DEPARTMENT OF EDUCATION
REPUBLIC OF SOUTH AFRICA

DEPARTEMENT VAN ONDERWYS
REPUBLIEK VAN SUID-AFRIKA

SENIOR CERTIFICATE EXAMINATION - 2004
SENIORSERTIFIKAAT-EKSAMEN - 2004

MATHEMATICS P1 : ALGEBRA
WISKUNDE V1 : ALGEBRA

HIGHER GRADE
HOËR GRAAD

OCTOBER/NOVEMBER 2004
OKTOBER/NOVEMBER 2004

301-1/1

Marks: 200
Punte : 200

3 Hours
3 Ure

This question paper consists of 9 pages, 1 graph paper and 1 formula sheet.
Hierdie vraestel bestaan uit 9 bladsye, 1 grafiekpapier en 1 formuleblad.

MATHEMATICS HG: Paper 1
Algebra

301 1 1 HG

INSTRUKSIES

Lees die volgende instruksies sorgvuldig deur voordat die vrae beantwoord word:

1. Hierdie vraestel bestaan uit **8** vrae. Beantwoord **AL** die vrae.
2. Toon duidelik **AL** die bewerkings, diagramme, grafieke, ensovoorts wat jy gebruik het om die antwoorde te bepaal.
3. 'n Goedgekeurde sakrekenaar (nie-programmeerbaar en nie-grafies) kan gebruik word, tensy anders vermeld.
4. Indien nodig, moet antwoorde tot **TWEE** desimale plekke afgerond word, tensy anders vermeld.
5. Die aangehegte grafiekpapier moet slegs vir **VRAAG 8** gebruik word. Maak dit los van die vraestel, vul jou eksamennommer en sentrumnommer daarop in en plaas dit **VOOR** in die antwoordeboek.
6. Nommer die antwoorde **PRESIES** soos die vrae genommer is.
7. Diagramme is nie noodwendig volgens skaal geteken nie.
8. Dit is tot jou eie voordeel om leesbaar te skryf en om die werk netjies aan te bied.
9. **'n Inligtingsblad met formules is ingesluit aan die einde van die vraestel.**

INSTRUCTIONS

Read the following instructions carefully before answering the questions:

1. This paper consists of **8** questions. Answer **ALL** the questions.
2. Clearly show **ALL** calculations, diagrams, graphs, et cetera you have used in determining the answers.
3. An approved calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to **TWO** decimal places, unless stated otherwise.
5. The attached graph paper must be used only for **QUESTION 8**. Detach it from your question paper, fill in your examination number and centre number and insert it in the **FRONT** of the answer book.
6. Number the answers **EXACTLY** as the questions are numbered.
7. Diagrams are not necessarily drawn to scale.
8. It is in your own interest to write legibly and to present the work neatly.
9. **An information sheet with formulae is included at the end of the question paper.**

VRAAG 1

1.1 1.1.1 Los op vir x: $\sqrt{x+6} = x$ (5)

1.1.2 Gebruik die oplossing vir x in VRAAG 1.1.1 om die waarde te bepaal van y waarvoor $\sqrt{y+1} = y-5$. (2)

1.2 Los op vir x:

1.2.1 $|4-x| = 4$ (2)

1.2.2 $\dfrac{4}{x-3} \leq 1$ (6)

1.2.3 $\dfrac{4}{(x-3)^2} < 1$ (5)

1.3 Gegee: $3x^2 + kx - k = 3x$

1.3.1 Bewys dat hierdie vergelyking rasionale wortels het vir alle rasionale waardes van k. (4)

1.3.2 Bepaal die wortels van die vergelyking as dit ook gegee word dat hulle gelyk is. (5)

1.4 Daar word in 'n wedloop van 'n atleet verwag om 10 km te hardloop en 50 km met 'n fiets af te lê. Siviwe hardloop teen 'n spoed van x km/h en ry met die fiets teen $(x + 31)$ km/h. Hy neem $\dfrac{10}{x}$ ure om die 10 km-gedeelte te hardloop.

1.4.1 Druk die tyd wat hy neem vir die 50 km-fietsrygedeelte in terme van x uit. (1)

1.4.2 Bereken die spoed (korrek tot **TWEE** desimale plekke) waarteen hy moet hardloop om die hele wedloop in 2 ure te voltooi. (7)

[37]

QUESTION 1

1.1 1.1.1 Solve for x: $\sqrt{x+6} = x$ (5)

1.1.2 Use the solution for x in QUESTION 1.1.1 to determine the value of y for which $\sqrt{y+1} = y-5$. (2)

1.2 Solve for x:

1.2.1 $|4-x| = 4$ (2)

1.2.2 $\frac{4}{x-3} \leq 1$ (6)

1.2.3 $\frac{4}{(x-3)^2} < 1$ (5)

1.3 Given: $3x^2 + kx - k = 3x$

1.3.1 Prove that this equation has rational roots for all rational values of k. (4)

1.3.2 Determine the roots of the equation if it is also given that they are equal. (5)

1.4 A race requires an athlete to run 10 km and cycle 50 km. Siviwe runs at a speed of x km/h and cycles at $(x + 31)$ km/h. He takes $\frac{10}{x}$ hours for the 10 km run.

1.4.1 Express the time he takes for the 50 km cycle in terms of x. (1)

1.4.2 Calculate the speed (correct to **TWO** decimal places) at which he must run to complete the entire race in 2 hours. (7)

[37]

VRAAG 2

2.1 Gegee: $f(x) = -\frac{1}{2}x(x+6)$ en $g(x) = |x|$

2.1.1 Skryf die koördinate van die x-afsnitte van die grafiek van f. (2)

2.1.2 Teken 'n sketsgrafiek van f. Dui duidelik die koördinate van die draaipunt sowel as die afsnitte met die asse aan. (6)

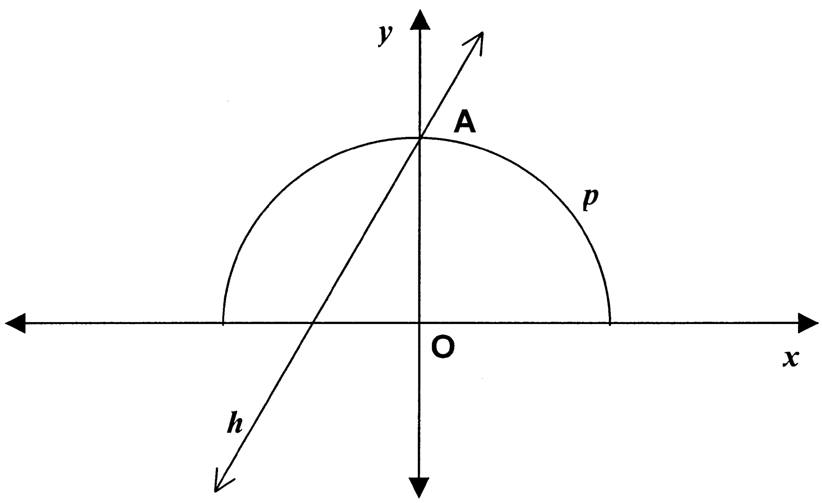
2.1.3 **Gebruik die grafiek** van VRAAG 2.1.2 om die waardes van k te bepaal waarvoor $-x(x+6) = 2k$ twee ongelyke negatiewe wortels het. (3)

2.1.4 Teken op dieselfde assestelsel as in VRAAG 2.1.2, 'n sketsgrafiek van g. (2)

2.1.5 Bereken die koördinate van die snypunte van die grafieke van f en g. (6)

2.1.6 Bepaal vervolgens die waardes van x waarvoor $f(x) > g(x)$. (2)

2.2 In die diagram hieronder sny die grafieke van die reguitlyn $h(x) = 2x + 4$ en 'n halfsirkel p, met middelpunt by die oorsprong, mekaar by die punt A op die y-as.



Bepaal:

2.2.1 Die vergelyking van die halfsirkel, p (2)

2.2.2 Die vergelyking van h^{-1}, die inverse van h, in die vorm $y = \ldots$ (2)

2.2.3 Die waarde van $h^{-1}(4)$ (1)
[26]

QUESTION 2

2.1 Given: $f(x) = -\frac{1}{2}x(x+6)$ and $g(x) = |x|$

2.1.1 Write the co-ordinates of the x-intercepts of the graph of f. (2)

2.1.2 Draw a sketch graph of f. Clearly indicate the co-ordinates of the turning point as well as the intercepts with the axes. (6)

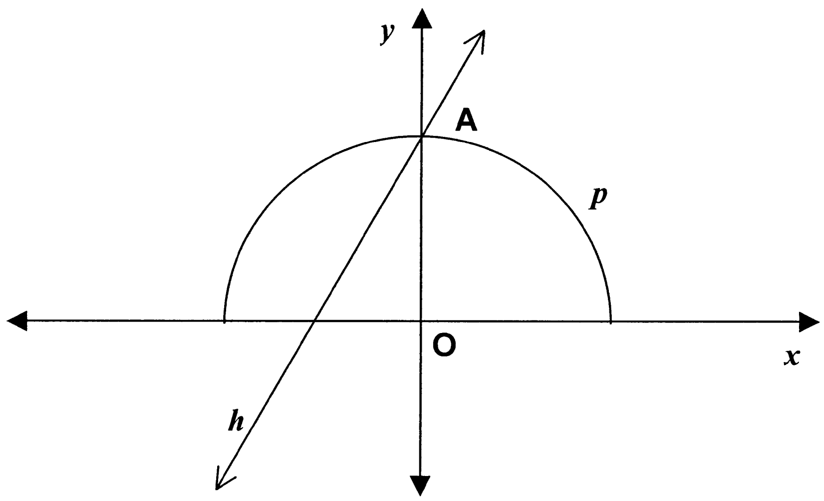
2.1.3 **Use the graph** of QUESTION 2.1.2 to determine the values of k for which $-x(x+6) = 2k$ has two unequal negative roots. (3)

2.1.4 Draw, on the same system of axes as in QUESTION 2.1.2, sketch graph of g. (2)

2.1.5 Calculate the co-ordinates of the points of intersection of the graphs of f and g. (6)

2.1.6 Hence, determine the values of x for which $f(x) > g(x)$. (2)

2.2 In the diagram below, the graphs of the straight line $h(x) = 2x + 4$ and a semi-circle p, having centre at the origin, intersect at point A on the y-axis.



Determine:

2.2.1 The equation of the semi-circle, p (2)

2.2.2 The equation of h^{-1}, the inverse of h, in the form $y = \ldots$ (2)

2.2.3 The value of $h^{-1}(4)$ (1)

[26]

VRAAG 3

Die polinoom $p(x) = 16x^3 - 4x^2 + 8$ kan uitgedruk word in die vorm

$p(x) = (2x-1).Q(x) + R$, waar $Q(x)$ 'n polinoom en R 'n konstante is.

3.1 Toon dat $R = 9$. (2)

3.2 Bepaal die res as $Q(x)$ gedeel word deur $(x + 1)$. (4)

3.3 Faktoriseer vervolgens, of op 'n ander wyse: $16x^3 - 4x^2 - 1$. (3)
[9]

VRAAG 4

4.1 Bewys dat $\log_a M = \dfrac{\log_b M}{\log_b a}$. (4)

4.2 Gegee dat $5^n = x$ en $n = \log_2 y$.

4.2.1 Skryf y in terme van n. (1)

4.2.2 Druk $\log_8 4y$ in terme van n uit. (4)

4.2.3 Druk 50^{n+1} in terme van x en y uit. (4)

4.3 Los op vir x, **sonder om 'n sakrekenaar te gebruik**:

4.3.1 $3^{x-1} + 3^{x+1} = \sqrt{300}$ (6)

4.3.2 $\log_{\frac{1}{2}} x + \log_{\frac{1}{2}}(x+1) \geq -1$ (9)

4.4 4.4.1 Teken 'n sketsgrafiek van $y = 2^{-x}$. Toon sy y-afsnit. (2)

4.4.2 Skryf die waardeversameling van hierdie funksie. (2)

4.5 Die intensiteit van 'n lig word tot 'n breuk van sy oorspronklike intensiteit verlaag nadat dit deur d meter mis getrek het. Hierdie breuk, F, word gegee deur die formule

$$F = 3^{-0,07d}$$

Bereken die afstand d (afgerond tot **TWEE** desimale plekke), as $F = \dfrac{1}{2}$. (4)
[36]

QUESTION 3

The polynomial $p(x) = 16x^3 - 4x^2 + 8$ can be expressed in the form

$p(x) = (2x-1).Q(x) + R$, where $Q(x)$ is a polynomial and R is a constant.

3.1 Show that $R = 9$. (2)

3.2 Determine the remainder when $Q(x)$ is divided by $(x + 1)$. (4)

3.3 Hence, or otherwise, factorise: $16x^3 - 4x^2 - 1$. (3)
[9]

QUESTION 4

4.1 Prove that $\log_a M = \dfrac{\log_b M}{\log_b a}$. (4)

4.2 Given that $5^n = x$ and $n = \log_2 y$.

4.2.1 Write y in terms of n. (1)

4.2.2 Express $\log_8 4y$ in terms of n. (4)

4.2.3 Express 50^{n+1} in terms of x and y. (4)

4.3 Solve for x, **without using a calculator**:

4.3.1 $3^{x-1} + 3^{x+1} = \sqrt{300}$ (6)

4.3.2 $\log_{\frac{1}{2}} x + \log_{\frac{1}{2}}(x+1) \geq -1$ (9)

4.4 4.4.1 Draw a sketch graph of $y = 2^{-x}$. Indicate its y-intercept. (2)

4.4.2 Write the range of this function. (2)

4.5 The intensity of a light is reduced to a fraction of its original intensity after passing through d metres of mist. This fraction, F, is given by the formula

$$F = 3^{-0,07d}$$

Calculate the distance d (rounded off to **TWO** decimal places), if $F = \dfrac{1}{2}$. (4)
[36]

VRAAG 5

5.1 5; x; y is 'n rekenkundige ry en x; y; 81 is 'n meetkundige ry. Alle terme in die rye is heelgetalle.
Bereken die waardes van x en y. (9)

5.2 Die som van 'n rekenkundige reeks is 100 maal die waarde van sy eerste term, terwyl die laaste term 9 maal die eerste term is.
Bereken die aantal terme in die reeks, as die eerste term nie gelyk aan nul is nie. (6)

5.3 Gegee die meetkundige reeks:
$2.(5)^5 + 2.(5)^4 + 2.(5)^3 + \ldots\ldots$

5.3.1 Toon dat die reeks konvergeer. (2)

5.3.2 Bereken die som tot oneindigheid van die reeks. (3)

5.3.3 Bereken die som van die eerste 8 terme van die reeks, korrek tot **TWEE** desimale plekke. (4)

5.3.4 Gebruik jou antwoorde op VRAAG 5.3.2 en VRAAG 5.3.3 om $\sum_{n=9}^{\infty} 2.(5)^{6-n}$ korrek tot **TWEE** desimale plekke te bepaal. (2)

[26]

VRAAG 6

6.1 Gegee: $g(x) = \dfrac{2x^2 + x - 1}{2x - 1}$

6.1.1 Vir watter waarde van x is $g(x)$ ongedefinieer? (1)

6.1.2 Die volgende tabel gee waardes van $g(x)$ soos x nader beweeg aan 0,5. Gee, korrek tot **DRIE** desimale plekke, die ontbrekende waarde van $g(0{,}501)$.

x	0	0,4	0,49	0,499	0,501	0,51	0,6
$g(x)$	1	1,4	1,49	1,499		1,51	1,6

(1)

6.1.3 Deur gebruik te maak van die tabel, of op 'n ander wyse, bepaal die waarde van:

$$\lim_{x \to 0{,}5} g(x)$$

(2)

6.2 Gegee: $f(x) = x^3$

6.2.1 Bereken die afgeleide, $f'(x)$, **vanaf eerste beginsels**. (6)

6.2.2 Skryf vervolgens die waarde van $f'(2)$. (1)

QUESTION 5

5.1 $5;\ x;\ y$ is an arithmetic sequence and $x;\ y;\ 81$ is a geometric sequence. All terms in the sequences are integers.
Calculate the values of x and y. (9)

5.2 The sum of an arithmetic series is 100 times the value of its first term, while the last term is 9 times the first term.
Calculate the number of terms in the series if the first term is not equal to zero. (6)

5.3 Given the geometric series:
$2.(5)^5 + 2.(5)^4 + 2.(5)^3 + \ldots\ldots$

5.3.1 Show that the series converges. (2)

5.3.2 Calculate the sum to infinity of the series. (3)

5.3.3 Calculate the sum of the first 8 terms of the series, correct to **TWO** decimal places. (4)

5.3.4 Use your answers to QUESTION 5.3.2 and QUESTION 5.3.3 to determine $\sum_{n=9}^{\infty} 2.(5)^{6-n}$ correct to **TWO** decimal places. (2)
[26]

QUESTION 6

6.1 Given: $g(x) = \dfrac{2x^2 + x - 1}{2x - 1}$

6.1.1 For which value of x is $g(x)$ undefined? (1)

6.1.2 The following table gives values of $g(x)$ as x approaches 0,5.
Give, correct to **THREE** decimal places, the missing value of $g(0{,}501)$.

x	0	0,4	0,49	0,499	0,501	0,51	0,6
$g(x)$	1	1,4	1,49	1,499		1,51	1,6

(1)

6.1.3 Using the table, or otherwise, determine the value of:

$$\lim_{x \to 0{,}5} g(x)$$ (2)

6.2 Given: $f(x) = x^3$

6.2.1 Calculate the derivative, $f'(x)$, **from first principles.** (6)

6.2.2 Hence, write the value of $f'(2)$. (1)

6.3 As $y = \dfrac{x^2 - 1}{\sqrt{x}}$, bepaal $\dfrac{dy}{dx}$. (4)

6.4 Gegee dat $f'(x) = 3x^2$ en $g'(x) = x$, bepaal

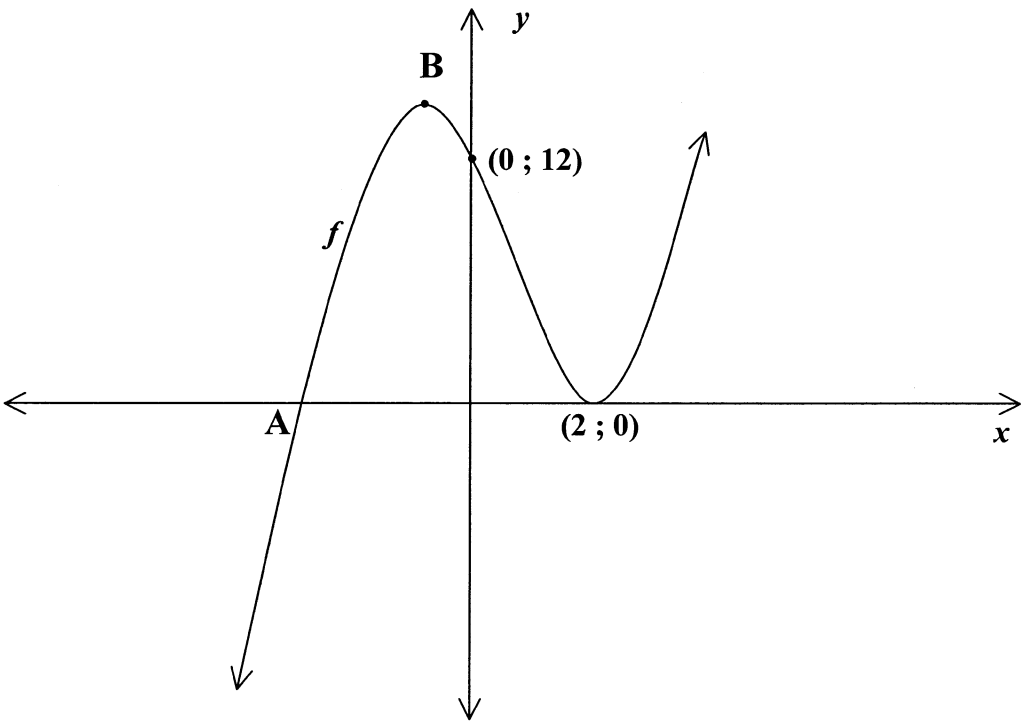
$\dfrac{d}{dx}[f(x) + 3.g(x)]$ (2)

6.5 Die lyn $g(x) = 5x + 1$ is 'n raaklyn aan die kromme van 'n funksie f by die punt waar $x = 2$.
Bereken die waarde van $f(2) + f'(2)$. (4)

[21]

VRAAG 7

7.1 Die sketsgrafiek hieronder toon die kromme van $f(x) = x^3 - x^2 - 8x + 12$.
Die kromme het 'n y-afsnit by (0 ; 12) en draaipunte by (2 ; 0) en B.
Die punt A is 'n x-afsnit.



7.1.1 Bereken die koördinate van A. (5)

7.1.2 Bereken die x-koördinaat van B. (4)

7.1.3 Skryf die waardes van x waarvoor $f'(x) > 0$. (3)

7.1.4 As $k < 0$, hoeveel reële wortels sal die vergelyking $x^3 - x^2 - 8x + 12 = k$ hê? (2)

6.3 If $y=\dfrac{x^2-1}{\sqrt{x}}$, determine $\dfrac{dy}{dx}$. (4)

6.4 Given that $f'(x)=3x^2$ and $g'(x)=x$, determine

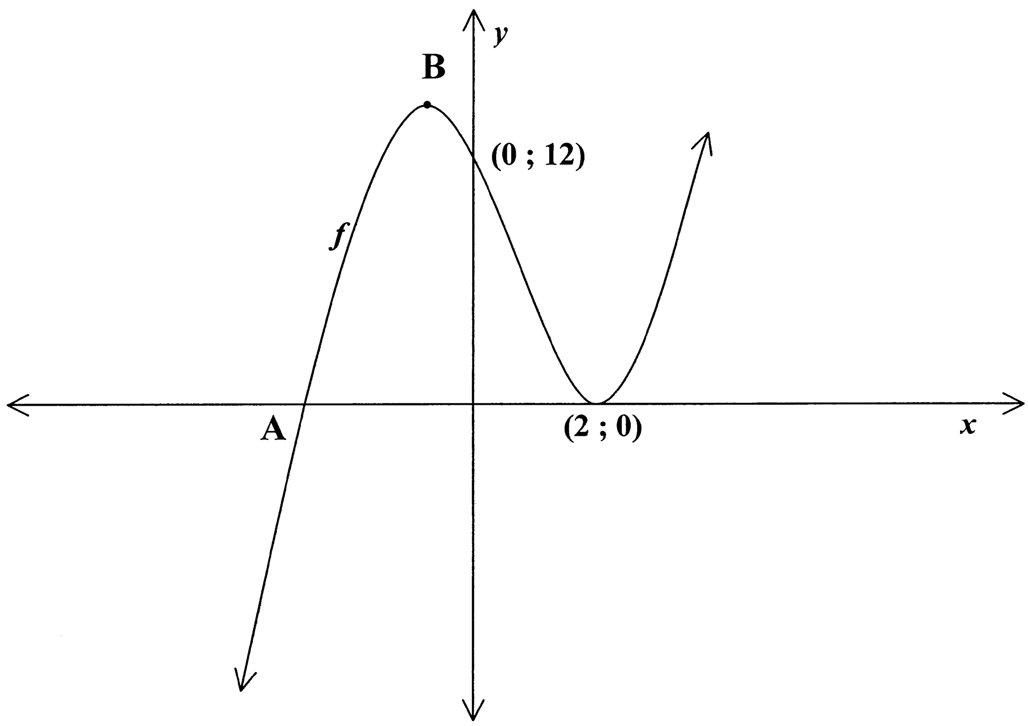
$\dfrac{d}{dx}[f(x)+3.g(x)]$ (2)

6.5 The line $g(x)=5x+1$ is a tangent to the curve of a function f at the point where $x=2$.
Calculate the value of $f(2)+f'(2)$. (4)

[21]

QUESTION 7

7.1 The sketch graph below shows the curve of $f(x)=x^3-x^2-8x+12$.
The curve has a y-intercept at (0 ; 12) and turning points at (2 ; 0) and B.
The point A is an x-intercept.



7.1.1 Calculate the co-ordinates of A. (5)

7.1.2 Calculate the x-co-ordinate of B. (4)

7.1.3 Write the values of x for which $f'(x)>0$. (3)

7.1.4 If $k<0$, how many real roots will the equation $x^3-x^2-8x+12=k$ have? (2)

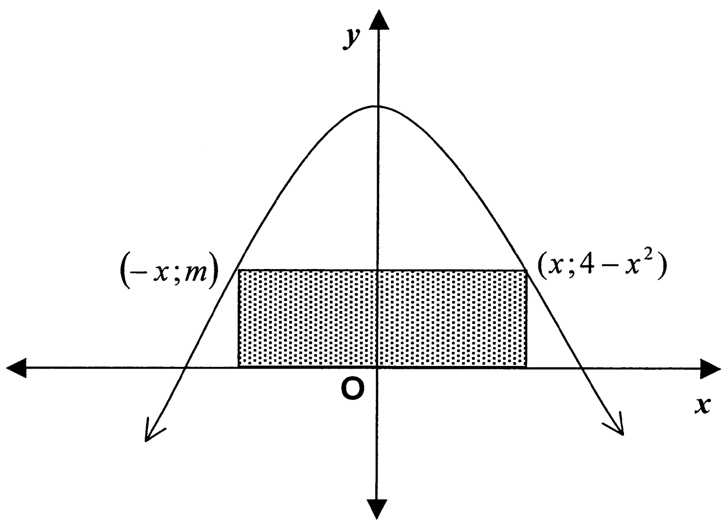
7.2 Die diepte (in meter) van water wat in 'n dam oorbly t ure nadat 'n sluishek oopgemaak is om water toe te laat om uit die dam te loop, word gegee deur die formule:

$$L = 28 - \frac{1}{9}t^2 - \frac{1}{27}t^3$$

7.2.1 Bereken die **gemiddelde** tempo waarteen die diepte verander in die eerste 3 ure. (4)

7.2.2 Bepaal die tempo waarteen die diepte verander na presies 2 ure. (3)

7.3 'n Reghoek het twee hoekpunte op die x-as en twee op die kromme met vergelyking $y = 4 - x^2$, soos in die skets hieronder getoon.



7.3.1 Druk m in terme van x uit. (1)

7.3.2 Toon dat die oppervlakte van die reghoek gegee word deur:
$A = 8x - 2x^3$ (2)

7.3.3 Bepaal die oppervlakte van die grootste reghoek wat op hierdie wyse geteken kan word. (5)

[29]

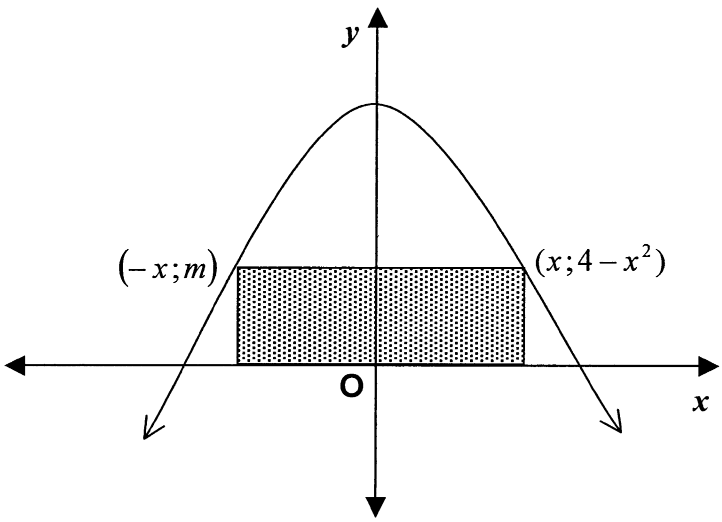
7.2 The depth (in metres) of water left in a dam t hours after a sluice gate has been opened to allow water to drain from the dam, is given by the formula:

$$L = 28 - \frac{1}{9}t^2 - \frac{1}{27}t^3$$

7.2.1 Calculate the **average** rate at which the depth changes in the first 3 hours. (4)

7.2.2 Determine the rate at which the depth changes after exactly 2 hours. (3)

7.3 A rectangle has two vertices on the x-axis and two on the curve with equation $y = 4 - x^2$, as shown in the sketch below.



7.3.1 Express m in terms of x. (1)

7.3.2 Show that the area of the rectangle is given by:
$A = 8x - 2x^3$ (2)

7.3.3 Determine the area of the largest rectangle which can be drawn in this manner. (5)

[29]

VRAAG 8

'n Groep studente beplan om x hamburgers en y hoenderburgers by 'n rugbywedstryd te verkoop. Hulle het vleis vir hoogstens 300 hamburgers en hoogstens 400 hoenderburgers. Elke burger van albei soorte word in 'n sakkie verkoop. Daar is 500 sakkies beskikbaar. Die aanvraag behoort sodanig te wees dat die aantal hoenderburgers wat verkoop word ten minste die helfte van die aantal hamburgers wat verkoop word, is.

8.1 Skryf die beperkingsongelykhede. (5)

8.2 Die gebied wat deur twee van hierdie beperkingsongelykhede bepaal word, word op die grafiekpapier, wat voorsien word, getoon.
Stel die oorblywende beperkingsongelykhede op die grafiekpapier voor. (3)

8.3 Arseer die gangbare gebied op die grafiekpapier. (2)

8.4 'n Wins van R3 word op elke hamburger wat verkoop word gemaak en R2 word op elke hoenderburger wat verkoop word, gemaak.
Skryf die vergelyking wat die algehele wins, P, in terme van x en y, voorstel. (2)

8.5 Die doel is om wins te maksimeer. Trek die soeklyn (doelfunksie) op die grafiekpapier deur dit as 'n stippellyn in sy optimum posisie te toon. (2)

8.6 Hoeveel van elke soort burger behoort verkoop te word om die wins te maksimeer? (2)
[16]

TOTAAL: 200

QUESTION 8

A group of students plan to sell x hamburgers and y chicken burgers at a rugby match. They have meat for at most 300 hamburgers and at most 400 chicken burgers. Each burger of both types is sold in a packet. There are 500 packets available. The demand is likely to be such that the number of chicken burgers sold is at least half the number of hamburgers sold.

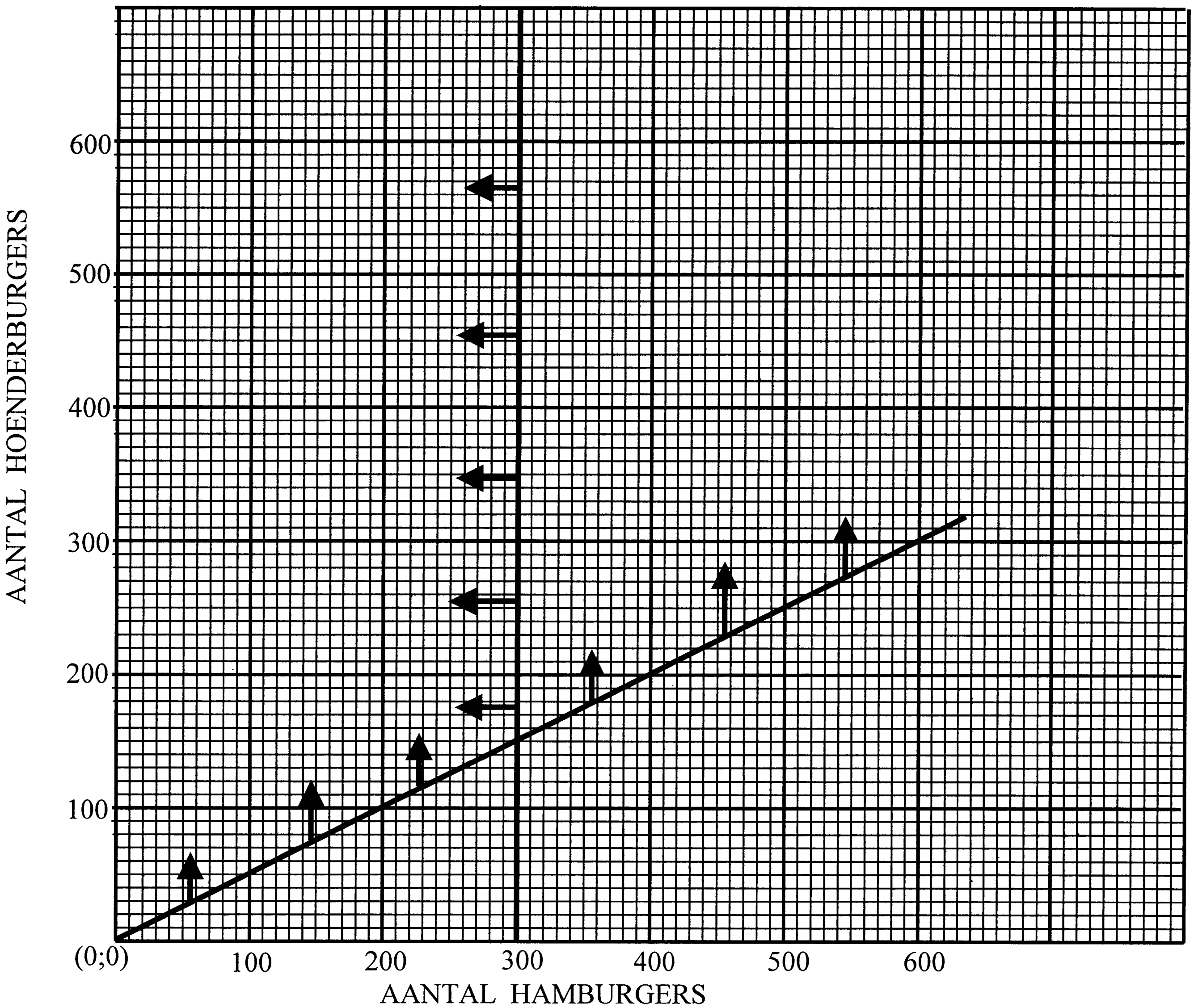
8.1	Write the constraint inequalities.	(5)
8.2	Two constraint inequalities are shown on the graph paper provided. Represent the remaining constraint inequalities on the graph paper.	(3)
8.3	Shade the feasible region on the graph paper.	(2)
8.4	A profit of R3 is made on each hamburger sold and R2 on each chicken burger sold. Write the equation which represents the total profit, P, in terms of x and y.	(2)
8.5	The objective is to maximise profit. Draw the search line (objective function) on the graph paper by indicating it as a dotted line in its optimum position.	(2)
8.6	How many, of each type of burger, should be sold to maximise profit?	(2) **[16]**

TOTAL: 200

VRAAG 8

EKSAMENNOMMER

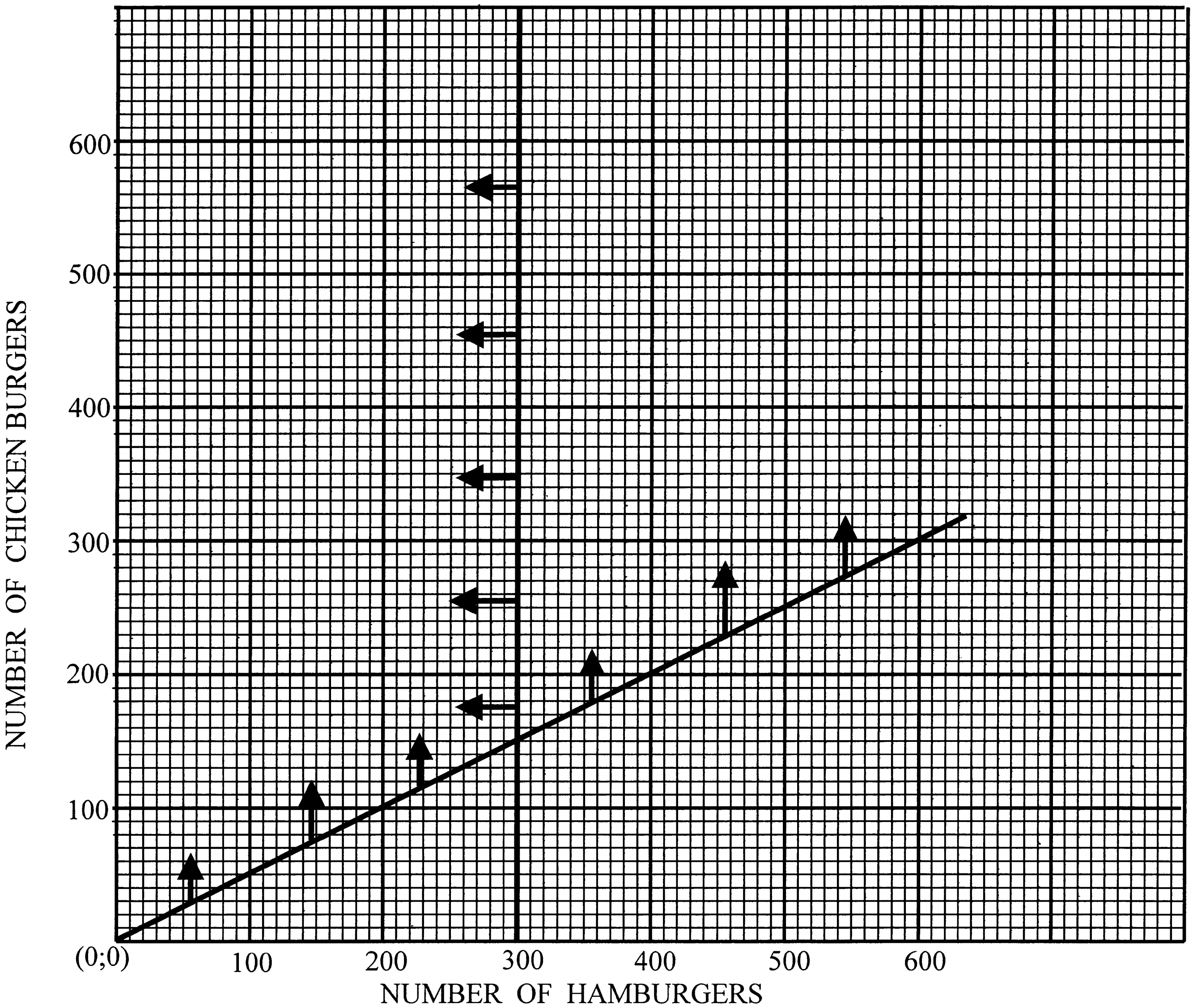
SENTRUMNOMMER



QUESTION 8

EXAMINATION NUMBER

CENTRE NUMBER



SENIOR CERTIFICATE EXAMINATION - 2004
SENIORSERTIFIKAAT-EKSAMEN - 2004

Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(a + l) \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = a.r^{n-1} \qquad S_n = \frac{a(1-r^n)}{1-r}\ (r \neq 1) \qquad S_n = \frac{a(r^n - 1)}{r-1}\ (r \neq 1)$$

$$S_\infty = \frac{a}{1-r}\ (r \neq 1)$$

$$A = P\left(1 + \frac{r}{100}\right)^n \qquad A = P\left(1 - \frac{r}{100}\right)^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2$$

$$(x-p)^2 + (y-q)^2 = r^2$$

In ΔABC: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \tfrac{1}{2}ab.\sin C$$