



DEPARTMENT OF EDUCATION REPUBLIC OF SOUTH AFRICA

DEPARTEMENT VAN ONDERWYS REPUBLIEK VAN SUID-AFRIKA

SENIOR CERTIFICATE EXAMINATION - 2005

SENIORSERTIFIKAAT-EKSAMEN - 2005

MATHEMATICS P1 : ALGEBRA

WISKUNDE V1 : ALGEBRA

STANDARD GRADE

STANDAARDGRAAD

FEBRUARY/MARCH 2005

FEBRUARIE/MAART 2005

301-2/1

MATHEMATICS SG: Paper 1

Algebra

Marks: 150



301 2 1 SG

**3 Hours
3 Ure**

**This question paper consists of 8 pages and 1 information sheet.
Hierdie vraestel bestaan uit 8 bladsye en 1 inligtingsblad.**



x05



INSTRUKSIES

Lees die volgende instruksies sorgvuldig deur voordat die vrae beantwoord word.

1. Hierdie vraestel bestaan uit **AGT** vrae. Beantwoord **AL** die vrae.
2. Toon duidelik **AL** die bewerkings, diagramme, grafieke, ensovoorts wat jy gebruik het om die antwoorde te bepaal.
3. 'n Goedgekeurde sakrekenaar (nie-programmeerbaar en nie-grafies) mag gebruik word, tensy anders vermeld.
4. Indien nodig, moet antwoorde tot **TWEE** desimale plekke afgerond word, tensy anders vermeld.
5. Grafiekpapier word **NIE** in hierdie vraestel benodig nie.
6. Nommer die antwoorde **PRESIES** soos die vrae genommer is.
7. Diagramme is nie noodwendig volgens skaal geteken nie.
8. Dit is tot jou eie voordeel om leesbaar te skryf en om die werk netjies aan te bied.
9. **'n Inligtingsblad met formules is ingesluit aan die einde van die vraestel.**

INSTRUCTIONS

Read the following instructions carefully before answering the questions.

1. This paper consists of **EIGHT** questions. Answer **ALL** the questions.
2. Clearly show **ALL** calculations, diagrams, graphs, et cetera you have used in determining the answers.
3. An approved calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to **TWO** decimal places, unless stated otherwise.
5. Graph paper is **NOT** required in this question paper.
6. Number the answers **EXACTLY** as the questions are numbered.
7. Diagrams are not necessarily drawn to scale.
8. It is in your own interest to write legibly and to present the work neatly.
9. **An information sheet with formulae is included at the end of this question paper.**

VRAAG 1

- 1.1 Gegee: $f(x) = x(x + 2) - 4$

Bepaal:

1.1.1 $f(-1)$ (2)

1.1.2 x as $f(x) = 0$ (Gee jou antwoord korrek tot TWEE desimale plekke.) (7)

- 1.2 Vir watter waardes van p sal die vergelyking nie-reële wortels hê:

$$3x^2 + 2x + 2 + p = 0 \quad (7)$$

- 1.3 Senami het die diskriminant van 'n kwadratiese vergelyking bereken en het die volgende bepaal:

$$\Delta = (2k - 9)(2k - 1)$$

Beskryf die aard van die wortels van die vergelyking as $k = 6$. (3)

- 1.4 Los op vir x en y as hulle die volgende vergelykings gelyktydig bevredig:

$$\begin{aligned} y + 7 &= 2x \\ x^2 + xy + y^2 &= 21 \end{aligned} \quad (8)$$

[27]

VRAAG 2

- 2.1 As $f(x) = ax^3 - 5x^2 - 2x + 5$ gedeel word deur $(x - 2)$, is die res -3 .

Vind die waarde van a . (5)

- 2.2 Los die volgende vergelyking op:

$$2x^3 - 3x^2 - 5x + 6 = 0 \quad (6)$$

[11]

QUESTION 1

- 1.1 Given: $f(x) = x(x + 2) - 4$

Determine:

1.1.1 $f(-1)$ (2)

1.1.2 x if $f(x) = 0$ (Give your answer correct to **TWO** decimal places.) (7)

- 1.2 For which values of p will the following equation have non-real roots:

$$3x^2 + 2x + 2 + p = 0 \quad (7)$$

- 1.3 Senami calculated the discriminant of a quadratic equation and determined the following:

$$\Delta = (2k - 9)(2k - 1)$$

Describe the nature of the roots of the equation if $k = 6$. (3)

- 1.4 Solve for x and y if they satisfy the following equations simultaneously:

$$\begin{aligned} y + 7 &= 2x \\ x^2 + xy + y^2 &= 21 \end{aligned} \quad (8)$$

[27]

QUESTION 2

- 2.1 If $f(x) = ax^3 - 5x^2 - 2x + 5$ is divided by $(x - 2)$, the remainder is -3 . Find the value of a . (5)

- 2.2 Solve the following equation:

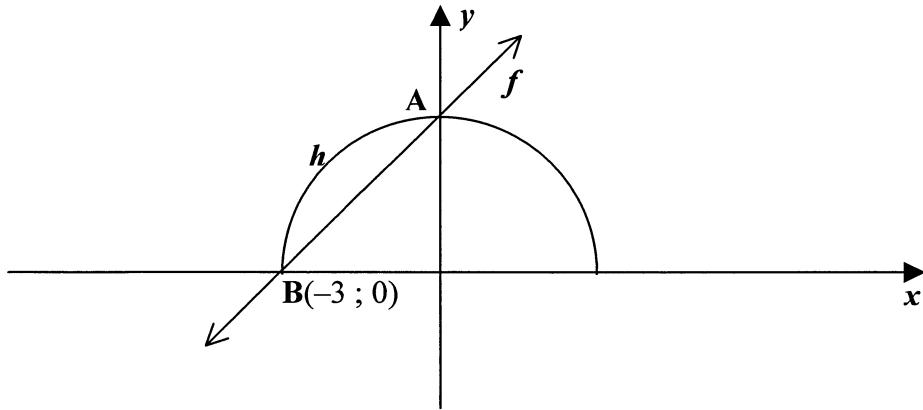
$$2x^3 - 3x^2 - 5x + 6 = 0 \quad (6)$$

[11]

VRAAG 3

3.1 Gegee: $f(x) = -x^2 + 4x - 3$

- 3.1.1 Bereken die x - en y -afsnitte van die grafiek van f . (4)
- 3.1.2 Bereken die koördinate van die draaipunt van f . (5)
- 3.1.3 Wat is die grootste moontlike waarde van $-x^2 + 4x - 3$? (1)
- 3.1.4 Teken 'n netjiese sketsgrafiek van f . Toon die koördinate van die afsnitte met die asse en die draaipunt van die grafiek. (5)
- 3.1.5 Teken op dieselfde assestelsel '**n reguitlyn**' wat jou sal help om die vergelyking $-x^2 + 4x - 3 = -1$ op te los. (2)
- 3.1.6 Gebruik die grafiek om die waardes van x te bepaal waarvoor $-x^2 + 4x - 3 \geq 0$. (2)
- 3.2 Die grafieke van 'n reguitlyn f en die halfsirkel h word hieronder geskets. A en $B(-3 ; 0)$ is afsnitte van die grafieke met die koördinaatasse. (2)



- 3.2.1 Bepaal die vergelyking van h . (2)
- 3.2.2 Bepaal die vergelyking van f . (2)

[25]

QUESTION 3

3.1

Given: $f(x) = -x^2 + 4x - 3$

3.1.1

Calculate the x - and y -intercepts of the graph of f . (4)

3.1.2

Calculate the co-ordinates of the turning point of f . (5)

3.1.3

What is the largest possible value of $-x^2 + 4x - 3$? (1)

3.1.4

Make a neat sketch graph of f . Indicate the co-ordinates of the intercepts on the axes and of the turning point of the graph. (5)

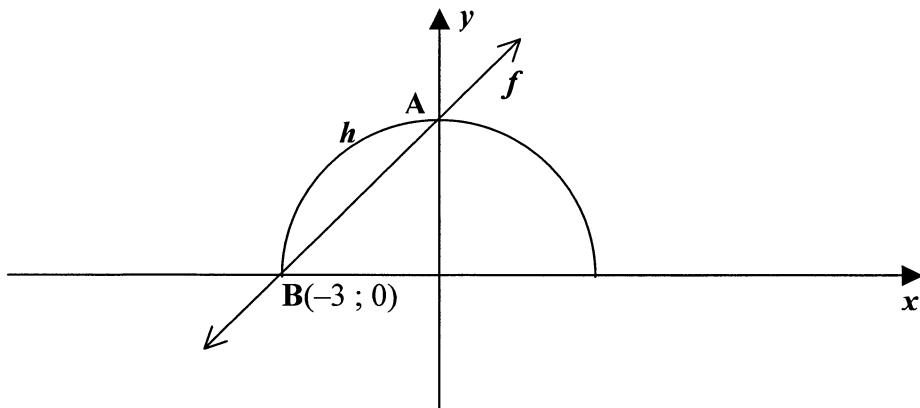
3.1.5

On the same system of axes, **draw a straight line** which will help you to solve the equation $-x^2 + 4x - 3 = -1$. (2)

3.1.6

Use the graph to determine the values of x for which $-x^2 + 4x - 3 \geq 0$. (2)

3.2

The graphs of a straight line f and the semi-circle h are sketched below. A and $B(-3 ; 0)$ are intercepts of the graphs on the co-ordinate axes. (2)

3.2.1

Determine the equation of h . (2)

3.2.2

Determine the equation of f . (2)

[25]

VRAAG 4

4.1 **Sonder om 'n sakrekenaar te gebruik,** bereken die waarde van elk van die volgende in sy eenvoudigste vorm:

4.1.1 $(3^{-1} + 2^{-1})^{-1}$ (3)

4.1.2 $\frac{9^{n-1} \cdot 27^{3-2n}}{81^{2-n}}$ (6)

4.1.3 $2\log 2 + \log 25$ (3)

4.1.4 $\frac{\sqrt{98} - \sqrt{8}}{\sqrt{50}}$ (4)

4.2 Los op vir x , **sonder om 'n sakrekenaar te gebruik:**

4.2.1 $2x^{\frac{3}{4}} = 16$ (4)

4.2.2 $3^x - 3^{x-2} = 24$ (6)

4.2.3 $\log x = \frac{\log 625}{\log 25}$ (4)
[30]

QUESTION 4

- 4.1 **Without using a calculator**, calculate the value of each of the following in its simplest form:

4.1.1 $(3^{-1} + 2^{-1})^{-1}$ (3)

4.1.2 $\frac{9^{n-1} \cdot 27^{3-2n}}{81^{2-n}}$ (6)

4.1.3 $2\log 2 + \log 25$ (3)

4.1.4 $\frac{\sqrt{98} - \sqrt{8}}{\sqrt{50}}$ (4)

- 4.2 Solve for x , **without using a calculator**:

4.2.1 $2x^{\frac{3}{4}} = 16$ (4)

4.2.2 $3^x - 3^{x-2} = 24$ (6)

4.2.3 $\log x = \frac{\log 625}{\log 25}$ (4)
[30]

VRAAG 5

5.1 Die volgende rekenkundige ry word gegee: $-1; 6; 13; \dots$

Bepaal:

5.1.1 Die 49^{ste} term (3)

5.1.2 Die som van die eerste 87 terme van die ry (3)

5.2 $20; 16; \dots$ is 'n meetkundige reeks.

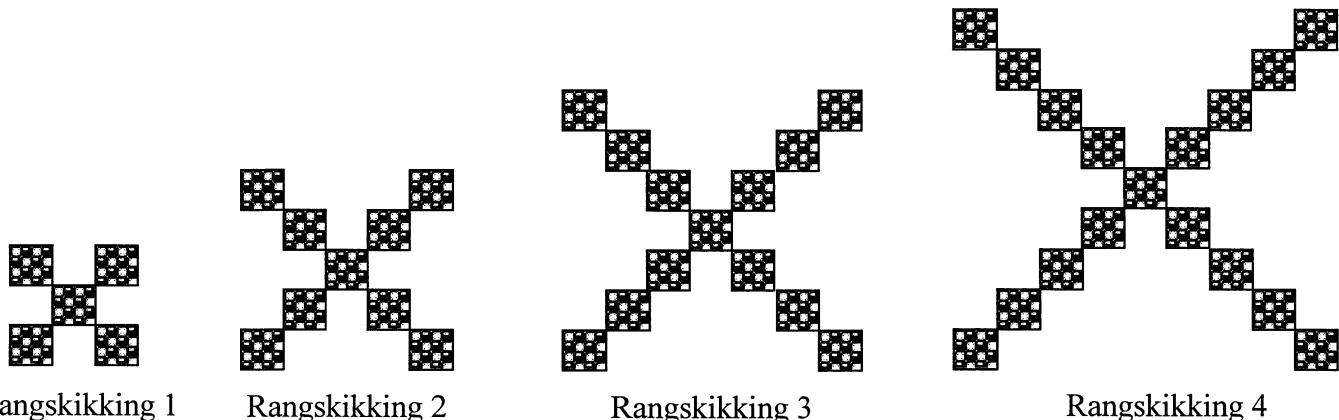
Bereken die som van die eerste tien terme. (4)

5.3 Die volgende is drie opeenvolgende terme van 'n meetkundige ry:

$$3x - 2 ; 2x + 2 ; 4x + 1 \quad (x \text{ is 'n natuurlike getal})$$

Bereken die waarde van x . (6)

5.4 Teëls word soos hieronder getoon gerangskik. Die eerste rangskikking het 5 teëls, die tweede rangskikking het 9 teëls, die derde rangskikking het 13 teëls en die vierde rangskikking het 17 teëls. Die rangskikkings volg hierdie patroon.



Bepaal, in terme van n , 'n formule vir die aantal teëls in die n^{de} -rangskikking. (3)

[19]

QUESTION 5

5.1 The following arithmetic sequence is given: $-1; 6; 13; \dots$

Determine:

5.1.1 The 49^{th} term (3)

5.1.2 The sum of the first 87 terms (3)

5.2 $20; 16; \dots$ is a geometric sequence.

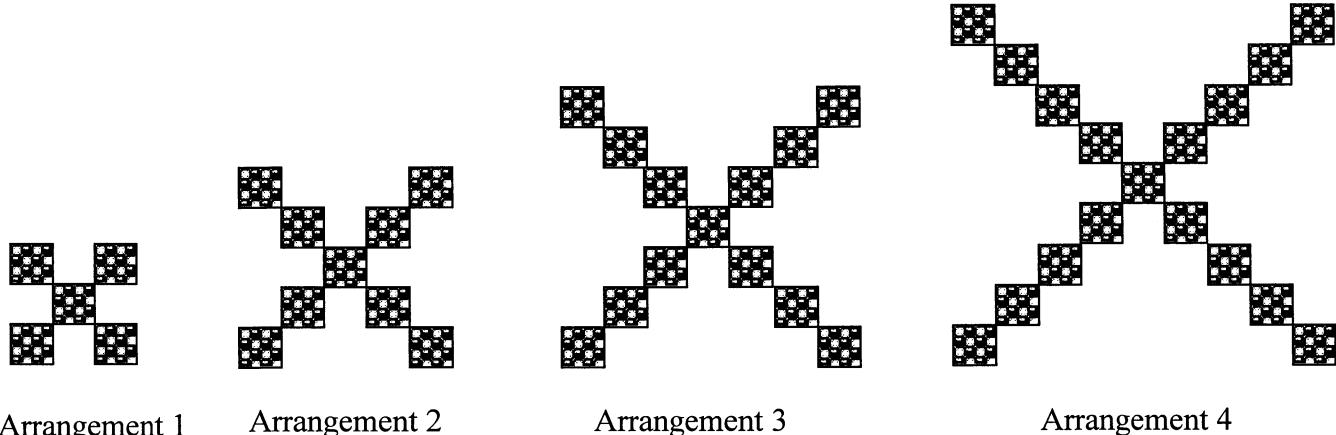
Calculate the sum of the first ten terms. (4)

5.3 The following are three consecutive terms of a geometric sequence:

$$3x - 2; 2x + 2; 4x + 1 \quad (x \text{ is a natural number})$$

Calculate the value of x . (6)

5.4 Tiles are arranged as shown below. The first arrangement has 5 tiles, the second arrangement has 9 tiles, the third arrangement has 13 tiles and the fourth arrangement has 17 tiles. The arrangements continue in this pattern.



Derive, in terms of n , a formula for the number of tiles in the n^{th} arrangement. (3)

[19]

VRAAG 6

Lees die advertensie hieronder sorgvuldig deur en beantwoord dan die vrae wat volg.
Rond jou antwoord af korrek tot **TWEE** desimale plekke.



In 3 jaar se tyd sal Thembu R12 500 vir 'n vakansie benodig. Hoeveel geld moet hy nou in **SOUTH AFRICAN BANK** belê as hy daardie bedrag aan die einde van die 3 jaar wil onttrek? [5]

VRAAG 7

7.1 Gebruik **eerste beginsels** om die afgeleide van $f(x)$ te bepaal as $f(x) = 4x^2$ (5)

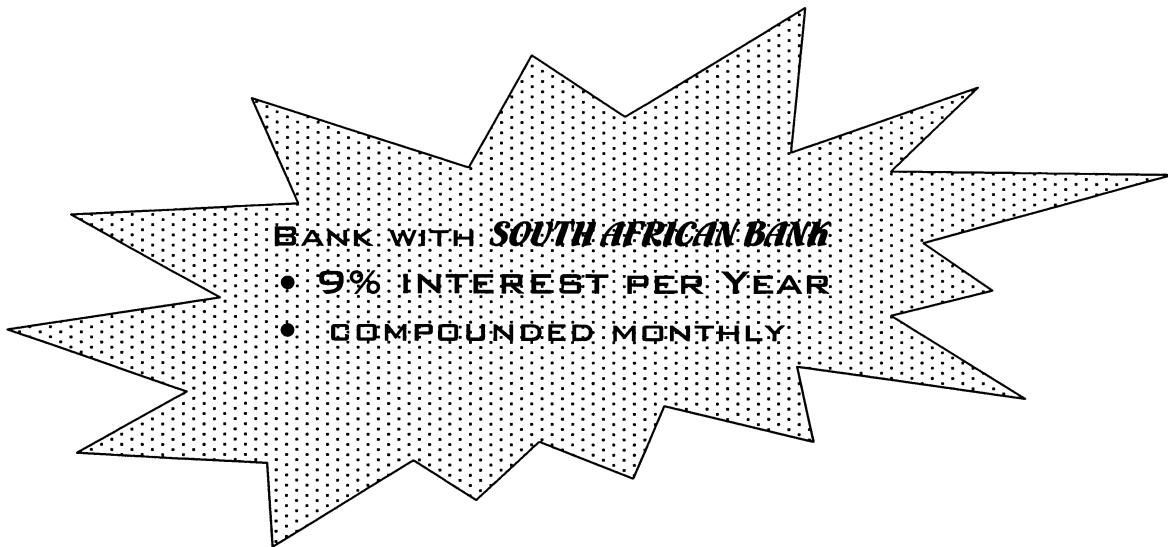
7.2 Gebruik differensiasie-reëls om die afgeleides van die volgende funksies te bepaal:

$$7.2.1 \quad y = 4x^3 + 12x^2 + 9x \quad (3)$$

$$7.2.2 \quad f(x) = -\frac{1}{x^4} + \sqrt{x} \quad (4)$$

QUESTION 6

Read the advertisement below carefully and then answer the question that follows.
Round off your answer correct to **TWO** decimal places.



In 3 years' time Thembu needs R12 500 for a vacation. How much money does he need to deposit now into **SOUTH AFRICAN BANK** in order to be able to withdraw that amount at the end of the 3 years? [5]

QUESTION 7

7.1 Use **first principles** to determine the derivative of $f(x)$ if $f(x) = 4x^2$ (5)

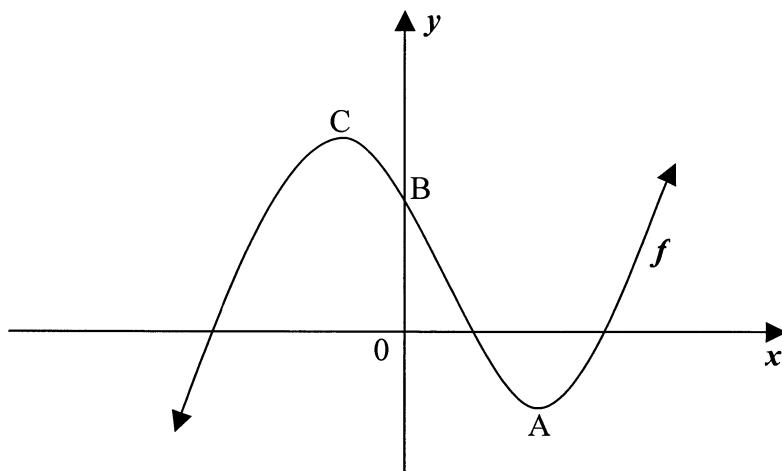
7.2 Use differentiation rules to determine the derivatives of the following functions:

7.2.1 $y = 4x^3 + 12x^2 + 9x$ (3)

7.2.2 $f(x) = -\frac{1}{x^4} + \sqrt{x}$ (4)

- 7.3 Die grafiek hieronder, nie volgens skaal geteken nie, verteenwoordig die funksie gegee deur:

$$f(x) = x^3 - 3x^2 - 9x + 25$$



- 7.3.1 Bepaal die koördinate van die draaipunte A en C. (8)
- 7.3.2 Gebruik die grafiek om vir x op te los as $f'(x) \leq 0$. (2)
- 7.3.3 Bepaal die gradiënt van die raaklyn aan die grafiek van f by die y -afsnit B. (5)
[27]

VRAAG 8

'n Bioloog beweer dat as 'n sekere tipe antibakterie by 'n kultuur gevoeg word, dan word die aantal bakterieë teenwoordig in die kultuur gegee deur die formule waar $b(t)$, in miljoene, die aantal bakterieë teenwoordig is by tyd t , gemeet in ure:

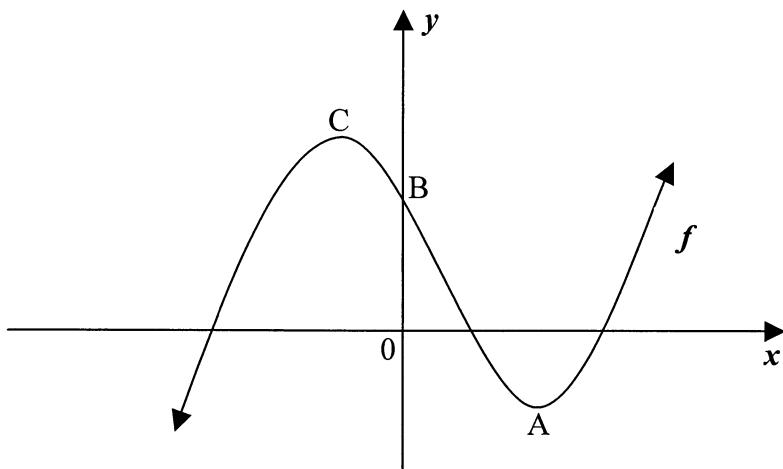
$$b(t) = -4t^2 + 60t + 1500$$

- 8.1 Hoeveel bakterieë was aan die begin teenwoordig? (3)
- 8.2 Op watter oomblik was die maksimum aantal bakterieë teenwoordig? (3)
[6]

TOTAAL: 150

- 7.3 The graph below, not drawn to scale, represents the function given by:

$$f(x) = x^3 - 3x^2 - 9x + 25$$



- 7.3.1 Determine the co-ordinates of the turning points A and C. (8)
- 7.3.2 Use the graph to solve for x if $f'(x) \leq 0$. (2)
- 7.3.3 Determine the gradient of the tangent to the graph of f at the y -intercept B. (5)
- [27]

QUESTION 8

A biologist states that when a certain type of antibacterium is introduced into a culture of bacteria, the number of bacteria present is given by the formula where $b(t)$, in millions, is the number of bacteria present at time t , measured in hours:

$$b(t) = -4t^2 + 60t + 1500$$

- 8.1 How many bacteria were present at the beginning? (3)
- 8.2 At what moment was the maximum number of bacteria present? (3)
- [6]

TOTAL: 150

Information Sheet (HG and SG)
Inligtingsblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n - 1)d \quad S_n = \frac{n}{2} (a + l) \quad S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$T_n = a \cdot r^{n-1} \quad S_n = \frac{a(1 - r^n)}{1 - r} \quad S_n = \frac{a(r^n - 1)}{r - 1} \quad S_\infty = \frac{a}{1 - r}$$

$$A = P \left(1 + \frac{r}{100} \right)^n \quad A = P \left(1 - \frac{r}{100} \right)^n$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$x^2 + y^2 = r^2$$

$$(x - p)^2 + (y - q)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

Information Sheet (HG and SG)
Inligtingsblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n - 1)d \quad S_n = \frac{n}{2} (a + l) \quad S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$T_n = a \cdot r^{n-1} \quad S_n = \frac{a(1 - r^n)}{1 - r} \quad S_n = \frac{a(r^n - 1)}{r - 1} \quad S_\infty = \frac{a}{1 - r}$$

$$A = P \left(1 + \frac{r}{100} \right)^n \quad A = P \left(1 - \frac{r}{100} \right)^n$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2} \right)$$

$$x^2 + y^2 = r^2$$

$$(x - p)^2 + (y - q)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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