



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NASIONALE SENIOR SERTIFIKAAT

GRAAD 12

WISKUNDE V2

NOVEMBER 2010

MEMORANDUM

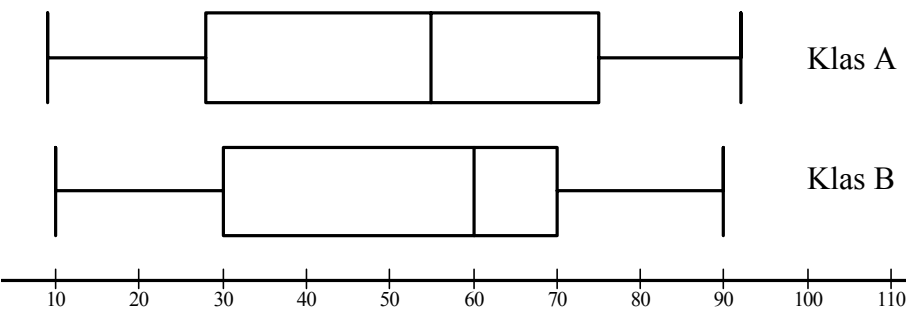
PUNTE: 150

Hierdie memorandum bestaan uit 34 bladsye.

NOTA:

- As 'n kandidaat'n vraag TWEE keer beantwoord, merk net die EERSTE poging.
- As 'n kandidaat 'n antwoord deurhaal en nie oordoen nie, merk die deurgehaalde antwoord.
- Konstante Akkuraatheid moet deurgaans in die memorandum toegepas word.

VRAAG 1

1.1	<p>Die Datastel</p> <p style="text-align: center;"> K_1 \downarrow 9 14 14 19 21 23 33 35 37 37 42 45 55 56 57 59 68 75 K_3 \downarrow 75 75 77 78 80 81 92 </p> <p> Min = 9 Maks = 92 K_3 = Boonste Kwartiel = 75 K_1 = Onderste Kwartiel = 28 (6^{de} getal is 23. 7^{de} getal is 33. Die getal in posisie 6,25 is $23 + \frac{1}{4}(33 - 23) = 25,5$ (Aanvaar $K_1 = 25,5$) K_2 = Mediaan = 55 Vyf getal opsomming is (9 ; 28 ; 55 ; 75 ; 92) OF (9 ; 25,5 ; 55 ; 75 ; 92) </p>	<p>✓ min & max</p> <p>✓ boonste kwartiel</p> <p>✓ onderste kwartiel</p> <p>✓ mediaan</p> <p>(4)</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Nota: Penaliseer met 1 as geen byskrifte of as getalle nie in stygende orde is nie</p> </div>
1.2	 <p style="text-align: right;">Klas A</p> <p style="text-align: right;">Klas B</p>	<p>✓ blok</p> <p>✓ spriet</p> <p>(2)</p>

Nota:

As die kandidaat die antwoord van 1.1 en 1.2 kombineer deur die korrekte blok en spriet diagram te teken in 1.2 en die getalle op die diagram inskryf (d.i. ons merk vraag 1.1 en 1.2 gekombineer), **maks 5 / 6 punte vir vraag 1.1 en 1.2**

As 'n kandidaat getalle in enige orde in 1.1 skryf en die blok en spriet diagram korrek teken, maar nie die getalle op die diagram aandui nie, **maks 5 / 6 punte vir vraag 1.1 en 1.2**

As 'n kandidaat slegs die blok en spriet diagram teken in 1.2 en nie waardes daarop aandui nie of 1.1 beantwoord, **maks 2 / 6 punte**

As die kandidaat twee diagramme (een in antwoordboek en een op diagramblad), merk die een op die **ANTWOORDBLAD**.

1.3	<p>Klas B Klas B het beter presteer omdat die helfte van die aantal leerders bokant 60% behaal het, terwyl die helfte van Klas A meer as 55% behaal het. Klas B het beter presteer omdat die helfte van die aantal leerders bokant 60% behaal het, terwyl die helfte van Klas A minder as 55% behaal het. Mediaan van Klas B > Mediaan van Klas A</p> <p>OF Klas B Klas B is meer skeefgetrek na links as Klas A.</p> <p>OF Klas A 25% van klas het 75% of meer behaal in Klas A terwyl 25% van die klas 70% of meer behaal het in Klas B. Hoogste Punt in Klas A > Hoogste Punt in Klas B.</p> <p>Nota: As kandidaat se antwoord: Kan nie bepaal watter klas beter presteer het nie, omdat nie genoeg informasie gegee is nie en nie presies weet waar die punte voorkom nie. maks 1 / 3</p> <p>Nota: As kandidaat slegs antwoord Klas A of Klas B sonder enige redes, 0 / 3 marks</p>	<p>✓ Klas B</p> <p>✓✓ mediaan Klas B > Mediaan Klas A (3)</p> <p>✓ Klas B ✓✓ Klas B meer skeefgetrek na links as A (3)</p> <p>Klas A ✓ hoogste A > hoogste B ✓ 25% van A bokant 75% en 25% van B bokant 70% (2)</p> <p>[9]</p>
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VRAAG 2

2.1	<table><thead><tr><th>EKSAMENPUNTE (x)</th><th>FREKWENSIE</th><th>KUMULATIEWE FREKWENSIE</th></tr></thead><tbody><tr><td>$30 \leq x < 40$</td><td>12</td><td>12</td></tr><tr><td>$40 \leq x < 50$</td><td>18</td><td>30</td></tr><tr><td>$50 \leq x < 60$</td><td>55</td><td>85</td></tr><tr><td>$60 \leq x < 70$</td><td>57</td><td>142</td></tr><tr><td>$70 \leq x < 80$</td><td>43</td><td>185</td></tr><tr><td>$80 \leq x < 90$</td><td>11</td><td>196</td></tr><tr><td>$90 \leq x < 100$</td><td>4</td><td>200</td></tr></tbody></table>	EKSAMENPUNTE (x)	FREKWENSIE	KUMULATIEWE FREKWENSIE	$30 \leq x < 40$	12	12	$40 \leq x < 50$	18	30	$50 \leq x < 60$	55	85	$60 \leq x < 70$	57	142	$70 \leq x < 80$	43	185	$80 \leq x < 90$	11	196	$90 \leq x < 100$	4	200	<div>✓ eerste 3 waardes</div> <div>✓ laaste 4 waardes</div> <div>(2)</div>
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2.2	<div><div>Kumulatiewe frekwensiegrafiek van eksamenpunte</div></div>	<div>✓ vorm (punte moet nie met 'n reguitlyn verbind word nie, nie met 'n liniaal verbind word nie)</div> <div>✓ ankerpunt (30 ; 0)</div> <div>✓ gebruik die boonste limiet</div> <div>✓ gebruik kumulatiewe frekwensie</div> <div>✓ as 4 of meer punte korrek afgesteek is</div> <div>(5)</div>																								
<div>Nota:</div> <div>As leerders die middelpunt van die interval en die kumulatiewe frekwensie gebruik maks 1 / 5 punte vir vorm</div> <div>As kandidate die onderste limiet en die kumulatiewe frekwensie gebruik maks 1 / 5 punte</div>																										
2.3	<div>200 – 165 = 35 learners</div> <div>OF</div>	<div><div>Nota:</div><div>Aanvaar enigeen van 34, 35 of 36</div></div> <div>✓ antwoord</div> <div>(1)</div>																								

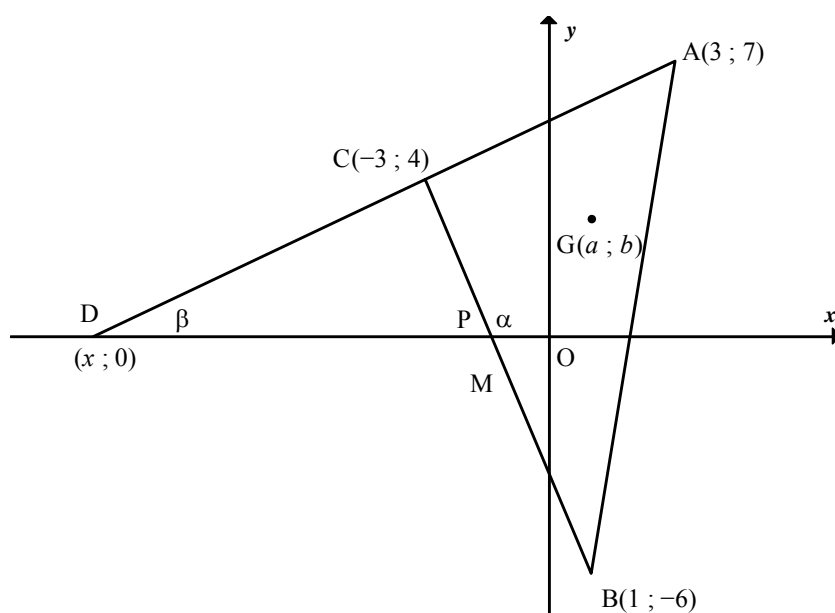
	$\frac{142 + 185}{2} = 163,5$ $200 - 163,5 = 36,5$	✓ antwoord (1) [8]
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VRAAG 3

3.1	<p>Mean</p> $= \frac{217 + 211 + 221 + 239 + 144 + 161 + 168 + 185 + 265 + 249 + 160 + 184}{12}$ $= \frac{2404}{12}$ $= 200,33$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Nota: Penaliseer 1 vir verkeerde afronding</p> </div>	<p>✓✓ antwoord (2)</p> <p>Slegs antwoord: Volpunte</p>																																										
3.2	<p>Met behulp van 'n sakrekenaar: $\sigma = 37,37$</p> <p>OF</p> <p>Pen en papier metode: gemiddelde (\bar{x}) = 200,33</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>x</th><th>$x - \bar{x}$</th><th>$(x - \bar{x})^2$</th></tr> </thead> <tbody> <tr><td>217</td><td>16,67</td><td>277,889</td></tr> <tr><td>211</td><td>10,67</td><td>113,848</td></tr> <tr><td>221</td><td>20,67</td><td>427,248</td></tr> <tr><td>239</td><td>38,67</td><td>1495,368</td></tr> <tr><td>144</td><td>-56,33</td><td>3173,0689</td></tr> <tr><td>161</td><td>-39,33</td><td>1546,848</td></tr> <tr><td>168</td><td>-32,33</td><td>1045,228</td></tr> <tr><td>185</td><td>-15,33</td><td>235,008</td></tr> <tr><td>265</td><td>64,67</td><td>4182,208</td></tr> <tr><td>249</td><td>48,67</td><td>2368,768</td></tr> <tr><td>160</td><td>-40,33</td><td>1626,508</td></tr> <tr><td>184</td><td>-16,33</td><td>266,668</td></tr> <tr> <td colspan="2">SOM</td><td>16758,666</td></tr> </tbody> </table> $\sigma = \sqrt{\frac{16758,6668}{12}} = 37,37$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Nota: Geen penalisering vir afronding</p> <p>Aanvaar 37</p> </div>	x	$x - \bar{x}$	$(x - \bar{x})^2$	217	16,67	277,889	211	10,67	113,848	221	20,67	427,248	239	38,67	1495,368	144	-56,33	3173,0689	161	-39,33	1546,848	168	-32,33	1045,228	185	-15,33	235,008	265	64,67	4182,208	249	48,67	2368,768	160	-40,33	1626,508	184	-16,33	266,668	SOM		16758,666	<p>✓✓✓ antwoord (3)</p> <p>✓ totaal</p> <p>✓ substitusie ✓ antwoord</p> <p>(3)</p>
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3.3	<p>200,33+1(37,37) = 237,70 liter</p> <p>Aanvaar enigiets insluitend en tussen 237 en 238 liter.</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Nota: As kandidaat se antwoord 200,33 – 1(37,37) = 162,96 liter Maks 1 / 2 punte</p> </div>	<p>✓metode ✓antwoord</p> <p>(2)</p> <p>[7]</p>																																										

VRAAG 4

4.1	Fly High	✓ antwoord (1)
4.2	$\frac{5120}{1000} \times 7,9$ = 40,45 Ja OF $\frac{5120}{1000} \times 8$ = 40,96 OF 8×5 = 40	✓ berekening ✓ lei na ‘n getal naby 40 (1)
4.3	<p>Ja. Die datapunte stel ‘n reguitlyn paslyn voor met negatiewe gradient, met Fly-High ‘n uitskieter .</p> <p>OF</p> <p>Ja. Swak negatiewe korrelasie. ($r = -0,2128075984$)</p> <p>OF</p> <p>Ja, Best Air en Best Fly en Alpha het ‘n hoë rekord vir betyds land en ‘n lae rekord vir verlore bagasie.</p> <p>As een of meer voorbeelde gebruik word om die antwoord JA te regverdig, gee een punt.</p> <p>Nota:</p> <p>As die kandidaat aandui “Best Air” en/of “Best Fly” en/of “Alpha” het hoë rekord vir betyds arriveer en lae verlore bagasie, Maks 1 / 2 punte</p>	<p>✓ Ja ✓ negatiewe gradient (2)</p> <p>✓ Ja ✓ negatiewe korrelasie (2)</p> <p>✓ Ja ✓ rede (2)</p>
4.4	<p>Alpha, 70% rekord vir betyds land en minste verlore bagasie</p> <p>OF</p> <p>Best Air, beste rekord vir betyds aankom (arriveer)</p>	<p>✓ Naam van maatskappy ✓ korrekte regverdiging (2) [6]</p>

VRAAG 5

5.1.1	$m_{AD} = m_{AC} = \frac{7-4}{3-(-3)} = \frac{3}{6} = \frac{1}{2}$ <p style="text-align: center;">OF</p> $m_{AD} = \frac{4-7}{-3-(x)} = \frac{-3}{-6} = \frac{1}{2}$ <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p>Nota: As kandidaat se antwoord</p> $m_{AD} = \frac{7}{3-x}$ <p>slegs 1 / 2 punte</p> </div>	<p>✓ substitusie van A en C in korrekte formule</p> <p>✓ antwoord</p> <p style="text-align: right;">(2)</p>
5.1.2	$m_{BC} = \frac{-6-4}{1-(-3)} = \frac{-10}{4} = \frac{-5}{2}$ <p style="text-align: center;">OF</p> $m_{BC} = \frac{4-(-6)}{-3-(1)} = \frac{10}{-4} = \frac{-5}{2}$	<p>✓ antwoord</p> <p style="text-align: right;">(1)</p>
5.2	$m_{AD} = \frac{1}{2} = \tan \hat{CDO}$ $\hat{CDO} = 26,56505\dots^\circ$ $m_{BC} = \frac{-5}{2} = \tan \alpha$ $\alpha = 111,814\,095^\circ$ $\hat{DCB} = 111,8014095\dots^\circ - 26,56505\dots^\circ$ $= 85,236359^\circ$ $= 85,24^\circ$ $\approx 85,2^\circ$ <p style="text-align: center;">OF</p>	<p>✓ 26,57°</p> <p>✓ 111,80°</p> <p>✓ antwoord</p> <p style="text-align: right;">(3)</p>

	$\tan \hat{CDO} = \frac{1}{2}$ $\hat{CDO} = 26,56505....^\circ$ $\tan(180^\circ - \alpha) = \frac{5}{2}$ $180^\circ - \alpha = 68,19859051...^\circ$ $\hat{DCB} = 180^\circ - (26,56505....^\circ + 68,19859051...^\circ)$ $= 85,236359^\circ$ $= 85,24^\circ$ <p>OF</p> $\hat{DCB} = \alpha - \hat{CDO}$ $\tan \hat{DCB} = \frac{m_{CB} - m_{CD}}{1 + m_{CB} \cdot m_{CD}}$ $= \frac{-\frac{5}{2} - \frac{1}{2}}{1 + \left(-\frac{5}{2}\right)\left(\frac{1}{2}\right)}$ $= 12$ $\hat{DCB} = 85,24^\circ$ <p>OF</p> $AC = \sqrt{45}$ $BC = \sqrt{116}$ $AB = \sqrt{173}$ $\cos \hat{ACB} = \frac{AC^2 + BC^2 - AB^2}{2AC \cdot BC}$ $= \frac{45 + 116 - 173}{2(\sqrt{45})(\sqrt{116})}$ $= -0,083045...$ $\hat{ACB} = 94,76...^\circ$ $\hat{DCB} = 180^\circ - 94,76...^\circ$ $= 85,24^\circ$ <p>OF</p>	<p>✓ 26,57°</p> <p>✓ 68,2°</p> <p>✓ antwoord (3)</p> <p>✓</p> $\tan \hat{DCB} = \frac{m_{CB} - m_{CD}}{1 + m_{CB} \cdot m_{CD}}$ <p>✓ substitusie</p> <p>✓ antwoord (3)</p> <p>✓ cos reël</p> <p>✓ substitusie in cos reël</p> <p>✓ antwoord (3)</p>
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$$D(-11; 0)$$

$$DC = \sqrt{80}$$

$$BC = \sqrt{116}$$

$$DB = \sqrt{180}$$

$$\begin{aligned}\cos \hat{DCB} &= \frac{DC^2 + BC^2 - DB^2}{2DC \cdot BC} \\ &= \frac{80 + 116 - 180}{2(\sqrt{80})(\sqrt{116})} \\ &= 0,08304547985\dots\end{aligned}$$

$$\hat{DCB} = 85,24^\circ$$

✓ cos reël

✓ substitusie in
cos reël

✓ antwoord

OF

Vergelyking AC: $2y = x + 11$

$$D(-11; 0)$$

$$C(-3; 4)$$

$$\begin{aligned}DC^2 &= (x_C - x_D)^2 + (y_C - y_D)^2 \\ &= (-3 + 11)^2 + (4 - 0)^2 \\ &= 80\end{aligned}$$

Vergelyking BC: $2y = -5x - 7$

$$P(-\frac{7}{5}; 0)$$

$$\begin{aligned}PC^2 &= (-3 + \frac{7}{5})^2 + (4 - 0)^2 \\ &= \frac{464}{25}\end{aligned}$$

$$\begin{aligned}DP^2 &= (-\frac{7}{5} + 11)^2 \\ &= \frac{2304}{25}\end{aligned}$$

In $\triangle DCP$: $DP^2 = DC^2 + CP^2 - 2DC \cdot CP \cdot \cos \hat{DCP}$

$$\frac{2304}{25} = \frac{2000}{25} + \frac{464}{25} - 2\left(\frac{\sqrt{2000}}{5}\right)\left(\frac{\sqrt{464}}{5}\right) \cdot \cos \hat{DCP}$$

$$\hat{DCP} = 85,23635\dots$$

$$\hat{DCP} = 85,24^\circ$$

✓ cos reël

✓ substitusie in cos reël

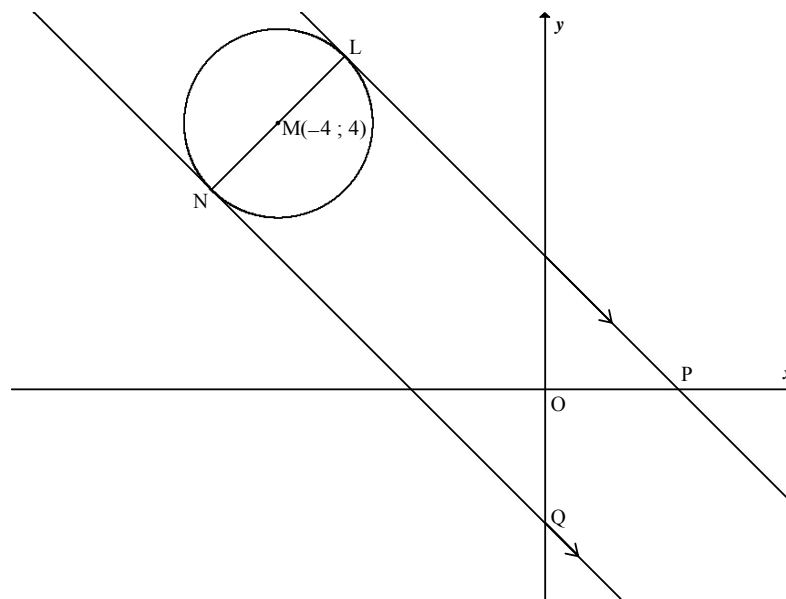
✓ antwoord

(3)

5.3	$y - 7 = \frac{1}{2}(x - 3)$ $y = \frac{1}{2}x + \frac{11}{2}$ $x - 2y + 11 = 0$ <p>OF</p> $y - 4 = \frac{1}{2}(x + 3)$ $y = \frac{1}{2}x + \frac{11}{2}$ $x - 2y + 11 = 0$ <p>OF</p> $y = \frac{1}{2}x + c$ $(7) = \frac{1}{2}(3) + c$ $c = \frac{11}{2}$ $y = \frac{1}{2}x + \frac{11}{2}$ $x - 2y + 11 = 0$ <p>OF</p> $D(-11 ; 0)$ $y - 0 = \frac{1}{2}(x + 11)$ $y = \frac{1}{2}x + \frac{11}{2}$ $x - 2y + 11 = 0$	<p>✓ substitusie van (3 ; 7) in $y - y_1 = m(x - x_1)$</p> <p>✓ antwoord in enige vorm (2)</p> <p>✓ substitusie van (-3 ; 4) in $y - y_1 = m(x - x_1)$</p> <p>✓ antwoord in enige vorm (2)</p> <p>✓ substitusie van (3 ; 7) in $y = mx + c$</p> <p>✓ antwoord in enige vorm (2)</p> <p>✓ substitusie van (-11 ; 0) in $y - y_1 = m(x - x_1)$</p> <p>✓ antwoord in enige vorm (2)</p>
5.4	$M(x; y) = \left(\frac{-3+1}{2}; \frac{4-6}{2} \right)$ $M(x; y) = (-1; -1)$	<p>✓ substitusie</p> <p>✓ antwoord (2)</p>

5.5	$m_{AM} = \frac{7 - (-1)}{3 - (-1)} = 2$ $y = 2x + c$ $-1 = 2(-1) + c$ $\therefore c = 1$ $y = 2x + 1$ <p>G lies on the line</p> $\therefore b = 2a + 1$ <p>OF</p> $\frac{7-b}{3-a} = \frac{b+1}{a+1}$ $(7-b)(a+1) = (b+1)(3-a)$ $7a + 7 - ab - b = 3b - ab + 3 - a$ $8a - 4b = -4$ $2a - b = -1$ $b = 2a + 1$ <p>OF</p> <p>Gebruik die punt $(-1 ; -1)$</p> $\frac{b+1}{a+1} = \frac{8}{4}$ $\frac{b+1}{a+1} = 2$ $b+1 = 2a+2$ $b = 2a+1$ <p>OF</p> <p>Gebruik die punt $(3 ; 7)$</p> $\frac{7-b}{3-a} = \frac{8}{4}$ $\frac{7-b}{3-a} = 2$ $7-b = 6-2a$ $b = 2a+1$	<p>✓ gradient = 2</p> <p>✓ substitusie $(-1 ; -1)$</p> <p>✓ $c = 1$</p> <p>✓ afleiding (4)</p> <p>✓ $\frac{7-b}{3-a}$</p> <p>✓ $\frac{b+1}{a+1}$</p> <p>✓ gelykstel aan</p> <p>✓ vereenvoudiging lei na $2a - b = -1$ (4)</p> <p>✓ substitusie OF $(-1 ; -1)$ into gradient</p> <p>✓ gradient = 2</p> <p>✓ gelykstel aan</p> <p>✓ vereenvoudiging lei na $b+1 = 2a+2$ (4)</p> <p>✓ substitusie van $(3 ; 7)$ in gradient</p> <p>✓ gradient = 2</p> <p>✓ equating</p> <p>✓ vereenvoudiging lei na $7-b = 6-2a$ (4)</p>
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5.6	<p> $GC = \sqrt{17}$ $GC^2 = 17$ $(a+3)^2 + (b-4)^2 = 17$ $(a+3)^2 + (2a+1-4)^2 = 17$ $a^2 + 6a + 9 + 4a^2 - 12a + 9 - 17 = 0$ $5a^2 - 6a + 1 = 0$ $(5a-1)(a-1) = 0$ $a = \frac{1}{5}$ or $a = 1$ $\therefore b = \frac{7}{5}$ or $b = 3$ </p> <p>OF</p> <p> $a = \frac{b-1}{2}$ $17 = (a+3)^2 + (b-4)^2$ $17 = \left(\left(\frac{b-1}{2} \right) + 3 \right)^2 + (b-4)^2$ $17 = \left(\frac{b+5}{2} \right)^2 + (b-4)^2$ $17 = \frac{b^2 + 10b + 25 + 4b^2 - 32b + 64}{4}$ $68 = 5b^2 - 22b + 89$ $0 = 5b^2 - 22b + 21$ $0 = (5b-7)(b-3)$ $\therefore b = \frac{7}{5}$ of $b = 3$ </p>	<p> ✓ afstand formule in terme van a en b ✓ substitusie van $b = 2a + 1$ </p> <p> ✓ standaardvorm ✓ faktore of korrekte substitusie in formule ✓ waardes van a ✓ waardes van b </p> <p>(6)</p> <p> ✓ $a = \frac{b-1}{2}$ ✓ afstandformule in terme van a en b ✓ substitusie van $a = \frac{b-1}{2}$ ✓ standaardvorm </p> <p> ✓ faktore of korrekte substitusie in formule </p> <p> ✓ waardes van b </p> <p>(6)</p> <p>[20]</p>
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VRAAG 6

6.1	$y = -x + 2$ $m_{LP} = -1$ $\therefore m_{LN} = \frac{-1}{-1} = 1$ $y = x + c$ $4 = -4 + c$ $\therefore c = 8$ $y = x + 8$ OF $y - 4 = 1(x + 4)$ $y = x + 8$	<div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> Nota: As kandidaat antwoord laat as $y - 4 = x + 4$ Maks 2 / 3 Slegs antwoord: Volpunte </div>	$\checkmark m_{LP} = -1$ $\checkmark m_{LN} = 1$ \checkmark vergelyking (3) $\checkmark m = 1$ \checkmark substitusie in $y - y_1 = m(x - x_1)$ \checkmark antwoord (3)
6.2	$x + 8 = -x + 2$ $2x = -6$ $x = -3$ $y = -3 + 8$ $y = 5$ $L(-3; 5)$	OF $y + x = 2 \dots\dots\dots(1)$ $y - x = 8 \dots\dots\dots(2)$ $2y = 10$ $\therefore y = 5$ $\therefore x = -3$ $L(-3; 5)$	$\checkmark x$ -waarde $\checkmark y$ -waarde (2) (vergelykings wat tot die waardes lei moet gebruik word)

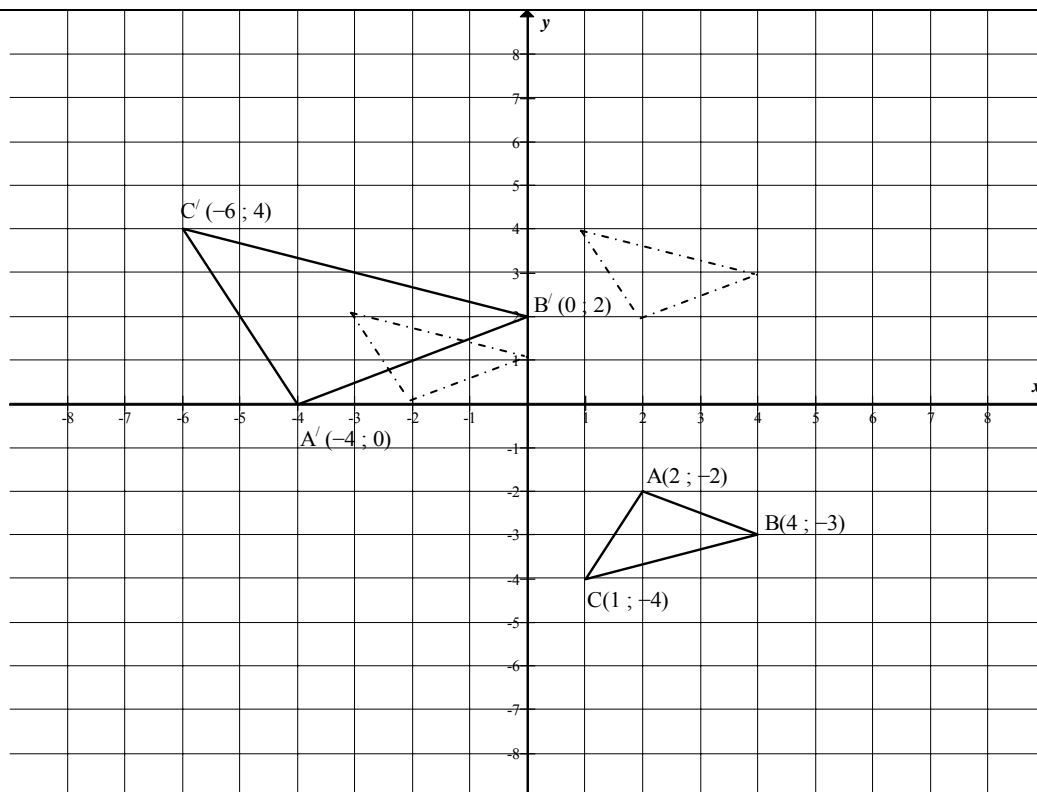
Nota:

Geen penalisering as antwoord in koördinaatvorm gelaat word nie

6.3	$(x+4)^2 + (y-4)^2 = r^2$ $(-3+4)^2 + (5-4)^2 = r^2$ $\therefore r^2 = 2$ $(x+4)^2 + (y-4)^2 = 2$ Vergelyking kan gelaat word as: $x^2 + 8x + y^2 - 8y + 30 = 0$	<div style="border: 1px solid black; padding: 5px;"> <p>Nota: As kandidaat slegs die afstandformule gebruik om die radius te bepaal $(-3+4)^2 + (5-4)^2 = r^2$ $\therefore r^2 = 2$ 2 / 3 punte</p> </div>	$\checkmark (x+4)^2 + (y-4)^2 = r^2$ \checkmark substitusie van $(-3; 5)$ $\checkmark r^2 = 2$ (3)
6.4	Laat N(x, y). Omdat M (-4;4) die middelpunt van LN is en L(-3;5) $\frac{x-3}{2} = -4; \frac{y+5}{2} = 4$ $\therefore x = -5; y = 3$ OF $y = x + 8$ $(x+4)^2 + (y-6)^2 = 2$ $(x+4)^2 + (x+8-4)^2 - 2 = 0$ $x^2 + 8x + 16 + x^2 + 8x + 16 - 2 = 0$ $2x^2 + 16x + 30 = 0$ $x^2 + 8x + 15 = 0$ $(x+5)(x+3) = 0$ $x = -3$ of $x = -5$ $y = 5$ of $y = 3$ $\therefore N(-5; 3)$	<div style="border: 1px solid black; padding: 5px;"> <p>Nota: Slegs antwoord: Volpunte</p> </div>	\checkmark M is die middelpunt van LN $\checkmark x = -5$ $\checkmark y = 3$ (3) \checkmark $(x+4)^2 + (x+8-4)^2 - 2 = 0$ $\checkmark x^2 + 8x + 15 = 0$ \checkmark both $x = -5$ and $y = 3$ (3)
6.5	$m_{NQ} = -1$ $y = -x + c$ $3 = -(-5) + c$ $c = -2$ $y = -x - 2$ OF $m_{NQ} = -1$ $y - 3 = -(x + 5)$ $y = -x - 2$ OF Vergelyking van LP is $x + y = 2$ NQ \parallel LP \therefore vergelyking van NQ is $x + y = k$ vir sommige $k \in R$ Maar N(-5; 3) is op NQ $\therefore x + y = -5 + 3 = -2$	<div style="border: 1px solid black; padding: 5px;"> <p>Note: Slegs antwoord: Volpunte</p> </div>	\checkmark gradient \checkmark substitusie van $(-5; 3)$ in $y = mx + c$ $\checkmark c = -2$ (3) \checkmark gradient \checkmark substitusie van $(-5; 3)$ in $y - y_1 = m(x - x_1)$ \checkmark vergelyking (3) $\checkmark x + y = k$ \checkmark substitusie van $(-5; 3)$ \checkmark vergelyking (3)

VRAAG 7

7.1



2 punte vir elke
diagram van die
transformasie
(6)

OF

As die kandidaat
eers die algemene
reel uitwerk

$(x; y) \rightarrow$
 $(2x - 8; -2y - 4)$
 ✓ $A'(-4; 0)$
 ✓ $B'(0; 2)$
 ✓ $C'(-6; 4)$
 1 punt vir elke
korrekte punt
afgesteek en
verbind

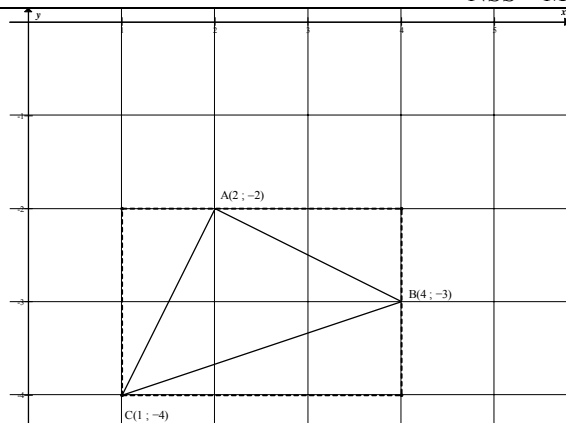
(6)

Nota:

- As die kandidaat slegs die korrekte driehoek teken sonder byskrifte, **volpunte**
- As die kandidaat punte korrek afsteek en nie die driehoek teken nie, **maks 5 / 6 punte**
- In die 3 sketse, as een van die drie hoekpunte verkeerd is, slegs **1 / 2** punte vir die verkeerde skets dan word KA toegepas.
- As die kandidaat die punte neerskryf en nie die punte afsteek nie en die driehoek teken, **maks 3 / 6 punte**
- As die hoekpunte korrek is maar nie benoem is nie en die punte is verbind, **maks 5 / 6 punte**
- As die hoekpunte korrek is maar nie benoem en nie verbind is nie **maks 4 / 6 punte**
- As 'n kandidaat eers 'n verkeerde formule gebruik
 Maks 1 punt vir die formule
 Maks 2 punte vir die berekening van A' , B' , C' koördinate (KA)
 1 punt vir afsteek van 3 hoekpunte
 1 punt vir voltooiing van driehoek en benoeming

7.2	<p> $(x; y) \rightarrow (x; -y)$ $(x; -y) \rightarrow (x - 4; -y - 2)$ $(x - 4; -y - 2) \rightarrow (2x - 8; -2y - 4)$ </p> <p>OF</p> <p> $(x; y) \rightarrow (2x - 8; -2y - 4)$ </p> <div style="border: 1px solid black; padding: 10px; margin-top: 10px;"> <p>Nota:</p> <ul style="list-style-type: none"> As die kandidaat antwoord $(x; y) \rightarrow (x; -y)$ $(x; y) \rightarrow (x - 4; y - 2)$ $(x; y) \rightarrow (2x; 2y)$ <p>Slegs 2 / 4 punte</p> <ul style="list-style-type: none"> As die kandidaat antwoord $(x; y) \rightarrow (x; -y)$ $(x; y) \rightarrow (x - 4; -y - 2)$ $(x; y) \rightarrow (2x - 8; -2y - 4)$ <p>4 / 4 marks</p> <ul style="list-style-type: none"> As die kandidaat se antwoord $(x; y) \rightarrow 2(x - 4; -y - 2)$ Slegs 3 / 4 punte </div>	<p>✓</p> <p>$(x; y) \rightarrow (x; -y)$</p> <p>✓ $(x; -y) \rightarrow$ $(x - 4; -y - 2)$</p> <p>✓✓ $(2x - 8; -2y - 4)$</p> <p>(4)</p>
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7.3



Area ΔABC = area van reghoek – som van 3 driehoek areas

$$= 6 - \left(1 + \frac{3}{2} + 1\right)$$

$$= \frac{5}{2}$$

$$\text{Area } \Delta A'B'C' = 2^2 \left(\frac{5}{2}\right)$$

$$= 10 \text{ eenhede}^2$$

OF

$$\text{Area } \Delta A'B'C' = 24 - \left(\left(\frac{1}{2} \cdot 6 \cdot 2 \right) + \frac{1}{2} (4)(2) + \frac{1}{2} (2)(4) \right)$$

$$= 24 - 6 - 4 - 4$$

$$= 10 \text{ eenhede}^2$$

OF

$$m_{AC} = 2 \text{ en } m_{AB} = -\frac{1}{2} \therefore \text{produk} = -1$$

$$\therefore \hat{CAB} = 90^\circ$$

$$AB = \sqrt{5}$$

$$AC = \sqrt{5}$$

$$\therefore \text{Area } \Delta ABC \text{ is } \frac{1}{2} (\sqrt{5})^2 = \frac{5}{2}$$

$$\therefore \text{Area } \Delta A'B'C' \text{ is } 4 \times \frac{5}{2} = 10 \text{ vierkante eenhede}$$

OF

$$\checkmark 6 - \left(1 + \frac{3}{2} + 1\right)$$

$$\checkmark \frac{5}{2}$$

$$\checkmark \checkmark 10$$

(4)

✓

$$\left(\left(\frac{1}{2} \cdot 6 \cdot 2 \right) + \frac{1}{2} (4)(2) + \frac{1}{2} (2)(4) \right)$$

$$\checkmark 24$$

$$\checkmark \checkmark 10$$

(4)

$$\checkmark AB = \sqrt{5} \text{ en } AC = \sqrt{5}$$

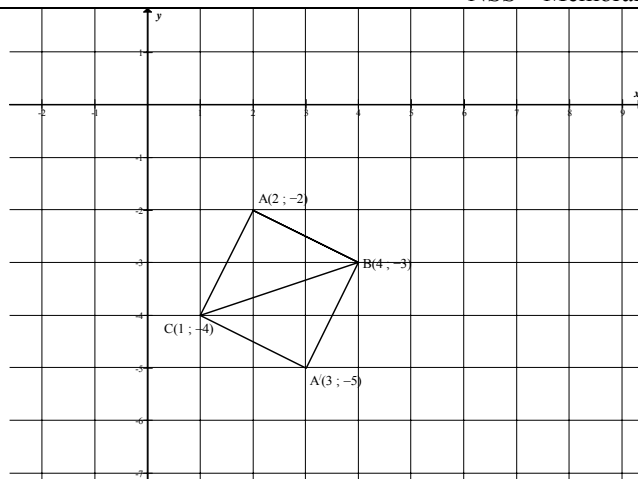
$$\checkmark \frac{5}{2}$$

$$\checkmark \checkmark 10$$

(4)

$$\checkmark A'B' = \sqrt{20}$$

$m_{A'C'} = -2$ and $m_{A'B'} = \frac{1}{2} \therefore \text{produk} = -1$ $\therefore \hat{C'AB'} = 90^\circ$ $A'B' = \sqrt{20}$ $A'C' = \sqrt{20}$ $\therefore \text{Area } \triangle A'B'C' \text{ is } \frac{1}{2}(\sqrt{20})^2 = 10 \text{ vierkante eenhede}$ OF $AB = \sqrt{5}$ $AC = \sqrt{5}$ $BC^2 = 10$ $BC = \sqrt{10}$ $\perp \text{hoogte} = \sqrt{\frac{10}{4}}$ $\text{Area } \triangle ABC = \frac{1}{2} \cdot \sqrt{\frac{10}{4}} \cdot \sqrt{10} = \frac{5}{2}$ $\text{Area } \triangle A'B'C' = 4 \times \frac{5}{2} = 10 \text{ vierkante eenhede}$ OF $AB = \sqrt{5}$ $AC = \sqrt{5}$ $BC^2 = 10$ $BC = \sqrt{10}$ $AC^2 + AB^2 = BC^2$ $\therefore \hat{CAB} = 90^\circ$ $\text{Area } \triangle ABC = = \frac{1}{2}(\sqrt{5})^2 = \frac{5}{2}$ $\text{Area } \triangle A'B'C' = 4 \times \frac{5}{2} = 10 \text{ vierkante eenhede}$ OF $\text{Area } \triangle ABC = \frac{1}{2}bc \sin A$ $= \frac{1}{2}(\sqrt{5})(\sqrt{5})\sin 90$ $= \frac{5}{2}$ $\text{Area } \triangle A'B'C' = 4 \times \frac{5}{2} = 10 \text{ vierkante eenhede}$ OF	$\checkmark A'C' = \sqrt{20}$ $\checkmark \checkmark 10$ (4) $\checkmark AB = \sqrt{5}$ en $AC = \sqrt{5}$ $\checkmark \frac{5}{2}$ $\checkmark \checkmark 10$ (4) $\checkmark AB = \sqrt{5}$ en $AC = \sqrt{5}$ $\checkmark \frac{5}{2}$ $\checkmark \checkmark$ antwoord $\checkmark AB = \sqrt{5}$ en $AC = \sqrt{5}$ $\checkmark \frac{5}{2}$ $\checkmark \checkmark$ antwoord (4)
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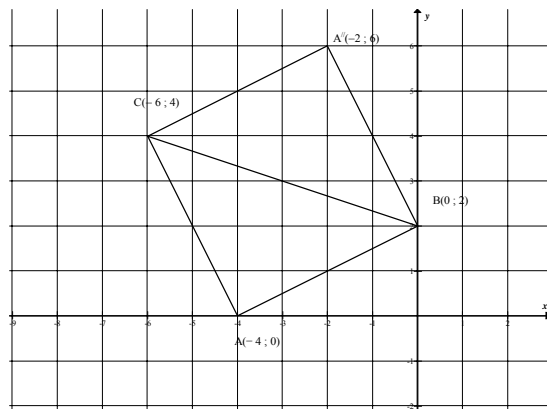
Reflekteer $\triangle ABC$ om CB en kry die vierkant $ABA'C$ met sy $\sqrt{5}$

Area van vierkant $= (\sqrt{5})(\sqrt{5}) = 5$

$$\text{Area } \triangle ABC = \frac{5}{2}$$

$$\text{Area } \triangle A'B'C' = 4 \times \frac{5}{2} = 10 \text{ vierkante eenhede}$$

OF



Reflekteer $\triangle A'B'C'$ om $C'B'$ en kry vierkant $A'B'A''C'$ met sy $\sqrt{20}$

Area van vierkant $= (\sqrt{20})(\sqrt{20}) = 20$

$$\text{Area } \triangle A'B'C' = \frac{1}{2} \times 20 = 10 \text{ vierkante eenhede}$$

- ✓ $AB = \sqrt{5}$ en $AC = \sqrt{5}$
- ✓ refleksie om vierkant te kry
- ✓ $\frac{5}{2}$
- ✓ antwoord

(4)

- ✓ $A'B' = \sqrt{20}$ en $A'C' = \sqrt{20}$
- ✓ refleksie om vierkant te kry
- ✓ 20
- ✓ antwoord

(4)

[14]

VRAAG 8

<p>8.1</p>	<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> $\begin{aligned} x' &= x \cos \alpha - y \sin \alpha \\ &= 2 \cos 75^\circ - 4 \sin 75^\circ \\ &= 2 \cos(30^\circ + 45^\circ) - 4 \sin(30^\circ + 45^\circ) \\ &= 2 \cos 30^\circ \cos 45^\circ - 2 \sin 30^\circ \sin 45^\circ - 4 \sin 30^\circ \cos 45^\circ - 4 \cos 30^\circ \sin 45^\circ \\ &= 2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - 4 \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - 4 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{2\sqrt{6} - 2\sqrt{2} - 4\sqrt{2} - 4\sqrt{6}}{4} \\ &= \frac{-2\sqrt{2} - 6\sqrt{6}}{4} \\ &= \frac{-\sqrt{6} - 3\sqrt{2}}{2} \quad \text{of} \quad -\frac{1}{\sqrt{2}}(3 + \sqrt{3}) \end{aligned}$ </div> <div style="width: 50%; border: 1px solid black; padding: 10px;"> <p>Nota: As die kandidaat 'n sakrekenaar gebruik d.i. gee 'n desimale antwoord maks 5 / 6 punte</p> <p>Verkeerde formule: maks 5 / 6 punte</p> </div> </div> <div style="margin-top: 20px;"> <p>OF</p> $\begin{aligned} x' &= x \cos \alpha + y \sin \alpha \\ &= 2 \cos(-75^\circ) + 4 \sin(-75^\circ) \\ &= 2 \cos(75^\circ) - 4 \sin(75^\circ) \\ &= 2 \cos(30^\circ + 45^\circ) - 4 \sin(30^\circ + 45^\circ) \\ &= 2 \cos 30^\circ \cos 45^\circ - 2 \sin 30^\circ \sin 45^\circ - 4 \sin 30^\circ \cos 45^\circ - 4 \cos 30^\circ \sin 45^\circ \\ &= 2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - 4 \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - 4 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{2\sqrt{6} - 2\sqrt{2} - 4\sqrt{2} - 4\sqrt{6}}{4} \\ &= \frac{-2\sqrt{2} - 6\sqrt{6}}{4} \\ &= \frac{-\sqrt{6} - 3\sqrt{2}}{2} \quad \text{of} \quad -\frac{1}{\sqrt{2}}(3 + \sqrt{3}) \end{aligned}$ </div>	<div style="display: flex; flex-direction: column; align-items: flex-end;"> <div style="margin-bottom: 20px;"> <p>✓ substitusie in anti-kloksgewyse formule</p> <p>✓ $75 = 30 + 45$</p> <p>✓ cos uitbreiding</p> <p>✓ sin uitbreiding</p> <p>✓ substitusie van spesiale hoeke</p> </div> <div> <p>✓ vereenvoudigde antwoord van x</p> </div> </div> <p style="text-align: right;">(6)</p>
		<div style="display: flex; flex-direction: column; align-items: flex-end;"> <div style="margin-bottom: 20px;"> <p>✓ substitusie in kloksgewyse formule</p> <p>✓ $75 = 30 + 45$</p> <p>✓ cos uitbreiding</p> <p>✓ sin uitbreiding</p> <p>✓ substitusie van Spesiale hoeke</p> </div> <div> <p>✓ vereenvoudigde antwoord van x</p> </div> </div> <p style="text-align: right;">(6)</p>

	<p>Bereken eers</p> $\begin{aligned}\cos 75^\circ &= \cos(30^\circ + 45^\circ) \\ &= \cos 30^\circ \cdot \cos 45^\circ - \sin 30^\circ \cdot \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$ <p>En</p> $\begin{aligned}\sin 75^\circ &= \sin(30^\circ + 45^\circ) \\ &= \sin 30^\circ \cdot \cos 45^\circ + \cos 30^\circ \cdot \sin 45^\circ \\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2}(1 + \sqrt{3})}{4}\end{aligned}$ $\begin{aligned}x' &= 2 \cos 75^\circ - 4 \sin 75^\circ \\ &= 2 \left(\frac{\sqrt{6} - \sqrt{2}}{4} \right) - 4 \left(\frac{\sqrt{2} + \sqrt{6}}{4} \right) \\ &= \frac{-2\sqrt{6} - 6\sqrt{2}}{4} \\ &= \frac{-\sqrt{6} - 3\sqrt{2}}{2}\end{aligned}$	<p>✓ $75 = 30 + 45$ ✓ cos uitbreiding</p> <p>✓ substitusie van spesiale hoeke in die eerste uitbreiding</p> <p>✓ sin uitbreiding</p> <p>✓ substitusie</p> <p>✓ vereenvoudigde antwoord van x (6)</p>
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8.2	<p> $x' = x \cos \beta - y \sin \beta$ $3 \cos \beta - \sin \beta = \frac{3 - \sqrt{3}}{2} \quad \dots (1)$ $y' = y \cos \beta + x \sin \beta$ $\cos \beta + 3 \sin \beta = \frac{1 + 3\sqrt{3}}{2} \quad \dots (1^*)$ $(1) \times 3 + (1)^*:$ $10 \cos \beta = 3 \left(\frac{3 - \sqrt{3}}{2} \right) + \frac{(1 + 3\sqrt{3})}{2}$ $= \frac{1}{2} (9 - 3\sqrt{3} + 1 + 3\sqrt{3})$ $= 10 \left(\frac{1}{2} \right)$ $\therefore \cos \beta = \frac{1}{2}$ $\therefore \beta = 60^\circ$ OF $(1) - 3(1)^*:$ $-10 \sin \beta = 3 \left(\frac{1}{2} \right) - \left(\frac{3 - \sqrt{3}}{2} \right)$ $= \frac{3 - \sqrt{3} - 3(1 + 3\sqrt{3})}{2}$ $= \frac{-10\sqrt{3}}{2}$ $\therefore \sin \beta = \frac{\sqrt{3}}{2}$ $\therefore \beta = 60^\circ$ OF $3 \cos \beta - \sin \beta = \frac{3 - \sqrt{3}}{2} \quad \dots (1)$ $\cos \beta = \frac{1 + 3\sqrt{3}}{2} - 3 \sin \beta \quad \dots (2)$ Vervang (2) in (1) $3 \left(\frac{1 + 3\sqrt{3}}{2} - 3 \sin \beta \right) - \sin \beta = \frac{3 - \sqrt{3}}{2}$ $\frac{3 + 9\sqrt{3}}{2} - 9 \sin \beta - \sin \beta = \frac{3 - \sqrt{3}}{2}$ $-10 \sin \beta = \frac{3 - \sqrt{3} - 3 - 9\sqrt{3}}{2}$ $-10 \sin \beta = \frac{-10\sqrt{3}}{2}$ $\sin \beta = \frac{\sqrt{3}}{2}$ $\beta = 60^\circ$ </p> <div data-bbox="805 1579 1209 1727" style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p>Nota: Slegs antwoord: maks 2 / 6 marks</p> </div>	<p> ✓ substitusie in x' ✓ substitusie in y' ✓ vereenvoudiging ✓ gelyktydige oplossing ✓ $\frac{\sqrt{3}}{2}$ of $\frac{1}{2}$ ✓ antwoord (6) </p> <p> ✓ vergelyking (1) ✓ vergelyking (2) ✓ gelyktydige oplossing ✓ vereenvoudiging ✓ $\sin \beta = \frac{\sqrt{3}}{2}$ ✓ antwoord (6) </p>
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OF

$$3 \cos \beta - \sin \beta = \frac{3 - \sqrt{3}}{2}$$

$$\text{en } \cos \beta + 3 \sin \beta = \frac{1 + 3\sqrt{3}}{2}$$

Probeer $\beta = 60^\circ$

$$3 \cos \beta - \sin \beta = 3 \left(\frac{1}{2} \right) - 3 \left(\frac{\sqrt{3}}{2} \right) = \frac{3 - \sqrt{3}}{2}$$

$$\cos \beta + 3 \sin \beta = \frac{1}{2} + 3 \left(\frac{\sqrt{3}}{2} \right) = \frac{1 + 3\sqrt{3}}{2}$$

$$\therefore \beta = 60^\circ$$

OF

$$\tan \alpha = \frac{1}{3}$$

$$\alpha = 18,43^\circ$$

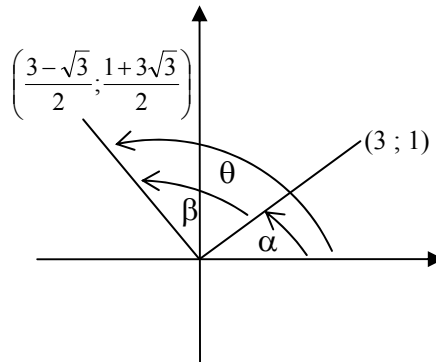
$$\tan \theta = \frac{\frac{1 + 3\sqrt{3}}{2}}{\frac{3 - \sqrt{3}}{2}}$$

$$= \frac{1 + 3\sqrt{3}}{3 - \sqrt{3}}$$

$$\theta = 78,43^\circ$$

$$\beta = 78,43^\circ - 18,43^\circ$$

$$= 60^\circ$$



$$\checkmark \checkmark \beta = 60^\circ$$

✓ substitusie
✓ vereenvoudiging

✓ substitusie
✓ vereenvoudiging

(6)

$$\checkmark \tan \alpha = \frac{1}{3}$$

$$\checkmark \alpha = 18,43^\circ$$

$$\checkmark \frac{1 + 3\sqrt{3}}{3 - \sqrt{3}}$$

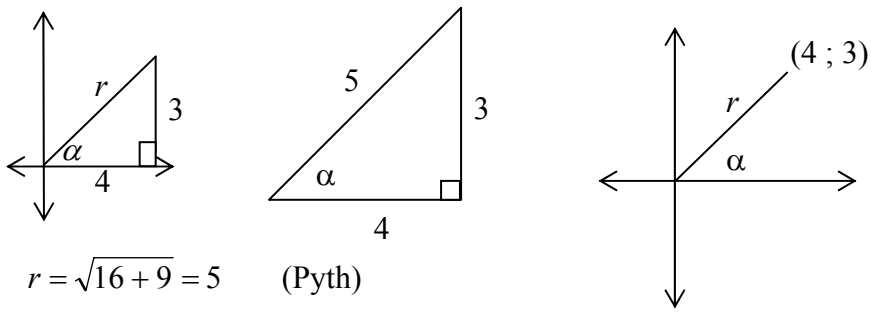
$$\checkmark \theta = 78,43^\circ$$

✓ vereenvoudiging
✓ 60°

OF

$x' = x \cos \beta - y \sin \beta$ $\frac{3 - \sqrt{3}}{2} \cos \beta - \frac{1 + 3\sqrt{3}}{2} \sin \beta = 3$ $3 \cos \beta - \sqrt{3} \cos \beta - \sin \beta - 3\sqrt{3} \sin \beta = 6$ $\frac{1 + 3\sqrt{3}}{2} \cos \beta + \frac{3 - \sqrt{3}}{2} \sin \beta = 1$ $\cos \beta + 3\sqrt{3} \cos \beta + 3 \sin \beta - \sqrt{3} \sin \beta = 2$ $3 \cos \beta + 9\sqrt{3} \cos \beta + 9 \sin \beta - 3\sqrt{3} \sin \beta = 6$ $3 \cos \beta - \sqrt{3} \cos \beta - \sin \beta - 3\sqrt{3} \sin \beta = 6$ $10\sqrt{3} \cos \beta + 10 \sin \beta = 0$ $\sin \beta = -\sqrt{3} \cos \beta$ $\cos \beta + 3\sqrt{3} \cos \beta + 3(-\sqrt{3} \cos \beta) - \sqrt{3}(-\sqrt{3} \cos \beta) = 2$ $4 \cos \beta = 2$ $\cos \beta = \frac{1}{2}$ $\beta = 60^\circ$	<p>✓ substitusie</p> <p>✓ substitusie</p> <p>✓ vereenvoudiging</p> <p>✓ $\sin \beta = -\sqrt{3} \cos \beta$</p> <p>✓ $\cos \beta = \frac{1}{2}$</p> <p>✓ antwoord</p> <p>(6)</p> <p>[12]</p>
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VRAAG 9

9.1	 <p>$r = \sqrt{16 + 9} = 5$ (Pyth)</p> <p>$\sin \alpha = \frac{3}{5}$</p> <p>Aanvaar : 0,6</p>	<p>✓ diagram</p> <p>✓ waarde van r</p> <p>✓ antwoord (3)</p>
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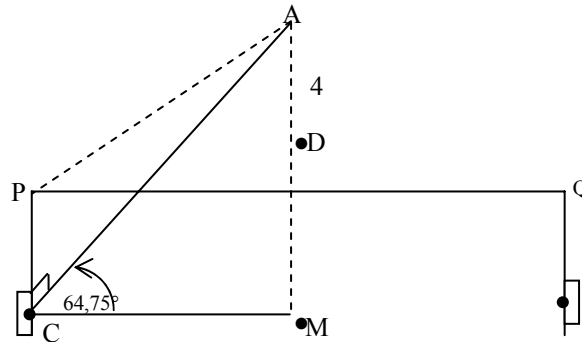
9.2	$\cos^2(90^\circ - \alpha) - 1$ $= \sin^2 \alpha - 1$ $= \left(\frac{3}{5}\right)^2 - \frac{25}{25}$ $= \frac{-16}{25}$ $= -0,64$	$\cos^2(90^\circ - \alpha) - 1$ $= \sin^2 \alpha - (\sin^2 \alpha + \cos^2 \alpha)$ $= -\cos^2 \alpha$ $= -\left(\frac{4}{5}\right)^2$ $= \frac{-16}{25}$ $= -0,64$	<p>✓ $\cos(90^\circ - \alpha) = \sin \alpha$</p> <p>✓ substitusie van $\sin \alpha = \frac{3}{5}$</p> <p>(2)</p>
9.3	$1 - \sin 2\alpha$ $= 1 - 2 \sin \alpha \cos \alpha$ $= 1 - 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$ $= 1 - \frac{24}{25}$ $= \frac{1}{25}$ <p>OF</p> $1 - \sin 2\alpha$ $= \sin^2 \alpha - 2 \sin \alpha \cos \alpha + \cos^2 \alpha$ $= (\sin \alpha - \cos \alpha)^2$ $= \left(\left(\frac{3}{5}\right) - \left(\frac{4}{5}\right)\right)^2$ $= \left(-\frac{1}{5}\right)^2$ $= \frac{1}{25}$	$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$ $\frac{24}{25}$ <p>✓ antwoord</p> <p>(3) [8]</p>	

VRAAG 10

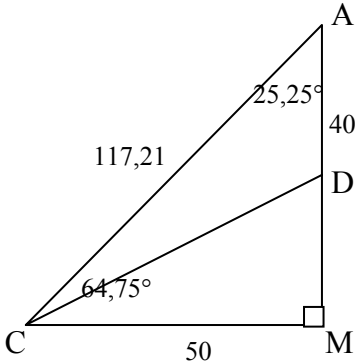
10.1	$\frac{\sin(90^\circ + \theta) + \cos(180^\circ + \theta)\sin(-\theta)}{\sin 180^\circ - \tan 135^\circ}$ $= \frac{\cos \theta + (-\cos \theta)(-\sin \theta)}{0 + 1}$ $= \cos \theta + \cos \theta \cdot \sin \theta$ $= \cos \theta(1 + \sin \theta)$	✓ $\cos \theta$ ✓ $-\cos \theta$ ✓ $-\sin \theta$ ✓ $0 + 1$ ✓ antwoord (5)
10.2	<p>Linkerkant = $\frac{4 \sin A \cos A \cos 2A \cdot \sin 15^\circ}{2 \sin A \cos A(1 - 2 \sin^2 A)}$</p> $= \frac{2 \cos 2A \cdot \sin 15^\circ}{\cos 2A}$ $= 2 \sin 15^\circ$ $= 2 \sin(45^\circ - 30^\circ)$ $= 2[\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ]$ $= 2\left[\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}\right]$ $= 2\left[\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right]$ $= \frac{\sqrt{6} - \sqrt{2}}{2} = RK$ <p>OF $= \frac{\sqrt{3} - 1}{\sqrt{2}}$</p> <p>OF For these values of x, $\cos 2x \neq 0$</p> $2 \cos x = \frac{\sin 2x}{\cos 2x}$ $= \frac{2 \sin x \cos x}{2 \cos^2 x - 1}$ $2 \cos x(2 \cos^2 x - 1) = 2 \sin x \cos x$ $2 \cos x(2(1 - \sin^2 x) - 1) = 2 \sin x \cos x$ $2 \cos x(1 - 2 \sin^2 x) - 2 \sin x \cos x = 0$ $2 \cos x(2 \sin^2 x + \sin x - 1) = 0$ $2 \sin^2 x + \sin x - 1 = 0 \quad \text{or} \quad \cos x = 0$ $(\sin x + 1)(2 \sin x - 1) = 0$ $\sin x = -1 \quad \text{or} \quad \sin x = \frac{1}{2} \quad \text{or} \quad \cos x = 0$ $x = \pm 90^\circ \quad \text{or} \quad x = 30^\circ$ <p>OF</p>	✓ $2 \sin A \cos A$ ✓ $1 - 2 \sin^2 A = \cos 2A$ ✓ $2 \sin 15^\circ$ ✓ $15 = 45 - 30$ of $15 = 60 - 45$ ✓ substitusie ✓ $2\left[\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right]$ (6)
		✓ $\frac{\sin 2x}{\cos 2x}$ ✓ $2 \sin x \cos x$ ✓ $2 \cos^2 x - 1$ ✓ $\cos x = 0$ ✓ factors ✓ equations ✓ $\pm 90^\circ$ ✓ 30° (8)

	$\text{Linkerkant} = \frac{4 \sin A \cos A \cos 2A \cdot \sin 15^\circ}{2 \sin A \cos A (1 - 2 \sin^2 A)}$ $= \frac{2 \cos 2A \cdot \sin 15^\circ}{\cos 2A}$ $= 2 \sin 15^\circ$ $= 2 \sin(60^\circ - 45^\circ)$ $= 2[\sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ]$ $= 2 \left[\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \right]$ $= 2 \left[\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \right]$ $= \frac{\sqrt{6} - \sqrt{2}}{2} = RK$ <p>OF</p> $\text{Linkerkant} = \frac{4 \sin A \cos A \cos 2A \cdot \sin 15^\circ}{2 \sin A \cos A (1 - 2 \sin^2 A)}$ $= \frac{2 \sin 2A \cos 2A \cdot \sin 15^\circ}{\sin 2A \cos 2A}$ $= 2 \sin 15^\circ$ $= 2 \sin(45^\circ - 30^\circ)$ $= 2[\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ]$ $= 2 \left[\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \right]$ $= 2 \left[\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \right]$ $= \frac{\sqrt{6} - \sqrt{2}}{2} = RK$	$\checkmark 2 \sin A \cos A$ $\checkmark 1 - 2 \sin^2 A = \cos 2A$ $\checkmark 2 \sin 15^\circ$ $\checkmark 15 = 45 - 30 \quad \text{OF}$ $15 = 60 - 45$ \checkmark substitusie $\checkmark 2 \left[\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \right]$ <p>(6)</p>
10.3	$6 \cos x - 5 = \frac{4}{\cos x}$ $6 \cos^2 x - 5 \cos x = 4$ $6 \cos^2 x - 5 \cos x - 4 = 0$ $(3 \cos x - 4)(2 \cos x + 1) = 0$ $\cos x = \frac{4}{3} \quad \text{of} \quad \cos x = \frac{-1}{2}$ <p>geen oplossing of $x = 120^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$</p> <p>of</p> $x = 240^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$	\checkmark standaardvorm \checkmark faktore \checkmark albei vergelykings $\checkmark 240^\circ + k \cdot 360^\circ$ $\checkmark 120^\circ + k \cdot 360^\circ$ $\checkmark k \in \mathbb{Z}$ <p>(6)</p>

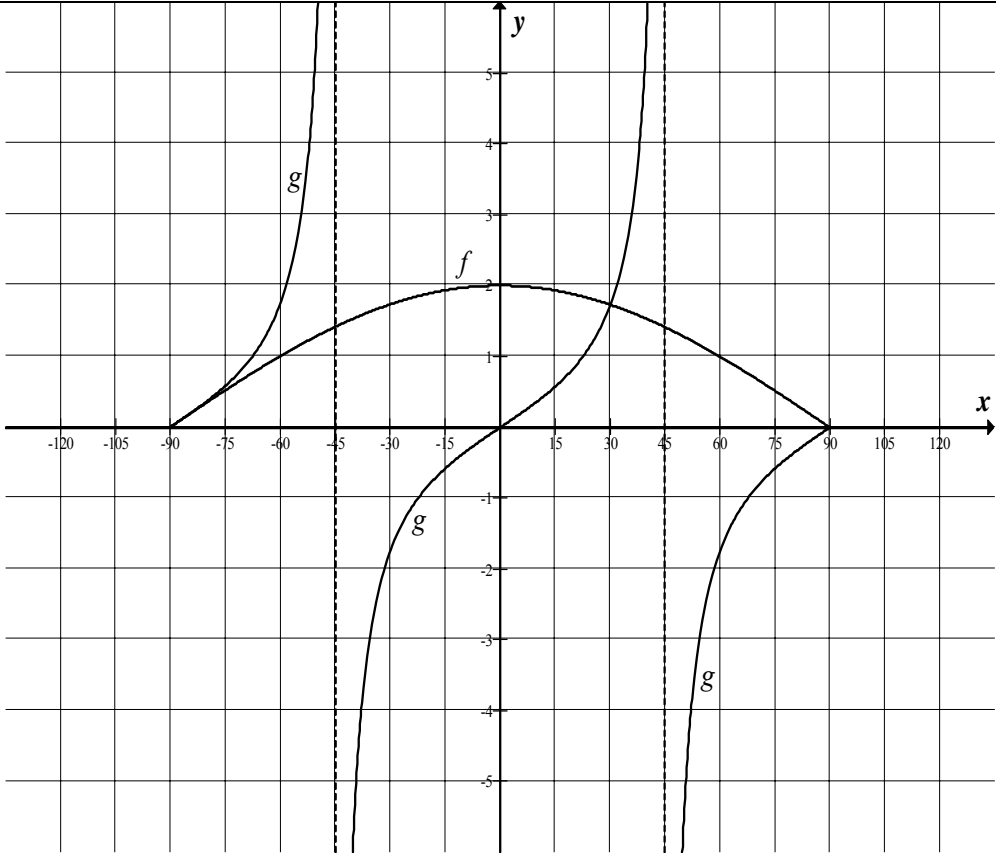
	<p>Alternatiewe oplossing vir $\cos x = \frac{-1}{2}$</p> <p>$x = k.360^\circ \pm 120^\circ \quad k \in \mathbb{Z}$</p> <div><p>Nota: As kandidaat $\pm k.360$ gebruik, dan is $k \in \mathbb{N}_0$</p></div>	<p>[17]</p>
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VRAAG 11

11.1	$\cos 64,75^\circ = \frac{50}{AC}$ $AC = \frac{50}{\cos 64,75^\circ}$ $= 117,21 \text{ m}$ <p>OF</p> $AC = \frac{50}{\cos 64,75^\circ}$ $\therefore AC = 117,2144026 \text{ m}$ $\therefore AC = 117,21 \text{ m}$ <p>OF</p> $\frac{50}{\sin 25,25^\circ} = \frac{AC}{\sin 90^\circ}$ $\therefore AC = \frac{50 \sin 90^\circ}{\sin 25,25^\circ}$ $\therefore AC = 117,21 \text{ m}$	<p>✓ substitusie in verhouding</p> <p>✓ AC onderwerp van die formule</p> <p>✓ antwoord (3)</p> <p>✓ substitusie in verhouding</p> <p>✓ AC onderwerp van formule</p> <p>✓ antwoord (3)</p> <p>✓ substitusie in sin reë</p> <p>✓ AC onderwerp van formule</p> <p>✓ antwoord (3)</p>
11.2	<p>PC is given to be $\frac{1}{2}(64) = 32 \text{ m}$</p> $\tan \hat{PAC} = \frac{32}{117,21}$ $\theta = 15,27^\circ$ <p>Nota: As die kandidaat die onafgeronde antwoord vir AC neem, dan is die antwoord $15,27^\circ$</p>	<p>✓ $\tan \hat{PAC}$</p> <p>✓ $\frac{32}{117,21}$</p> <p>✓ antwoord (3)</p>

11.3	<p> $CD^2 = 117,21^2 + 40^2 - 2(117,21)(40) \cos 25,25$ $= 6857,289092$ $\therefore CD = 82,81 \text{ m}$ </p> <p>OF</p>  <p> $AM = AC \sin 64,75^\circ$ OF $AM = CM \tan 64,75^\circ$ OF $AM = AC \cos 25,25^\circ$ $= 106,0111876$ $= 50 \tan 64,75^\circ$ $= 117,21 \cdot \cos 25,25^\circ$ $= 106,01$ $= 106,01$ $= 106,01$ $DM = 106,01 - 40$ $= 66,01$ $CD^2 = CM^2 + DM^2$ $= (50)^2 + (66,01)^2$ $= 6857,3201$ $CD = 82,81 \text{ meter}$ </p> <p>OF</p> <p> $AM = AC \sin 64,75^\circ$ OF $AM = CM \tan 64,75^\circ$ OF $AM = AC \cos 25,25^\circ$ $= 106,0111876$ $= 50 \tan 64,75^\circ$ $= 117,21 \cdot \cos 25,25^\circ$ $= 106,01$ $= 106,01$ $= 106,01$ $DM = 106,01 - 40$ $= 66,01$ $DC^2 = (50)^2 + (66,01)^2 - 2(50)(66,01) \cdot \cos 90^\circ$ $= 6857,3201$ $CD = 82,81 \text{ meter}$ </p> <p>OF</p> <p> $\sin 64,75^\circ = \frac{40 + x + 32}{117,21}$ $x = 34,01$ $CD^2 = CM^2 + DM^2$ $= (50)^2 + (32 + 34,01)^2$ $= 6857,3201$ $CD = 82,81 \text{ meter}$ </p>	<p> \checkmark cos reël $\checkmark\checkmark$ substitusie \checkmark antwoord (4) </p> <p>\checkmark AM = 106,01</p> <p> \checkmark DM = 66,01 \checkmark Pythagoras </p> <p>\checkmark antwoord (4)</p> <p>\checkmark AM = 106,01</p> <p> \checkmark DM = 66,01 \checkmark cosreël </p> <p>\checkmark antwoord (4)</p> <p>[10]</p>
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VRAAG 12

12.1		<p>tan 2x: ✓asimptote (45° & -45°) ✓x-afsnitte ✓middel kurwe ✓eind kurwes</p> <p>2 cos x: ✓y-afsnit ✓x-afsnitte</p> <p>(6)</p> <p>Nota: As die kandidaat buite die interval teken, GEEN PENALISERING.</p>
12.2	<p>Vir hierdie waardes van x, $\cos 2x \neq 0$</p> $2 \cos x = \frac{\sin 2x}{\cos 2x}$ $= \frac{2 \sin x \cos x}{1 - 2 \sin^2 x}$ $1 - 2 \sin^2 x = \sin x \text{ of } \cos x = 0$ $2 \sin^2 x + \sin x - 1 = 0 \text{ of } \cos x = 0$ $(\sin x + 1)(2 \sin x - 1) = 0$ $\sin x = -1 \text{ of } \sin x = \frac{1}{2} \text{ of } \cos x = 0$ $x = \pm 90^\circ \text{ of } x = 30^\circ$ <p>OF</p>	<p>✓ $\frac{\sin 2x}{\cos 2x}$ ✓ $2 \sin x \cos x$ ✓ $1 - 2 \sin^2 x$</p> <p>✓ $\cos x = 0$</p> <p>✓ faktore ✓ vergelykings ✓ $\pm 90^\circ$ ✓ 30°</p> <p>(8)</p>

	$2 \cos x = \frac{\sin 2x}{\cos 2x}$ $= \frac{2 \sin x \cos x}{2 \cos^2 x - 1}$ $\cos x (2 \cos^2 x - 1 - \sin x) = 0$ $\cos x = 0 \quad \text{of} \quad 2(1 - \sin^2 x) - 1 - \sin x = 0$ $\cos x = 0 \quad \text{of} \quad 2 - 2 \sin^2 x - \sin x - 1 = 0$ $2 \sin^2 x + \sin x - 1 = 0 \quad \text{of} \quad \cos x = 0$ $(\sin x + 1)(2 \sin x - 1) = 0 \quad \text{of} \quad \cos x = 0$ $\sin x = -1 \quad \text{of} \quad \sin x = \frac{1}{2} \quad \text{of} \quad \cos x = 0$ $x = \pm 90^\circ \quad \text{of} \quad x = 30^\circ$ $2 \cos x = \frac{\sin 2x}{\cos 2x}$ $2 \cos x \cdot \cos 2x = \sin 2x$ $2 \cos x \cdot \cos 2x - 2 \sin x \cdot \cos x = 0$ $2 \cos x (\cos 2x - \sin x) = 0$ $\cos 2x = \sin x \quad \text{of} \quad 2 \cos x = 0$ $1 - 2 \sin^2 x = \sin x$ $2 \sin^2 x + \sin x - 1 = 0$ $(\sin x + 1)(2 \sin x - 1) = 0$ $\sin x = -1 \quad \text{of} \quad \sin x = \frac{1}{2} \quad \text{of} \quad \cos x = 0$ $x = \pm 90^\circ \quad \text{of} \quad x = 30^\circ$ <p>OF</p>	$\checkmark \frac{\sin 2x}{\cos 2x}$ $\checkmark 2 \sin x \cos x$ $\checkmark 2 \cos^2 x - 1$ $\checkmark \cos x = 0$ $\checkmark \text{faktore}$ $\checkmark \text{vergelykings}$ $\checkmark \pm 90^\circ$ $\checkmark 30^\circ$ <p>(8)</p>
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	$2 \cos x = \frac{\sin 2x}{\cos 2x}$ $2 \cos x \cdot \cos 2x = \sin 2x$ $2 \cos x \cdot \cos 2x - 2 \sin x \cdot \cos x = 0$ $2 \cos x (\cos 2x - \sin x) = 0$ $\cos 2x = \sin x \quad \text{of} \quad 2 \cos x = 0$ $\cos 2x = \cos(90^\circ - x) \quad x = 90^\circ$ $2x = \pm(90^\circ - x) + k \cdot 360^\circ$ $3x = 90^\circ + k \cdot 360^\circ$ $x = 30^\circ + k \cdot 120^\circ$ $x = -90^\circ \quad \text{of} \quad x = 30^\circ$ of $x = -90^\circ + k \cdot 360^\circ$ $x = -90^\circ$	
12.3	$0^\circ < x < 45^\circ$ OF $-90^\circ < x < -45^\circ$	✓✓kritieke waardes ✓✓notasie (4)
12.4	Periode = $2(360^\circ) = 720^\circ$	✓✓ antwoord (2)
12.5	$x = -45^\circ + 25^\circ = -20^\circ$ $x = 45^\circ + 25^\circ = 70^\circ$ OF $2(x - 25^\circ) = -90^\circ$ $2(x - 25^\circ) = 90^\circ$ $2x - 50^\circ = -90^\circ$ $2x - 50^\circ = 90^\circ$ $2x = -40^\circ$ en $2x = 140^\circ$ $x = -20^\circ$ $x = 70^\circ$	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> Nota: Slegs antwoord: Volpunte </div> ✓ $x = -20^\circ$ ✓ $x = 70^\circ$ (2)
		[22]

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