education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

FEBRUARY/MARCH 2010

MARKS: 150

TIME: 3 hours

**This question paper consists of 11 pages, 3 diagram sheets and 1 information sheet.**

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.   This question paper consists of 12 questions. Answer ALL the questions.

2.   Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers. Answers only will not necessarily be awarded full marks.

3.   An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4.   If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5.   Diagrams are NOT necessarily drawn to scale.

6.   THREE diagram sheets for answering QUESTION 2.1.1, QUESTION 3.1, QUESTION 3.2, QUESTION 3.4 and QUESTION 11.2 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.

7.   Number the answers correctly according to the numbering system used in this question paper.

8.   It is in your own interest to write legibly and to present the work neatly.

QUESTION 1

The graph below shows the monthly maximum temperatures in a certain city.
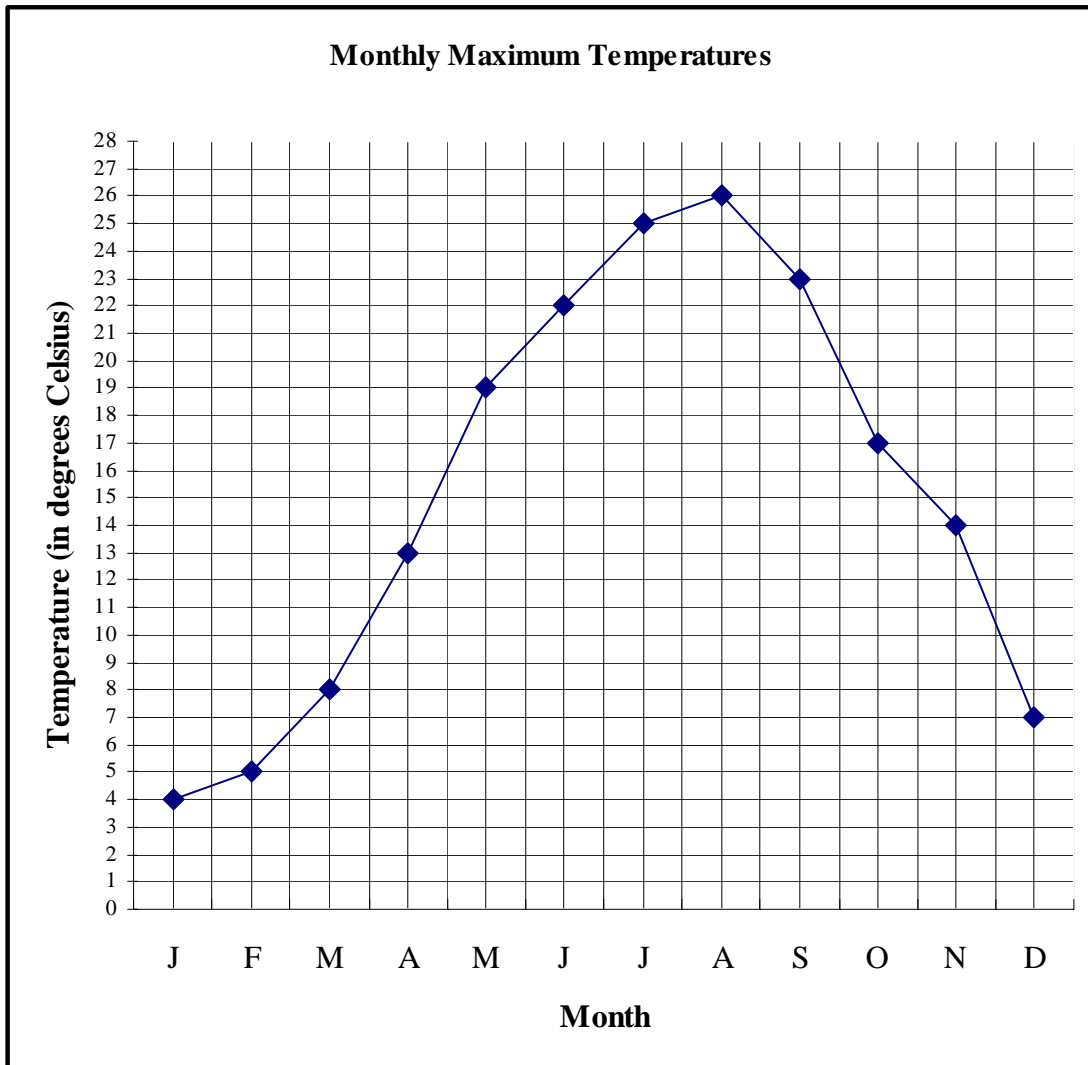
1.1   What is the range of the monthly maximum temperatures?   (2)

1.2   Calculate the mean monthly maximum temperature.   (3)

1.3   Calculate the standard deviation of the monthly maximum temperature.   (2)

1.4   It is predicted that one hundred years from now, global warming is likely to increase
        the city's monthly maximum temperature by $5°\,C$ in December, January and
        February. It will also result in an increase of $1°\,C$ in the other months of the year.

        1.4.1   By how much does the mean increase?   (2)

        1.4.2   Describe the effect that the predicted increases in temperature will have
                   on the standard deviation. Justify your answer.   (2)

                                                                                                            [11]

QUESTION 2

2.1   A study was conducted amongst soccer players in South Africa. The number of goals scored in a season was measured against the number of hours each player trained each week. The table below displays the results.

Hours trained (x)	17	22	20	14	19	21	15	18	16
Number of goals (y)	11	14	13	10	8	13	11	12	11

2.1.1   Draw a scatter plot for the data on DIAGRAM SHEET 1.   (3)

2.1.2   One of the data points is an outlier. Indicate this point on the diagram sheet with the letter A.   (1)

2.1.3   Use your graph to predict the number of goals scored by a player who trains for 10 hours a week.   (2)

2.2   Learners at Phambili High School travel from three different neighbourhoods, neighbourhood A, B and C. The table below shows the number of learners from each neighbourhood, and their mean travelling times from home to school.

Neighbourhood	**A**	**B**	**C**
Number of learners	135	225	200
Mean travelling time (in min.)	24	32	x

The mean travelling time for learners living in neighbourhood C is the same as the mean travelling time for all 560 learners.

Calculate the mean travelling time for neighbourhood C.   (4)

[10]

QUESTION 3

The time taken (to the nearest minute) for a certain task to be completed was recorded on 48 occasions and the following data was obtained.

Time (in minutes)	Frequency
$11 \leq t < 15$	6
$15 \leq t < 19$	9
$19 \leq t < 23$	13
$23 \leq t < 27$	12
$27 \leq t \leq 30$	8

3.1 Complete the cumulative frequency table on DIAGRAM SHEET 2. (1)

3.2 Draw an ogive (cumulative frequency curve) for the given data on DIAGRAM SHEET 2. (4)

3.3 Determine, from the ogive, the median, lower quartile and upper quartile for the data. (3)

3.4 Draw a box and whisker diagram of the data on DIAGRAM SHEET 3. (2)

3.5 Comment on the spread of the time taken to complete the task. (1)

[11]

QUESTION 4

The diagram below shows the points P(0 ; 2) and Q(4 ; 0). Point A is the midpoint of PQ. The line AB is perpendicular to PQ and intersects the *x*-axis at G and the *y*-axis at B.
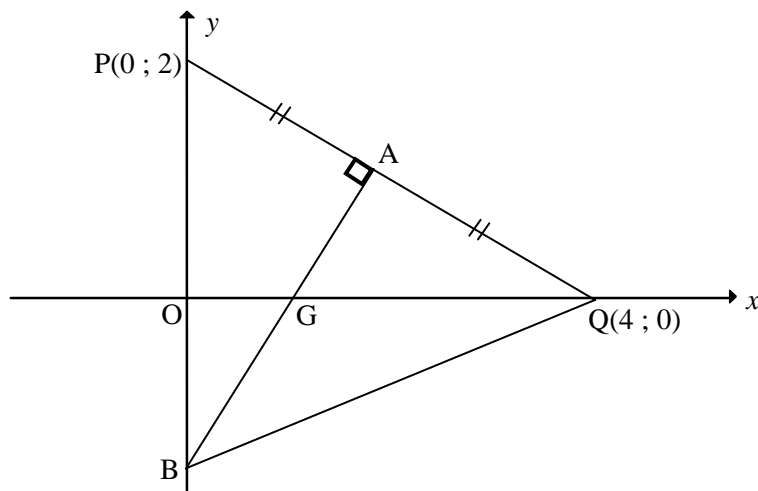

4.1   Show that the gradient of PQ is $-\dfrac{1}{2}$.   (1)

4.2   Determine the coordinates of A.   (2)

4.3   Determine the equation of the line AB.   (5)

4.4   Calculate the length of BQ.   (3)

4.5   Show that ΔBPQ is isosceles.   (2)

4.6   If PBQR is a rhombus, determine the coordinates of R.   (3)
[16]

QUESTION 5

The straight line AB has the equation $5y - 3x - 5 = 0$. Another straight line CD is drawn to intersect AB at P(5 ; 4) such that the acute angle between AB and CD is 45°.
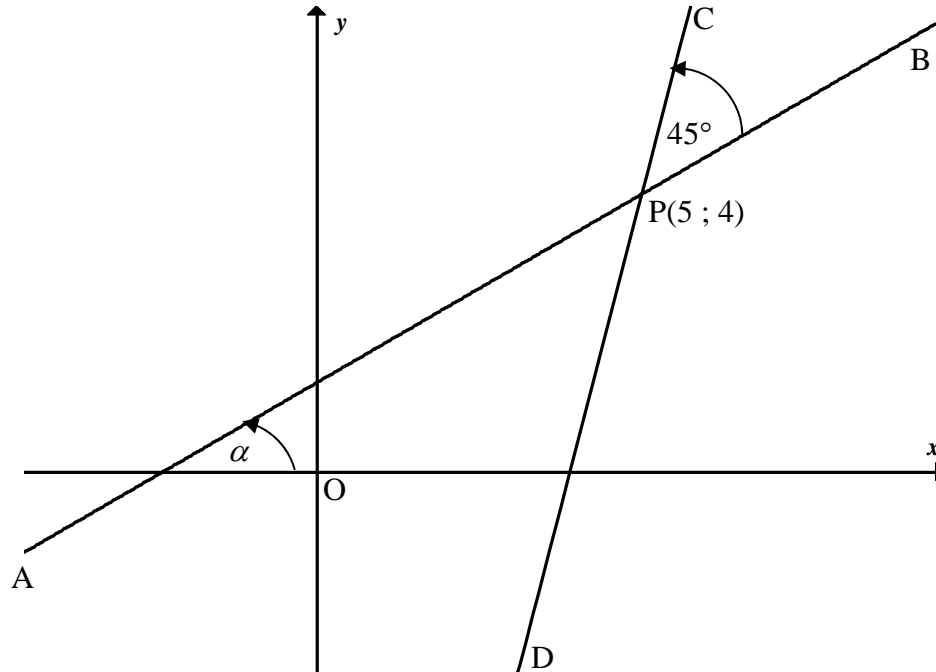
5.1   Determine the gradient of the line CD.   (5)

5.2   Hence, or otherwise, determine the equation of the line CD.   (2)

[7]

QUESTION 6

6.1   Determine the centre and radius of the circle with the equation $x^2 + y^2 + 8x + 4y - 38 = 0$.   (4)

6.2   A second circle has the equation $(x - 4)^2 + (y - 6)^2 = 26$. Calculate the distance between the centres of the two circles.   (2)

6.3   Hence, show that the circles described in QUESTION 6.1 and QUESTION 6.2 intersect each other.   (3)

6.4   Show that the two circles intersect along the line $y = -x + 4$.   (4)

[13]

QUESTION 7

7.1   The point P(– 2 ; 5) lies in a Cartesian plane. Determine the coordinates of P$'$, the
        image of P, if:

        7.1.1   P is reflected across the line $y = x$ (1)

        7.1.2   P has been rotated about the origin through 90° in a clockwise direction   (2)

7.2   KUHLE has undergone two transformations to obtain K$''$U$''$H$''$L$'$E$''$.
        K$''$(2 ; 7), U$''$(3 ; 9), H$''$(4 ; 8), L$''$(5 ; 9) and E$''$(6 ; 7) are the coordinates of the
        vertices of K$''$U$''$H$''$L$''$E$''$.
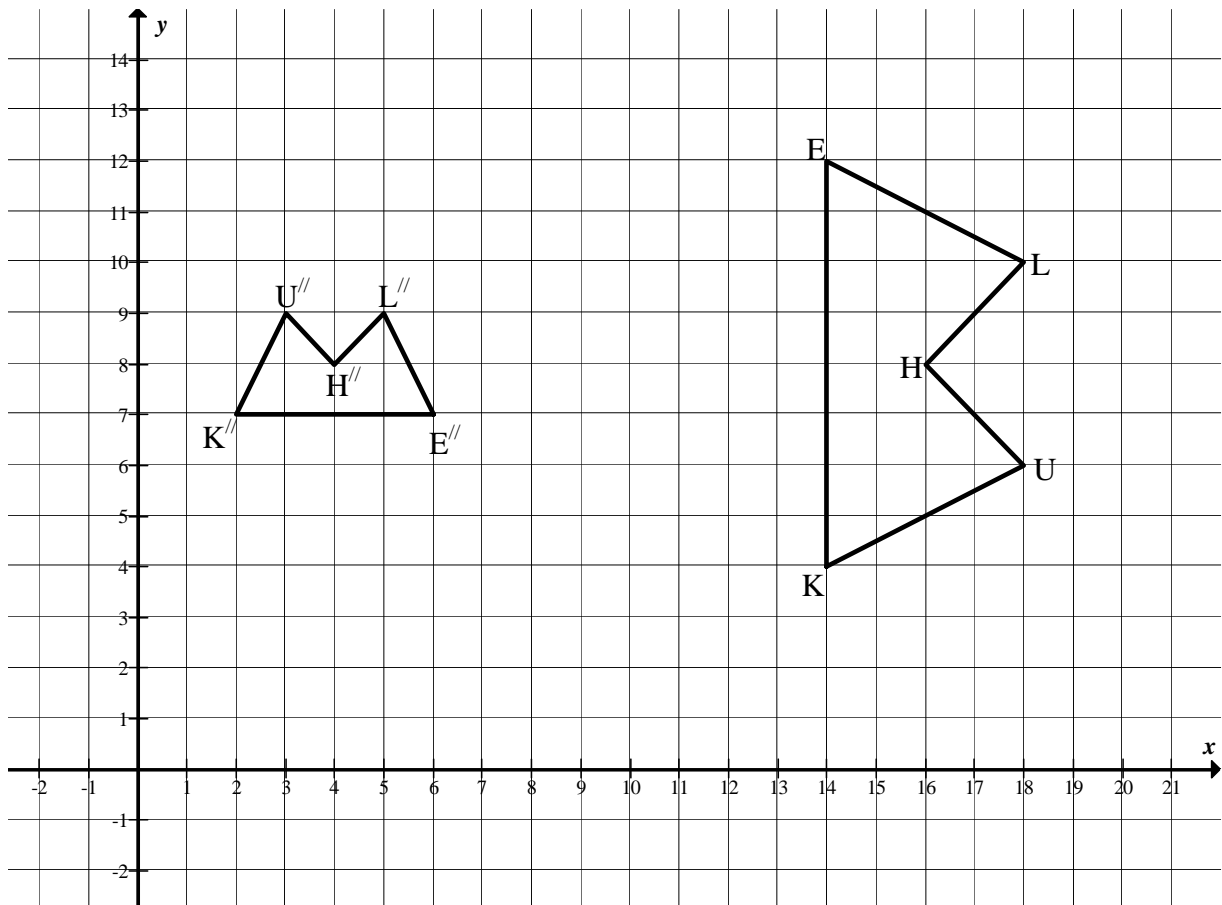
        7.2.1   Describe, in words, two transformations of KUHLE (in the order which
                     they occurred), to obtain K$''$U$''$H$''$L$'$E$''$.   (4)
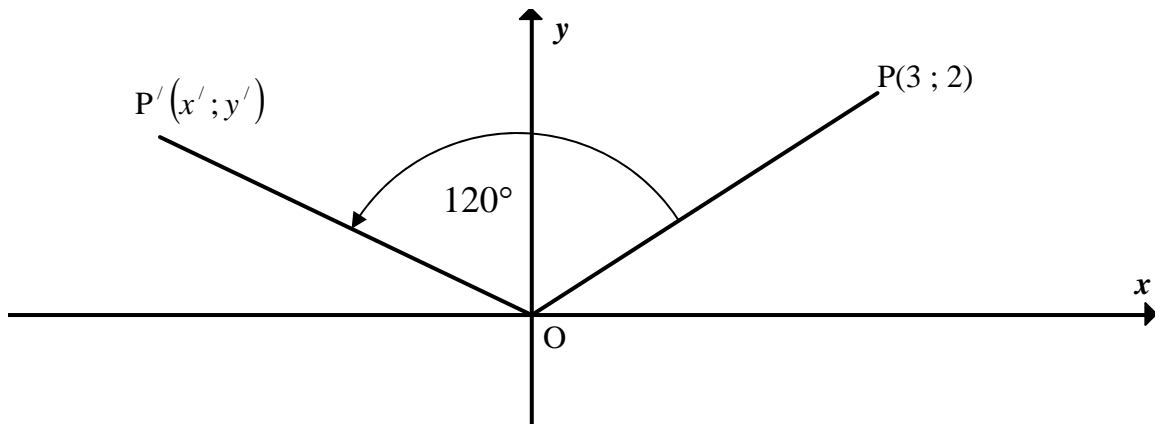
        7.2.2   Write down TWO possible sets of coordinates for H$'$, the image of H after
                     the first transformation.   (2)

        7.2.3   Determine: area of KUHLE : area of K$''$U$''$H$''$L$''$E$''$.   (2)
                                                                                                          [11]

QUESTION 8

(You may NOT use a calculator to answer this question.)

8.1   The point P(3 ; 2) is rotated about the origin through an angle of 120° in an
        anticlockwise direction. Determine $x^{/}$ and $y^{/}$, the coordinates of $P^{/}$.   (6)

8.2   The same rotation sends a point Q into (–2 ; 0). Determine the coordinates of Q.   (4)
                                                                                                  [10]

QUESTION 9

9.1   If $4\tan\theta = 3$ and $180° < \theta < 360°$, determine with the aid of a diagram:

        9.1.1 $\sin\theta + \cos\theta$ (4)

        9.1.2 $\tan 2\theta$ (5)

9.2   9.2.1   Show that: $\dfrac{\cos(360° - x)\tan^2 x}{\sin(x - 180°)\cos(90° + x)} = \dfrac{1}{\cos x}$ (5)

        9.2.2   Hence, calculate without the use of a calculator, the value of:

                  $\dfrac{\cos 330° \tan^2 30°}{\sin(-150°)\cos 120°}$ (Leave your answer in surd form.)   (2)
                                                                                                  [16]

QUESTION 10

10.1   If sin 36° cos 12° = p and cos 36° sin 12° = q, determine in terms of p and q the value of:

      10.1.1   sin 48°   (3)

      10.1.2   sin 24°   (3)

      10.1.3   cos 24°   (3)

10.2   Show that $\sin^2 20° + \sin^2 40° + \sin^2 80° = \dfrac{3}{2}$

      (**HINT**: 40° = 60° − 20° and 80° = 60°+ 20°)   (7)

10.3   10.3.1   Prove: $\dfrac{\sin^4 x + \sin^2 x \cos^2 x}{1 + \cos x} = 1 - \cos x$ (4)

      10.3.2   For which values of x is $\dfrac{\sin^4 x + \sin^2 x \cos^2 x}{1 + \cos x} = 1 - \cos x$ NOT true?   (2)

                                                                [22]

QUESTION 11

Given: $f(x) = 1 + \sin x$ and $g(x) = \cos 2x$

11.1   Calculate the points of intersection of the graphs f and g for $x \in [180° \,; 360°]$.   (7)

11.2   Draw sketch graphs of f and g for $x \in [180° \,; 360°]$ on the same system of axes provided on DIAGRAM SHEET 3.   (4)

11.3   For which values of x will $f(x) \le g(x)$ for $x \in [180° \,; 360°]$?   (3)

                                                                [14]

QUESTION 12

In the diagram below A, B and C are three points in the same horizontal plane. D is vertically above B and E is vertically above C. The angle of elevation of E from D is $\theta°$. F is a point on EC such that DF ∥ BC.

$\hat{BAC} = \alpha$, $\hat{ACB} = \beta$ and AC = b metres.
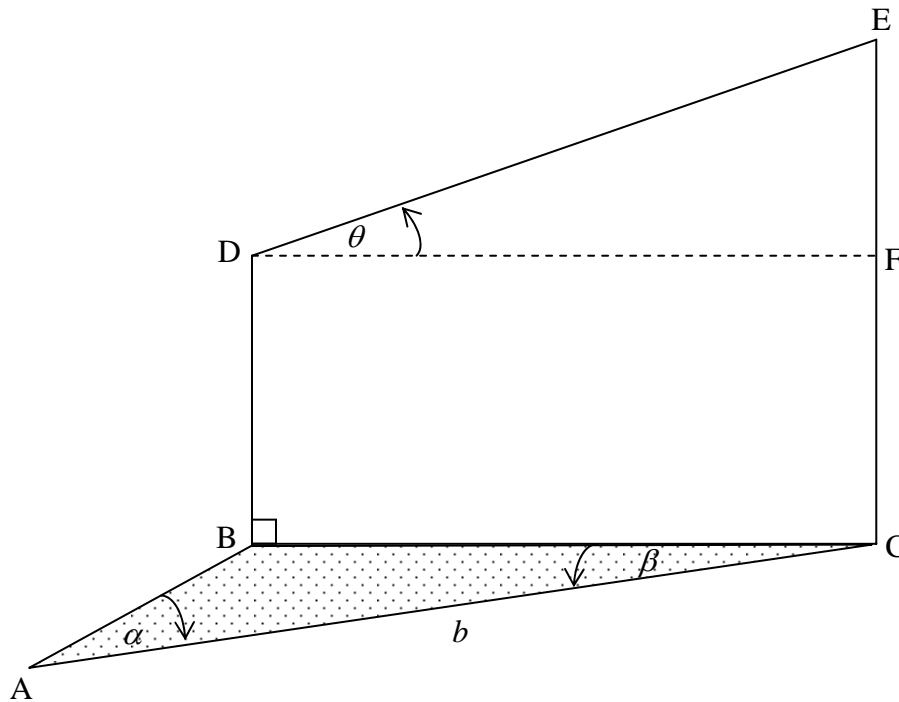
12.1 Prove that $DE = \dfrac{b \sin\alpha}{\sin(\alpha + \beta)\cos\theta}$ (6)

12.2 Calculate DE if $b = 2\,000$ metres, $\alpha = 43°$, $\beta = 36°$ and $\theta = 27°$. (3)

[9]

TOTAL: 150

CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 2.1.1
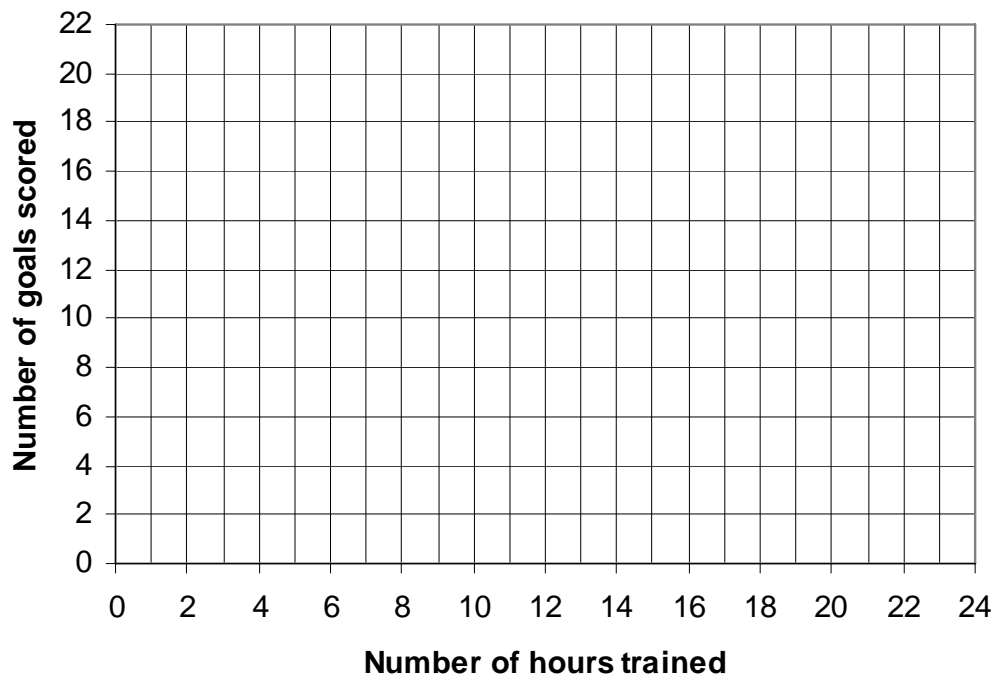
CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 2

QUESTION 3.1

Time (in minutes)	Frequency	Cumulative frequency
$11 \leq t < 15$	60	
$15 \leq t < 19$	9	
$19 \leq t < 23$	13	
$23 \leq t < 27$	12	
$27 \leq t \leq 30$	8	

QUESTION 3.2
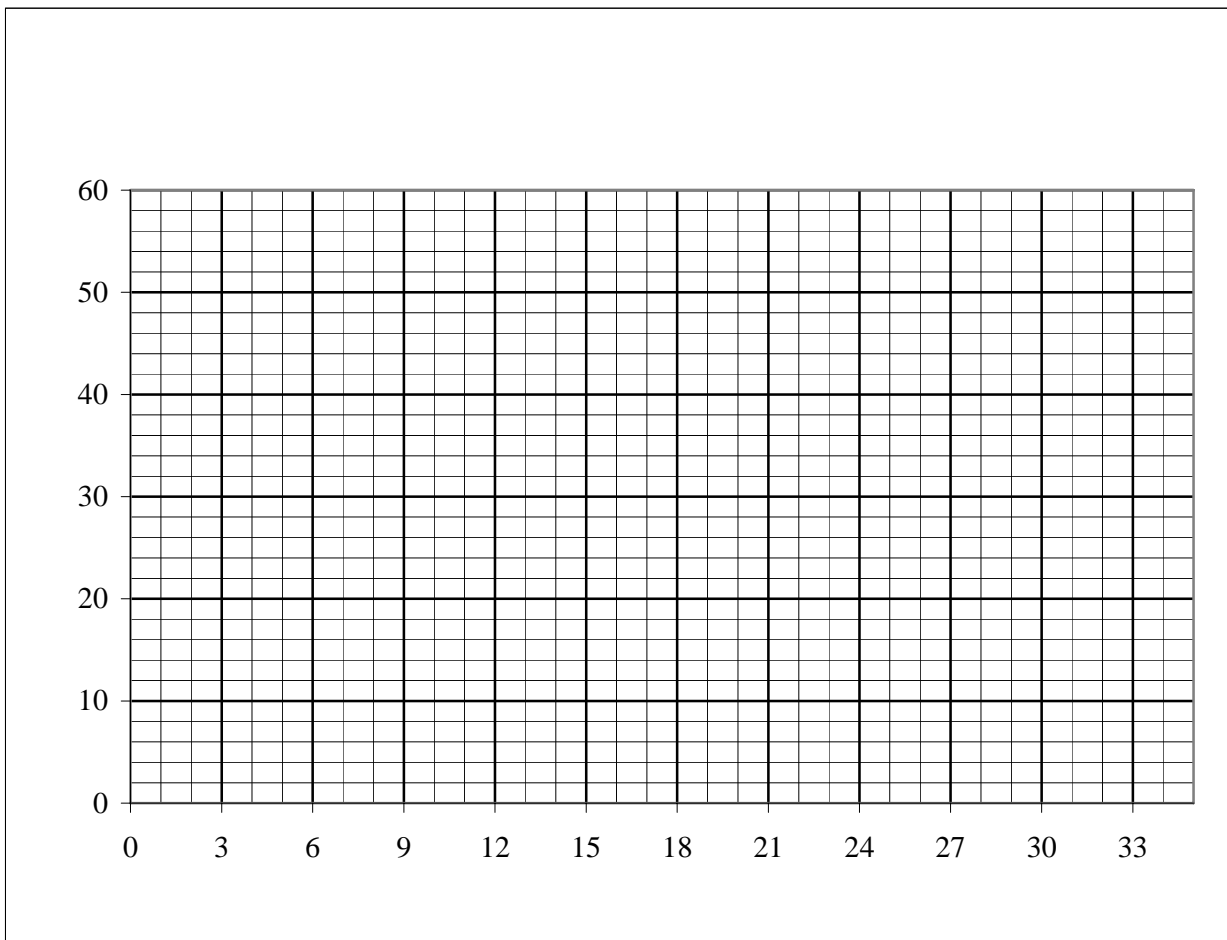
CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 3

QUESTION 3.4

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0    10    20    30    40    50    60    70    80    90    100
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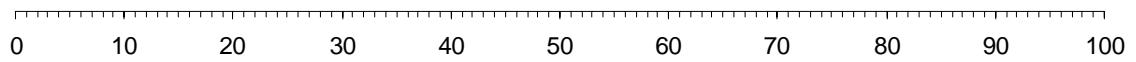
QUESTION 11.2
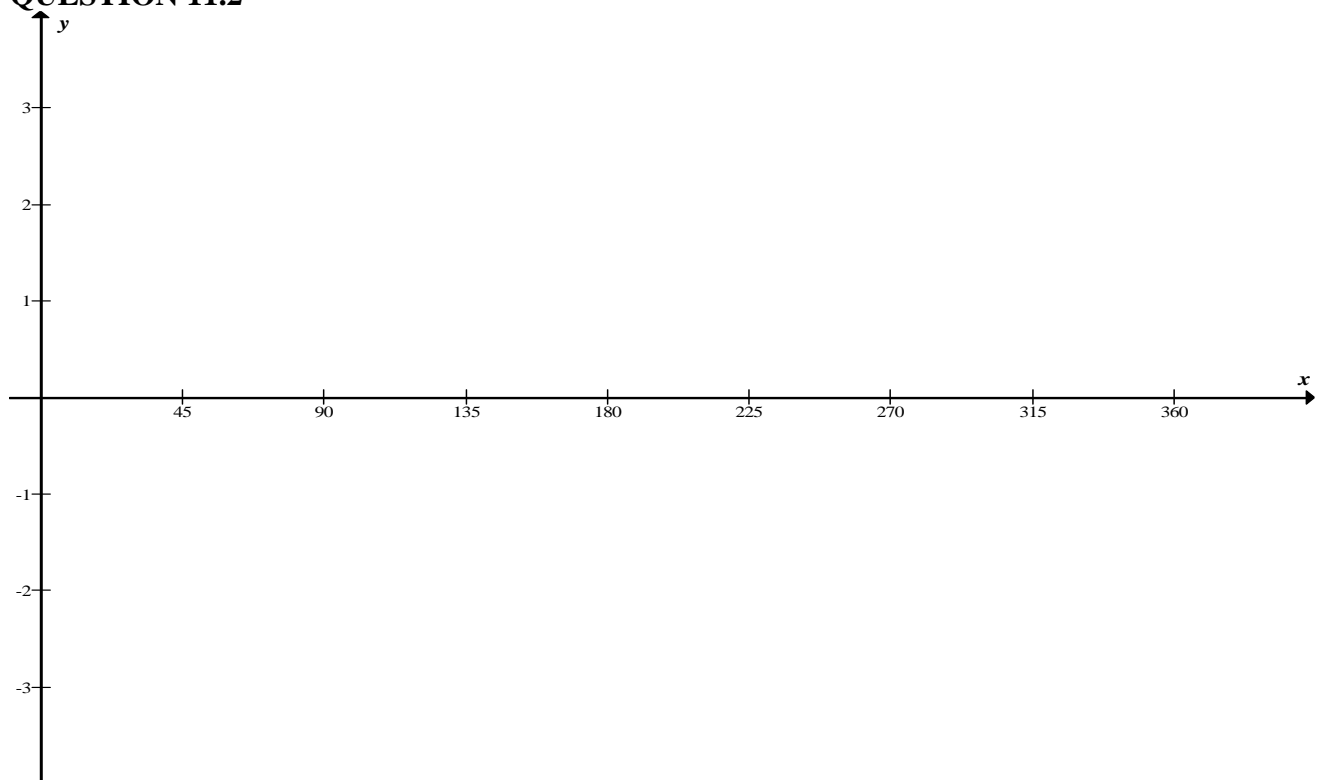
INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \quad ; \qquad r \neq 1 \qquad S_\infty = \frac{a}{1 - r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$(x ; y) \to (x\cos\theta + y\sin\theta ; y\cos\theta - x\sin\theta) \qquad (x ; y) \to (x\cos\theta - y\sin\theta ; y\cos\theta + x\sin\theta)$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$