education

Department of Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATION - 2007

MATHEMATICS P2

HIGHER GRADE

FEBRUARY/MARCH 2007

301-1/2

MATHEMATICS HG: Paper 2

MARKS: 200

TIME: 3 HOURS

301 1 2E HG

This question paper consists of 11 pages, 1 formula sheet and a diagram sheet of 6 pages.

X05

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consist of 9 questions, a formula sheet and diagram sheets.
2. Use the formula sheet to answer this question paper.
3. Detach the diagram sheets from the question paper and place them inside your ANSWER BOOK.
4. The diagrams are not drawn to scale.
5. Answer ALL the questions.
6. Number ALL the answers correctly and clearly.
7. ALL the necessary calculations must be shown.
8. Non–programmable calculators may be used, unless otherwise stated.
9. The number of decimal digits to which answers must be rounded off will be stated in the question where necessary.

ANALYTICAL GEOMETRY

NOTE: – USE ANALYTICAL METHODS IN THIS SECTION.
– CONSTRUCTION AND MEASUREMENT METHODS NOT BE USED.

QUESTION 1

In the diagram alongside,

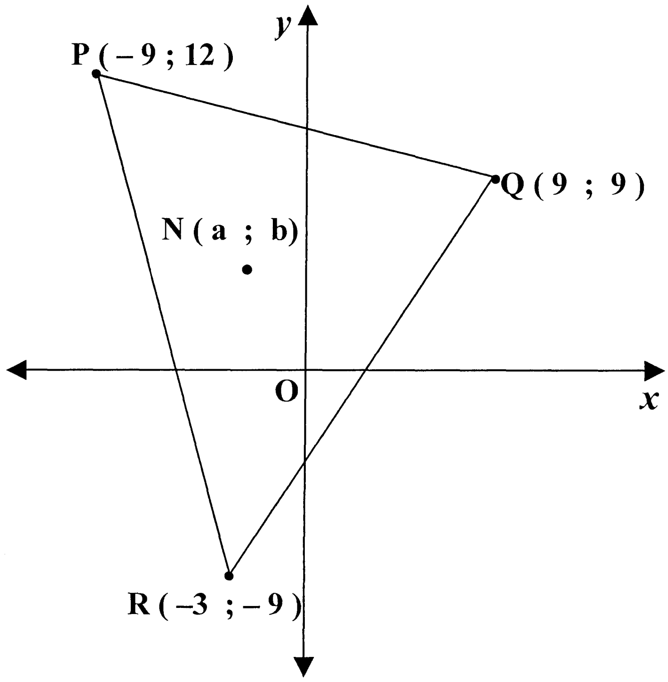
P (– 9 ; 12), Q (9 ; 9) and

R (– 3 ; – 9) are the vertices

of Δ PQR.

N (a ; b) is a point in the second

quadrant.

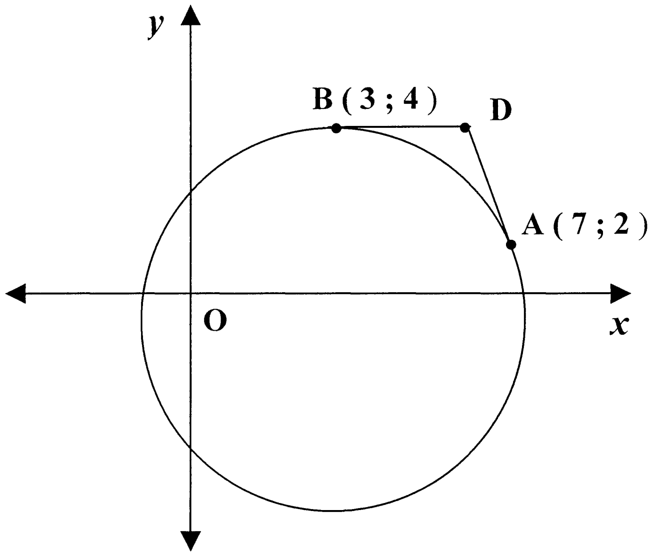


1.1	Calculate the gradient of PQ.	(2)
1.2	Calculate the size of $\hat{Q}$, rounded off to TWO decimal digits.	(5)
1.3	Determine the co-ordinates of M, the midpoint of QR .	(2)
1.4	Determine the equation of median PM .	(4)
1.5	Determine the co-ordinates of N if P, N and M are collinear and QN = $5\sqrt{5}$ units.	(10)
1.6	Write down the equation of the straight line passing through N and which is parallel to the y-axis.	(2)

[25]

QUESTION 2

2.1 In the diagram alongside, a circle passes through points A (7 ; 2) and B (3 ; 4). AD is a tangent to the circle.



2.1.1 Prove that the point (3 ; – 1) is the centre of the circle. (5)

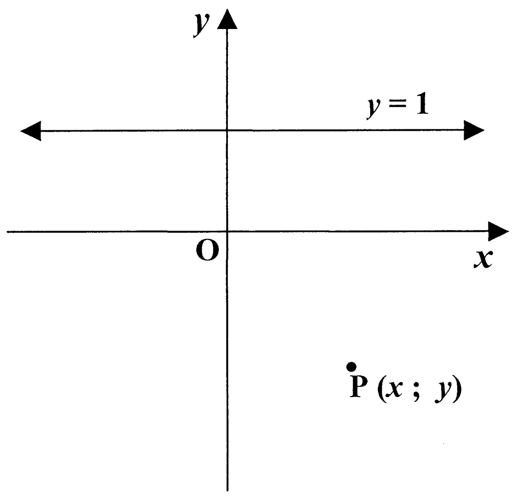
2.1.2 Hence, determine the equation of the circle. (3)

2.1.3 Determine the equation of tangent AD . (5)

2.1.4 Determine the co-ordinates of D if DB is a tangent to the circle at B . (6)

2.2 In the diagram alongside, locus point P (x ; y) and straight line $y = 1$ are shown.

The square of the distance from P to the straight line is three units more than the square of the distance from P to origin O.



Determine the equation of the locus of P (x ; y). (6)

[25]

TRIGONOMETRY

QUESTION 3

Answer this question without the use of a calculator.

3.1 If $\cos 25° = \sqrt{1-t^2}$, express each of the following in terms of t:

3.1.1 $\sin 25°$ (2)

3.1.2 $\cot 115°$ (2)

3.1.3 $\sin 50°$ (4)

3.2 Simplify:

$$\frac{\sin(1530°-x)\,.\,\sec(x-360°)\,.\,\tan(x-180°)}{\sin(-x)\,.\,\mathrm{cosec}(x-90°)}$$

(7)
[15]

QUESTION 4

Given: $f(x) = -\sin x$ and $g(x) = 1 + \tan x$

4.1 Use the set of axes provided on the diagram sheet to draw sketch graphs of the curves of f and g for $x \in [-90° \ ; \ 270°]$.
Show clearly all the intercepts with the axes and the co-ordinates of all turning points. Represent the asymptotes using dotted lines. (9)

4.2 Use the graphs to answer the following:

4.2.1 Indicate on the graphs the value(s) of x for which $1 + \tan x + \sin x = 0$, using letters A, B, ... (3)

4.2.2 Determine the value(s) of $x \in [-90° \ ; \ 270°]$ for which $g(x) - f(x) = 1$ (2)

4.2.3 Determine the value(s) of $x \in [-90° \ ; \ 90°]$ for which $f(x) \,.\, g(x) \leq 0$ (4)
[18]

QUESTION 5

5.1 Determine the general solution of $\sin\theta = \cos\theta$ (4)

5.2 For which real value(s) of m will

$\cos\theta = m + 1$ have real solutions? (2)

5.3 Given the identity: $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

5.3.1 Prove the identity. (5)

5.3.2 Hence, or otherwise, determine the solution of

$\sin 3\theta = -\operatorname{cosec}\theta$ for $\theta \in [-180° \,;\, 0°]$ (8)

5.4 Prove the identity:

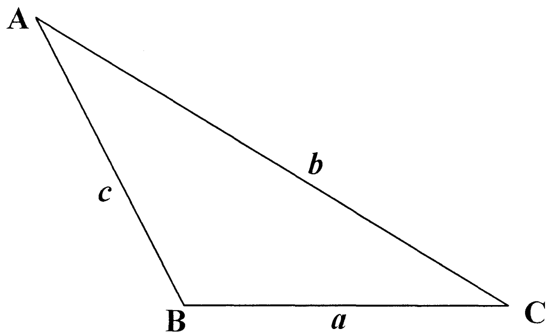
$2\cot 2\theta \,.\, \tan\theta = 2 - \sec^2\theta$ (6)

[25]

QUESTION 6

6.1 In the diagram alongside, Δ ABC is given with $\hat{B}$ obtuse.

Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove that:



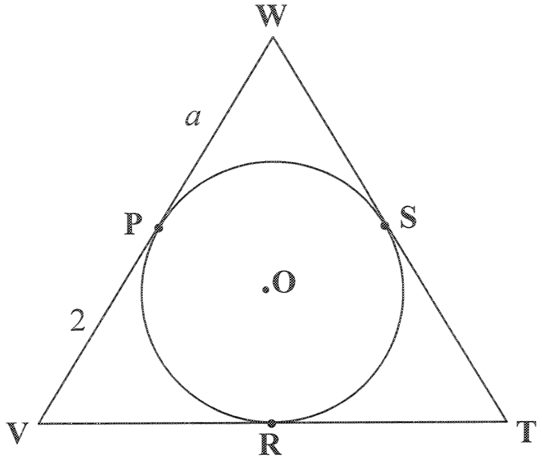
$$b^2 = a^2 + c^2 - 2(a)(c)\cos B$$ (6)

6.2 In the diagram alongside, circle PSR with centre O is the inscribed circle of Δ WTV.

WV = WT

WP = a units

PV = 2 units



Determine:

6.2.1 cos V in terms of a (7)

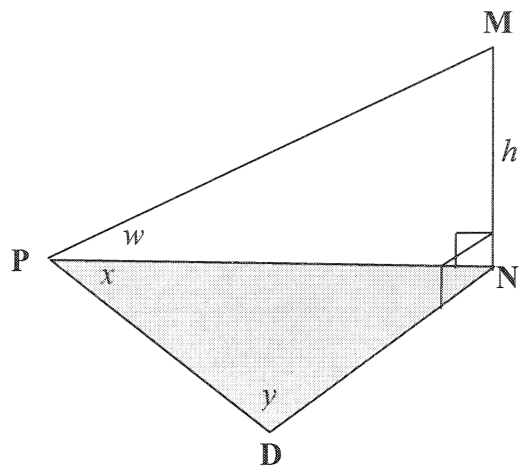
6.2.2 the length of the radius of the circle if $a = 3$ (rounded off to ONE decimal digit). (5)

6.3 In the diagram alongside, MN represents a vertical mast.

P, D and N are points in the same horizontal plane.

MN = h units, $D\hat{P}N = x$ and $N\hat{D}P = y$

The angle of elevation of M from P is w.



6.3.1 Prove that $PD = \dfrac{h.\sin(x+y)}{\sin y\ .\ \tan w}$ (5)

6.3.2 Calculate the length of MN, rounded off to the nearest metre, if $x = 60°$, $y = 50°$, $w = 64°$ and PD = 70 m. (2)

[25]

EUCLIDEAN GEOMETRY

N0TE: – DIAGRAMS FOR PROVING THEORY MAY BE USED ON THE DIAGRAM SHEETS.
– DETACH THE DIAGRAM SHEETS FROM THE QUESTION PAPER AND PLACE THEM IN YOUR ANSWER BOOK.
– GIVE A REASON FOR EACH STATEMENT, UNLESS OTHERWISE STATED.

QUESTION 7

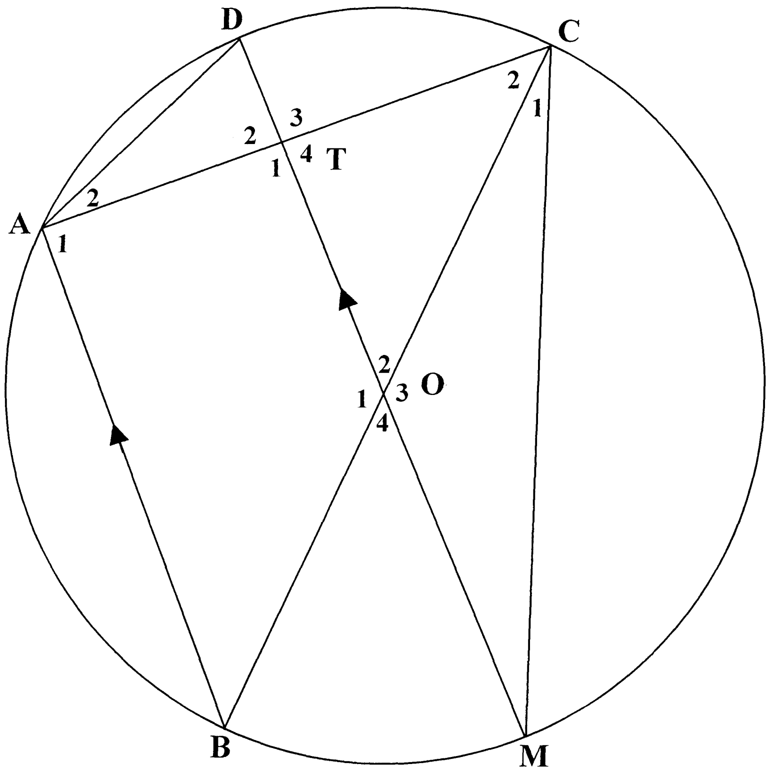
In the diagram below, O is the centre of circle DABMC.

BC and DM are diameters.

AC and DM intersect at T.

OT = 3 DT

AB || DM



7.1 Prove that T is the midpoint of AC. (4)

7.2 Determine the length of MC in terms of DT. (6)

7.3 Express $\hat{D}$ in terms of $\hat{O}_2$. (5)

[15]

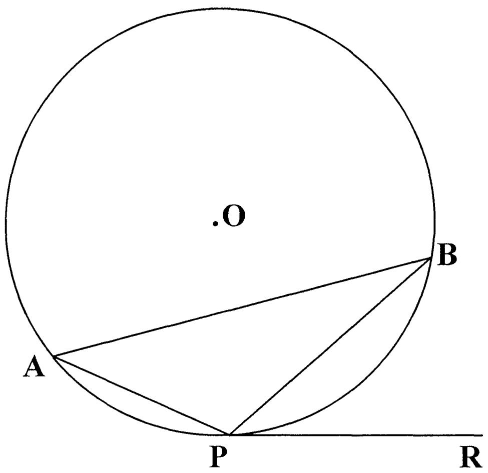
QUESTION 8

8.1 In the diagram alongside,
O is the centre of circle APB.

PR is a straight line drawn through the endpoint of chord PB.

Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove the theorem which states that :

If $B\hat{P}R = \hat{A}$, then PR is a tangent to the circle at P.



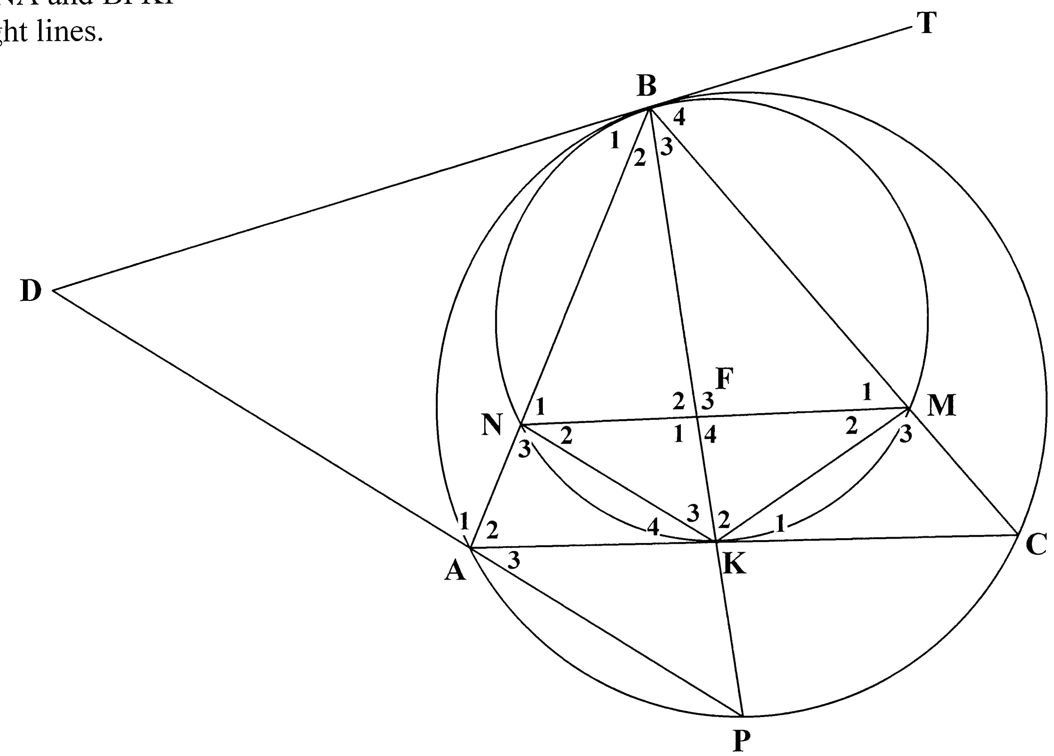
(6)

8.2 In the diagram below, TBD is a tangent to circles BAPC and BNKM at B.

AKC is a chord of the larger circle and is also a tangent to the smaller circle at K.

Chords MN and BK intersect at F.
PA is produced to D.

BMC, BNA and BFKP are straight lines.



Prove that :

8.2.1 MN || CA (4)

8.2.2 Δ KMN is isosceles (3)

8.2.3 $\frac{BK}{KP} = \frac{BM}{MC}$ (7)

8.2.4 DA is a tangent to the circle passing through points A, B and K. (5)

[25]

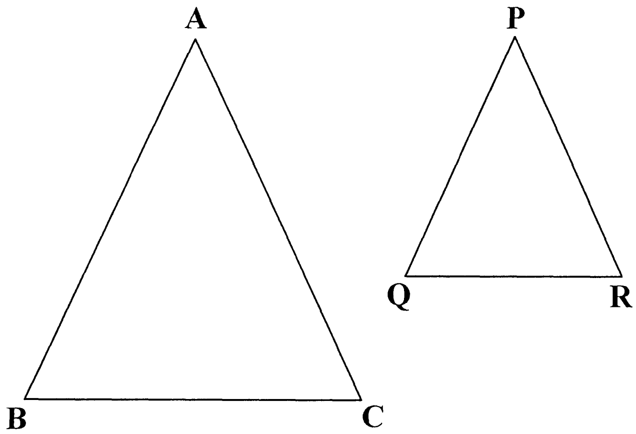
QUESTION 9

9.1 In the diagram alongside, Δ ABC and Δ PQR are given.

Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove the theorem which states that :

If $\hat{A} = \hat{P}$, $\hat{B} = \hat{Q}$ **and** $\hat{C} = \hat{R}$,

then $\dfrac{AB}{PQ} = \dfrac{BC}{QR}$

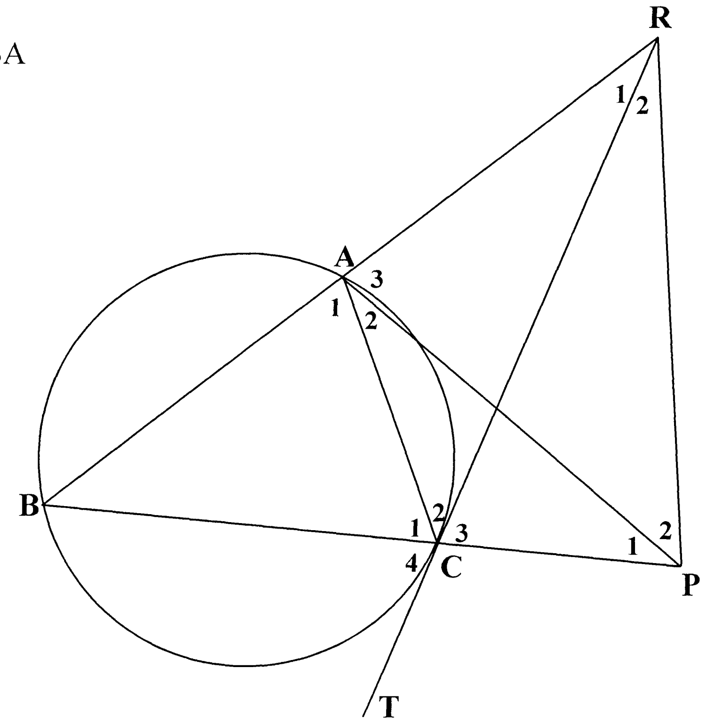


(7)

9.2 In the diagram alongside, chord BA and tangent TC of circle ABC are produced to meet at R.

BC is produced to P with RC = RP

AP is not a tangent.



9.2.1 Prove that :

(a) ACPR is a cyclic quadrilateral (5)

(b) Δ CBA ||| Δ RPA (6)

(c) $RC = \dfrac{CB \cdot RA}{AC}$ (2)

(d) RB . AC = RC . CB (4)

9.2.2 Hence, prove that $RC^2 = RA \cdot RB$ (3)

[27]

TOTAL: 200

Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(a + T_n)$ $\quad S_n = \frac{n}{2}(a + \ell)$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = a.r^{n-1}$ $\quad S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$ $\quad S_n = \frac{a(r^n - 1)}{r-1} \quad (r \neq 1)$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

$A = P\left(1 + \frac{r}{100}\right)^n$ $\quad A = P\left(1 - \frac{r}{100}\right)^n$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$(x_3 \,;\, y_3) = \left(\frac{x_1 + x_2}{2} \,;\, \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2$$

$$(x-p)^2 + (y-q)^2 = r^2$$

In Δ ABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \tfrac{1}{2}ab.\sin C$$

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SENIOR CERTIFICATE EXAMINATION/**SENIORSERTIFIKAAT-EKSAMEN**
MATHEMATICS HG/**WISKUNDE HG**
PAPER II/VRAESTEL II
FEBRUARY/MARCH/**FEBRUARIE/MAART**

DIAGRAM SHEET/DIAGRAMVEL

INSTRUCTION

This diagram sheet must be handed in with your answer book. Ensure that your details are complete.

INSTRUKSIE

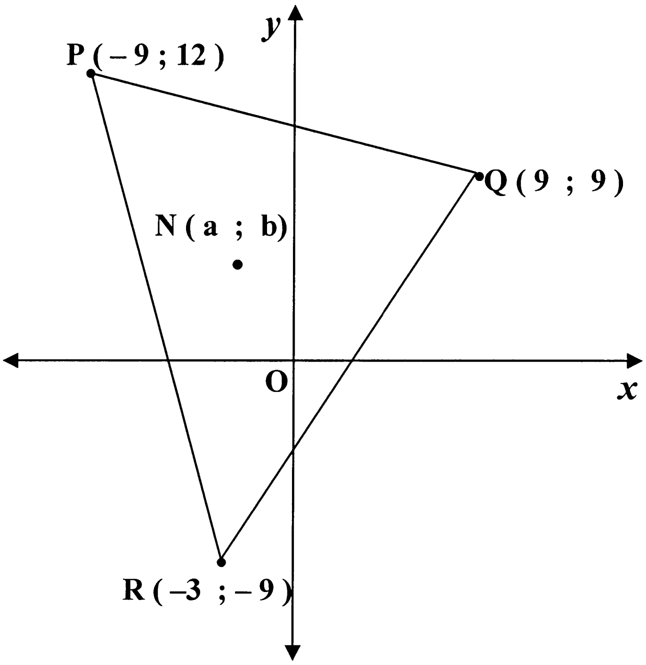
Hierdie diagramvel moet saam met jou antwoordeboek ingelewer word. Maak seker dat jou besonderhede volledig ingevul is.

EXAMINATION NUMBER
EKSAMENNOMMER

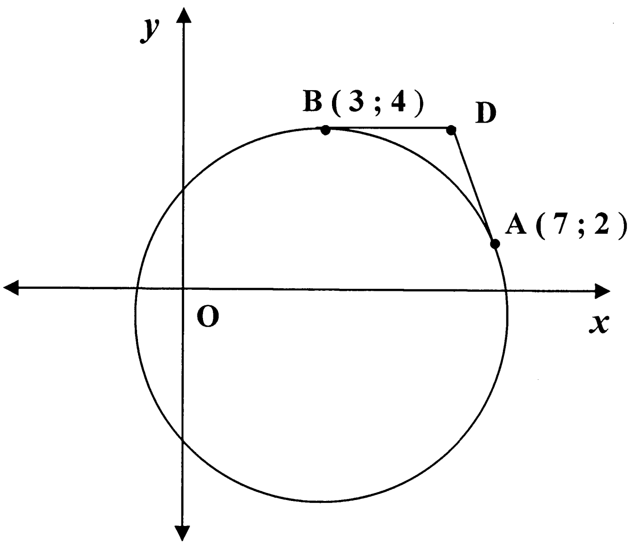
CENTRE NUMBER
SENTRUMNOMMER

EXAMINATION NUMBER
EKSAMENNOMMER

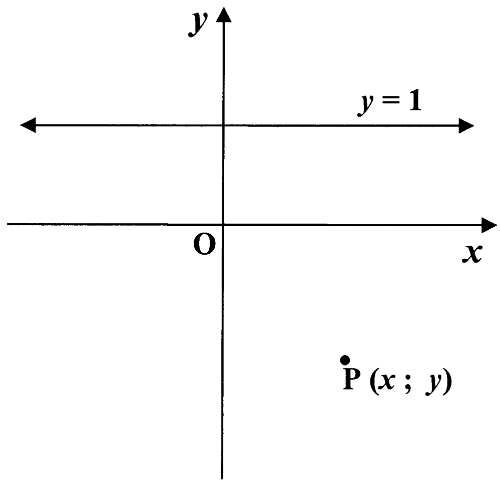
QUESTION 1/VRAAG 1



QUESTION 2.1/VRAAG 2.1



QUESTION 2.2/ VRAAG 2.2



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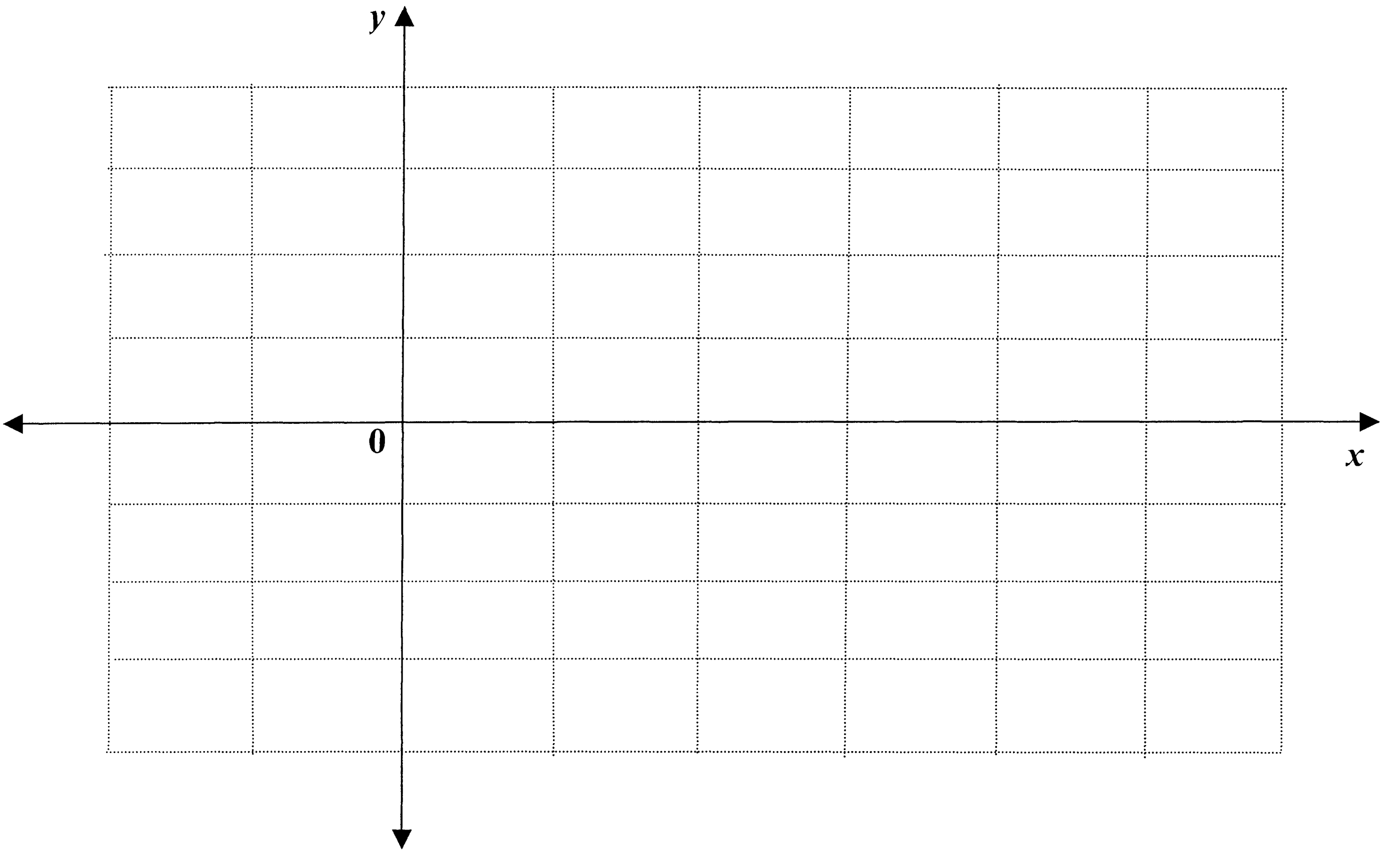
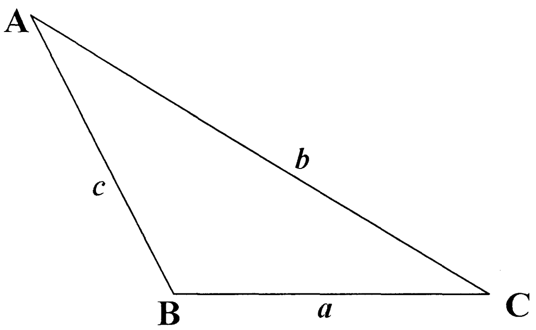
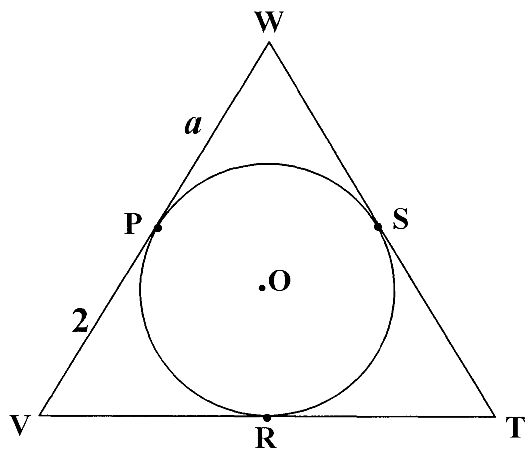
QUESTION 4.2/VRAAG 4.2



QUESTION 6. 1/VRAAG 6.1



QUESTION 6.2/VRAAG 6.2



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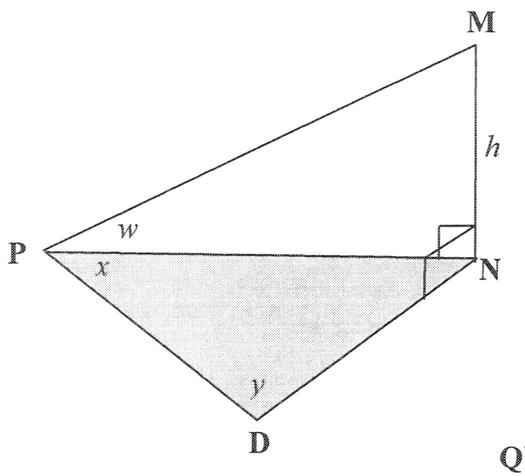
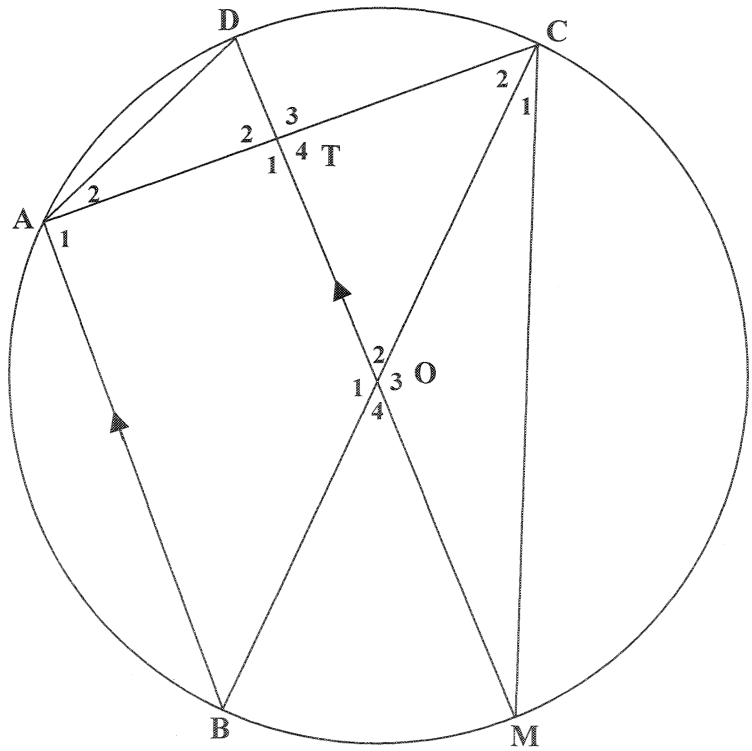
QUESTION 6.3/VRAAG 6.3



QUESTION 7 /VRAAG 7



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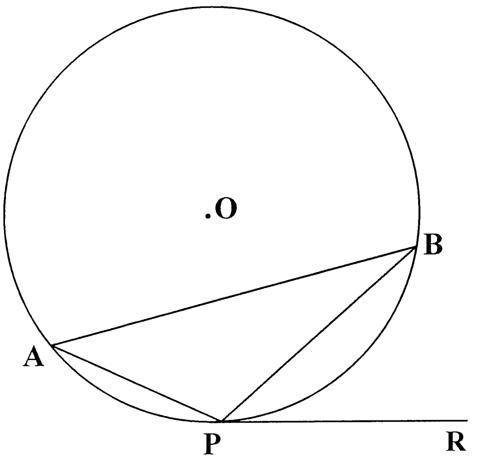
QUESTION 8.1/VRAAG 8.1



QUESTION 8.2/ VRAAG 8.2

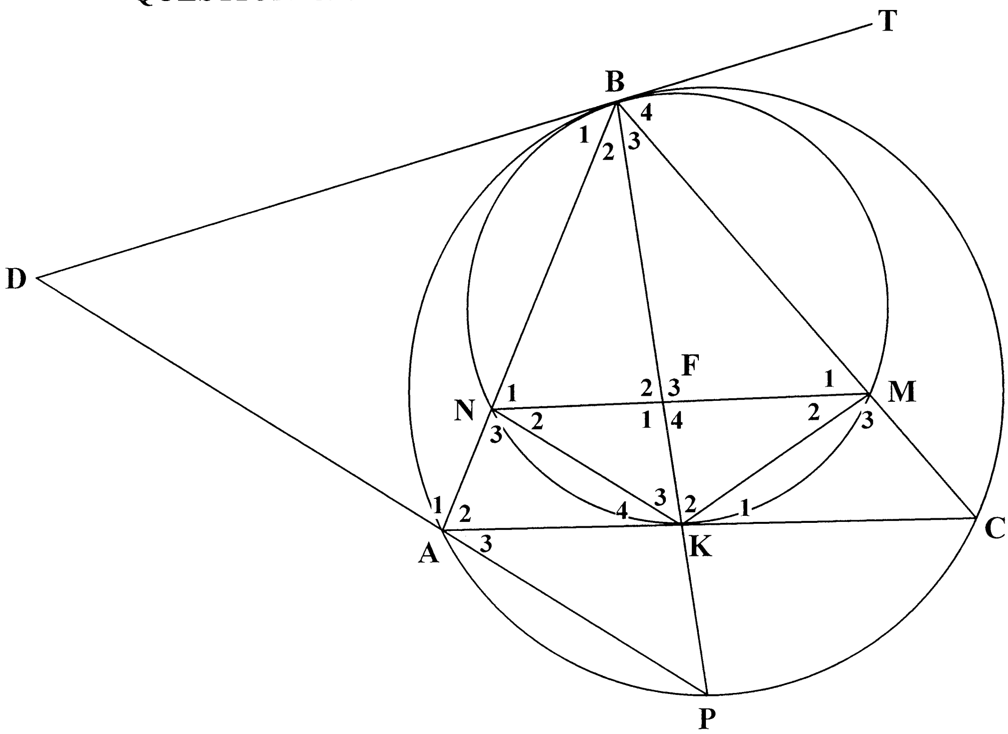


DIAGRAM SHEET/DIAGRAMVEL

EXAMINATION NUMBER
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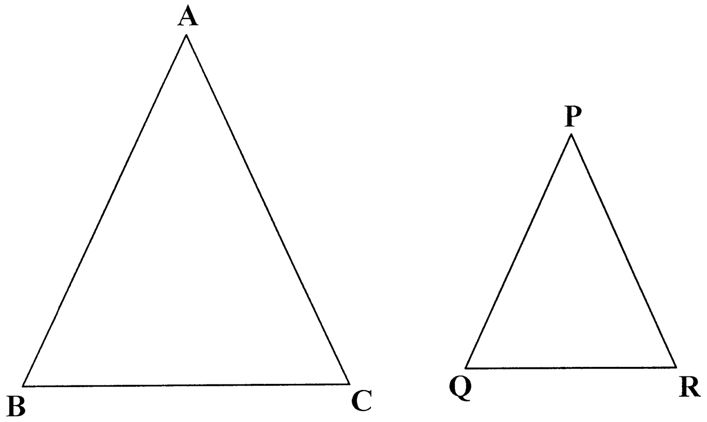
QUESTION 9.1/VRAAG 9.1



QUESTION 9.2/VRAAG 9.2