx}{dt} = 0.03$

$\frac{dy}{dx} = \frac{6}{(2x-1)^2}$

At $(2, 9)$

$\frac{dy}{dx} = \frac{6}{3^2}$

$= \frac{2}{3}$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$= (0.03) \times \frac{2}{3}$

$\frac{dy}{dt} = 0.02$

~~Rate~~:

The y-co-ordinate is increasing at 0.02 units/s

11

$OP = \sqrt{(8-0)^2 + (-6-0)^2}$

$= \sqrt{64+36}$

$= 10$

Equation of $C_1 \Rightarrow x^2 + y^2 = 10^2$

$x^2 + y^2 = 100$

1

ii) $\overrightarrow{OP} = 2\overrightarrow{OQ}$

$\begin{pmatrix} 8 \\ -6 \end{pmatrix} = 2\overrightarrow{OQ}$

$\overrightarrow{OQ} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$Q = (4, -3)$

$|\overrightarrow{OQ}| = \sqrt{4^2 + (-3)^2}$

$= 5$

Ans: $(x-4)^2 + (y+3)^2 = 5^2$

$x^2 - 8x + 16 + y^2 + 6y + 9 = 25$

Ans $x^2 + y^2 - 8x + 6y = 0$. #

iii) Gradient of OP $= \dfrac{-6-0}{8-0}$

$= -\dfrac{3}{4}$

Gradient of AB $= \dfrac{4}{3}$

Eq^n of AB $\Rightarrow y = \dfrac{4}{3}x + c$

At $(4, -3)$

$-3 = \dfrac{16}{3} + c$

$c = -8\dfrac{1}{3}$

$y = \dfrac{4}{3}x - \dfrac{25}{3} \Rightarrow$ ~~3y = 4x + 7.~~

2

Sub (2) into (1)

$$x^2 + \left(\frac{4}{3}x - \frac{25}{3}\right)^2 = 100 \qquad \times 9$$

$$9x^2 + (4x^2 - 100x + 625) = 900$$

$$\cancel{13x^2 - 56x - 275 = 0}$$

$$13x^2 - 100x - 275 = 0$$

$$x^2 + \left(\frac{4x - 25}{3}\right)^2 = 100$$

$$x^2 + \frac{16x^2 - 200x + 625}{9} = 100$$

$$9x^2 + 16x^2 - 200x + 625 = 900$$

$$25x^2 - 200x - 275 = 0$$

$$x^2 - 8x - 11 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{8 \pm \sqrt{64 - 4(1)(-11)}}{2(1)}$$

$$= \frac{8 + \sqrt{108}}{2} \quad \text{or} \quad \frac{8 - \sqrt{108}}{2}$$

$$= \frac{8 + 6\sqrt{3}}{2} \quad \text{or} \quad \frac{8 - 6\sqrt{3}}{2}$$

$$\begin{array}{l|l} 2 & 108 \\ 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

Ans $= 4 + 3\sqrt{3}$ or $4 - 3\sqrt{3}$