

GAUTENG DEPARTMENT OF EDUCATION
GAUTENG DEPARTEMENT VAN ONDERWYS

SENIOR CERTIFICATE EXAMINATION
SENIORSER TIFIKAAT-EKSA MEN

ADDITIONAL MATHEMATICS HG
ADDISIONELE WISKUNDE

POSSIBLE ANSWERS OCT / NOV 2006

SECTION / AFDELING A
VERPLIGTEND
CALCULUS

QUESTION / VRAAG 1

$$1.1.1 \quad \theta = \frac{3\pi}{2} \div 3 = \frac{\pi}{2} \quad k \quad (2)$$

$$1.1.2 \quad \begin{aligned} \text{On} / \text{Op} &= \frac{1}{2} r^2 \theta \\ &= \frac{1}{2} \cdot 9 \cdot \frac{\pi}{4} = \frac{9\pi}{8} \quad k \end{aligned} \quad (2)$$

$$1.1.3 \quad \begin{aligned} \text{On} / \text{Op} \Delta &= \frac{1}{2} r^2 \sin \theta \\ &= \frac{1}{2} r^2 \sin \frac{\pi}{2} \quad k \\ &= \frac{1}{2} r^2 = 4,5 \quad k \end{aligned}$$

$$\therefore \text{On shaded part} / \text{Op gesk. deel} = \frac{9\pi}{4} - 4,5$$

$$\sim 2,57$$

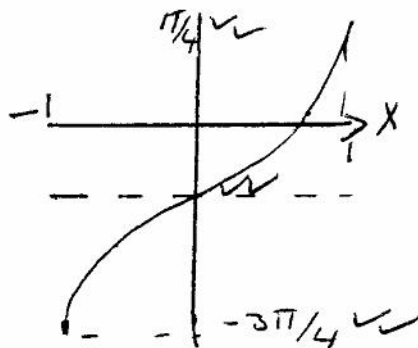
(6)
[10]

QUESTION / VRAAG 2

$$2.1.1 \quad -\frac{p}{2} \leq \arcsin / \text{bg sin } x \leq \frac{p}{2} \quad k$$

$$\therefore -\frac{3\pi}{4} \leq \arcsin / \text{bg sin } x - \frac{\pi}{4} \leq \frac{\pi}{4} \quad k \quad (4)$$

2.1.2



(6)

$$2.2.1 \quad \arcsin / \text{bg sin } \left(\cos \frac{\pi}{6} \right) \quad k$$

$$= \arcsin / \text{bg sin } \left(-\frac{\sqrt{3}}{2} \right) \quad k$$

$$= -\frac{\pi}{3} \quad k \quad (6)$$

$$2.2.2 \quad \arccos / \text{bg cos } \left(\sin \frac{\pi}{6} \right) \quad k$$

$$= \arccos / \text{bg cos } \left(\frac{1}{2} \right) \quad k$$

$$= \pi - \frac{\pi}{3} \quad k$$

$$= \frac{2\pi}{3} \quad k \quad (8)$$

$$2.2.3 \quad \text{Set } \arctan / \text{Stel bg tan } \frac{5}{12} = A$$

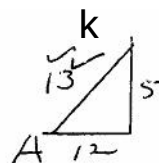
$$\therefore \tan A = \frac{5}{12} \quad k$$

$$\cos 2A \quad k$$

$$= \cos^2 A - \sin^2 A \quad k$$

$$= \frac{144}{169} - \frac{25}{169} \quad k$$

$$= \frac{119}{169} \quad k$$

(10)
[34]

QUESTION / VRAAG 3

3.1 $\lim_{x \rightarrow 0^-} f(x) = 2$ $\lim_{x \rightarrow 0^+} f(x) = |-2| = 2$ $f(0) = 3$
 $\therefore$ Discontinue: removable / Diskontinu: verwyderbaar (8)

3.2 No, because discontinuous / Nee, want nie kontinuu (2)

3.3 $f(x) = x - 2$ if / as $x = 2$
 $-x + 2$ if / as $0 < x < 2$
 $\lim_{x \rightarrow 2^-} f(x) = -1$ and / en $\lim_{x \rightarrow 2^+} f'(x) = 1$
 $\therefore$ Not diff. at $x = 2$. / Nie diff.baar by $x = 2$ nie. (8)

[18]

QUESTION / VRAAG 4

4.1 $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} \cdot \frac{1 + \cos x}{1 + \cos x}$ $\left(\frac{0}{0}, \therefore \text{L'H} \right)$
 $= \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x^2(1 + \cos x)}$ or / of $\lim_{x \rightarrow 0} \frac{\sin x}{2x}$ $\left(\frac{0}{0}, \therefore \text{L'H} \right)$
 $= \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2(1 + \cos x)}$ $= \lim_{x \rightarrow 0} \frac{\cos x}{2}$
 $= \frac{1}{2}$ (8)

4.2 $\lim_{x \rightarrow \infty} \frac{x \sqrt{9 - \frac{3}{x} + \frac{2}{x^2}}}{x \left(9 - \frac{5}{x} \right)}$
 $= \frac{1}{3}$ (6)

[14]

QUESTION / VRAAG 5

$$x_i = \frac{2i}{n} k$$

$$x_i = \frac{2i}{n} k$$

$$f(x_i) = 2 \left(\frac{2i}{n} \right)^2 + 1 k$$

$$= \frac{8i^2}{n^2} + 1 k$$

$$f(x_i) \cdot x_i = \frac{16i^2}{n^3} + \frac{2}{n} k$$

$$\sum_{i=1}^n f(x_i) \cdot x_i = \sum_{i=1}^n \left(\frac{16i^2}{n^3} + \frac{2}{n} \right) k$$

$$= \frac{16}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{2}{n} \cdot n k$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \cdot x_i = \frac{8}{3} \cdot 2 + 2$$

$$= 7 \frac{1}{3} k$$

(18)

[18]

QUESTION / VRAAG 6

$$6.1 \quad f'(x) = \frac{\cos x \cdot \cos x - \sin x \cdot -\sin x}{\cos^2 x} k$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} k$$

$$= \frac{1}{\cos^2 x} k$$

$$= \sec^2 x k$$

(8)

$$6.2.1 \quad f'(x) = \frac{\sec^2 x \cdot \sec^2 x - \tan x \cdot 2 \sec x \cdot \sec x \tan x}{\sec^4 x} \quad k$$

$$f'\left(\frac{\pi}{6}\right) = \frac{\sec^4\left(\frac{\pi}{6}\right) - 2 \tan^2\left(\frac{\pi}{6}\right) \sec^2 \frac{\pi}{6}}{\sec^4\left(\frac{\pi}{6}\right)} \quad k$$

$$= 1 - \frac{2 \cdot \frac{1}{3}}{\frac{4}{3}} \quad k$$

$$= \frac{1}{2} \quad k$$

(12)

OR / OF

$$\begin{aligned} f(x) &= \tan x \cdot \cos^2 x \quad k \\ f'(x) &= \sec^2 x \cdot \cos^2 x + \tan x \cdot 2 \cos x \cdot -\sin x \\ f'\left(\frac{\pi}{6}\right) &= 1 + -2 \sin^2\left(\frac{\pi}{6}\right) \quad k \\ &= 1 - 2 \left(\frac{1}{4}\right) \quad k \\ &= \frac{1}{2} \quad k \end{aligned}$$

$$6.2.2 \quad \frac{dy}{dx} = \frac{k}{3(\arccos / \arcsin \cos(2x+1))^2} \cdot \frac{1}{\sqrt{1 - (2x+1)^2}} \cdot 2 \quad k$$

(6)

$$\begin{aligned} 6.3 \quad f'(x) &= \frac{k}{3(1+x^2)^2} \cdot 2x \quad k \\ f''(x) &= \frac{k}{6[2(1+x^2) \cdot 2x \cdot x + (1+x^2)^2 \cdot 1]} \\ &= \frac{k}{6[(1+x^2)(4x^2 + 1 + x^2)]} \\ &= \frac{k}{6(1+x^2)(5x^2 + 1)} \end{aligned}$$

(10)
[36]

QUESTION / VRAAG 7

$$\begin{aligned} 7.1 \quad & \int_2^6 3(2x-3)^{-1/2} dx \quad k \\ &= \frac{3(2x-3)^{1/2} \cdot 2}{2 \quad k} \Big|_2^6 \\ &= 3[\sqrt{2 \cdot 6 - 3} - \sqrt{2 \cdot 2 - 3}] \quad k \\ &= 3(\sqrt{9} - 1) \\ &= 6 \quad k \end{aligned}$$

(10)

$$\begin{aligned}
 7.2 \quad & -\frac{1}{2} \int [\cos(4x-1) - \cos(-2x+3)] dx \\
 & = -\frac{1}{2} \left[\sin(4x-1) \cdot \frac{1}{4} - \sin(-2x+3) \cdot \frac{1}{2} \right] + k
 \end{aligned}
 \tag{10}$$

[20]

QUESTION / VRAAG 8

$$\begin{aligned}
 8.1 \quad g'(x) &= -\left((1+x^2)^{-2} \cdot 2x - \frac{1}{1+x^2} \right) k \\
 &= \frac{-2x-1-x^2}{(1+x^2)^2} k \\
 &= \frac{-(x+1)^2}{(1+x^2)^2} k
 \end{aligned}
 \tag{10}$$

$$\begin{aligned}
 8.2 \quad A_0 &= 1 \\
 A_1 &= 0,7146 \quad k \\
 A_2 &= 0,7468 \quad k \\
 A_3 &= 0,7472 \\
 A_4 &= 0,7472 \quad k \\
 x &= 0,747 \quad k
 \end{aligned}
 \tag{8}$$

[18]

QUESTION / VRAAG 9

$$\begin{aligned}
 9.1 \quad L &= \int_0^{\frac{3}{2}} \sqrt{1 + \left(\frac{x^{\frac{1}{2}}}{k}\right)^2} \cdot dx \quad k & f'(x) &= x^{\frac{1}{2}} \\
 &= \left(\frac{k}{1+x}\right)^{\frac{3}{2}} \cdot \frac{2}{3} \Bigg|_0^{\frac{3}{2}} \\
 &= \frac{2}{3} \left[\left(\frac{k}{1+\frac{3}{2}}\right)^{\frac{3}{2}} - \left(\frac{k}{1+0}\right)^{\frac{3}{2}} \right] \\
 &= \frac{2}{3} \left[2,5^{\frac{3}{2}} - 1 \right] \quad k \\
 &= 1,969 \quad k
 \end{aligned} \tag{12}$$

$$\begin{aligned}
 9.2.1 \quad f\left(\frac{3}{2}\right) &= \frac{2}{3} \left(\frac{3}{2}\right)^{\frac{3}{2}} \quad k \\
 \text{Circum. / Omtrek} &= \frac{3}{2} + 1,969 + 1,225 \\
 &= 4,694 \quad k
 \end{aligned} \tag{6}$$

$$\begin{aligned}
 9.2.2 \quad V &= \pi \int_0^{\frac{3}{2}} [f'(x)]^2 dx - \pi \int_0^{\frac{3}{2}} (f(x))^2 dx \\
 &= \pi \int_0^{\frac{3}{2}} \left(\frac{k}{x^{\frac{1}{2}}}\right)^2 dx - \pi \int_0^{\frac{3}{2}} \frac{4}{9} x^3 dx \\
 &= \pi \left[\frac{1}{2} \frac{k}{x^2} - \frac{1}{9} \frac{k}{x^4} \right] \Bigg|_0^{\frac{3}{2}} \quad k \\
 &= p (0,5625) \\
 &= 1,767 \quad k
 \end{aligned} \tag{14}$$

[32]

TOTAL FOR SECTION A / TOTAAL VIR AFDELING A: [200]

SECTION / AFDELING B
QUESTION / VRAAG 10

$$10.1.1 \quad C'(x) = \frac{(400x + 150)(x+1) - (200x^2 + 150x + 80)}{(x+1)^2} \quad \text{k} \quad (6)$$

$$10.1.2 \quad C'(10) = \frac{(400 \cdot 10 + 150)(10+1) - (200 \cdot 10^2 + 150 \cdot 10 + 80)}{(10+1)^2} \quad \text{k}$$

$$= R198,93 \quad \text{k} \quad (4)$$

$$10.2 \quad \frac{C(10) - C(0)}{10} \quad \text{k}$$

$$= \frac{1961,82 - 80}{10} \quad \text{k}$$

$$= R188,18 \quad \text{k} \quad (8)$$

[18]

QUESTION / VRAAG 11

$$11.1 \quad \begin{array}{ccccccccccc} T_0 & T_1 & T_4 & T_6 & & T_8 & T_9 & T_{12} \\ | & | & | & | & m & | & | & | \\ 20\,000 & 5\,000 & x & 2x & & 12\,000 & 3x & 31\,308,34 \end{array} \quad (4)$$

$$11.2 \quad T_{10}: 31\,308,34 + 3x(1,01125)^3 + x(1,01125)^8 = 12\,000(1,01125)^4 + 2x(1,01125)^6 + 5\,000(1,01125)^{11} + 20\,000(1,01125)^{12}$$

$$x = R2\,000,00$$

Points / Punte: $i = 0,01125$ k

right time / regte tyd k

equation cor. / vergelyking reg k

comb. form / saamg: formule k

answ. / antw. k

months / maande k

(12)
[16]

QUESTION / VRAAG 12

$$12.1 \quad \begin{aligned} (1,03)^k &= (1+i)^3 \quad k \\ i &= 0,0099 \dots \quad k \end{aligned} \quad (6)$$

$$12.2 \quad F = \frac{1000(1,0099^{120} - 1)}{0,0099...} \cdot \frac{k}{(1,0099)^{120}} = 228424.70(1,0099)^{120} = R230\,686,10 \cdot k \quad (12)$$

QUESTION / VRAAG 13

$$13.1 \quad \frac{k}{350\,000} = x \left(1 - \left(\frac{\frac{k}{16}}{1\,200} + 1 \right) \right)^{-240} \quad \frac{16}{1\,200} \text{ form/vorm} = 0,013$$

OR/ OF

$$350\,000 = x(1,013)^{-1} \left(\frac{1 - 1,013^{-240}}{1 - 1,013^{-1}} \right)$$

$$= R4\,869,40 \quad (12)$$

$$13.2 \quad UB = 350\,000(1,013)^{120} - \frac{4\,869,40}{0,013}((1,013)^{120} - 1) \quad \text{k}$$

OR / OF

$$\begin{aligned}
 UB &= \frac{4\,869,40\,(1 - 1,013^{\frac{0}{120}})}{,013} \\
 &= R290\,687,68 \\
 &(290\,687,42 - exact / presies)
 \end{aligned}
 \tag{12}$$

$$13.3 \quad 290\,700 = \frac{20\,000(1 - 1,0375^{-n})}{0,0375} \quad k \text{ form}$$

$$n = \frac{\log 0,4549375}{\log 1,0375} \quad k : \log s$$

$$n = 21 + 1 \text{ last / laaste}$$

(14)

$$13.4 \quad 290\,688(1,0375)^{22} - \frac{20\,000(1,0375^{21} - 1)(1,0375)}{0,0375}$$

$$= R7\,939,45$$

OR

$$\frac{290\,700(1,0375)^{22} - 20\,000(1,0375^{21} - 1)}{0,0375} \quad k \text{ form}$$

$$= R7\,966,42 \quad k$$

(10)

[48]

TOTAL FOR SECTION B / TOTAAL VIR AFDELING B: [100]

SECTION / AFDELING C

QUESTION / VRAAG 14

ANALITIESE MEE TKUNDE

$$14.1 \quad (x+2)^2 + (y-1)^2 = 16 \text{ en } (x-2)^2 + (y-4)^2 = 9$$

$$\therefore x^2 + 4x + 4 + y^2 - 2y + 1 = 16 \quad \therefore x^2 - 4x + 4 + y^2 - 8y + 16 = 9$$

$$\therefore x^2 + 4x + y^2 - 2y - 11 = 0 \quad (1) \quad \therefore x^2 - 4x + y^2 - 8y + 11 = 0 \quad (2)$$

$$(1) - (2): 8x + 6y - 22 = 0$$

$$\therefore \text{Equation of common chord / Vgl. van gemeenskaplike koord: } 4x - 3y - 11 = 0 \quad (3) \quad (6)$$

14.2 From / Uit (3): $y = \frac{11-4x}{3}$ and state (substitute) in / en stel in (1).

$$\therefore x^2 + 4x + \left(\frac{11-4x}{3}\right)^2 - 2\left(\frac{11-4x}{3}\right) - 11 = 0$$

$$\therefore 25x^2 - 28x - 44 = 0$$

$$\therefore (25x+22)(x-2) = 0$$

$$\therefore x = -\frac{22}{25} \quad \text{of } x = 2$$

$$y = \frac{121}{25} \quad \text{of } y = 1$$

$$\therefore \text{points of intersection / snypunte: } (2;1) \text{ and / en } \left(-\frac{22}{25}; \frac{121}{25}\right)$$

$$\therefore \text{Length / Lengte} = \sqrt{\left(2 + \frac{22}{25}\right)^2 + \left(1 - \frac{121}{25}\right)^2}$$

$$= \sqrt{\frac{14\,400}{625}}$$

$$= \frac{120}{25}$$

$$= 4,8 \text{ units / eenhed e}$$

(18)

[24]

QUESTION / VRAAG 15

15.1 Distance from / Afstand van $P(x;y)$ to axis / na riglyn $x = -a$ is $x - (-a) = x + a$

Distance from / Afstand van P to focus / na fokus $(a;0)$ is $\sqrt{y^2 + (x-a)^2}$

Acc. def of parabola / Volgens def. van parabool: $x + a = \sqrt{y^2 + (x-a)^2}$

$$\therefore (x+a)^2 = y^2 + (x-a)^2$$

$$\therefore y^2 = (x+a)^2 - (x-a)^2$$

$$= x^2 + 2ax + a^2 - x^2 + 2ax - a^2$$

$$= 4ax$$

(8)

15.2 $y = 4(\frac{1}{2})x \therefore y = 2x$

(2)

15.3 $y^2 = 16x = 4(4)x \therefore a = 4$

Equa of tangen t at / Vgl. van raaklyn by (1; 4) is $4y = 8(x+1)$

$$\therefore 4y = 8x + 8$$

$$\therefore y = 2x + 2$$

Normal / Normaal $m = -1/2$

$$\therefore \text{Vgl: } y - 4 = -1/2 (x-1)$$

$$\therefore 2y - 8 = -x + 1$$

$$\therefore 2y = -x + 9$$

(12)

[22]

QUESTION / VRAAG 16

16.1 $\overrightarrow{AB} = [-4; 2; 4]$ and / en $\overrightarrow{DC} = [-4; 2; 4]$

Beca use / Omdat $\overrightarrow{AB} = \overrightarrow{DC}$ is ABC D a / ? par allelogra m and / en

$$|\overrightarrow{AB}| = \sqrt{(-4)^2 + 2^2 + 4^2} = \sqrt{36} = 6 \text{ and / en}$$

$$|\overrightarrow{AD}| = \sqrt{(-4)^2 + (-4)^2 + (-2)^2} = \sqrt{36} = 6$$

$\therefore$ Adjacent sides are equ al / Aangr ense nde sye is gelyk

$\therefore$ ABC D is a rhombus / ? ruit

$$\text{and / en } \overrightarrow{AB} \cdot \overrightarrow{AD} = [-4, 2, 4] \cdot [-4, -4, -2]$$

$$= 16 - 8 - 8$$

$$= 0$$

$$\therefore \overrightarrow{AB} \text{ perpendicular on / is loodreg op } \overrightarrow{AD}$$

$\therefore$ ABC D is a square / ? v ierkan t.

(14)

16.2 *Point / Punt P(1; 9; 6) and normal / en normaal [1; -2, 2]*

$$\therefore x = 1 + t; y = 9 - 2t; z = 6 + 2t$$

(6)

16.3 *Line 1 intersects level P, where / Lyn l sny die vlak P, waar*

$$(1 + t) - 2(9 - 2t) + 2(6 + 2t) = 4$$

$$\therefore 1 + t - 18 + 4t + 12 + 4t = 4$$

$$\therefore 9t = 9$$

$$\therefore t = 1$$

$\therefore$ Q is the poin t / die punt (2, 7, 8)

(6)

[26]

QUESTION / VRAAG 17

17.1 $A(1, 0, 3)$ and / en $B(0, -1, 4) \therefore \overrightarrow{AB} = [-1, -1, 1]$

$$\therefore x = 1 - t, y = -t, z = 3 + t \quad (6)$$

17.2 *Lines intersect where / Lyne sny waar*

$$1 - t = 2 + S, \quad -t = 1 + 2S, \quad 3 + t = 3 + S$$

$$\therefore 1 - t = 2 + t \quad \therefore -t = 1 + 2t \quad \therefore t = D$$

$$\therefore -1 = 2t \quad \therefore -1 = 3t$$

$$\therefore -\frac{1}{2} = t \quad \therefore -\frac{1}{3} = t$$

$\therefore$ No point of intersection / Geen snypunt nie! (14)

17.3 *Distance from / Afstand van $A(1, 0, 3)$ to the level / na die vlak $3x - 2y + x = 7$*

$$= \left| \frac{3(1) - 2(0) + 3 - 7}{\sqrt{3^2 + (-2)^2 + 1^2}} \right|$$

$$= \left| \frac{-1}{\sqrt{14}} \right| = \frac{1}{\sqrt{14}} \quad (6)$$

17.4 It is the distance between the two lines. / Dit is die afstand tussen die twee lyne. (2)
[28]

TOTAL FOR SECTION C / TOTAAL VIR AFDELING C: [100]

SECTION / AFDELING D
QUESTION / VRAAG 18
ALGEBRA

18.1 $\frac{11x^2+3x-1}{2x^3+x^2} = \frac{11x^2+3x-1}{x^2(2x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x+1} \quad \text{k}$

$\therefore 11x^2+3x-1 = A x(2x+1) + B(2x+1) + Cx^2 \quad \text{k}$

Set / Stel $x = 0 \quad \therefore -1 = B \quad \therefore B = -1 \quad \text{k}$

Set / Stel $x = -\frac{1}{2} \quad \therefore 11(-\frac{1}{2})^2 + 3(-\frac{1}{2}) - 1 = C(-\frac{1}{2})^2 \quad \text{k}$

$$\therefore +\frac{11}{4} - \frac{3}{2} - 1 = \frac{C}{4}$$

$$\therefore 11 - 6 - 4 = C = 1 \quad \text{k}$$

and / en $11 = 2A + C \quad \therefore 2A = 10 \quad \therefore A = 5 \quad \text{k}$

$\therefore \frac{5}{x} + \frac{-1}{x^2} + \frac{1}{2x+1} \quad \text{k}$

(14)

18.2 $a + b + c = -12$

$ab + ac + bc = 4 \quad \text{k}$

$abc = 6$

$\frac{abc}{6} = 1 \quad \text{k}$

$\frac{a+b+c}{ab+ac+bc} = -\frac{12}{4} = -3 \quad \text{k}$

$\therefore \text{sum / som: } 1 - 3 - 3 = -5 \quad \text{k}$

$S/p: 1 \cdot -3 + 1 \cdot -3 + -3 \cdot -3 = 3 \quad \text{k}$

$\text{product / produk: } 1 \cdot -3 \cdot -3 = 9 \quad \text{k}$

$\therefore x^3 + 5x^2 + 3x - 9 \quad \text{k}$

(14)

[48]

18.3 Prove that / Bewys dat $\sum_{k=1}^n k \cdot \log x = \frac{n}{2} \log x^{n+1}$

(i) For / Vir $n = 1$: $LK = \log x$ and / en $RK = \frac{1}{2} \log x^2$

$$\begin{aligned} \therefore LK &= RK &= \frac{1}{2} \cdot 2 \log x \\ & &= \log x \end{aligned}$$

(ii) For / Vir $n = k$: Assume the statement is true for $n = k$ d.i. .
Veronderstel die bewering is waar vir $n = k$ d.i. .

$$\log x + 2 \log x + 3 \log x \dots + k \cdot \log x = \frac{k}{2} \log x^{k+1}$$

(iii) Vir $n = k + 1$:

$$\begin{aligned} LS / LK &= \log x + 2 \log x + 3 \log x \dots + k \cdot \log x + (k+1) \log x \\ &= \frac{k}{2} \log x^{k+1} + (k+1) \log x \\ &= \frac{k}{2} \cdot (k+1) \log x + (k+1) \cdot \log x = \left(\frac{k}{2} + 1 \right) (k+1) \log x \\ &= \frac{(k+2)(k+1)}{2} \log x \end{aligned}$$

$$\begin{aligned} \text{and RS / en RK} &= \frac{k+1}{2} \log x^{k+2} \\ &= \frac{(k+1)}{2} (k+2) \log x \end{aligned}$$

$$\therefore LS / LK = RS / RK$$

(iv) Statement is true for / Bewering is waar vir $n = k + 1$ if it is true for / as dit waar is vir $n = k$

$\therefore$ Where for all / Waar vir alle $n \in \mathbb{N}$. k (20)

QUESTION / VRAAG 19

19.1.1 **HCF of / GGD van** $x^4 - 5x^3 + yx^{hf} - 5x + 6$ **and / en** $x^3 + x^{2g} + x + 1$

$$\begin{array}{r}
 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad | \quad \begin{array}{r} 1 \quad -6 \\ 1 \quad -5 \quad +7 \quad -5 \quad +6 \\ \underline{1 \quad 1 \quad 1 \quad 1} \\ -6 \quad 6 \quad -6 \quad 6 \quad \text{k} \\ \underline{-6 \quad -6 \quad -6 \quad -6} \\ 12 \quad 0 \quad 12 \end{array}
 \end{array}$$

$$\begin{array}{r}
 12 \quad 0 \quad 12 \quad | \quad \begin{array}{r} \frac{1}{2} \quad \frac{1}{2} \\ 1 \quad 1 \quad 1 \quad 1 \\ \underline{1 \quad 0 \quad 1} \\ 1 \quad 0 \quad 1 \quad \text{k} \\ \underline{1 \quad 0 \quad 1} \end{array}
 \end{array}$$

or $x^2 + 1$

$$\therefore \text{HCF} / \text{GGD} = 12x^2 + 12 \quad (16)$$

19.1.2 $f = (x-6) \left(\frac{x}{12} + \frac{1}{12} \right) (12x^2 + 12) + (12x^2 + 12) \text{ k}$

$$= (12x^2 + 12) \left[(x-6) \left(\frac{x}{12} + \frac{1}{12} \right) + 1 \right] \text{ k}$$

$$= (12x^2 + 12) \left(\frac{x^2}{12} + \frac{x}{12} - \frac{6x}{12} - \frac{6}{12} + 1 \right) \text{ k}$$

$$= (x^2 + 1)(x^2 + x - 6x - 6 + 12)$$

$$= (x^2 + 1)(x^2 - 5x + 6) \text{ k}$$

$$= (x^2 + 1)(x-3)(x-2) \text{ k} \quad (8)$$

OR / OF

$$\begin{array}{r}
 1 \quad 0 \quad 1 \quad | \quad \begin{array}{r} 1 \quad -5 \quad 6 \\ 1 \quad -5 \quad 7 \quad -5 \quad 6 \\ \underline{1 \quad 0 \quad 1} \\ -5 \quad 6 \quad -5 \quad \text{k} \\ \underline{-5 \quad 0 \quad -5} \\ 6 \quad 0 \quad 6 \end{array} \\
 \text{k}
 \end{array}$$

$$\begin{aligned}
 \therefore f(x) &= (x^2 + 1) (x^2 - 5x + 6) \text{ k} \\
 &= (x^2 + 1) (x-3) (x-2) \text{ k}
 \end{aligned}$$

19.2 $-1 - \sqrt[3]{2}$ also zero point / ook nulpunt
 $(x+1+\sqrt[3]{2})(x+1-\sqrt[3]{2})$ a factor / ? faktor
 $= x^2 + 2x - 1$ k

$$\begin{array}{r|rrrrrr} 1 & 2 & -1 & 1 & -4 & 1 & k \\ & & & 1 & -2 & -8 & 6 & -1 \\ \hline & & & 1 & 2 & -1 & & \\ & & & & -4 & -7 & 6 & k \\ & & & & -4 & -8 & 4 & \\ \hline & & & & & 1 & 2 & -1 \end{array}$$

$$\begin{aligned} f(x) &= (x^2 + 2x - 1)(x^2 - 4x + 1) \\ &= (x+1+\sqrt[3]{2})(x+1-\sqrt[3]{2})(x-2-\sqrt[3]{3})(x-2+\sqrt[3]{3}) \\ x^2 - 4x &= -1 \end{aligned}$$

$$(x-2)^2 = 3 \text{ or else / of anders}$$

$$x = 2 \pm \sqrt[3]{3}$$

(18)

[34]

QUESTION / VRAAG 20

20.1
$$f'(x) = \frac{k(x^2+5) - 5x(2x)}{(x^2+5)^2} k$$

$$= \frac{5x^2 + 25 - 10x^2}{(x^2+5)^2} = \frac{-5x^2 + 25}{(x^2+5)^2} = \frac{-5(x^2-5)}{(x^2+9)^2}$$

(6)

20.2 If / Stel $f'(x) = 0$

$$\therefore \frac{-5(x^2-5)}{(x^2+5)^2} = 0 \quad \therefore x^2 - 5 = 0 \quad k$$

$$\therefore x = \pm\sqrt{5} \quad k$$

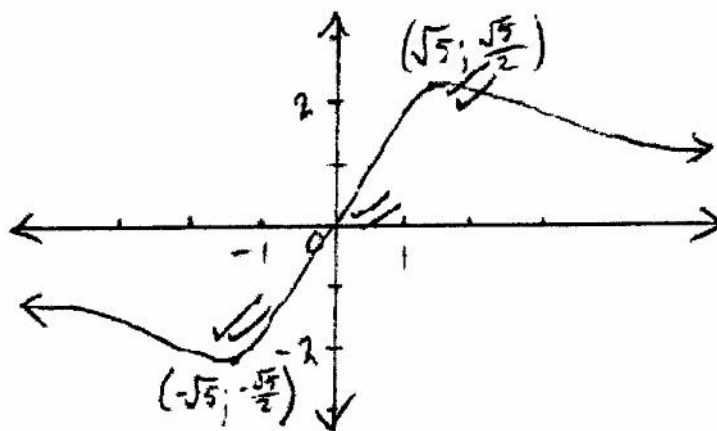
$$\text{or / of } \pm 2,236$$

$$\begin{aligned} \therefore f(\pm\sqrt{5}) &= \frac{-5\sqrt{5}}{10} \text{ and / en} & f(\pm\sqrt{5}) &= \frac{5\sqrt{5}}{10} \\ &= \frac{-\sqrt{5}}{2} & &= \frac{\sqrt{5}}{2} \quad k \end{aligned}$$

$$\text{and / en } \left(\pm\sqrt{5}; \frac{\pm\sqrt{5}}{2} \right)$$

(8)

20.3



(6)

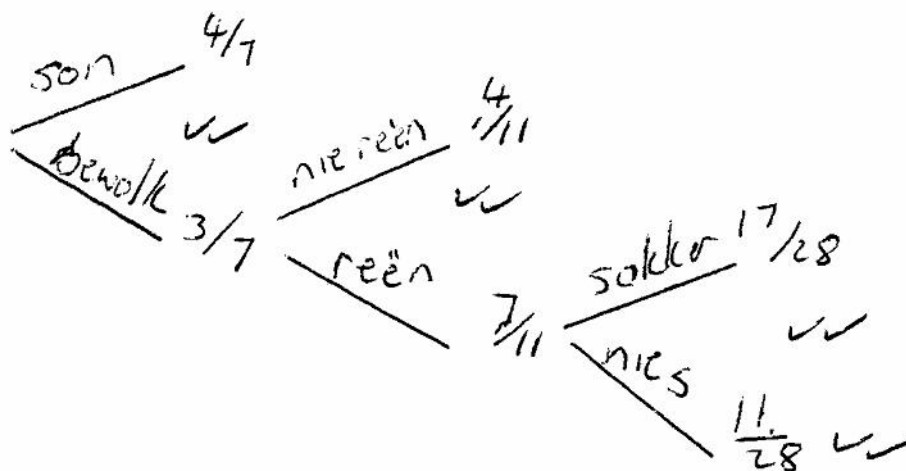
[20]

TOTAL FOR SECTION D / TOTAAL VIR AFDELING D: [100]

SECTION E / AFDELING E

QUESTION / VRAAG 21

21.1



(8)

21.2 $P(\text{soccer} / \text{sokker}) = 1 - P(\text{no soccer} / \text{nie sokker})$

$$= \frac{3}{7} \times \frac{4}{11} \times \frac{17}{28} = 0,0946$$

(6)

[14]

QUESTION / VRAAG 22

$$22.1 \quad \frac{{}^k 6!}{{}^k 3!} = 120 \quad {}^k k \quad (6)$$

$$22.2 \quad \frac{{}^k 5!}{{}^k 2!} + \frac{{}^k 5!}{{}^k 3!} = 80$$

$$\text{waarsk} = \frac{80}{{}^k 120} = \frac{2}{3} = 0,6667 \quad (8)$$

[14]

QUESTION / VRAAG 23

$$23.1 \quad \binom{x}{2} = 45$$

$$\frac{x!}{{}^k 2!(x-2)!} = 45 \quad x \frac{{}^k}{(x-1)} = 90$$

$$x^2 - x - 90 = 0 \quad \text{any method / enige metode}$$

$$(x-10)(x+9) = 0 \quad \text{could also "hit and miss" / kan ook "probeer en tref"}$$

$$x = 10 \quad {}^k k \quad (6)$$

$$23.2.1 \quad \frac{{}^k}{\binom{9}{5}} \left(\frac{{}^k}{\binom{11}{20}} \right)^5 \left(\frac{{}^k}{\binom{9}{20}} \right)^4 = 0,2600 \quad (8)$$

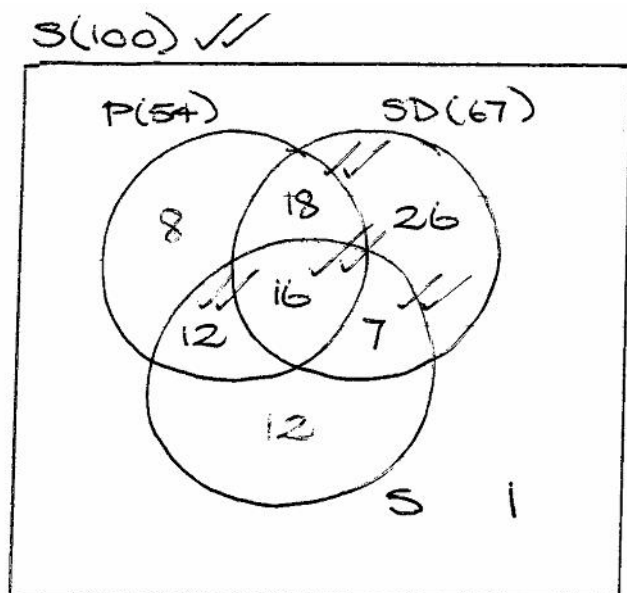
$$23.2.2 \quad \frac{{}^k}{\binom{11}{5}} \left(\frac{{}^k}{\binom{9}{4}} \right)^k = 0,3466$$

$$\binom{20}{9} {}^k k \quad (8)$$

[22]

QUESTION / VRAAG 24

24.1



(10)

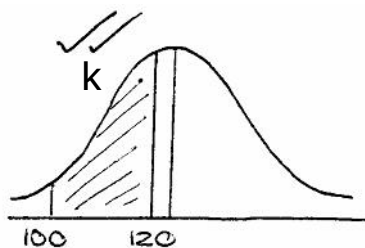
24.2.1 12% m

(4)
[18]

QUESTION / VRAAG 25

25.1

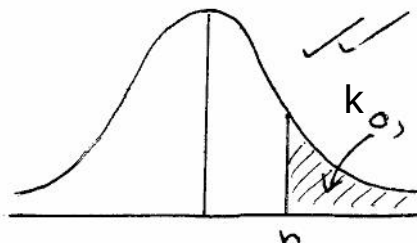
$$\begin{aligned}
 &P(110 < x < 120) \\
 &= P(-1,86 < Z < -0,43) \\
 &= P(0,43 < Z < 1,86) \\
 &= 0,4686 - 0,1664 \\
 &= 0,3022
 \end{aligned}$$



(12)

25.2

$$\begin{aligned}
 &P(X > h) = 0,0123 \\
 &\therefore P(0 < Z < Z_1) = 0,4877 \\
 &\therefore Z_1 = 2,25 \\
 &x = \mu + z\sigma \\
 &= 123 + 2,25 \cdot 7 \\
 &= 138,75 \\
 &\therefore n = 139
 \end{aligned}$$

(10)
[22]

QUESTION / VRAAG 26

26.1 $P = \frac{44}{50} \quad k$
 $= 0,88 \quad k$ (4)

26.2 **96% conf. int. /-vertroue nsinterval**

$$= \left(p \pm 2,05 \sqrt{\frac{p(1-p)}{n}} \right) \quad - \text{correct formula / korrekte formule}$$

$$= \left(\begin{matrix} 0,88 \\ k \end{matrix} \pm 2,05 \sqrt{\frac{0,88(1-0,88)}{50}} \right) k$$

$$= \begin{pmatrix} 0,786 \\ k \end{pmatrix}; \begin{pmatrix} 0,974 \\ k \end{pmatrix} \quad (10)$$

[14]

TOTAL FOR SECTION E / TOTAAL VIR AFDELING E: [100]

TOTAL / TOTAAL: 400