

MATHAMATICS P2 HG

ANALYTICAL GEOMETRY		[25]
QUESTION 1		
1.1	$\frac{x-1}{2} = 2 \quad \text{and} \quad \frac{y+0}{2} = 2$ $x-1 = 4 \quad \therefore y = 4 \quad \checkmark_A$ $\therefore x = 5 \quad \checkmark_A$ $\therefore D(5; 4) \quad (2)$	
1.2	$m_{AD} = \tan \beta = \frac{4-0}{5+1} = \frac{4}{6} = \frac{2}{3} \quad \checkmark_M \quad \checkmark_{CA}$ $\beta = 33,7^\circ \quad \checkmark_{CA}$ $m_{CD} = \tan \alpha = \frac{4+2}{5-2} = \frac{6}{3} = 2 \quad \checkmark_{CA}$ $\alpha = 63,4^\circ \quad \checkmark_{CA}$ $\theta = 63,4^\circ - 33,7^\circ$ $= 29,7^\circ \quad \checkmark_{CA} \quad (6)$ <p>OR</p> $m_{AE} = \tan \beta = \frac{2-0}{2+1} = \frac{2}{3} \quad \checkmark_M \quad \checkmark_{CA}$	
1.3	$3AB = DC$ $9[(x+1)^2 + (y+0)^2] = [(5-2)^2 + (4+2)^2] \quad \checkmark_M \quad \checkmark_A$ $9[(x+1)^2 + y^2] = 9 + 36$ $9[(x+1)^2 - y^2] = 45 \quad \dots\dots(1)$ $(x+1)^2 + y^2 = 5 \quad \checkmark_A$ $AB \parallel DC \quad \therefore \frac{y-0}{x+1} = 2 \quad \checkmark_{CA}$ $y = 2x + 2 \quad \dots\dots(2) \quad \checkmark_{CA}$ <p>Subst. (2) into (1):</p> $(x+1)^2 + (2x+2)^2 = 5 \quad \checkmark_{CA}$ $x^2 + 2x + 1 + 4x^2 + 8x + 4 = 5$ $5x^2 + 10x = 0 \quad \checkmark_{CA}$ $5x(x+2) = 0$ $x = 0 \text{ or } x = -2 \quad \checkmark_{CA}$ <p>NA</p> <p>If $x = -2, y = -2$</p> $\therefore B(-2; -2) \quad \checkmark_{CA} \quad (9)$	
1.4	<p>Eq. of perp. bisector of BC: $x = 0$</p> <p>Midpt of AC: $(\frac{-1+2}{2}; \frac{0-2}{2}) \quad \checkmark_{CA}$</p> $= (\frac{1}{2}; -1)$ $m_{AC} = \frac{0+2}{-1-2} = -\frac{2}{3} \quad \checkmark_A$ $\therefore m_{\text{perp.bisector}} = \frac{3}{2} \quad \checkmark_{CA}$ <p>Eq. perp. bisector: $y + 1 = \frac{3}{2}(x - \frac{1}{2}) \quad \checkmark_{CA}$</p> $y = \frac{3}{2}x - \frac{3}{4} - 1$ $= \frac{3}{2}x - \frac{7}{4} \quad \checkmark_{CA}$ <p>H: For $x = 0$: $y = \frac{3}{2}(0) - \frac{7}{4} = -1\frac{3}{4} \quad \checkmark_{CA}$</p> $\therefore H(0; -\frac{7}{4}) \quad (8)$	<p>Eq. of AB: $y - 0 = 2(x + 1)$</p> $y = 2x + 2 \quad \checkmark_{CA} \quad \checkmark_A \quad \checkmark_A$ <p>Midpt of AB: $(\frac{-1-2}{2}; \frac{0-2}{2}) = (-\frac{3}{2}; -1)$</p> $m_{AB} = 2 \quad \checkmark_A$ $\therefore m_{\text{perp.bisector}} = -\frac{1}{2} \quad \checkmark_{CA}$ <p>Eq. perp. bisector: $y + 1 = -\frac{1}{2}(x + \frac{3}{2}) \quad \checkmark_{CA}$</p> $y = -\frac{1}{2}x - \frac{7}{4} \quad \checkmark_{CA}$ <p>H: $-\frac{1}{2}x - \frac{7}{4} = \frac{3}{2}x - \frac{7}{4}$</p> $-2x - 7 = 6x - 7$ $-8x = 0$ $x = 0 \quad \text{and} \quad y = -1\frac{3}{4} \quad \checkmark_{CA}$ $\therefore H(0; -\frac{7}{4}) \quad (8)$

QUESTION 2		[22]
2.1.1	$\begin{aligned} NP^2 &= (2 - 4)^2 + (3 - 5)^2 \\ &= 4 + 4 \\ &= 8 \quad \checkmark_{CA} \end{aligned}$ <p>Equation of circle N: $(x - 2)^2 + (y - 3)^2 = 8$ \checkmark_{CA}</p> <p style="text-align: right;">(4)</p>	
2.1.2	$\begin{aligned} m_{PN} &= \frac{5-3}{4-2} = \frac{2}{2} = 1 \quad \checkmark_A \\ \therefore m_{PT} &= -1 \quad \checkmark_A \\ \text{Equation PT: } y - 5 &= -1(x - 4) \quad \checkmark_{CA} \\ y &= -x + 9 \quad \checkmark_{CA} \\ \text{x-intercept: } x &= 9 \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad \checkmark_{CA} \\ \therefore T(9; 0) & \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad \checkmark_{CA} \end{aligned}$ <p style="text-align: right;">$\checkmark_{CA} \quad \checkmark_M$</p>	<p>OR</p> <p>T lies on x-axis: $(t; 0)$</p> $\begin{aligned} NP^2 + PT^2 &= NT^2 \quad \checkmark_A \\ 8 + [(5)^2 + (4-t)^2] &= (3-0)^2 + (2-t)^2 \quad \checkmark_A \\ 8 + 25 + 16 - 8t + t^2 &= 9 + 4 - 4t + t^2 \quad \checkmark_{CA} \\ 4t &= 36 \\ t &= 9 \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad \checkmark_{CA} \end{aligned}$ <p>$\therefore T(9; 0)$</p> <p style="text-align: right;">(5)</p>
2.1.3	$\begin{aligned} r^2 &= (9-4)^2 + (-5)^2 = 50 \\ \text{Area of circle} &= \pi r^2 \quad \checkmark_M \\ &= \pi(50) \quad \checkmark_{CA} \\ &= 157 \text{ units}^2 \quad \checkmark_{CA} \end{aligned}$ <p style="text-align: right;">(5)</p>	
2.2.1	$\begin{aligned} m_{PQ} &= \frac{y-4}{x-2} \quad \checkmark_M \quad \text{and} \quad m_{PR} = \frac{y+4}{x+4} \quad \checkmark_A \\ \therefore \frac{y-4}{x-2} \cdot \frac{y+4}{x+4} &= -1 \quad \checkmark_M \\ y^2 - 16 &= -(x^2 + 2x - 8) \quad \checkmark_{CA} \\ y^2 &= -x^2 - 2x + 24 \quad \checkmark_{CA} \end{aligned}$ <p style="text-align: right;">(5)</p>	<p>OR</p> $\begin{aligned} \text{Midpt QR} &= \left(\frac{2-4}{2}; \frac{4-4}{2} \right) \quad \checkmark_M \\ &= (-1; 0) \quad \checkmark_A \\ r &= \sqrt{(2+1)^2 + (4)^2} \\ &= \sqrt{25} = 5 \quad \checkmark_{CA} \\ \therefore (x+1)^2 + (y)^2 &= 25 \quad \checkmark_{CA} \quad \checkmark_M \end{aligned}$
2.2.2	$\begin{aligned} &\checkmark_{CA} \quad \checkmark_M \quad \checkmark_{CA} \\ \text{For } &\therefore -6 \leq x \leq 4 \end{aligned}$ <p style="text-align: right;">(3)</p>	

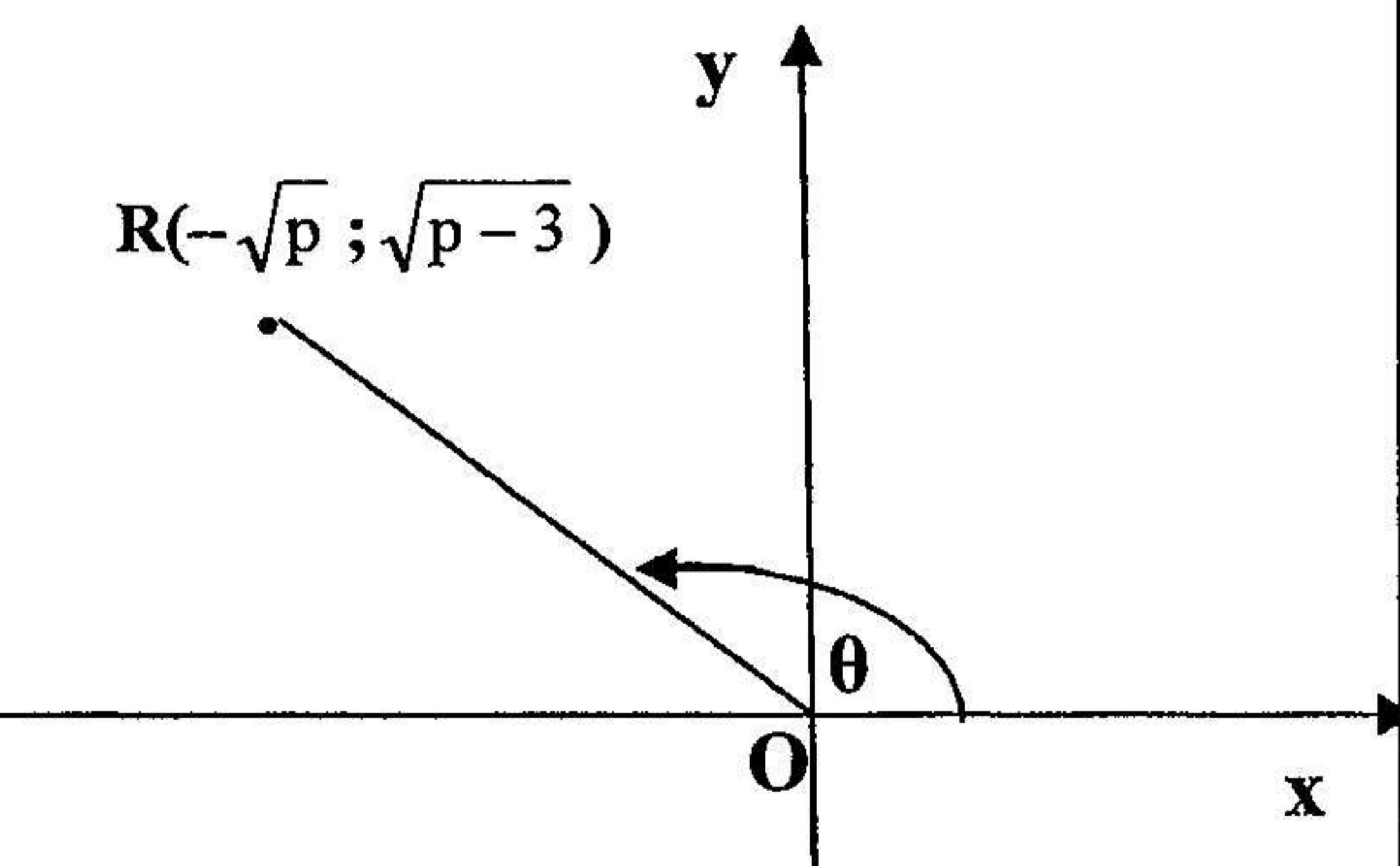
TRIGONOMETRY

QUESTION 3

[26]

3.1.
$$\begin{aligned} & \frac{\cos^2 208^\circ}{\tan 242^\circ \cdot \cos 28^\circ} \cdot \cosec 928^\circ \cdot \cot (-120^\circ) \cdot (\sec 30^\circ) \\ &= \frac{\cos^2 28^\circ}{\tan 62^\circ \cdot \cos 28^\circ} \cdot (-\cosec 28^\circ) \cdot \cot 60^\circ \cdot \frac{2}{\sqrt{3}} \cdot \sqrt{3} \\ &= -\frac{\cos 28^\circ}{\cot 28^\circ \cdot \sin 28^\circ} \cdot \frac{1}{\sqrt{3}} \cdot \frac{2}{\sqrt{3}} \\ &= -\frac{\cos 28^\circ}{\cos 28^\circ \cdot \sin 28^\circ} \cdot \frac{2}{3} \quad \checkmark_{CA} \\ &= -\frac{2}{3} \quad \checkmark_{CA} \end{aligned} \quad (10)$$

3.2.1
$$\begin{aligned} r^2 &= (-\sqrt{p})^2 + (\sqrt{p-3})^2 \quad \checkmark_M \\ &= p + p - 3 = 2p - 3 \\ r &= \sqrt{2p-3} \quad \checkmark_A \\ \sin^2 2 &= \left(\frac{\sqrt{p-3}}{\sqrt{2p-3}} \right)^2 \quad \checkmark_{CA} \\ &= \frac{p-3}{2p-3} \quad \checkmark_{CA} \end{aligned} \quad (4)$$



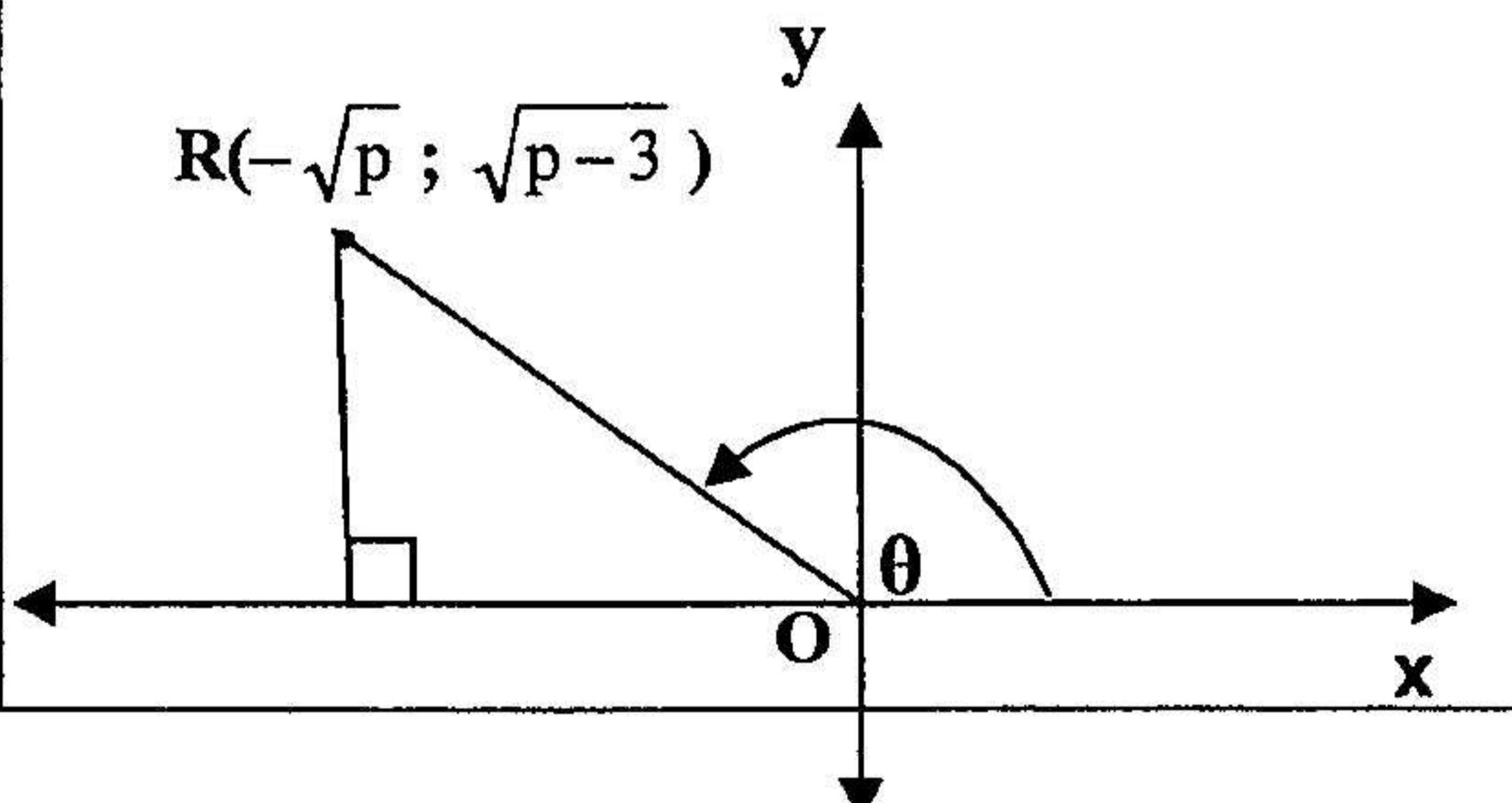
3.2.2 Def. for: $2p-3 \neq 0$ and $p-3 \geq 0$
 $p \neq \frac{3}{2}$ and $p \geq 3 \quad \checkmark_M$
 $\therefore p \geq 3 \quad (2)$



3.2.3
$$\begin{aligned} \sec \alpha &= -\sec \theta \quad \checkmark_A \\ &= -\left(\frac{\sqrt{2p-3}}{-\sqrt{p}} \right) \checkmark_{CA} \\ &= \frac{\sqrt{2p-3}}{\sqrt{p}} \quad \checkmark_{CA} \end{aligned} \quad (3)$$

3.2.4
$$\begin{aligned} \cos(\theta - 30^\circ) &= \cos \theta \cdot \cos 30^\circ + \sin \theta \cdot \sin 30^\circ \quad \checkmark_A \\ &= -\frac{\sqrt{6}}{3} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{3} \cdot \frac{1}{2} \\ &= -\frac{\sqrt{18}}{6} + \frac{\sqrt{3}}{6} \quad \checkmark_{CA} \\ &= \frac{-3\sqrt{2} + \sqrt{3}}{6} \quad \checkmark_A \end{aligned} \quad (7)$$

$$\begin{aligned} r^2 &= (\sqrt{6})^2 + (\sqrt{3})^2 \quad \text{or } r = \sqrt{2.6-3} \\ &= 6 + 3 = 9 \quad r = 3 \end{aligned}$$



QUESTION 4

[28]

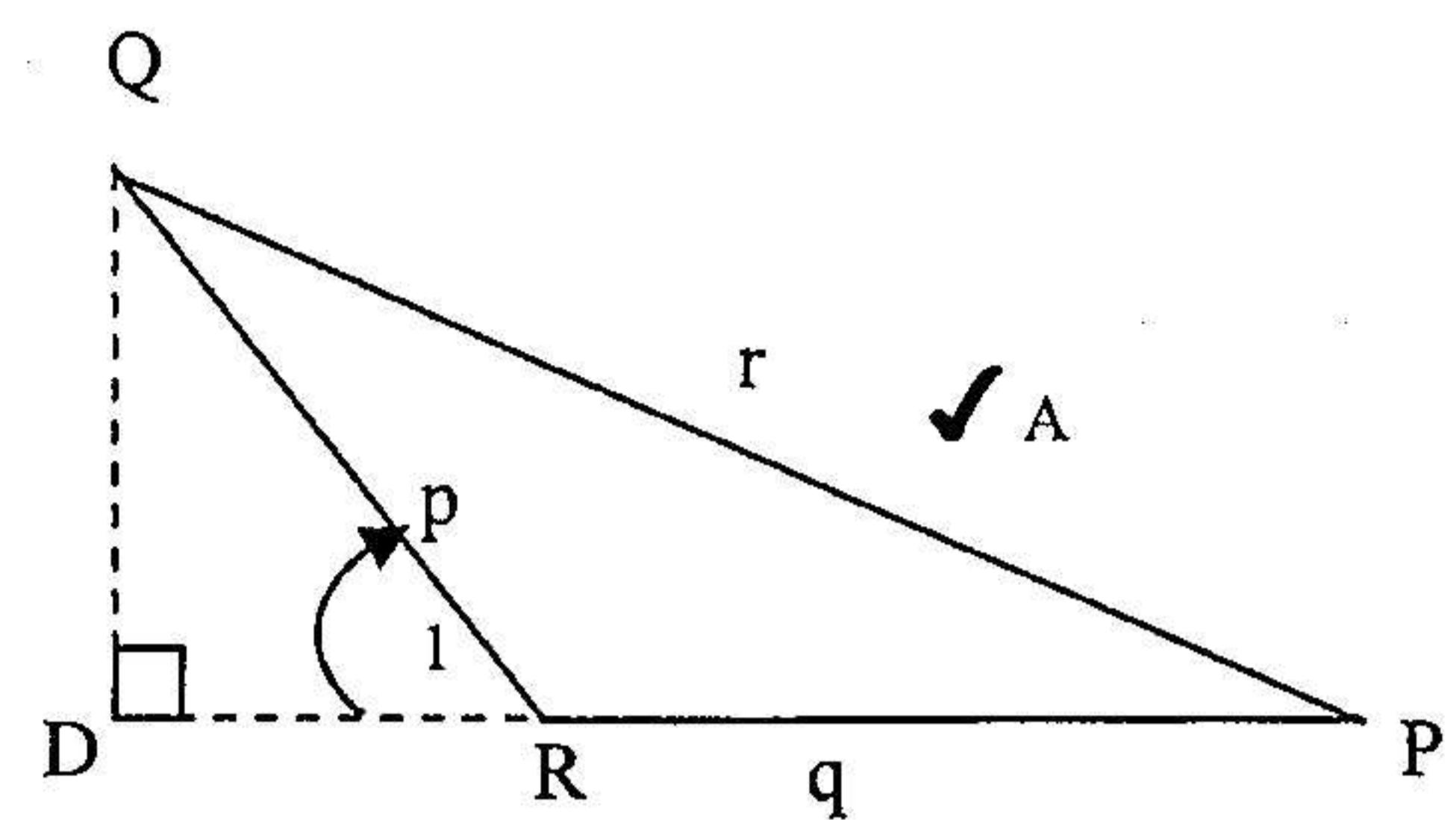
4.1.1	$\tan x = \sin 2x$ $\frac{\sin x}{\cos x} = 2 \sin x \cdot \cos x$ $\sin x = 2 \sin x \cdot \cos^2 x$ $0 = 2 \sin x \cdot \cos^2 x - \sin x$ $0 = \sin x (2 \cos^2 x - 1) \checkmark A$ $\therefore \sin x = 0 \text{ or } 2 \cos^2 x - 1 = 0 \checkmark CA$ $x = k \cdot 180^\circ \text{ or } \cos x = \pm \sqrt{\frac{1}{2}} \checkmark CA$ $k \in \mathbb{Z} \checkmark A \quad x = \pm 45^\circ + k \cdot 90^\circ \checkmark CA \quad (9)$	<p>OR could be obtained from the graph.</p> $0 = \sin x (2 \cos^2 x - 1) \checkmark A$ $0 = \sin x (\cos 2x) \checkmark A$ $\therefore \sin x = 0 \text{ or } 1 - 2 \sin^2 x = 0 \checkmark CA$ $x = k \cdot 180^\circ \text{ or } \sin x = \pm \sqrt{\frac{1}{2}} \checkmark CA$ $k \in \mathbb{Z} \checkmark A \quad x = \pm 45^\circ + k \cdot 90^\circ \checkmark CA$
4.1.2	$\checkmark CA \quad x = -180^\circ; \quad \checkmark CA \quad x = -135^\circ; \quad \checkmark CA \quad x = -45^\circ \quad (3)$	
4.2	<p>g: x-intercept $\checkmark A$ period $\checkmark A$ turning points $\checkmark A$ shape $\checkmark A$</p> <p>f: both asymptotes $\checkmark A$ $45^\circ, -135^\circ, -45^\circ \checkmark A$ intercepts $\checkmark A$ shape $\checkmark A$</p>	(8)
4.3.1	$\therefore \tan x \geq \sin 2x$ $x \in [-135^\circ; -90^\circ] \quad \text{or} \quad x \in [-45^\circ; 0^\circ]$ $\checkmark M \quad \checkmark A$	
4.3.2	$x \in (-45^\circ; 0^\circ) \quad \text{and} \quad x \in (0^\circ; 45^\circ)$ $\checkmark A \quad \checkmark M$	$\checkmark M \quad \checkmark A \quad \checkmark M$

Question 5		[11]
5.1	$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$ ✓ M	(1)
5.2.1	<p>LHS: $\begin{aligned} \frac{1 + \cos 2A}{\cos 2A} &= \frac{1 + (2 \cos^2 A - 1)}{\cos^2 A - \sin^2 A} \checkmark_A \\ &= \frac{2 \cos^2 A}{\cos^2 A - \sin^2 A} \checkmark_{CA} \end{aligned}$</p> <p>RHS: $\begin{aligned} \frac{\tan 2A}{\tan A} &= \frac{2 \tan A}{(1 - \tan^2 A) \cdot \tan A} \checkmark_M \\ &= \frac{2}{(1 - \frac{\sin^2 A}{\cos^2 A})} \checkmark_A \\ &= \frac{2 \cos^2 A}{\cos^2 A - \sin^2 A} = \text{LHS} \quad (5) \end{aligned}$</p>	<p><u>Alternate Method:</u></p> <p>LHS: $\begin{aligned} \frac{1 + \cos 2A}{\cos 2A} &= \frac{1 + 2 \cos^2 A - 1}{\cos^2 A - \sin^2 A} \checkmark_A \\ &= \frac{2 \cos^2 A}{\cos^2 A - \sin^2 A} \div \frac{\cos^2 A}{\cos^2 A} \checkmark_A \\ &= \frac{2}{1 - \tan^2 A} \times \frac{\tan A}{\tan A} \checkmark_{CA} \\ &= \frac{\tan 2A}{\tan A} = \text{RHS.} \quad (5) \end{aligned}$</p>
5.2.2	<p>Identity is undefined for: $\tan A = 0$ ✓ M or $\cos 2A = 0$ ✓ A $A = k \cdot 90^\circ$ ✓ A $2A = 90^\circ + k \cdot 180^\circ$ $k \in \mathbb{Z}$ ✓ A $A = 45^\circ + k \cdot 90^\circ$ ✓ CA (5)</p>	

QUESTION 6

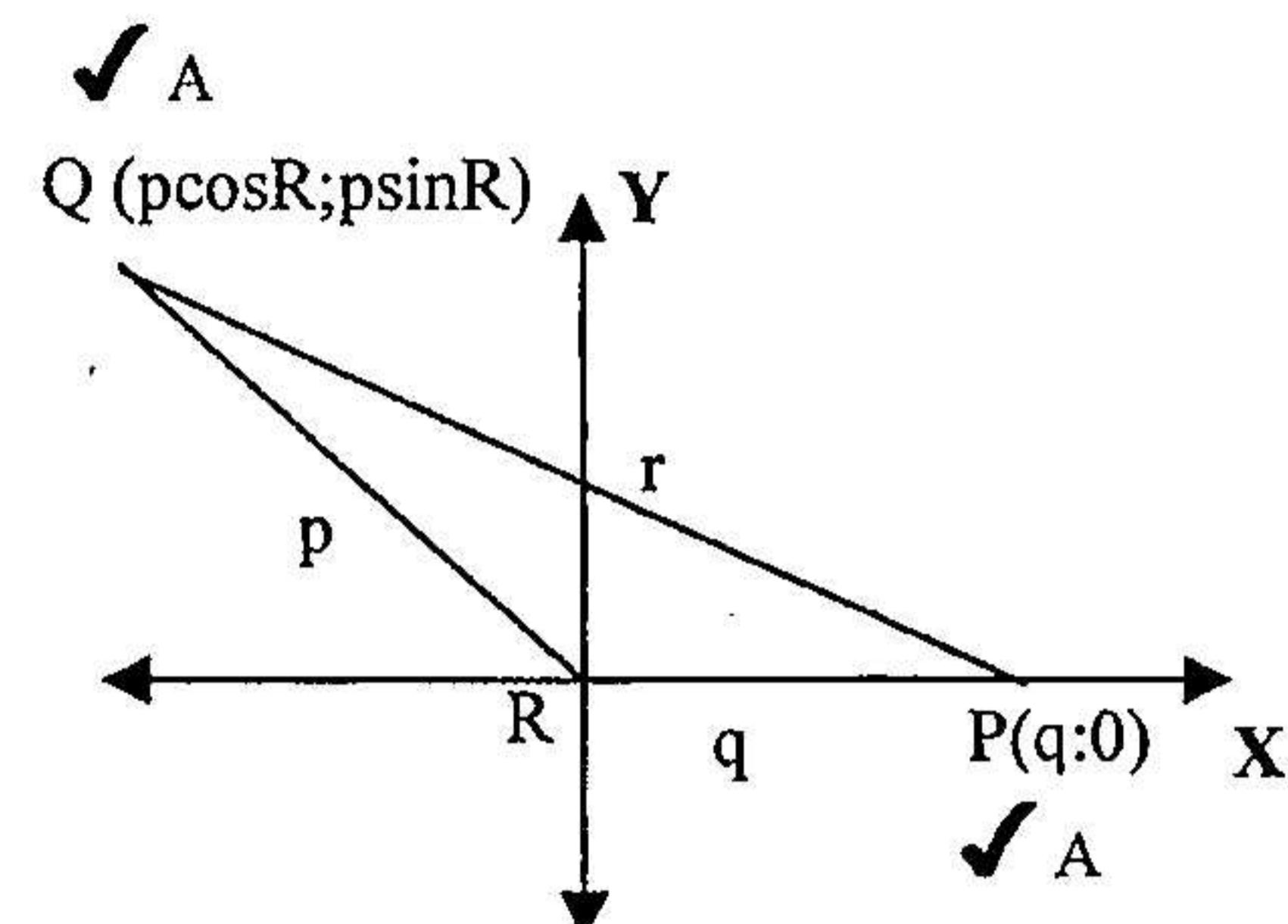
[22]

6.1	<p>Draw $QD \perp PR$ produced.</p> $\begin{aligned} r^2 &= DP^2 + QD^2 \quad \checkmark A \quad (\text{Pythagoras}) \\ &= (DR + q)^2 + QD^2 \quad \checkmark A \\ &= DR^2 + 2q \cdot DR + q^2 + QD^2 \\ &= p^2 + q^2 + 2q \cdot DR \quad \checkmark CA \quad (\text{Pythagoras}) \end{aligned}$ <p>but $\frac{DR}{p} = \cos R_1 = \cos(180^\circ - R) = -\cos R \quad \checkmark CA$</p> $\therefore DR = -p \cos R \quad \checkmark A$ $\therefore r^2 = p^2 + q^2 - 2pq \cos R \quad (6)$
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**OR**

Draw $\triangle PQR$ with R at the origin and RP on the x -axis. $\checkmark A$

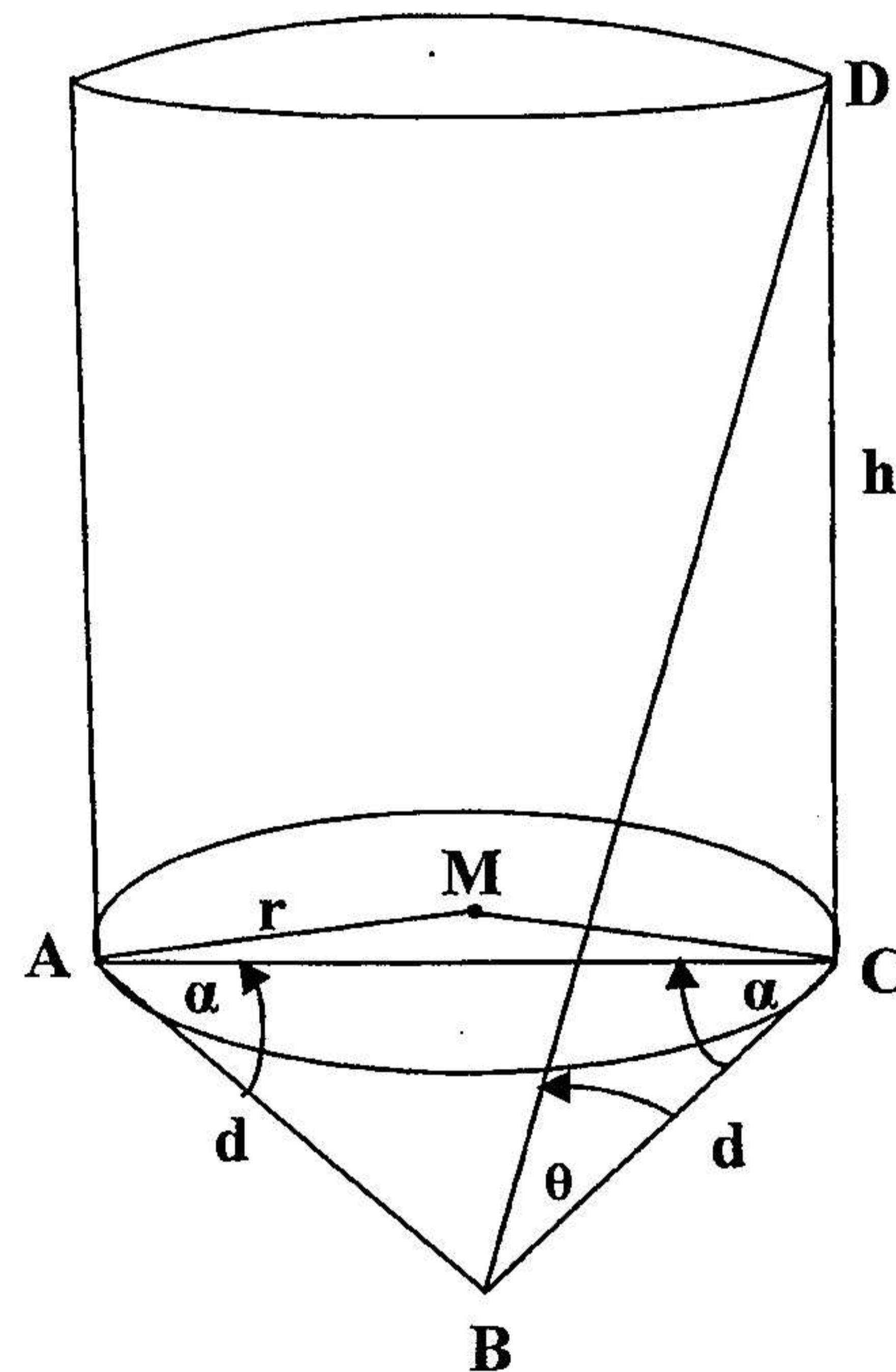
$$\begin{aligned} r^2 &= (p \cos R - q)^2 + (p \sin R)^2 \quad (\text{distance formula}) \quad \checkmark CA \\ &= p^2 \cos^2 R - 2pq \cos R + q^2 + p^2 \sin^2 R \quad \checkmark CA \\ &= p^2 (\cos^2 R + \sin^2 R) - 2pq \cos R + q^2 \quad \checkmark A \\ &= p^2 + q^2 - 2pq \cos R \quad (\cos^2 R + \sin^2 R = 1) \end{aligned}$$



6.2.1	$\begin{aligned} AC^2 &= d^2 + d^2 - 2d \cdot d \cdot \cos(180^\circ - 2\alpha) \quad \checkmark M \quad \checkmark A \\ &= 2d^2 + 2d^2 \cos 2\alpha \quad \checkmark A \\ &= d^2(2 + 2\cos 2\alpha) \quad \checkmark A \\ \therefore AC &= d \sqrt{2 + 2\cos 2\alpha} \end{aligned}$
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(4)

6.2.2	<p>In $\triangle DCB$: $\frac{d}{h} = \cot \theta$</p> $d = h \cot \theta \quad \checkmark M$ <p>In $\triangle ABC$ $\frac{AC}{\sin(180^\circ - 2\alpha)} = \frac{d}{\sin \alpha} \quad \checkmark M \quad \checkmark A$</p> $\begin{aligned} \therefore AC &= \frac{h \cot \theta \cdot \sin(180^\circ - 2\alpha)}{\sin \alpha} \\ &= \frac{h \cdot \cot \theta \cdot \sin 2\alpha}{\sin \alpha} \quad \checkmark CA \\ &= \frac{h \cdot \cot \theta \cdot 2 \sin \alpha \cdot \cos \alpha}{\sin \alpha} \quad \checkmark CA \\ &= 2h \cdot \cos \alpha \cdot \cot \theta \quad (5) \end{aligned}$
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**OR**

AMCB is a kite
 $\therefore AP = PC, \hat{P}_1 = 90^\circ \quad \checkmark M$

In $\triangle PBC$:

$$\frac{PC}{d} = \cos \alpha \quad \checkmark M$$

$$\therefore PC = d \cos \alpha \quad \checkmark A$$

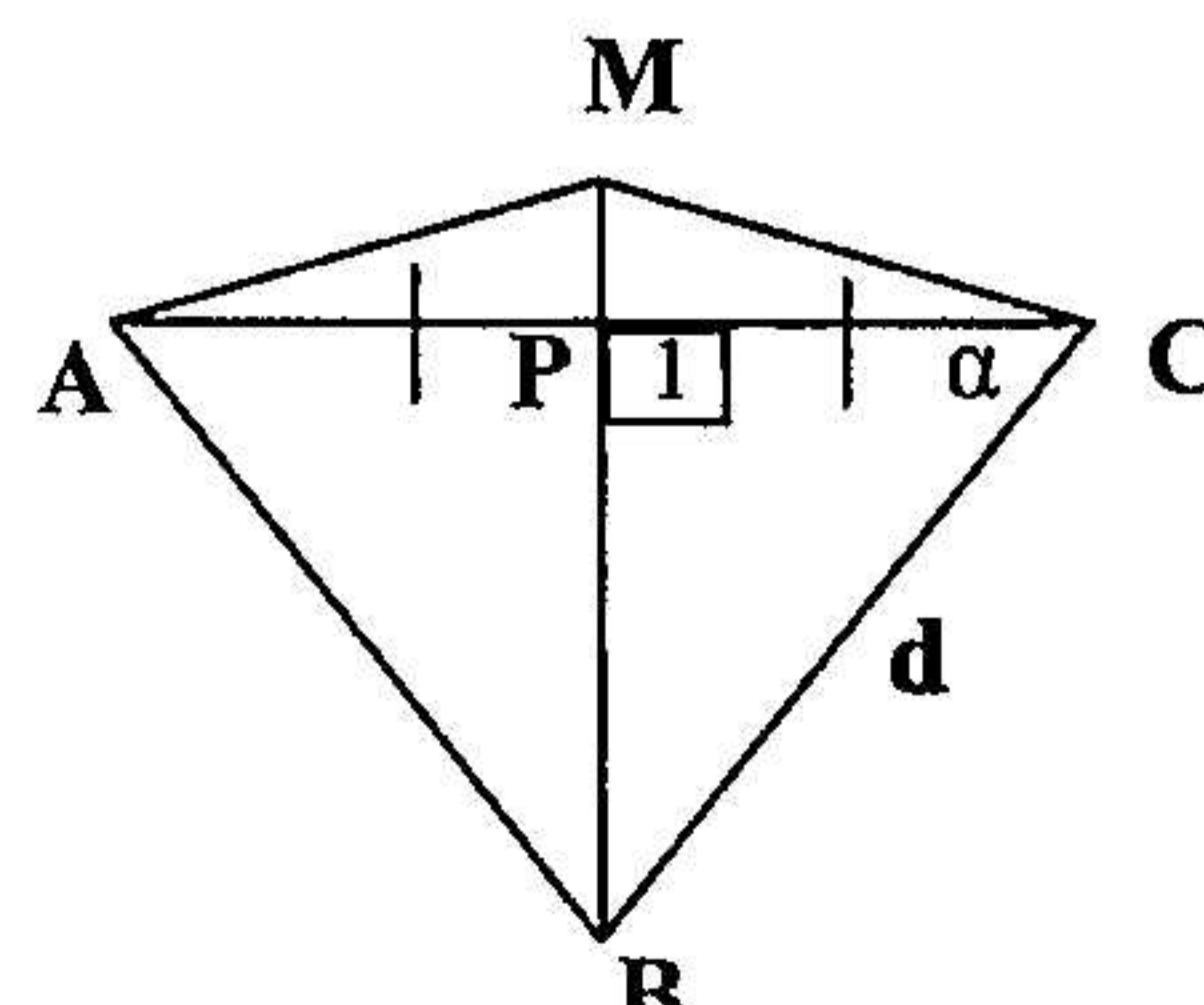
In $\triangle DCB$: $\frac{d}{h} = \cot \theta$

$$d = h \cot \theta \quad \checkmark CA$$

$$\therefore AC = 2 PC = 2 \cdot d \cos \alpha \quad \checkmark CA$$

$$= 2h \cdot \cos \alpha \cdot \cot \theta$$

(5)



6.2.3

AMCB is a kite. $\therefore \hat{P}_1 = 90^\circ$

$$\therefore \hat{B}_1 = 90^\circ - \alpha \quad \checkmark_M$$

$$\hat{M}C\hat{B} = 90^\circ \quad \checkmark_A \quad BC \text{ is a tangent}$$

$$\therefore \hat{M}_1 = 90^\circ - (90^\circ - \alpha)$$

$$= \alpha \quad \checkmark_A$$

$$\therefore \frac{r}{d} = \cot \alpha \quad \checkmark_M$$

$$r = d \cot \alpha \quad \checkmark_A$$

$$= 10,3 (\cot 54^\circ)$$

$$= 7,5 \text{ or } 7,48 \quad \checkmark_{CA}$$

$$\text{Volume} = \pi r^2 h$$

$$= \pi (10,3 \cot 54^\circ) (36) \text{ m}^3$$

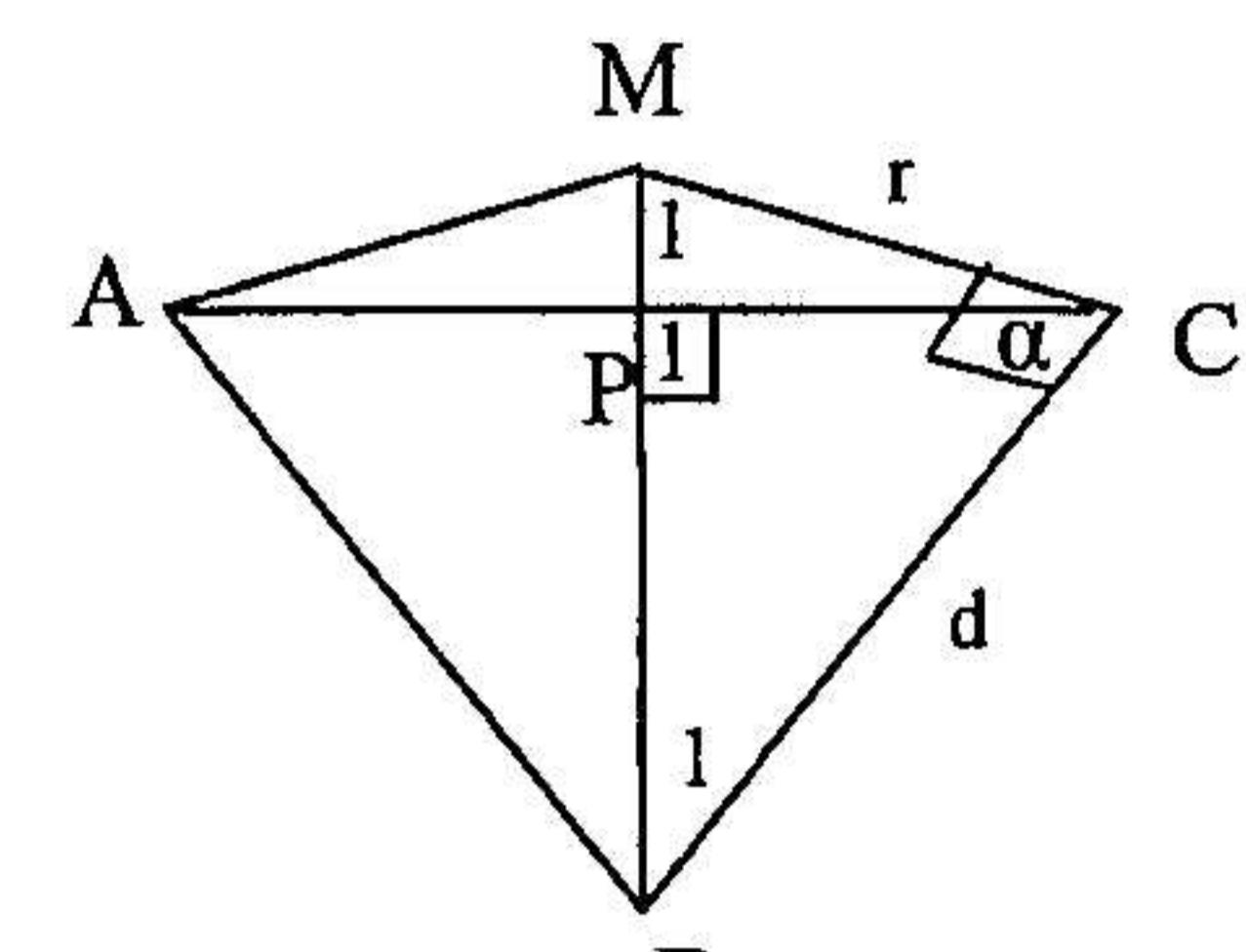
$$= 6334 \text{ m}^3 \quad \checkmark_{CA}$$

(7)

ORABCM is a cyclic quadrilateral \checkmark_M

$$(\hat{M}\hat{A}\hat{B} = \hat{M}\hat{C}\hat{B} = 90^\circ) \quad \checkmark_A$$

$$\therefore \hat{B}\hat{M}\hat{C} = \hat{M}_1 = \alpha \quad \checkmark_A \quad (\angle \text{s same segment})$$



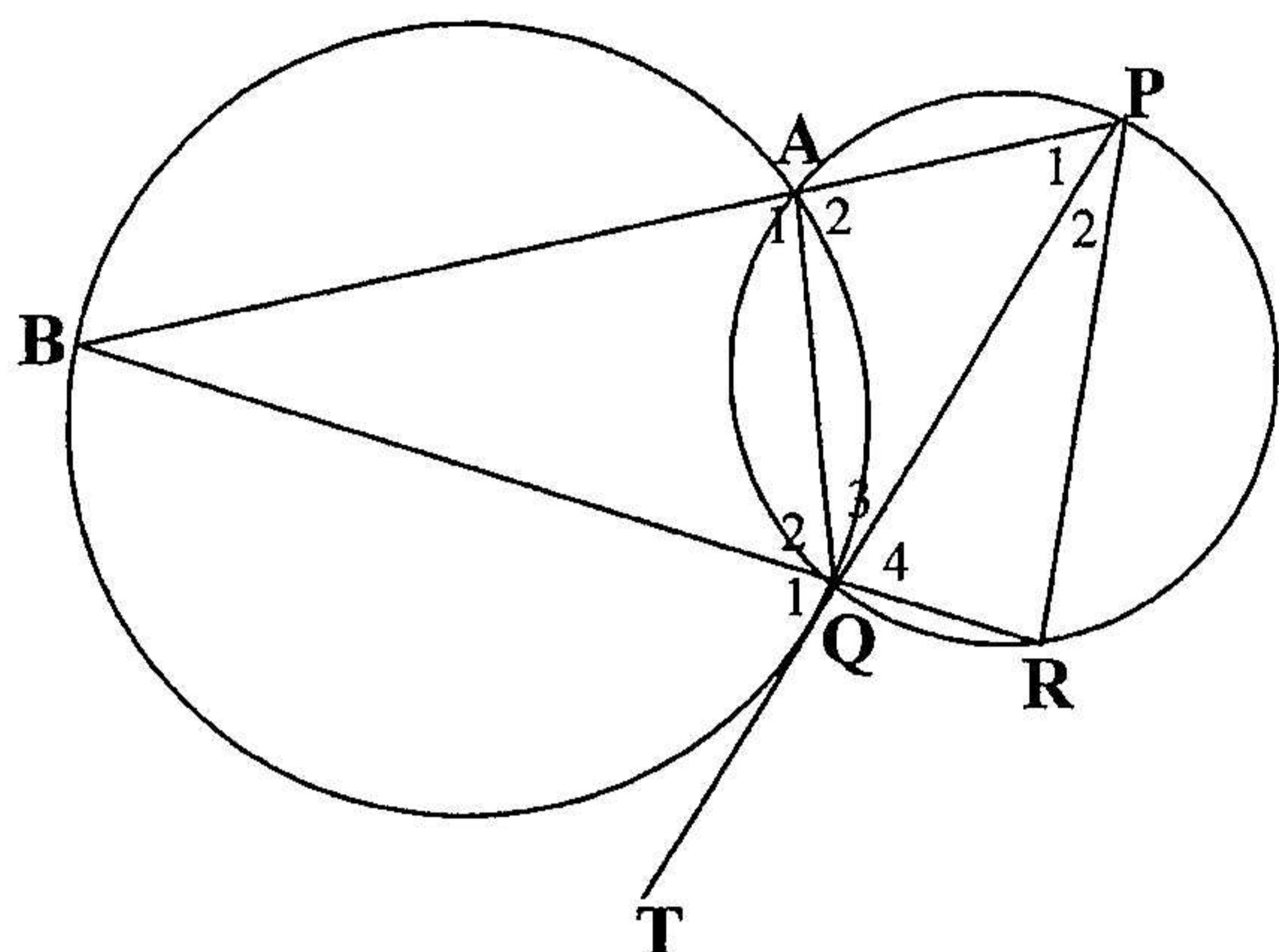
[if $r = 7,5$: volume = 6362;
if $r = 7,48$: volume = 6328]

QUESTION 7		[24]
7.1.1	<p>If a line is drawn through the end point of a chord making with the chord an angle equal to the angle in the alternate. ✓_M</p> <p>segment, then the line is a tangent to the circle. (2)</p>	
7.1.2	<p>Constr. Draw a tangent FP at F. ✓_M</p> <p>Proof:</p> <p>$\hat{D}FP = \hat{E}$ ✓_S (\angle between tan and chord) ✓_R</p> <p>But $\hat{DFG} = \hat{E}$ (Given)</p> <p>$\therefore \hat{DFP} = \hat{DFG}$ ✓_S</p> <p>and this is only true if FP and FG coincide.</p> <p>$\therefore FG$ is a tangent to circle FDE at F ✓_S</p> <p>(5)</p>	
7.2		
7.2.1	<p>$\hat{W}_1 = 90^\circ$ ✓_S (line from centre to midpt of chord) ✓_R</p> <p>But $\hat{S}_3 + \hat{S}_4 = 90^\circ$ ✓_S (radius \perp tangent) ✓_R</p> <p>$\hat{W}_1 + \hat{S}_3 + \hat{S}_4 = 180^\circ$</p> <p>$\therefore SB // RP$ (co-interior \angles) ✓_R</p> <p>(5)</p>	<p>or</p> <p>$\hat{W}_2 = 90^\circ$ (line from centre to midpt of chord) ✓_R</p> <p>But $\hat{S}_3 + \hat{S}_4 = 90^\circ$ ✓_S (radius \perp tangent) ✓_R</p> <p>$\therefore \hat{W}_2 = \hat{S}_3 + \hat{S}_4$</p> <p>$\therefore SB // RP$ (corresp. \angles equal) ✓_R</p> <p>(5)</p>

7.2.2		
7.2.2	<p>In $\triangle APS$ and $\triangle RWS$:</p> $\hat{P}_1 + \hat{P}_2 + \hat{P}_3 = 90^\circ \quad (\angle \text{ in semi-circle}) \checkmark \text{S/R}$ $= \hat{W}_4 \quad \checkmark \text{S/R} \quad (\text{line from centre to mdpt chord})$ $\hat{A} = \hat{R}_2 \quad (\angle \text{s same segment}) \checkmark \text{S/R}$ $\therefore \hat{S}_3 = \hat{S}_2 \quad (\text{sum } \angle \text{s of } \triangle)$ $\therefore \triangle APS \equiv \triangle RWS \quad (\angle \angle \angle) \checkmark \text{S/R}$ $\therefore \frac{PS}{AS} = \frac{WS}{RS} \quad \checkmark \text{S}$ <p>but $\triangle RWS \equiv \triangle PWS$ $(S\angle S) \quad \checkmark \text{S/R}$</p> $\therefore RS = PS \quad \checkmark \text{S}$ $\therefore \frac{RS}{AS} = \frac{WS}{RS} \quad \checkmark \text{S}$ $\therefore RS^2 = WS \cdot AS \quad (8)$	<p>OR</p> <p>In $\triangle APS$ and $\triangle PWS$:</p> $\hat{P}_1 + \hat{P}_2 + \hat{P}_3 = 90^\circ \quad (\angle \text{ in semi-circle}) \checkmark \text{S/R}$ $= \hat{W}_1 \quad \checkmark \text{S/R} \quad (\text{line from centre to mdpt chord})$ $\hat{S}_3 = \hat{S}_3 \quad (\text{Common}) \checkmark \text{S/R}$ $\hat{A} = \hat{P}_3 \quad (\text{sum } \angle \text{s of } \triangle)$ $\therefore \triangle APS \equiv \triangle PWS \quad (\angle \angle \angle) \checkmark \text{S/R}$ $\therefore \frac{PS}{WS} = \frac{AS}{PS} \quad \checkmark \text{S}$ <p>but $\triangle RWS \equiv \triangle PWS$ $(S\angle S) \quad \checkmark \text{S/R}$</p> $\therefore RS = PS \quad \checkmark \text{S}$ $\therefore \frac{RS}{AS} = \frac{WS}{RS} \quad \checkmark \text{S}$ $\therefore RS^2 = WS \cdot AS \quad (8)$ <p>OR</p> <p>In $\triangle APS$ and $\triangle PWS$:</p> $\hat{P}_1 + \hat{P}_2 + \hat{P}_3 = 90^\circ \quad (\angle \text{ in semi-circle})$ $= \hat{W}_1 \quad \checkmark \text{S/R} \quad (\text{line from centre to mdpt chord})$ $\therefore \triangle APS \equiv \triangle PWS \quad (\text{perp. from rt } \angle \text{ to hyp}) \checkmark \text{R}$ $\therefore \frac{PS}{WS} = \frac{AS}{PS} \quad \checkmark \text{S}$ <p>but $\triangle RWS \equiv \triangle PWS$ $(S\angle S) \quad \checkmark \text{S/R}$</p> $\therefore RS = PS \quad \checkmark \text{S}$ $\therefore \frac{RS}{AS} = \frac{WS}{RS} \quad \checkmark \text{S}$ $\therefore RS^2 = WS \cdot AS \quad (8)$
7.2.3	$RS^2 = RW^2 + WS^2 \quad \checkmark \text{S} \quad (\text{Pythagoras}) \checkmark \text{R}$ <p>and $RS^2 = WS \cdot AS$ (Proven 7.2.2)</p> $\therefore WS \cdot AS = RW^2 + WS^2 \quad \checkmark \text{S}$ $\therefore AS = \frac{RW^2}{WS} + WS \quad \checkmark \text{S} \quad (4)$	

QUESTION 8**[18]**

8.1



- 8.1.1 $\hat{R} = \hat{A}_1$ ✓ S (ext. \angle of cyclic quad.) ✓ R
 $\hat{Q}_4 = \hat{Q}_1$ (vert. opp. \angle s) ✓ S/R
 But $\hat{A}_1 = \hat{Q}_1$ (\angle between tang and chord) ✓ R
 $\therefore \hat{R} = \hat{Q}_4$ ✓ S
 $\therefore PQ = PR$ (5)

8.1.2 In ΔPAQ and ΔPQB

$$\begin{aligned}\hat{P}_1 &= \hat{P}_1 \quad (\text{common}) \quad \checkmark_{S/R} \\ \hat{Q}_3 &= \hat{B} \quad (\angle \text{ between tan/ chord}) \quad \checkmark_{S/R} \\ \hat{A}_2 &= \hat{Q}_2 + \hat{Q}_3 \quad (\text{sum } \angle's \Delta) \quad \checkmark_{S/R} \\ \therefore \Delta PAQ &\parallel\!\!\!\Delta PQB \quad \checkmark_{S} \quad (\angle, \angle, \angle) \\ \frac{PA}{PQ} &= \frac{PQ}{PB} \quad \checkmark_{S} \\ \text{but } PQ &= PR \quad \checkmark_{S} \quad (\text{proven 8.1.1}) \\ \therefore \frac{PA}{PR} &= \frac{PR}{PB} \quad \checkmark_{S}\end{aligned}$$

which forms the
geometrical sequence PA, PR and PB . (7)

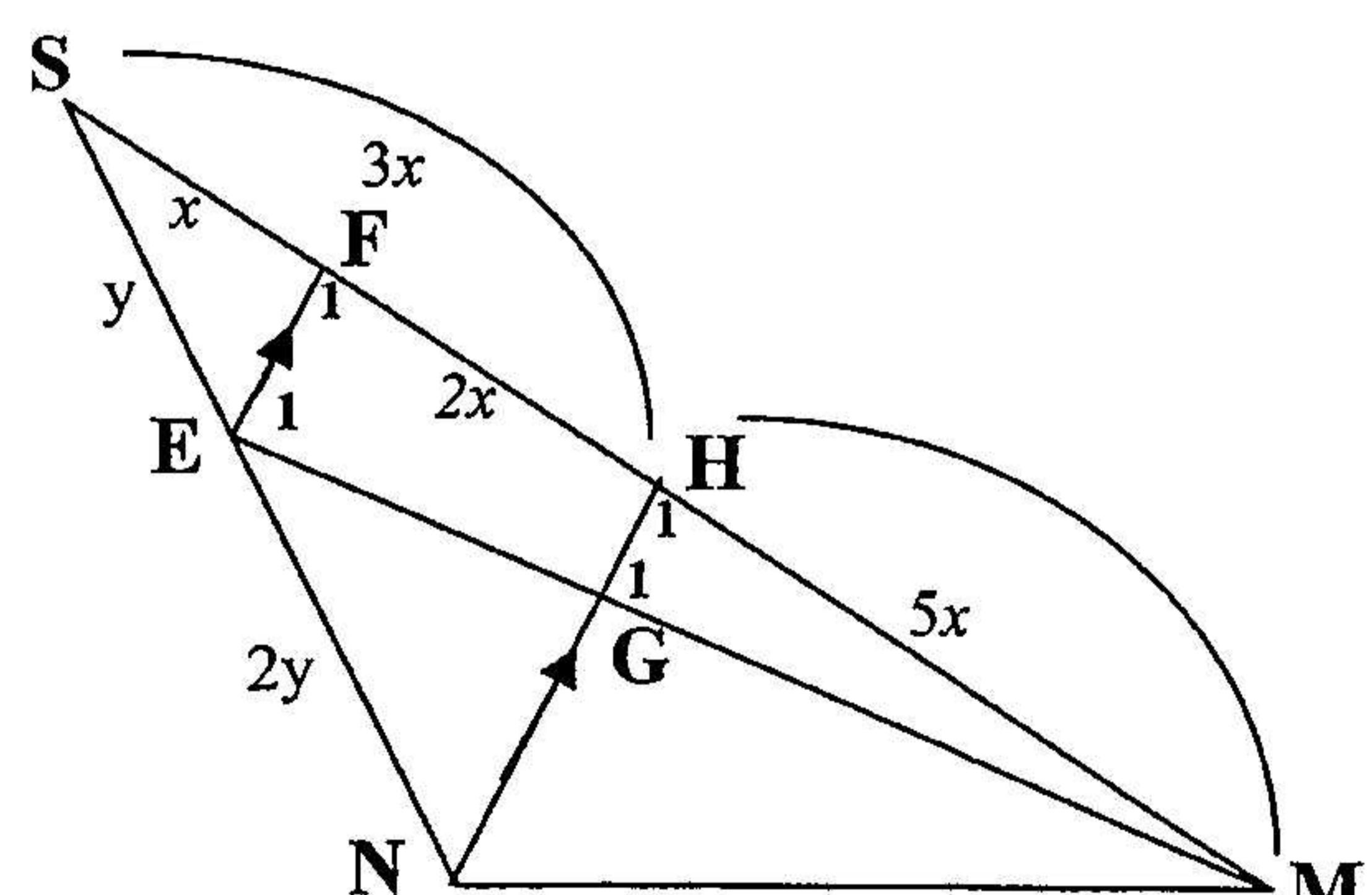
8.2 In ΔSNH :

$$\begin{aligned}\frac{SF}{FH} &= \frac{SE}{EN} \quad (\text{line } // \text{ one side of } \Delta) \quad \checkmark_{S/R} \\ &= \frac{1}{2} \quad \checkmark_{S} \quad (\text{given } 2SE = EN)\end{aligned}$$

Let SF = x, then FH = 2x and HM = 5x

In ΔGHM and ΔEFM :

$$\begin{aligned}\hat{GMH} &= \hat{EMF} \quad (\text{common}) \quad \checkmark_{S/R} \\ \hat{G}_1 &= \hat{E}_1 \quad (\text{corresp. } \angle\text{s } EF // NH) \quad \checkmark_{S/R} \\ \therefore \hat{H}_1 &= \hat{F}_1 \quad (\text{sum } \angle\text{s } \Delta / \text{corr. } \angle\text{s } EF // NH) \\ \therefore \Delta GHM &\parallel\!\!\!\Delta EFM \quad (\angle \angle \angle) \quad \checkmark_{S/R} \\ \therefore \frac{GH}{EF} &= \frac{HM}{FM} = \frac{5x}{7x} = \frac{5}{7} \quad \checkmark_{S}\end{aligned}\quad (6)$$



QUESTION 9

[24]