DEPARTMENT OF EDUCATION
REPUBLIC OF SOUTH AFRICA

DEPARTEMENT VAN ONDERWYS
REPUBLIEK VAN SUID-AFRIKA

SENIOR CERTIFICATE EXAMINATION - 2005
SENIORSERTIFIKAAT-EKSAMEN - 2005

MATHEMATICS P1 : ALGEBRA
WISKUNDE V1 : ALGEBRA

HIGHER GRADE
HOËR GRAAD

FEBRUARY/MARCH 2005
FEBRUARIE/MAART 2005

301-1/1

MATHEMATICS HG: Paper 1
Algebra

301 1 1 HG

Marks: 200
Punte : 200

3 Hours
3 Ure

This question paper consists of 8 pages, 1 sheet of graph paper and 1 information sheet.
Hierdie vraestel bestaan uit 8 bladsye, 1 vel grafiekpapier en 1 inligtingsblad.

X05

INSTRUKSIES AAN KANDIDATE

Lees die volgende instruksies sorgvuldig deur voordat die vrae beantwoord word:

1. Hierdie vraestel bestaan uit **NEGE** vrae. Beantwoord **AL** die vrae.

2. Toon duidelik **AL** die berekeninge, diagramme, grafieke, ensovoorts wat jy gebruik het om die antwoorde te bepaal.

3. 'n Goedgekeurde sakrekenaar (nie-programmeerbaar en nie-grafies) mag gebruik word, tensy anders vermeld.

4. Indien nodig, moet antwoorde tot **TWEE** desimale plekke afgerond word, tensy anders vermeld.

5. Die aangehegte grafiekpapier moet slegs vir **VRAAG 9** gebruik word. Maak dit los van die vraestel, vul jou eksamen- en sentrumnommer daarop in en plaas dit **VOOR** in die antwoordeboek.

6. Nommer die antwoorde **PRESIES** soos die vrae genommer is.

7. Diagramme is nie noodwendig volgens skaal geteken nie.

8. Dit is tot jou eie voordeel om leesbaar te skryf en om die werk netjies aan te bied.

9. **'n Inligtingsblad met formules is ingesluit aan die einde van die vraestel.**

INSTRUCTIONS TO CANDIDATES

Read the following instructions carefully before answering the questions:

1. This question paper consists of **NINE** questions. Answer **ALL** the questions.

2. Clearly show **ALL** calculations, diagrams, graphs, et cetera you have used in determining the answers.

3. An approved calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to **TWO** decimal places, unless stated otherwise.

5. The attached sheet of graph paper must be used only for **QUESTION 9**. Detach it from your question paper, fill in your examination number and centre number and insert it in the **FRONT** of the answer book.

6. Number the answers **EXACTLY** as the questions are numbered.

7. Diagrams are not necessarily drawn to scale.

8. It is in your own interest to write legibly and to present the work neatly.

9. **An information sheet with formulae is included at the end of the question paper.**

VRAAG 1

1.1 Gegee: $k+5=\frac{14}{k}$

1.1.1 Los op vir *k*. (3)

1.1.2 Los vervolgens vir x op as $\sqrt{x+5}+5=\frac{14}{\sqrt{x+5}}$. (3)

1.2 Los op vir x:

1.2.1 $|5-x|=9$ (3)

1.2.2 $\frac{2x-2}{x-3}>3$ (5)

1.3 Los vir x en y gelyktydig op as:

$\frac{1}{x}+\frac{1}{y}=3$ en

$x-y=\frac{1}{2}$ (8)

[22]

VRAAG 2

2.1 Gegee: $4x-5=k(x^2-1)$
Bepaal die waardes van k waarvoor die vergelyking gelyke wortels sal hê. (6)

2.2 Bewys dat die wortels van die vergelyking $x^2+p=(p+1)x$ rasionaal is vir alle rasionale waardes van *p*. (5)

[11]

VRAAG 3

3.1 Vind die konstantes p en r sodat die polinoom x^3+px+r 'n res van -9 laat as dit deur $(x+1)$ gedeel word en 'n res van -1 as dit deur $(x-1)$ gedeel word. (6)

3.2 Vind die waardes van x waarvoor $3-7x+5x^2-x^3=0$. (6)

[12]

QUESTION 1

1.1 Given: $k + 5 = \frac{14}{k}$

1.1.1 Solve for k. (3)

1.1.2 Hence solve for x if $\sqrt{x+5} + 5 = \frac{14}{\sqrt{x+5}}$. (3)

1.2 Solve for x:

1.2.1 $|5 - x| = 9$ (3)

1.2.2 $\frac{2x-2}{x-3} > 3$ (5)

1.3 Solve for x and y simultaneously if:

$\frac{1}{x} + \frac{1}{y} = 3$ and

$x - y = \frac{1}{2}$ (8)

[22]

QUESTION 2

2.1 Given: $4x - 5 = k(x^2 - 1)$
Determine the values of k for which the equation will have equal roots. (6)

2.2 Prove that the roots of the equation $x^2 + p = (p+1)x$ are rational for all rational values of p. (5)

[11]

QUESTION 3

3.1 Find the constants p and r so that the polynomial $x^3 + px + r$ has a remainder -9 when it is divided by $(x+1)$ and a remainder of -1 when it is divided by $(x-1)$. (6)

3.2 Find the values of x for which $3 - 7x + 5x^2 - x^3 = 0$. (6)

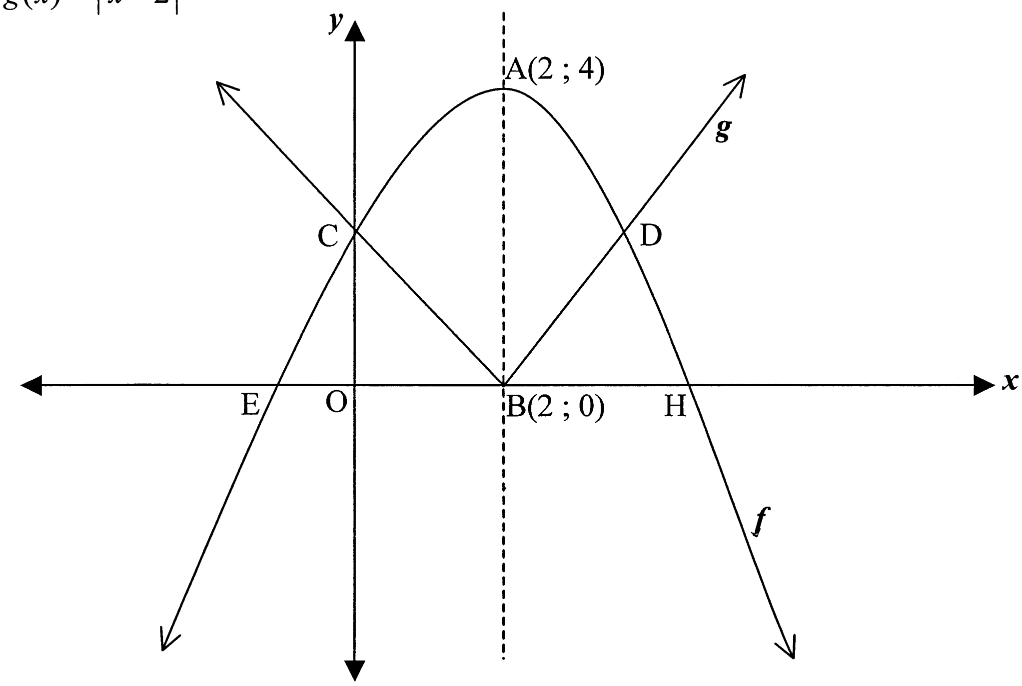
[12]

VRAAG 4

4.1 Die onderstaande figuur stel die grafieke van die volgende funksies voor:

$f(x) = ax^2 + bx + c$ en

$g(x) = |x - 2|$



A(2 ; 4) is die draaipunt van f en C is die y-afsnit van beide f en g. D is die spieëlbeeld van C ten opsigte van AB, die simmetrie-as van f. E en H is die x-afsnitte van f. Die koördinate van B is (2 ; 0).

4.1.1 Skryf die koördinate van C en D. (3)

4.1.2 Toon dat $a = -\frac{1}{2}$, $b = 2$ en $c = 2$. (5)

4.1.3 Bepaal die koördinate van E en H korrek tot **TWEE** desimale plekke. (4)

4.1.4 Gebruik jou grafiek om die waardes van x te bepaal waarvoor:

(a) $f(x) \leq g(x)$ (2)

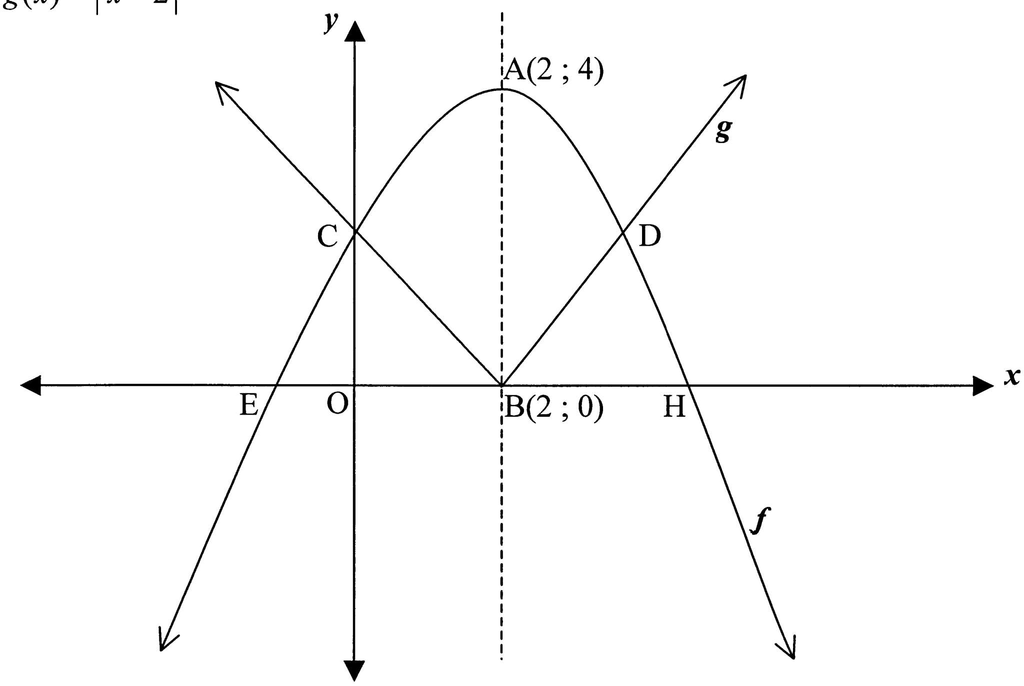
(b) $f(x).g(x) \leq 0$ (4)

QUESTION 4

4.1 The figure below represents the graphs of the following functions:

$f(x) = ax^2 + bx + c$ and

$g(x) = |x - 2|$



A(2 ; 4) is the turning point of f and C is the y-intercept of both f and g. D is the mirror image of C with respect to AB, the axis of symmetry of f. E and H are the x-intercepts of f. The co-ordinates of B are (2 ; 0).

4.1.1 Write the co-ordinates of C and D. (3)

4.1.2 Show that $a = -\frac{1}{2}$, $b = 2$ and $c = 2$. (5)

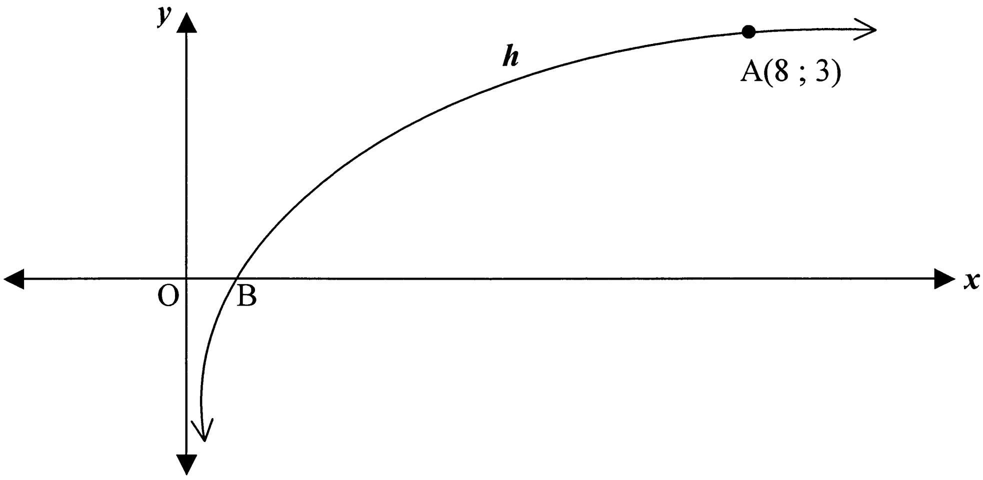
4.1.3 Determine the co-ordinates of E and H correct to **TWO** decimal places. (4)

4.1.4 Use your graph to determine the values of x for which:

(a) $f(x) \le g(x)$ (2)

(b) $f(x).g(x) \le 0$ (4)

4.2 Die figuur toon die grafiek van $h(x) = \log_a x$.
Die punt A(8 ; 3) lê op die kromme en B is die x-afsnit van h.



4.2.1 Bepaal die waarde van a. (3)

4.2.2 Skryf die koördinate van B neer. (1)

4.2.3 Bepaal die vergelyking van h^{-1}, die inverse van h, in die vorm $h^{-1}(x) = ...$ (2)

4.2.4 Bepaal die waarde van $h^{-1}(3)$. (1)

4.2.5 Skryf die definisieversameling van h. (1)
[26]

VRAAG 5

5.1 Toon, **sonder om 'n sakrekenaar te gebruik,** dat:

5.1.1 $\sqrt{\frac{8}{3}} + 5\sqrt{\frac{2}{3}} - \sqrt{\frac{1}{6}} = \frac{13}{2}\sqrt{\frac{2}{3}}$ (4)

5.1.2 $\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243} = \log 2$ (4)

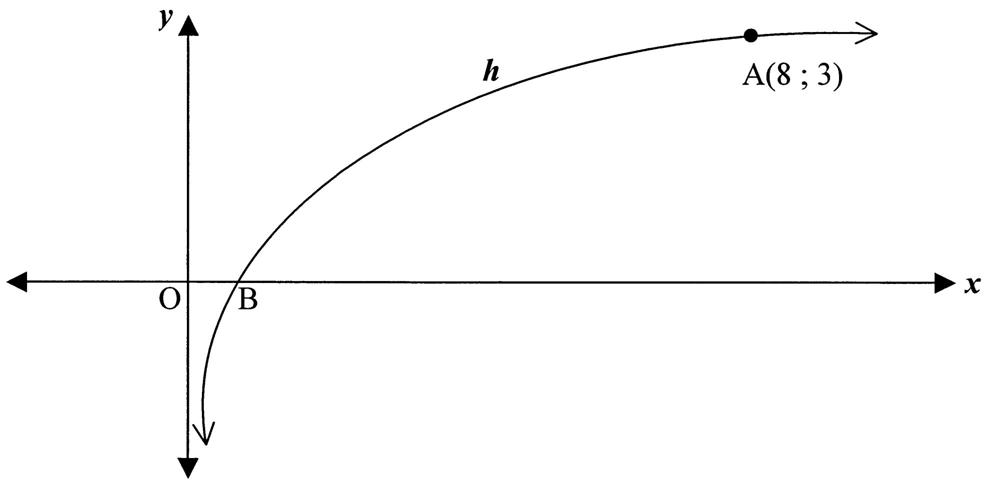
5.2 Los op vir x, **sonder om 'n sakrekenaar te gebruik**:

5.2.1 $2^{2x+1} - 3.2^x + 1 = 0$ (5)

5.2.2 $\log_2 9x + \log_4 x^2 = 2$ (6)

5.2.3 $\log_{\frac{1}{2}} 3x > 2$ (5)

4.2 The figure shows the graph of $h(x) = \log_a x$.
The point A(8 ; 3) lies on the curve and B is the x-intercept of h.



4.2.1 Determine the value of a. (3)

4.2.2 Write the co-ordinates of B. (1)

4.2.3 Determine the equation of h^{-1}, the inverse of h, in the form $h^{-1}(x) = ...$ (2)

4.2.4 Determine the value of $h^{-1}(3)$. (1)

4.2.5 Write the domain of h. (1)
[26]

QUESTION 5

5.1 Verify, **without using a calculator,** that:

5.1.1 $\sqrt{\frac{8}{3}} + 5\sqrt{\frac{2}{3}} - \sqrt{\frac{1}{6}} = \frac{13}{2}\sqrt{\frac{2}{3}}$ (4)

5.1.2 $\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243} = \log 2$ (4)

5.2 Solve for x, **without using a calculator**:

5.2.1 $2^{2x+1} - 3.2^x + 1 = 0$ (5)

5.2.2 $\log_2 9x + \log_4 x^2 = 2$ (6)

5.2.3 $\log_{\frac{1}{2}} 3x > 2$ (5)

5.3 Los op vir x korrek tot **TWEE desimale plekke**:

$2^x.3^{x+1} = 10$ (5)

[29]

VRAAG 6

6.1 Die som van die eerste n terme van 'n rekenkundige ry word gegee deur:

$S_n = \frac{5}{2}n^2 + \frac{7}{2}n$

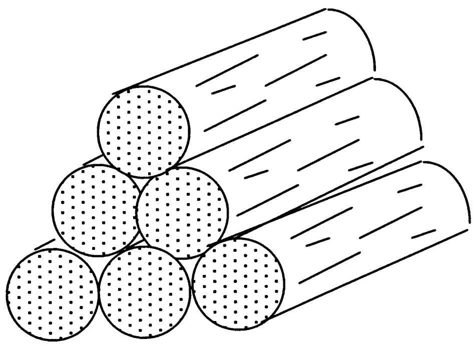
Bepaal:

6.1.1 Die eerste term (3)

6.1.2 Die gemene verskil (5)

6.1.3 Die tiende term (2)

6.2 'n Saagmeul het 'n driehoekige stapel houtblokke, 1 blok in die boonste ry, 2 blokke in die tweede, en so aan. Die boonste drie rye word hieronder getoon.



Daar is 630 blokke in die stapel.

Vind:

6.2.1 Die aantal rye in die stapel (5)

6.2.2 Die aantal blokke in die onderste ry (1)

6.3 Vind die waarde van n as:

$\sum_{k=1}^{n} 3^k = 1\,092$ (6)

5.3 Solve for x correct to **TWO decimal places**:

$2^x.3^{x+1} = 10$ (5)

[29]

QUESTION 6

6.1 The sum of the first n terms of an arithmetic sequence is given by:

$S_n = \frac{5}{2}n^2 + \frac{7}{2}n$

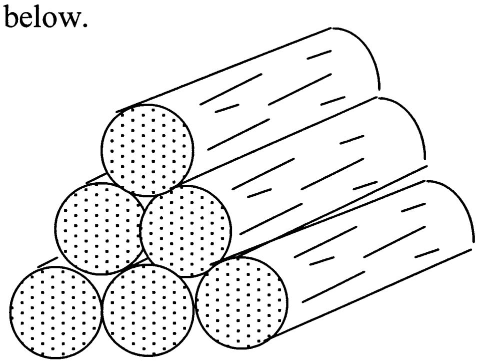
Determine:

6.1.1 The first term (3)

6.1.2 The common difference (5)

6.1.3 The tenth term (2)

6.2 A lumber mill has a triangular stack of logs, 1 log in the top row, 2 logs in the second, and so on. The top three rows are shown below.



There are 630 logs in the stack.

Find:

6.2.1 The number of rows in the stack (5)

6.2.2 The number of logs in the bottom row (1)

6.3 Find the value of n if:

$\sum_{k=1}^{n} 3^k = 1\,092$ (6)

6.4 Die n^{de} term van 'n ry word gegee deur
$T_n = 3(m-1)^{n+1}$
waar m 'n reële getal is, $m \neq 1$.

6.4.1 Toon dat die ry meetkundig is. (3)

6.4.2 Bepaal die waardes van m waarvoor die reeks $\sum_{n=1}^{\infty} 3(m-1)^{n+1}$ konvergeer.
Bereken (in terme van m) die som tot oneindigheid van hierdie reeks. (6)
[31]

VRAAG 7

7.1 Bepaal $f'(x)$ vanaf **eerste beginsels** as:
$f(x) = x^2 - 6x$ (6)

7.2 Bepaal $\frac{dy}{dx}$ as:

7.2.1 $y = (2x)^2 - \frac{1}{3x}$ (3)

7.2.2 $y = \frac{2\sqrt{x} - 5}{\sqrt{x}}$ (4)

7.3 Gegee: $f(x) = 2x^3 - 5x^2 - 4x + 3$
Die x-afsnitte van f is: $(-1\,;0)$, $\left(\frac{1}{2}\,;0\right)$ en $(3\,;0)$.

7.3.1 Bepaal die koördinate van die draaipunte van f. (6)

7.3.2 Teken 'n netjiese sketsgrafiek van f. Toon duidelik die koördinate van die snypunte met die asse, asook die koördinate van die draaipunte. (5)

7.3.3 Vir watter waardes van k sal die vergelyking $f(x) = k$, presies twee reële wortels hê? (4)

7.3.4 Bepaal die vergelyking van die raaklyn aan die grafiek van $f(x) = 2x^3 - 5x^2 - 4x + 3$ by die punt waar $x = 1$. (6)
[34]

6.4 The n^{th} term of a sequence is given by
$T_n = 3(m-1)^{n+1}$
where m is a real number, $m \neq 1$.

6.4.1 Show that the sequence is geometric. (3)

6.4.2 Determine the values of m for which the series $\sum_{n=1}^{\infty} 3(m-1)^{n+1}$ converges.
Calculate (in terms of m) the sum to infinity of this series. (6)
[31]

QUESTION 7

7.1 Determine $f'(x)$ from **first principles** if:
$f(x) = x^2 - 6x$ (6)

7.2 Determine $\frac{dy}{dx}$ if:

7.2.1 $y = (2x)^2 - \frac{1}{3x}$ (3)

7.2.2 $y = \frac{2\sqrt{x} - 5}{\sqrt{x}}$ (4)

7.3 Given: $f(x) = 2x^3 - 5x^2 - 4x + 3$
The x-intercepts of f are: $(-1\,;0)$, $\left(\frac{1}{2}\,;0\right)$ and $(3\,;0)$.

7.3.1 Determine the co-ordinates of the turning points of f. (6)

7.3.2 Draw a neat sketch graph of f. Clearly indicate the co-ordinates of the intercepts with the axes, as well as the co-ordinates of the turning points. (5)

7.3.3 For which values of k will the equation $f(x) = k$, have exactly two real roots? (4)

7.3.4 Determine the equation of the tangent to the graph of
$f(x) = 2x^3 - 5x^2 - 4x + 3$ at the point where $x = 1$. (6)
[34]

VRAAG 8

8.1 Nadat 'n bietjie navorsing gedoen is, het 'n vervoermaatskappy bepaal dat die tempo waarteen petrol verbruik word deur een van sy groot voertuie, wat teen 'n gemiddelde spoed van x km per uur reis, gegee word deur:

$P(x) = \frac{55}{2x} + \frac{x}{200}$ liter per kilometer

8.1.1 Neem aan dat die petrol R4,00 per liter kos en die drywer R18,00 per uur (reistyd) verdien. Lei nou af dat die totale koste, C, in rand, vir 'n 2 000 km-rit gegee word deur:

$C(x) = \frac{256\,000}{x} + 40x$ (6)

8.1.2 Bepaal vervolgens die gemiddelde spoed wat gehandhaaf moet word om 'n minimum koste vir 'n 2 000 km-rit uit te werk. (5)

8.2 Gedurende 'n eksperiment verander die temperatuur T (in grade Celsius), met tyd t (in ure), volgens die formule: $T(t) = 30 + 4t - \frac{1}{2}t^2; \quad t \in [0;10]$.

8.2.1 Bepaal 'n uitdrukking vir die tempo van verandering van temperatuur met tyd. (2)

8.2.2 Gedurende watter tydinterval was die temperatuur besig om te daal? (4)

[17]

VRAAG 9

Fashion-cards is 'n klein maatskappy wat twee soorte kaarte, tipe X en tipe Y maak. Met die beskikbare arbeid en materiaal, kan die maatskappy nie meer as 150 kaarte van tipe X en nie meer as 120 kaarte van tipe Y per week maak nie. In totaal kan hulle nie meer as 200 kaarte per week maak nie.

Daar is 'n bestelling vir minstens 40 tipe X-kaarte en 10 tipe Y-kaarte per week.

Fashion-cards maak 'n wins van R5 vir elke tipe X-kaart wat verkoop word en R10 vir elke tipe Y-kaart.

Laat die aantal tipe X-kaarte x en die aantal tipe Y-kaarte y wees wat per week vervaardig word.

9.1 Een van die beperkingsongelykhede wat die beperkings hierbo voorstel, is $x \leq 150$. Noem die ander beperkingsongelykhede. (4)

9.2 Stel die beperkings grafies voor en skakeer die gangbare gebied. (6)

9.3 Skryf die vergelyking wat die wins P (die doelfunksie) voorstel, in terme van x en y. (2)

9.4 Teken op jou grafiek, 'n reguit lyn wat jou sal help om te bepaal hoeveel van elke tipe weekliks gemaak moet word om die maksimum wins te verkry. (2)

9.5 Bereken die maksimum weeklikse wins. (4)

[18]

TOTAAL: 200

QUESTION 8

8.1 After doing some research, a transport company has determined that the rate at which petrol is consumed by one of its large carriers, travelling at an average speed of x km per hour, is given by:

$P(x) = \frac{55}{2x} + \frac{x}{200}$ litres per kilometre

8.1.1 Assume that the petrol costs R4,00 per litre and the driver earns R18,00 per hour (travelling time). Now deduce that the total cost, C, in rands, for a 2 000 km trip is given by:

$C(x) = \frac{256\,000}{x} + 40x$ (6)

8.1.2 Hence determine the average speed to be maintained to effect a minimum cost for a 2 000 km trip. (5)

8.2 During an experiment the temperature T (in degrees Celsius), varies with time t (in hours), according to the formula: $T(t) = 30 + 4t - \frac{1}{2}t^2$; $t \in [0;10]$.

8.2.1 Determine an expression for the rate of change of temperature with time. (2)

8.2.2 During which time interval was the temperature dropping? (4)

[17]

QUESTION 9

Fashion-cards is a small company that makes two types of cards, type X and type Y. With the available labour and material, the company can make not more than 150 cards of type X and not more than 120 cards of type Y per week. Altogether they cannot make more than 200 cards per week.

There is an order for at least 40 type X cards and 10 type Y cards per week.

Fashion-cards makes a profit of R5 for each type X card sold and R10 for each type Y card.

Let the number of type X cards be x and the number of type Y cards be y, manufactured per week.

9.1 One of the constraint inequalities which represents the restrictions above is $x \leq 150$. Write the other constraint inequalities. (4)

9.2 Represent the constraints graphically and shade the feasible region. (6)

9.3 Write the equation that represents the profit P (the objective function), in terms of x and y. (2)

9.4 On your graph, draw a straight line which will help you to determine how many of each type must be made weekly to produce the maximum profit. (2)

9.5 Calculate the maximum weekly profit. (4)

[18]

TOTAL: 200

VRAAG 9

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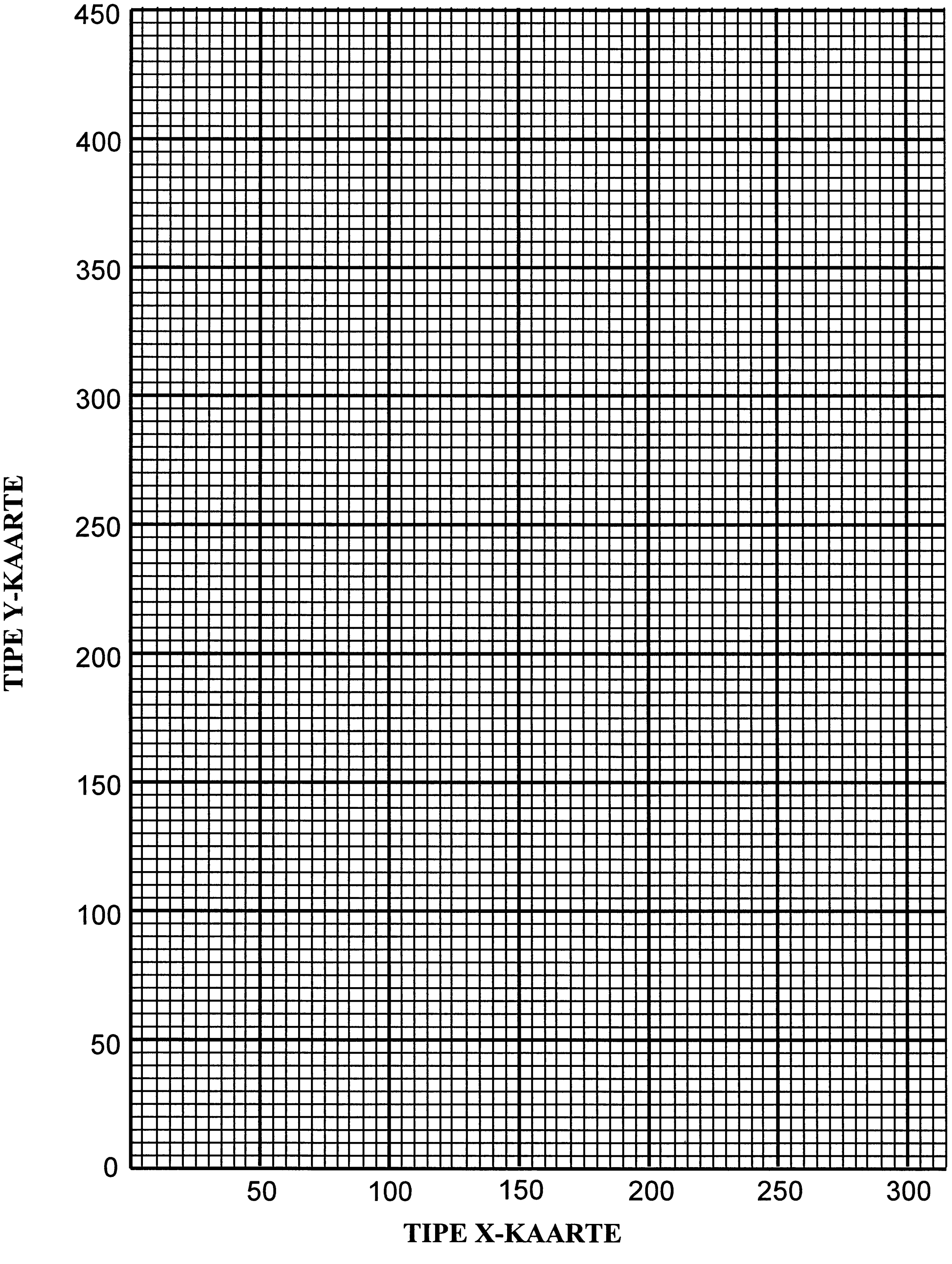
SENTRUMNOMMER														



QUESTION 9

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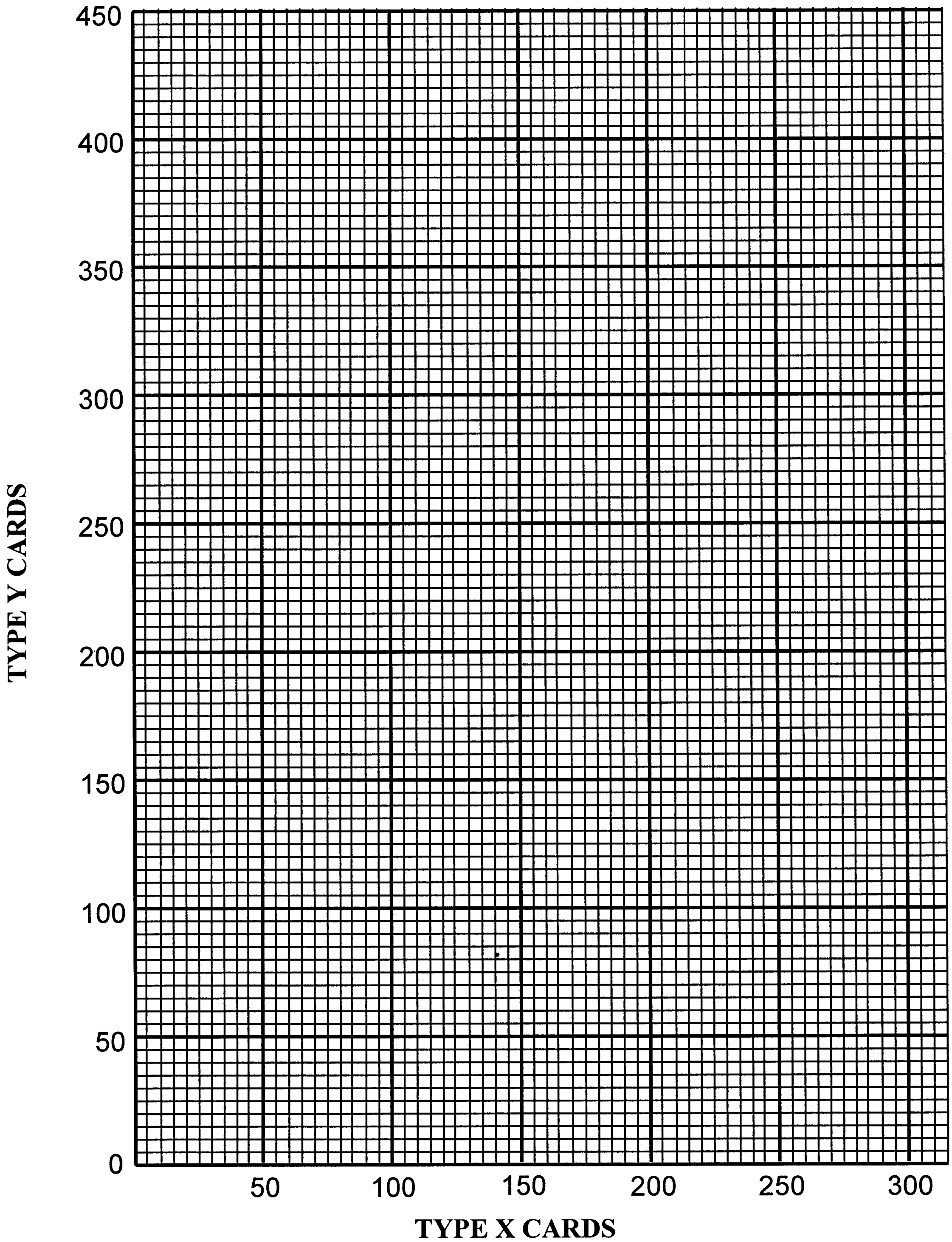

Information Sheet (HG and SG)
Inligtingsblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}(a + l)$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = a.r^{n-1}$ $S_n = \frac{a(1-r^n)}{1-r}$ (r ≠ 1) $S_n = \frac{a(r^n - 1)}{r-1}$ (r ≠ 1)

$S_\infty = \frac{a}{1-r}$ (r ≠ 1)

$A = P\left(1 + \frac{r}{100}\right)^n$ $A = P\left(1 - \frac{r}{100}\right)^n$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$y = mx + c$

$y - y_1 = m(x - x_1)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \tan\theta$

$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$x^2 + y^2 = r^2$

$(x-p)^2 + (y-q)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta ABC = \frac{1}{2}ab.\sin C$

Information Sheet (HG and SG)
Inligtingsblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n - 1)d \qquad S_n = \frac{n}{2}(a + l) \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = a.r^{n-1} \qquad S_n = \frac{a(1 - r^n)}{1 - r} \quad (r \neq 1) \quad S_n = \frac{a(r^n - 1)}{r - 1} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1 - r} \quad (r \neq 1)$$

$$A = P\left(1 + \frac{r}{100}\right)^n \qquad A = P\left(1 - \frac{r}{100}\right)^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2$$

$$(x - p)^2 + (y - q)^2 = r^2$$

In ΔABC: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$