GAUTENG DEPARTMENT OF EDUCATION

SENIOR CERTIFICATE EXAMINATION

ADDITIONAL MATHEMATICS HG

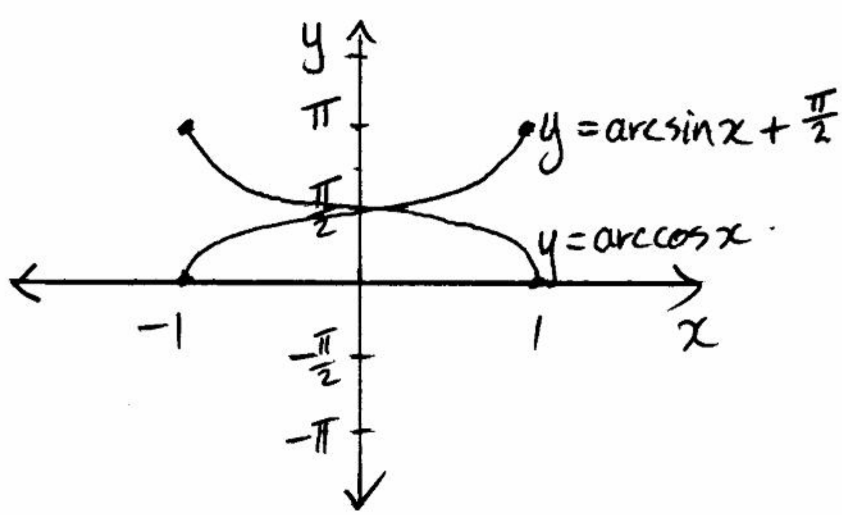
Possible Answers / Moontlike Antwoorde
Feb / Mar / Maart 2006

SECTION A

COMPULSORY
CALCULUS

QUESTION 1

1.1

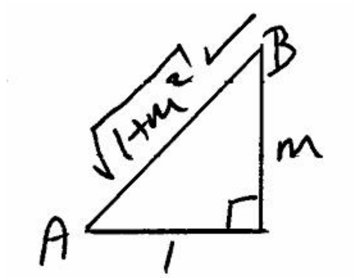


For each graph: Form
End points
Increasing
or decreasing (12)

1.2.1 $\arccos\left(\frac{-\sqrt{3}}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ (6)

1.2.2 Let $\arctan m = A$ and $\arctan \frac{1}{m} = B$

$\therefore \tan A = m$ $\therefore \tan B = \frac{1}{m}$



$\therefore \sin(A + B) = \sin A.\cos B + \cos A.\sin B$ (F)

$= \frac{m}{\sqrt{1+m^2}} \cdot \frac{m}{\sqrt{1+m^2}} + \frac{1}{\sqrt{1+m^2}} \cdot \frac{1}{\sqrt{1+m^2}}$ (Subst)

$= \frac{m^2+1}{1+m^2}$

$= 1$ (Answ) (12)

[30]

QUESTION 2

2.2.1 If differentiable, also continuous
$\therefore 2p + 2q - 4 = 4q - 2p + p$
$\therefore 3p + 2q = 6$ ... (1)
Diff: $p = 4q - p$
$\therefore p = 2q$ ... (2)
Solve (1) and (2): $p = 2$; $q = 1$ (12)

2.2.1 $\lim_{x \to 3^-} g(x) = 9 - 4 = 5$ and $\lim_{x \to 3^+} g(x) = 3 + 2 = 5$

$\therefore \lim_{x \to 3} g(x) = 5$ but $g(3) = 4$

$\therefore \lim_{x \to 3} g(x) \neq g(3)$ $\therefore$ Not continuous

$\therefore$ Removable discontinuity (10)

2.2.2 No, because not continuous (4)

[26]

QUESTION 3

3.1.1 $\lim_{n \to x} \dfrac{n^3\left(\dfrac{2}{n^3} + \dfrac{1}{n^2} - 4\right)}{n^3\left(\dfrac{5}{n^3} + 2\right)} = \dfrac{-4}{2} = -2$ (4)

3.1.2 $|x-9| = \begin{cases} x-9 \text{ if } x \geq 9 \\ -x+9 \text{ if } x < 9 \end{cases}$

$\therefore \lim_{x \to 9^-} \dfrac{(x-9)(x+9)}{-x+9} = -18$ and $\lim_{x \to 9^+} \dfrac{(x-9)(x+9)}{x-9} = 18$

$\therefore$ limit does not exist. (10)

3.2.1 $f'(x) = \lim_{h \to 0} \dfrac{\dfrac{1}{\sqrt{2-3(x+h)}} - \dfrac{1}{\sqrt{2-3x}}}{h}$

$= \lim_{h \to 0} \left[\dfrac{\sqrt{2-3x} - \sqrt{2-3x-3h}}{\sqrt{2-3x-3h}.\sqrt{2-3x}} \text{ x } \dfrac{\sqrt{2-3x} + \sqrt{2-3x-3h}}{\sqrt{2-3x} + \sqrt{2-3x-3h}} \text{ x } \dfrac{1}{h} \right]$

$= \lim_{h \to 0} \left[\dfrac{2-3x-(2-3x-3h)}{\sqrt{2-3x-3h}.\sqrt{2-3x}\left(\sqrt{2-3x} + \sqrt{2-3x-3h}\right)} \text{ x } \dfrac{1}{h} \right]$

$= \lim_{h \to 0} \left[\dfrac{3h}{\sqrt{2-3x-3h}.\sqrt{2-3x}\left(\sqrt{2-3x} + \sqrt{2-3x-3h}\right)} \text{ x } \dfrac{1}{h} \right]$

$= \dfrac{3}{2(2-3x)^{\frac{3}{2}}}$ (10)

3.2.2 $g'(x) = \dfrac{2x(x^5 - x^3 + x) - (x^2+1)(5x^4 - 3x^2 + 1)}{(x^5 - x^3 + x)^2}$

$\therefore g'(1) = \dfrac{2.1(1) - 2(5-3+1)}{(1)^2} = -4$ (12)

3.3 $f'(x) = 40(1-5x)^{39} . (-5)$

$f''(x) = 40.39\ (1-5x)^{38}.(-5)^2$

$f^n(x) = \dfrac{40!}{(40-n)!}(1-5x)^{40-n}\ (-5)^n$ (10)

[46]

QUESTION 4

4.1 $h(x) = \frac{1}{2}x - 3\sin x = 0$

$\therefore h'(x) = \frac{1}{2} - 3\cos x$

$$\therefore a_{n+1} = a_n - \frac{\frac{1}{2}a_n - 3\sin a_n}{\frac{1}{2} - 3\cos a_n}$$

Begin with $a_1 = 3$ (6)

4.2 At B: $f(x) = g(x)$

$\therefore f(x) - g(x) = 0$

$\therefore \frac{1}{2}x - (-3\sin x) = 0$

Let $k(x) = \frac{1}{2}x + 3\sin x = 0$

$\therefore k'(x) = \frac{1}{2} + 3\cos x$

$$\therefore a_{n+1} = a_n - \frac{\frac{1}{2}a_n + 3\sin a_n}{\frac{1}{2} + 3\cos a_n}$$

(6)

4.3 At C: $h'(x) = 0$

$\therefore \frac{1}{2} + 3\cos x = 0$

Let $p(x) = \frac{1}{2} + 3\cos x$

$\therefore p'(x) = -3\sin x$

$$\therefore a_{n+1} = a_n - \frac{\frac{1}{2} + 3\cos a_n}{-3\sin a_n}$$

(6)

[18]

QUESTION 5

5.1 $\Delta x_i = \frac{4-0}{n} = \frac{4}{n}$; $x_i = 0 + i\left(\frac{4}{n}\right) = \frac{4i}{n}$

$$f(x_i) = \frac{16i}{n} - \frac{16i^2}{n^2}; \quad f(x_i).?\,x_i = \frac{64i}{n^2} - \frac{64i^2}{n^3}$$

$$\therefore \sum_{n=1}^{n} f(x_i)\,?\,x_i = \frac{64}{n^2}\sum_{i=1}^{n} i - \frac{64}{n^3}\sum_{i=1}^{n} i^2$$

$$= \frac{64}{n^2}\cdot\frac{n}{2}(n+1) - \frac{64}{n^3}\cdot\frac{n}{6}(2n+1)(n+1)$$

$$= 32\left(1+\frac{1}{n}\right) - \frac{32}{3}\left(2+\frac{1}{n}\right)\left(1+\frac{1}{n}\right)$$

$$\therefore \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i).?\,x_i = 32 - \frac{64}{3}$$

$$= 32 - 21\frac{1}{3} = 10\frac{2}{3}$$

(20)

5.2 $V = \pi\int_0^4 (4x - x^2)^2\,dx$

$$= \pi\int_0^4 (16x^2 - 8x^3 + x^4)dx$$

$$= \pi\left[\frac{16x^3}{3} - \frac{8x^4}{4} + \frac{x^5}{5}\right]_0^4$$

$$= \pi\left[\frac{16.4^3}{3} - 2.(4)^4 + \frac{4^5}{5} - 0\right] = 34,13\pi$$

(12)

[32]

QUESTION 6

6.1.1 $\int \sin\theta\,(1-\cos\theta)^{-½}\,d\theta$ or Let $1-\cos\theta = u$ $\therefore \sin\theta.d\theta = du$

$= 2(1-\cos\theta)^{½} + C$ $\quad\therefore d\theta = \frac{du}{\sin\theta}$

$$\therefore \int u^{-½}\,du = \frac{u^{½}}{½} + C$$

$$= 2(1-\cos\theta)^{½} + C$$

(8)

6.1.2 $9t^2 + 6t + 2 = (3t + 1)^2 + 1$

$\therefore \int \frac{1}{1+(3t+1)^2}\,dt$

$= ?\ \arctan(3t + 1) + C$ (8)

6.2.1 $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin 2x - 2\cos^2 x)\,dx$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} [\sin 2x - 2.½(1+\cos 2x)]dx$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} [\sin 2x - 1 - \cos 2x)]dx$

$= [-½\cos 2x - x - ½\sin 2x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= -½\cos\pi - \frac{\pi}{2} - ½\sin\pi - \left(-½\cos\frac{\pi}{2} - \frac{\pi}{4} - ½\sin\frac{\pi}{2}\right)$

$= ½ - \frac{\pi}{2} - 0 + 0 + \frac{\pi}{4} + ½$

$= 1 - \frac{\pi}{4}$ (16)

6.2.2 Distance: $g(x) - f(x) = \sin 2x - 2\cos^2 x = h(x)$

Max where $h'(x) = 0$

$\therefore 2\cos 2x - 4\cos x(-\sin x) = 0$

$\therefore \cos 2x + 2\sin x.\cos x = 0$

$\therefore \cos 2x - \sin 2x = 0$

$\therefore \tan 2x = -1$

$\therefore\ 2x = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\therefore\ x = \frac{3\pi}{8}$

and h"(x) $= 2(-\sin 2x).2 + 4\cos^2 x - 4\sin^2 x$

$= 4\cos^2 x - 4.\ \sin 2x - 4\sin^2 x$

$= 4(\cos 2x - \sin^2 x) - 4\sin^2 x$

$= 4\cos 2x - 4\sin 2x$ } or direct

$= 4\cos\frac{3\pi}{4} - 4\sin\frac{3\pi}{4}$

$= 4\left(\frac{-\sqrt{2}}{2}\right) - 4\left(\frac{\sqrt{2}}{2}\right)$

$= -2\sqrt{2} - 2\sqrt{2} = -4\sqrt{2} < 0$ (or -5,6...)

$\therefore$ Maximum! (16)

[48]

TOTAL FOR SECTION A: [200]

SECTION B

QUESTION 7

7.1 The profit (2)

7.2 $\int_0^{200} (30 - mq - 10)dq = 100$

$\therefore \left(30q - \frac{mq^2}{2} - 10q\right)\Big|_0^{200} = 100$

$\therefore$ 4 000 – 20 000m = 100 (subst)

$\therefore$ m = 0,195 (12)

[14]

QUESTION 8

8.1 $1\ 000(1{,}13)^5$ = R1 842,44 (4)

8.2 1 842,44 = A(1 + 5.0,13)

$\therefore$ A = R1 116,63 (6)

[10]

QUESTION 9

T_0	T_{13}			T_{16}	T_{18}
R30 000	x			x	x

At T18: $i = \frac{0{,}125}{12}$

$$\underbrace{x + x\,(1 + i)^{24} + x\,(1 + i)^{60}} = 30\,000\,(1 + i)^{216}$$

$\therefore\ x(4{,}144582188) = 281338{,}7481$

$\therefore\ = \text{R}67\,881{,}09$ **[12]**

QUESTION 10

10.1 $\text{R}8\,000\,(1 + i)^5 = \text{R}13\,480$

$\therefore\ i = 11\%$ (4)

10.2 $\text{R}13\,480 - \text{R}8\,000\,(1 - 0{,}11)^5 = \text{R}9\,012{,}75$ (6)

10.3 $\text{R}9\,012{,}75\,(1{,}0075)\text{PV} = \frac{x(1{,}0075^{58} - 1)}{0{,}0075}$

At T58: $\therefore\ x = \text{R}122{,}76$ (10)

[20]

QUESTION 11

11.1 Interest: $\text{R}725\,000\,(0{,}01) = \text{R}7\,250$

$\therefore$ No, interest more than what I pay.

OR: $\text{R}725\,000 = \text{R}6\,000\left(\frac{1 - 1{,}01^{-n}}{0{,}01}\right)$

$\therefore\ 1{,}01^{-n} = -0{,}208\ \ldots$ no answer. (6)

11.2 $\text{R}72\,500 = \text{R}8\,000\left(\frac{1 - 1{,}01^{-n}}{0{,}01}\right)$

$\therefore\ 1{,}01^{-n} = 0{,}09375$

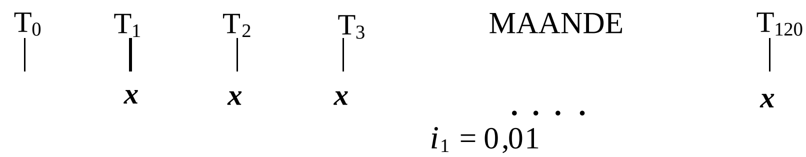
$\therefore -n = \frac{\log 0{,}09375}{\log 1{,}01} = -237{,}89$

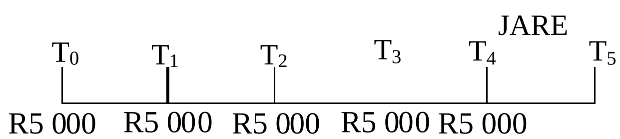
$\therefore$ 237 months + 1 last payment. (10)

11.3 $$\text{OB}: R725\,000\,(1{,}01)^{120} - R8\,000\left(\frac{1{,}01^{120}-1}{0{,}01}\right) \text{ Formula}$$

$$= R552\,470{,}98$$ (10)

11.4





$\therefore$ Convert interest rate to annual effective. / *Herlei rentekoers na jaarliks effektief.*

$$1 + i = \left(1 + \frac{i^{(12)}}{12}\right)^{12}$$

$$\therefore\ i = 1{,}01^{12} - 1$$

$$\therefore\ i = 0{,}12682503\ .\ .\ .$$

<u>At To</u>:

$$R552\,471{,}00 = \frac{x\left[1-1{,}01^{-120}\right]}{0{,}01\ \text{F}} + \frac{5\,000\left[1-1{,}126^{-4}\ ...\right]}{0{,}126\ ...\ \text{F}} + 5\,000$$

$$\therefore R532\,499{,}9965 = \frac{x\left[1-1{,}01^{-120}\right]}{0{,}01}$$

$$\therefore\ x = \underline{R7\,639{,}83} \longrightarrow$$

(18)

[44]

TOTAL FOR SECTION B: [100]

SECTION C

QUESTION 12

12.1 $ae = \sqrt{5}$ and $b = a\sqrt{1 - e^2}$

$\therefore a\left(\frac{\sqrt{5}}{3}\right) = \sqrt{5}$ $= 3\sqrt{1 - \left(\frac{\sqrt{5}}{3}\right)^2}$

$\therefore \frac{a}{3} = 1$ $= 3\sqrt{1 - \frac{5}{9}}$

$\therefore a = 3$ $= 3\sqrt{\frac{4}{9}} = 3 \text{ x } \frac{2}{3} = 2$ (12)

12.2 Equation of normal: $\frac{y - y_1}{x - x_1} = \frac{a^2 y_1}{b^2 x_1}$

$$\therefore \frac{y - \sqrt{\frac{8}{3}}}{x - \sqrt{3}} = \frac{9\sqrt{\frac{8}{3}}}{4\sqrt{3}}$$

$$\therefore 4\sqrt{3}\left(y - \frac{2\sqrt{2}}{3}\right) = \frac{18\sqrt{2}}{\sqrt{3}}\left(x - \sqrt{3}\right)$$

$$\therefore 4\sqrt{3}y - 8\sqrt{2} = \frac{18\sqrt{2}}{\sqrt{3}}x - 18\sqrt{2}$$

$$\therefore y = \frac{3\sqrt{2}}{2}x - \frac{5\sqrt{2}}{2\sqrt{3}} \left(\text{or } \frac{3}{\sqrt{2}}x - \frac{5}{\sqrt{6}}\right)$$ (14)

12.3 For the normal: $m_1 = \frac{3\sqrt{2}}{2}$

And for the diameter through $P\left(\sqrt{3}; \sqrt{\frac{8}{3}}\right)$ and (0;0)

$m_2 = \frac{\sqrt{8}}{3}$

$\therefore$ Angle $\theta = \text{bgtan}\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$

$= \arctan\left(\frac{\frac{3\sqrt{2}}{2} - \frac{\sqrt{8}}{3}}{1 + \frac{3\sqrt{2}\sqrt{8}}{6}}\right)$

$\therefore \theta = \arctan 0{,}3928371\ldots$

$\therefore \theta = 21{,}45°$ (14)

[40]

QUESTION 13

13.1 Substitute $x = at^2$ and $y = 2at$ in $y^2 = 4ax$

$\therefore$ LHS $= (2at)^2$ and RHS $= 4a(at^2)$

$= 4a^2t^2$ $\quad$ $= 4a^2t^2$

$\therefore$ LHS = RHS (6)

13.2 Gradient of parabola: $2y.\frac{dy}{dx} = 4a$

$\therefore \frac{dy}{dx} = \frac{2a}{y}$

At the point: $(at^2; 2at)$: $m = \frac{2a}{2at} = \frac{1}{t}$

$\therefore$ Equation of tangent: $y - 2at = \frac{1}{t}(x - at^2)$

$\therefore ty - 2at^2 = x - at^2$

$\therefore x - ty + at^2 = 0$ (10)

13.3 Equation of RQPT: $x - ty + at^2 = 0$

At R: $y = 0 \quad \therefore x = -at^2 \quad \therefore R(-at^2; 0)$ and $F(a; 0)$

$\therefore RF^2 = (a + at^2)^2$ and $PF^2 = (at^2 - a)^2 + (2at)^2$

$= a^2 + 2a^2t^2 + a^2t^4$ $\quad = a^2t^4 - 2a^2t^2 + a^2 + 4a^2t^2$

$= a^2t^4 + 2a^2t^2 + a^2$

$\therefore RF = PF$

$\therefore \hat{R}_1 = \hat{P}_2$ ... angles opp. equal sides

but $\hat{R}_1 = \hat{P}_1$ ...corresp.<[e]; PA ∥ RF

$\therefore \hat{P}_1 = \hat{P}_2$ (18)

[34]

QUESTION 14

14.1.1 AB has direction numbers 2; -1; 1
AC has direction numbers 1; 2; 0

If a, b and c are direction numbers of a line l, normal to the plane then

$2a - b + c = 0$ ... (i) $(AB \perp l_1)$

and $a + 2b = 0$ ... (ii) $(AC \perp l_1)$

ii + 2i: $5a + 2c = 0$

$\therefore a = -\frac{2}{5}c$...

$\therefore a = -2;\ b = -1$ and $C = 5$

$\therefore$ Equation of plane is $-2(x-0) + 1(y+1) + 5(z-1) = 0$

$\therefore 2x - y - 5z + 4 = 0$

(12)

14.1.2 Substitute (-1; 2; 0) into the equation

$\therefore$ LHS $= 2(-1) - 2 - 5(0) + 4 = 0 =$ RHS

$\therefore$ D lies on the same plane as A, B and C. (2)

14.1.3 Direction numbers for BC: 1; -3; 1
Direction numbers for DA: 1; -3; 1 $\quad \therefore$ BC ∥ DA

Direction numbers for CD: -2; 1; -1 $\therefore$ CD ∥ AB

$\therefore$ ABCD is a parallelogram ... opp. sides ∥ (6)

14.2 Direction numbers for AP: 1; 3; 2

$\therefore$ Equation for AP: $\dfrac{x-1}{1} = \dfrac{y-2}{3} = \dfrac{z-3}{2}$

[Alt: $x = 1 + k$; $y = 2 + 3k$; $z = 3 + 2k$] (2)

14.3 $d = \left|\dfrac{2(1) - 2 - 5(3) + 4}{\sqrt{2^2 + (-1)^2 + (-5)^2}}\right| = \left|\dfrac{-11}{\sqrt{30}}\right| = \dfrac{11}{\sqrt{30}}$ (4)

[26]

TOTAL FOR SECTION C: [100]

SECTION D

QUESTION 15

15.1 If $P(x) \in Z[x]$ and $a + b\sqrt{c}$, $a, b, c \in Q$, is an irrational zero of $P(x)$, then its conjugate surd $a - b\sqrt{c}$ will also be a zero of $P(x)$. (6)

15.2 Given $-1 + 2\sqrt{3}$; $-1-2\sqrt{3}$ also a zero $(x+1-2\sqrt{3})(x+1+2\sqrt{3})$ a factor
$= x^2 + 2x - 11$

			1	3	3	2
1 2 -11	1	5	-2	-25	-29	-22
	1	2	-11			
		3	9	-25		
		3	6	-33		
			3	8	-29	
			3	6	-33	
				2	4	-22
				2	4	-22

$\therefore f(x) = (x^2+2x-11)(x^3+3x^2+3x+2)$

↓

$x = -2$ is a zero

-2	1	3	3	2
	1	1	1	0

$\therefore f(x) = (x^2 + 2x - 11)(x + 2)(x^2 + x + 1)$ (20)

15.3 $2x^3 - 5x^2 + 10x - 5$, etc. (4)

[30]

QUES TION 16

16.1 Let n =1: 8 – 7 + 6 = 7 $\therefore$ True for n = 1.

Assu me statemen t true for n = k:

$\therefore$ $8^k - 7k + 6$ is divisible by 7

Let n = k+1: $8^{k+1} - 7(k + 1) + 6$ Subst.

$= 8.8^k - 7k + 6 - 7$

$= 8(8^k - 7k + 6) + 7.7k - 49$ ... divis ible by 7

$\therefore$ If true for n = k, it's also true for n = k+1

$\therefore$ True for all $n \in N$. (16)

16.2 $\alpha+\beta+\gamma=\frac{-3}{4}$

$\alpha\beta+d\gamma+\beta\gamma=\frac{1}{2}$

$\alpha\beta\gamma=\frac{-1}{4}$

$\therefore \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}=\frac{\frac{1}{2}}{\frac{-1}{4}}=-2$

$\frac{1}{\alpha\beta}+\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}=\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}=\frac{-\frac{3}{4}}{-\frac{1}{4}}=3$

$\frac{1}{\alpha\beta\gamma}=\frac{1}{-\frac{1}{4}}=-4$

$\therefore$ Sum : -2 + 3 -4 = -3

Sum / produc t: -2.3 + 2.4 + (-3)(4) = -10

Produ ct: (-2)(3)(-4) = 24

$\therefore x^3 + 3x^2 - 10x - 24$ (18)

[34]

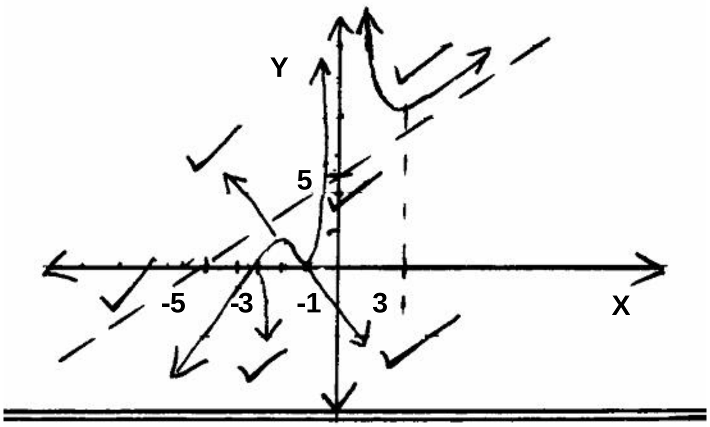
QUESTION 17

17.1 Vertical: x = 0
Oblique: y = x + 5 (4)

17.2 $\frac{7}{x}+\frac{3}{x^2}=0 \quad \therefore 7x=-3$

$\therefore x=-\frac{3}{7}$ (4)

17.3



(12)
[20]

QUESTION 18

$$\frac{3x^2+8x-3}{(x+1)^2\,(x-1)}=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-1}$$

$\therefore\ 3x^2 + 8x - 3 = A(x+1)(x-1) + B(x-1) + C(x+1)^2$

$x = 1$: $8 = 4C \quad \therefore C = 2$

$x = -1$ $-8 = -2B \quad \therefore B = 4$

x^2: $3 = A+C \quad \therefore A = 1$

$$\therefore \frac{1}{x+1}+\frac{4}{(x+1)^2}+\frac{2}{x-1}$$

[16]

TOTAL FOR SECTION D: [100]

SECTION E

QUESTION 19

19.1 Number of possible wins: $\binom{49}{6} = 13983816$

50%: $\binom{49}{6} \div 2 = 6991908$

... x R5: R34 959 540 (8)

19.2

19.2.1 $\frac{3}{10}$ (or 0,3) (2)

19.2.2 $\frac{2}{9}$ (or $0,\dot{2}$) (2)

19.2.3 $\dfrac{\binom{3}{1}\overbrace{\binom{4}{1}\binom{2}{1}}\binom{1}{0}}{\binom{10}{3}} = 0,2$ (8)

19.3 $0,6x = 0,6 + x - 0,88$

$\therefore x = 0,7$ (4)

[24]

QUESTION 20

20.1 $\binom{30}{20}\left(\frac{1}{2}\right)^{20}\left(\frac{1}{2}\right)^{10} = 0,02798$ (8)

20.2 $\binom{30}{30}\left(\frac{1}{2}\right)^{30}\left(\frac{1}{2}\right)^{0} = \left(\frac{1}{2}\right)^{30} = \frac{1}{2^{30}}$ (4)

20.3 $P(X = 1) = 1 - P(X = 0)$

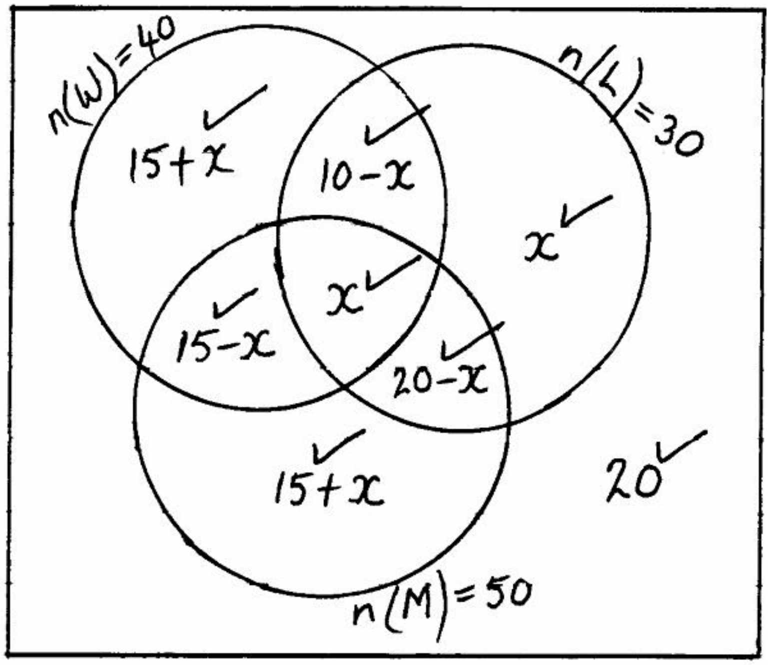
$= 1 - \binom{30}{0}\left(\frac{1}{2}\right)^{30}\left(\frac{1}{2}\right)^{0}$

$= 0,999 \ldots$ (6)

[18]

QUESTION 21

21.1 n(S) = 100



(16)

21.2 $15 + x + 10 - x + 15 - x + x + 20 - x + 15 + x + x = 80$

$\therefore$ <u>$x = 5$</u>

[or: .... + 20 = 100] (4)

[20]

QUESTION 22

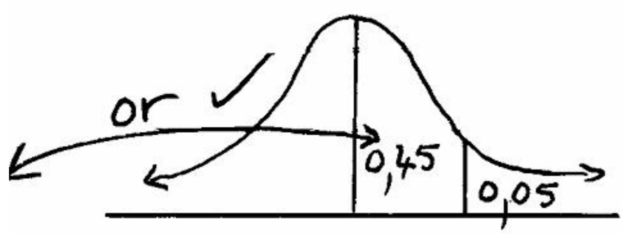
22.1 $X \sim n\,(100; (15)^2)$

$\therefore P(Z \geq a) = 0{,}05$

$\therefore P(Z \leq a) = 0{,}95$

$\therefore \dfrac{X - 100}{15} \leq 1{,}645$

$\therefore X \leq 124{,}675$



(10)

22.2 5% of 1 500 = 75 (4)

22.3 $\dfrac{85 - 100}{15} = -1 \;\therefore P(Z \leq -1)$

$= 0{,}5 - 0{,}3413 = 0{,}1587$

$\therefore 15{,}87\%$ (8)

[22]

QUESTION 23

23.1 $\frac{490}{500} = 0{,}98$ (4)

23.2 $p \pm 1{,}96\sqrt{\frac{p(1-p)}{n}}$

$= 0{,}98\ ^{F}\pm 1{,}96\sqrt{\frac{(0{,}98).(0{,}02)}{500}}$

$= 0{,}98 \pm 0{,}0122 \ldots$

$\therefore$ (0,968; 0,992) (12)

[16]

TOTAL FOR SECTION E: [100]

TOTAL: 400