Question 1

1.1. $\lim_{x\to\infty} \dfrac{x^2(3-\frac{2}{x}+\frac{1}{x^2})}{-4x^2} = -\frac{3}{4}$ ✓✓ (4)

1.2. $\lim_{x\to0} \dfrac{x^2+2x}{3\sin 2x} = \lim_{x\to0}\dfrac{x^2}{3\sin 2x} + \lim_{x\to0}\dfrac{2x}{3\sin 2x} = 0+\frac{1}{3} = \frac{1}{3}$ ✓✓ (8)

OR $\lim_{x\to0}\dfrac{2x+2}{3\cos 2x \cdot 2} = \dfrac{2}{6} = \dfrac{1}{3}$ ✓✓ (L'H ✓ $\frac{0}{0}$)

2.1.1. $\lim_{x\to1^-} f(x) = \tan 1$ ✓ $\lim_{x\to1^+} f(x) = 2$ ✓✓ (6)

∴ Not continuous ✓✓, jump.

2.1.2. $\lim_{x\to0^-} f(x) = 0$ ✓ $\lim_{x\to0^+} f(x) = 0$ and $f(0) = 0$ ✓✓ (6)

∴ continuous ✓✓

2.2.1. Not diff'b. because not cont. ✓✓ (2)

2.2.2. $\lim_{x\to0^-} f'(x) = 1$ ✓✓ $\lim_{x\to0^+} f'(x) = \sec^2 0 = 1$ ✓✓ (6)

∴ diff.b. ✓✓

2.3 $\lim_{x\to-2^-} f(x) = 4p$ $\lim_{x\to-2^+} f(x) = -2$ ✓✓ (8)

∴ $4p = -2 \Rightarrow p = -\frac{1}{2}$ ✓✓

$f(-2) = 2q = -2$ ✓✓ ∴ $q = -1$ ✓✓

Question 3.

3.1. $r^2 = 3^2+(-4)^2 = 25$ ✓✓ ∴ $r = 5$ ✓✓ (4)

3.2.1. $r\theta = 5$ ∴ $5\theta = 6{,}435$ ∴ $\theta = 1{,}287$

3.2.2. $A = \frac{1}{2}r^2\theta = \frac{1}{2}\cdot 25 \cdot 1{,}287$ ✓✓ (6)

$= 16{,}0875$ ✓✓

Question 4.

4.1. $\text{bg}\cos\left(-\frac{1}{\sqrt{2}}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ ✓✓ (4)

4.2. $\text{bg}\sin\left(\cos\frac{4\pi}{3}\right) = \text{bg}\sin\left(-\cos\frac{\pi}{3}\right)$ ✓✓

$= \text{bg}\sin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ ✓✓ (6)

4.3. $\cos\left(\text{bg}\sin\frac{1}{p}\right)$ Let $A = \arcsin\frac{1}{p}$

∴ $\sin A = \frac{1}{p}$ ✓✓

$x = \sqrt{p^2-1}$ ✓✓ (8)

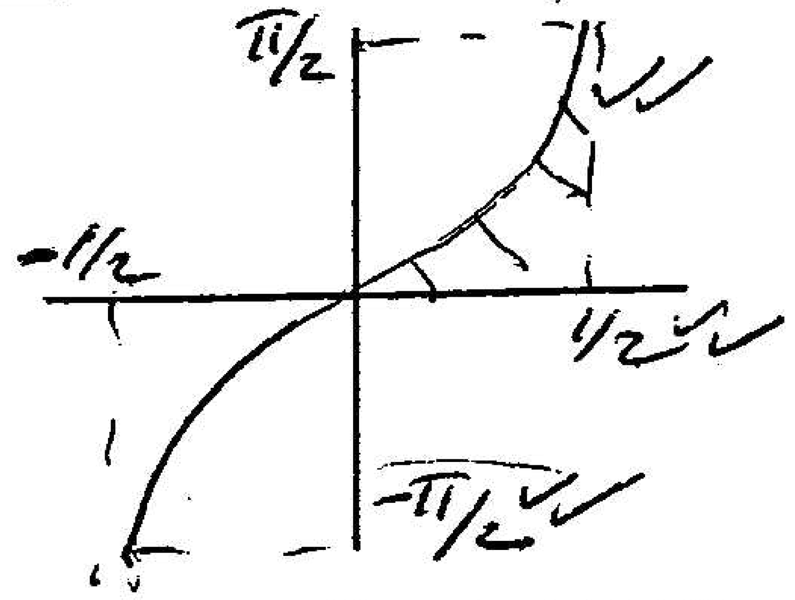
∴ $\cos\left(\arcsin\frac{1}{p}\right) = \dfrac{\sqrt{p^2-1}}{p}$ ✓✓

Questions 5

5.1. $y = \arcsin 2x$ (6)

$2x = \sin y$

$x = \frac{1}{2}\sin y$



5.2. $F'(x) = \text{bg}\sin 2x + x \cdot \dfrac{1}{\sqrt{1-4x^2}}\cdot 2 + \frac{1}{2}(1-4x^2)^{-\frac{1}{2}}\cdot(-8x)$ ✓✓ (12)

$= \arcsin 2x$ ✓✓

5.3.1 $\int_0^{\frac{1}{2}} \arcsin 2x\,dx = x\arcsin 2x - \frac{1}{2}\sqrt{1-4x^2}\,\Big|_0^{\frac{1}{2}}$ ✓✓ (8)

$= \left(\frac{1}{2}\arcsin 1 - \frac{1}{8}\cdot 0\right) - \left(0 - \frac{1}{2}\sqrt{1}\right)$ ✓✓

$= \frac{\pi}{4} - \frac{1}{2}$ ✓✓

5.3.2 see graph. ✓✓ (2)

p2

Question 6

$\Delta x_i = \frac{3}{n}$ ✓ $x_i = \frac{3i}{n}$ ✓ $f(x_i) = \frac{36i^2}{n^2} + \frac{12i}{n} + 1$ ✓

$f(x_i).\Delta x_i = \frac{108i^2}{n^3} + \frac{36i}{n^2} + \frac{3}{n}$ ✓

$\sum_{i=1}^{n} f(x_i).\Delta x_i = \sum \frac{108i^2}{n^3} + \sum \frac{36i}{n^2} + \sum \frac{3}{n}$ ✓

$= \frac{108}{n^3} \cdot \frac{n}{6}(2n+1)(n+1) + \frac{36}{n^2} \cdot \frac{n}{2}(n+1) + \frac{3}{n} \cdot n$ ✓✓

$= 36 + \frac{54}{n} + \frac{108}{6n^2} + 18 + \frac{18}{n} + 3$ ✓ (20)

$RS = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x_i = 57$ ✓

Question 7

(4)

7.1. $f(1) = 1 - 1 + \cos^2 1 > 0$ ✓ ∴ cuts between 1 and 2
$f(2) = 1 - 2 + \cos^2 2 < 0$ ✓

7.2. $f'(x) = -1 + 2\cos x.(-\sin x)$ ✓

∴ $a_{n+1} = a_n - \frac{1 - a_n + \cos^2 a_n}{-1 - 2\sin a_n \cos a_n}$ ✓ $\left(a_n + \frac{1 - a_n + \cos^2 a_n}{1 + \sin 2a_n}\right)$

$a_0 = 1$
$a_1 = 1,152$ ✓
$a_2 = 1,159$ ✓
$a_3 = 1,159$ ∴ $x = 1,16$ ✓ (10)

(10)

7.3 $f'(x) = -1 + 2\cos x(-\sin x) = 0$ ✓

∴ $\sin 2x = -1$ ✓

$2x = \frac{3\pi}{2}$ ✓ $x = \frac{3\pi}{4}$ ✓

Question 8

8.1. $\frac{(1-4x^2)^{1/2}}{1/2 \cdot -4} + k$ ✓ (8)

8.2. $\frac{2\arcsin(2x)}{2} + k$ ✓ (6)

Question 9

9.1. $\int f(x)dx = \frac{(1+\sin x)^{3/2}}{3/2} + k$ ✓ (6)

9.2. $V = \pi\int_0^{\pi/2} \cos^2 x(1+\sin x)dx$ ✓

$= \pi\left(\int_0^{\pi/2} \cos^2 x\,dx + \int \cos^2 x.\sin x\,dx\right)$

$= \pi\left(\int_0^{\pi/2} \frac{1}{2}(1 + 2\cos 2x)dx + \int \cos^2 x.\sin x\,dx\right)$

$= \pi\left[\left(\frac{1}{2}x - 2\frac{\sin 2x}{2}\right) + \frac{\cos^3 x(-1)}{3}\right]_0^{\pi/2}$ (16)

$= \pi\left[\left(\frac{1}{2}\frac{\pi}{2} - \sin\pi - \frac{1}{3}\cos^3\frac{\pi}{2}\right) - \left(0 - 0 - \frac{1}{3}\cos^3 0\right)\right]$

$= \pi\left[\frac{\pi}{4} - 0 - 0 + \frac{1}{3}\right] = 3,51$ ✓

Question 10

10.1. $r^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{b}{2}\right)^2$ ✓ ∴ $b^2 = 4r^2 - x^2$

$= \frac{x^2}{4} + \frac{b^2}{4}$ ✓ ∴ $S = k\sqrt{4r^2 - x^2}.x^3$ (8)

10.2. $S = k(0,16 - x^2)^{1/2}.x^3$

$S' = \frac{1}{2}k(0,16 - x^2)^{-1/2}.-2x.x^3 + k(0,16 - x^2)^{1/2}.3x^2 = 0$

∴ $-x^4 + 3x^2(0,16 - x^2) = 0$ ✓

$-x^4 + 0,48x^2 - 3x^4 = 0$

$-4x^4 + 0,48x^2 = 0$ (12)

$4x^2(x^2 - 0,12) = 0$

$x = 0$ or $x^2 = 0,12$

$x = 0,35$ ✓

Section B

Question 11

11.1.1. $400 \times 24 = R9600$ (2)

1.1.2. $9600 = A(1 + 0{,}13 \cdot 2)$

$A = R7619{,}05$ (4)

1.2.1 $7619.05 = x\,\dfrac{1 - (1 + \frac{0{,}145}{12})^{-24}}{\frac{0{,}145}{12}}$ form.

$x = R\,367.61$ (10)

11.2.2. $R\,8822{,}64$

1.3. Mr. Terrafirm pays R777.36 more. This is because with simple interest it is interest on the whole amount for the whole time, while with compound interest it is interest on the balance outstanding (4)

Question 12

2.1. $(1+i)^3 = (1{,}03)^1$ (6)

$i = 0{,}0099\ldots$

$i^{(12)} = ~~11{,}88\%~~ \; 0{,}99\%$

2.2. $475\,000 = 10\,000\left(\dfrac{1 - 1{,}0099^{-n}}{0{,}0099}\right)$ form.

$1{,}0099^{-n} = 0{,}52975$

$-n = \dfrac{\log 0{,}52975}{\log 1{,}0099}$ $\therefore n = 64 +$ laaste. (14)

2.3. $OB(1+i)^1 = \left[475000(1{,}0099)^{64} - 10000\left(\dfrac{1{,}0099^{64}-1}{0{,}0099}\right)\right](1{,}0099)$ (10)

$= R4951.61$

2.4 $OB = \left[475\,000(1{,}0099)^{24} - 10000\left(\dfrac{1{,}0099^{24}-1}{0{,}0099}\right)\right](1{,}0099)^{12}$ (10)

$- R373\,973.86$

Question 13.

13.1. $750\,000(1{,}1275)^{8} = R1\,366\,608.18$ (4)

13.2. $14500\left(\dfrac{(1+i)^{5}-1}{i}\right)(1+i)^{9,5}$ $\quad (1{,}01)^{12} = (1+i)$

$i^{(4)} = 12{,}68\ldots\%$ 10

$= R\,99.117.90$

OR

$14500(1{,}01)^{6}\left[\dfrac{1{,}01^{60}-1}{0{,}01}\right]$

13.3 $1184210.21 + 99117.90 = R1283328.11$ (6)

and $0{,}975 \times 1366608.18 = 1332442.98$

No.

Question 14.

14.1. $-0{,}03(q-10)^2 + 3 = aq^2 + 0{,}08q + 1{,}44$ $\quad q = 4$

$1{,}92 = 16a + 1{,}76$ (6)

$16a = 0{,}16$

$\therefore a = 0{,}01$

14.2. Maks W $R' = C'$

$\therefore -0{,}06(q-10) = 0{,}02q + 0{,}08$

$-0{,}06q + 0{,}6 = 0{,}02q + 0{,}08$

$0{,}08q = 0{,}52$

$q = 6{,}5$

$\therefore W = -0{,}03(6{,}5-10)^2 + 3 - (0{,}01 \cdot 36 + 0{,}08 \cdot 6 + 1{,}44)$ (12)

$= 0{,}24$ mil.

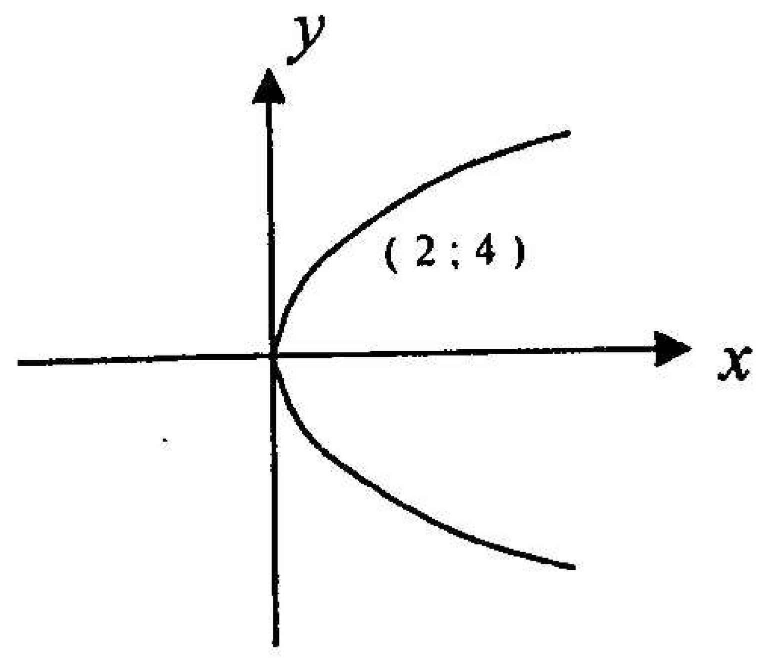
ANALYTICAL GEOMETRY 2

Question 15

15.1 $x = 2t^2 \; ; \; y = 4t$

15.1.1 $8x = y^2$

15.1.2



15.1.3 $y' = \dfrac{4}{\sqrt{8x}}$

$\tan\left(\dfrac{3\pi}{4}\right) = \dfrac{4}{\sqrt{8x}}$

$-1 = \dfrac{4}{\sqrt{8x}}$

$-\sqrt{8x} = 4$

$8x = 16$

$x = 2$

ie at $(2 \; ; \; 4)$

$y - 4 = -(x - 2)$

$y = -x + 6$

15.2 Focus $(2 \; ; \; 0)$ directrix $x = 18$

$ae = 2$ $\qquad \dfrac{a}{e} = 18$

$\therefore 18e^2 = 2$

$e^2 = \dfrac{1}{9}$

$e = \dfrac{1}{3}$ $\qquad \therefore a = 6$

$e^2 = 1 - \dfrac{b^2}{a^2}$

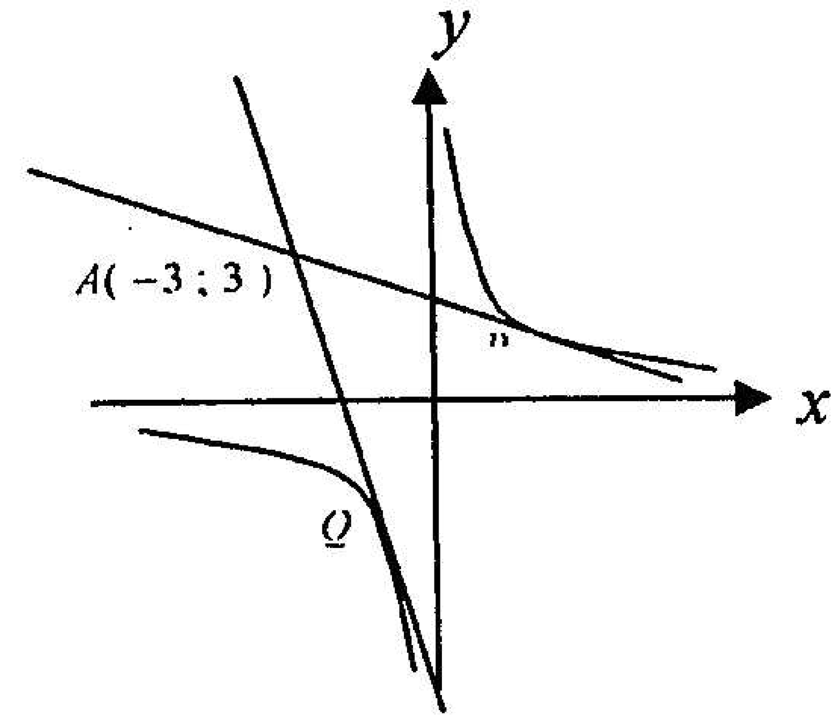
$\therefore \dfrac{1}{9} = 1 - \dfrac{b^2}{36}$

$4 = 36 - b^2$

$b^2 = 32$

$\therefore \dfrac{x^2}{36} + \dfrac{y^2}{32} = 1$

15.3



Let the tangent cut at $P\left(a \; ; \; \dfrac{16}{a}\right)$

$y = 16x^{-1}$

$\dfrac{dy}{dx} = -16x^{-2}$

$m/_{tgt} = \dfrac{-16}{a^2}$

$y - 3 = \dfrac{-16}{a^2}(x + 3)$

Subst: $\left(a \; ; \; \dfrac{16}{a}\right)$

$\dfrac{16}{a} - 3 = \dfrac{-16}{a^2}(a + 3)$

$\dfrac{16}{a} - 3 = \dfrac{-16}{a} - \dfrac{48}{a^2}$

$16a - 3a^2 = -16a - 48$

$3a^2 - 32a - 48 = 0$

$(3a + 4)(a - 12) = 0$

$a = \dfrac{-4}{3}$ or $a = 12$

$\therefore P\left(12 \; ; \; \dfrac{4}{3}\right) \; Q\left(\dfrac{-4}{3} \; ; \; -12\right)$

Question 16

16.1 $A(3 \; ; -2 \; ; 6)$ $3x + 4y - 5z = 21$

$3^2 + 4^2 + (-5)^2 = 9 + 16 + 25 = 50$

$\dfrac{3x}{\sqrt{50}} + \dfrac{4y}{\sqrt{50}} - \dfrac{5z}{\sqrt{50}} = \dfrac{21}{\sqrt{50}}$

$d_1 = \left|\dfrac{9}{5\sqrt{2}} - \dfrac{8}{5\sqrt{2}} - \dfrac{30}{5\sqrt{2}} - \dfrac{21}{5\sqrt{2}}\right|$

$d_1 = \left|\dfrac{-50}{\sqrt{50}}\right| = \sqrt{50} = 7{,}071$ units p5

16.2 $P_1 : x + 2y - z = 5$

$L_1 : \frac{x - 11}{-4} = \frac{y + 2}{2} = \frac{z + 8}{5}$

$L_2 : \frac{x - 1}{1} = \frac{y + 2}{-3} ; z = 7$

16.2.1 From L_1 :

$2x - 22 = -4y - 8$ and $5x - 55 = -4z - 32$

$2x + 4y = 14$ and $5x + 4z = 23$

$x + 2y = 7$

Plane 2: $x + 2y - 7 + \beta(5x + 4z - 23) = 0$

$(1 + 5\beta)x + 2y + 4\beta z = 7 + 23\beta$ *

From L_2: $-3x + 3 = y + 2 ; z = 7$

$y + 3x = 1 ; z = 7$

Plane 3: $y + 3x - 1 + \alpha(z - 7) = 0$

$\therefore 3x + y + \alpha z = 1 + 7\alpha$

$\therefore 6x + 2y + 2\alpha z = 2 + 14\alpha$

when $\alpha = 2$ and $\beta = 1$

we have the plane:

$6x + 2y + 4z = 30$ by *

ie: $3x + y + 2z = 15$

16.2.2 Plane 1: $x + 2y - z = 5$ —①

$3x + y + 2z = 15$ —②

2 ×②−①: $5x + 5z = 25$

$x + z = 5$ ie $x = 5 - z$

2 ×①+②: $5x + 5y = 25$

$x + y = 5$ ie $x = 5 - y$

$\therefore x = 5 - y = 5 - z$

Question 17

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad y = mx + c$

17.1 Consider the tgt at $(x_1 ; y_1)$:

$$\frac{d}{dx}\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = \frac{d}{dx}(1)$$

$$\frac{2x}{a^2} + \frac{2y}{b^2} y' = 0$$

$$y' = \frac{-2x}{a^2} \div \frac{2y}{b^2}$$

$$= \frac{-xb^2}{ya^2}$$

at $(x_1 ; y_1) : m = \frac{-b^2x_1}{a^2y_1}$

$y_1 = mx_1 + c$

$\therefore c = y_1 - mx_1$

$\therefore c = y_1 + \frac{b^2x_1^2}{a^2y_1}$

$\therefore cy_1 = y_1^2 + \frac{b^2x^2}{a^2}$

$\therefore \frac{cy_1}{b^2} = \frac{y_1^2}{b^2} + \frac{x_1^2}{a^2}$

But $\frac{y_1^2}{b^2} + \frac{x_1^2}{a^2} = 1$ (ellipse)

$\therefore \frac{cy_1}{b^2} = 1$

$\therefore cy_1 = b^2 \qquad \therefore y_1 = \frac{b^2}{c}$

But $m = \frac{-b^2x}{a^2y_1}$

$= \frac{b^2x_1}{a^2\left(\frac{b^2}{c}\right)}$

$= \frac{-cx_1}{a^2}$

$\therefore x_1 = \frac{-a^2m}{c}$

$y_1 = mx_1 + c$

$\therefore c = y_1 - mx_1$

$c = \frac{b^2}{c} - m\left(\frac{-a^2m}{c}\right)$

$c = \frac{b^2}{c} + \frac{a^2m^2}{c}$

$c^2 = b^2 + a^2m^2$

17.2 $A(3 ; 4) \qquad \frac{x^2}{16} + \frac{y^2}{25} = 1$

$a = 4 \qquad b = 5$

$c^2 = 16m^2 + 25$

$y_1 = mx_1 + \sqrt{16m^2 + 25}$

$4 = 3m + \sqrt{16m^2 + 25}$

$4 - 3m = \sqrt{16m^2 + 25}$

$16 - 24m + 9m^2 = 16m^2 + 25$

$\therefore 0 = 7m^2 + 24m + 9$

$0 = (7m + 3)(m + 3)$

$m = \frac{-3}{7}$ or $m = -3$

$y = \frac{-3}{7}x + \sqrt{\frac{1369}{49}}$

$y = \frac{-3}{7}x + \frac{37}{7}$

$y = -3x + \sqrt{169}$

$y = -3x + 13$

Section D

Question 18

Let $n=1$: LHS $=1$ RHS $=1$ ∴ True for $n=1$

Assume true for $n=k$: $1^2+2^2+\dots+k^2=\frac{k(k+1)(2k+1)}{6}$

Let $n=k+1$: LHS $=1^2+2^2+\dots+k^2+(k+1)^2$ RHS $=\frac{(k+1)(k+2)(2k+3)}{6}$

$=\frac{k(k+1)(2k+1)}{6}+(k+1)^2$

$=\frac{(k+1)(2k^2+k+6k+6)}{6}$ ∴ LHS = RHS

$=\frac{(k+1)(2k+3)(k+2)}{6}$ (16)

∴ If true for $n=k$, then also true for $n=k+1$

∴ True for all $n\in N$.

Question 19

$\left.\begin{array}{l}\alpha+\beta=-b\\ \alpha\beta=c\end{array}\right\}$ New: sum: $1-\alpha^2-\beta^2$

$=1-(\alpha+\beta)^2+2\alpha\beta$

$=1-b^2+2c$

product: $1.(-\alpha^2-\beta^2)$ (12)

$=-(\alpha+\beta)^2+2\alpha\beta$

$=-b^2+2c$

∴ New: $x^2-(1-b^2+2c)x-b^2+2c$

Question 20

$\frac{2}{x(x^2+3x+2)}=\frac{2}{x(x+2)(x+1)}\equiv\frac{A}{x}+\frac{B}{x+2}+\frac{C}{x+1}$

$2=A(x+2)(x+1)+Bx(x+1)+Cx(x+2)$ (14)

$x=0$: $2=2A$ ∴ $A=1$

$x=-1$: $2=-C$ ∴ $C=-2$

$x=-2$: $2=2B$ ∴ $B=1$

∴ $\frac{1}{x}+\frac{1}{x+2}-\frac{2}{x+1}$

Question 21.

21.1. If $p(x)\in Z[x]$ and ∃ a prime nu. p such that

i) leading coefficient is not divis. by p

ii) all other " are divis. by p (6)

iii) constant is not divis. by p^2

Then $p(x)$ is irreducible in $Z[x]$ (or $Q[x]$)

21.2. Possible p: 2, 3, 5, 7

Not: 2 then $n+2$ is div. by 2   (8)

Not 3 140 not div. by 3.

Not 5 42 not div. by 5

7

Question 22.

22.1. If $\frac{r}{s}$ in simplest form, $r,s\in Z$, is a zero of $P(x)\in Z[x]$, then r and s are factors of the leading coeff. and the constant respectively.   (6)

22.2. Possible zeros: $\pm1,\pm2,\pm3,\pm6$ (6)

$f(\pm1)\neq0$, $f(\pm2)\neq0$, $f(\pm3)\neq0$, $f(\pm6)\neq0$ ∴ No rat. zeros.

22.3. $\pm\sqrt2$ also a zero. ∴ $(x-\sqrt2)(x+\sqrt2)$ a factor $=x^2-2$

```
            1   0  -2   0  -3
1   0  -2 | 1   0  -4   0   1   0   6
            1   0  -2
                   -2   0   1
                   -2   0   4
                           -3   0   6
                           -3   0   6
```

∴ $(x^2-2)(x^4-2x^2-3)$ (12)

$=(x^2-2)(x^2-3)(x^2+1)$: Z

$=(x-\sqrt2)(x+\sqrt2)(x-\sqrt3)(x+\sqrt3)(x^2+1)$ (2) R

$=(x-\sqrt2)(x+\sqrt2)(x-\sqrt3)(x+\sqrt3)(x-\sqrt{-1})(x+\sqrt{-1})$: C (2)

Question 23

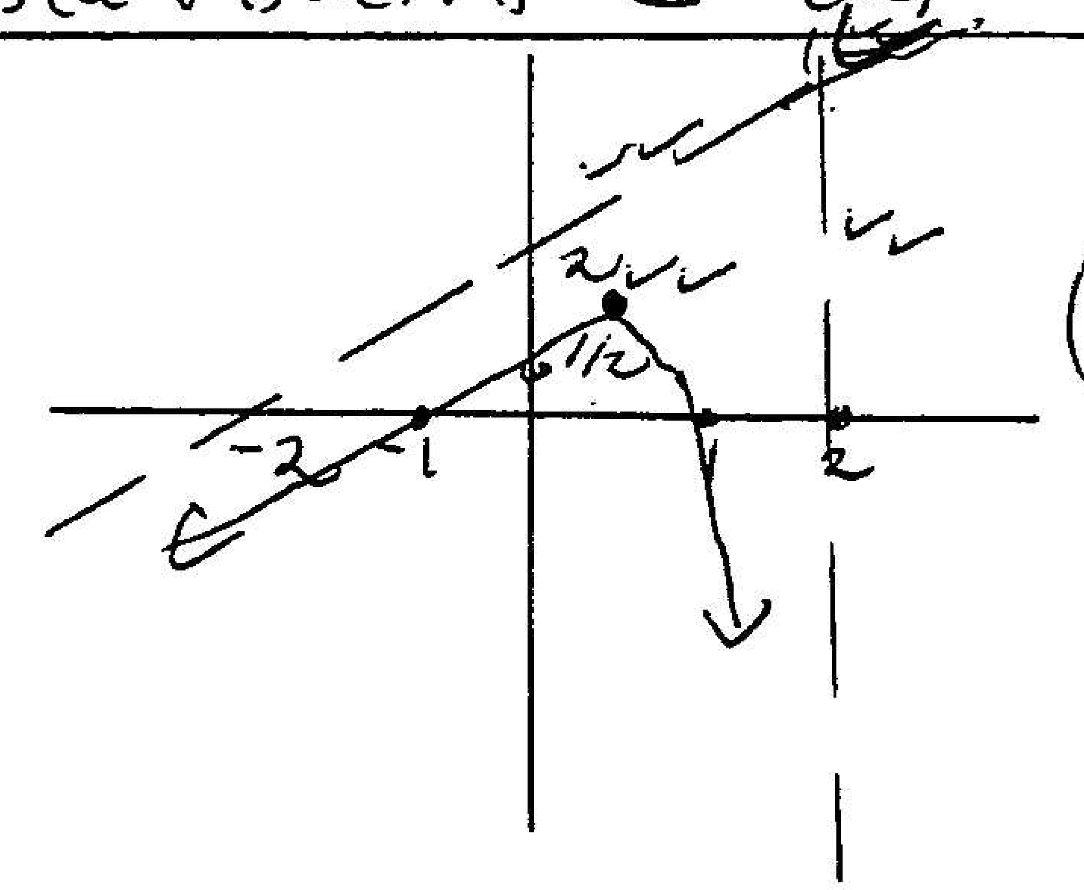
y int: $y=1/2$

x int: $x=\pm1$

vert. asympt: $x=2$

oblique:
```
       1   2
1 -2 | 1   0  -1
       1  -2
           2
```

$y=x+2$ (16)



Section E

Additional Mathematics

Question 24

24.1.1 $\dfrac{\binom{5}{3}\cdot\binom{7}{0}\cdot\binom{3}{0}}{\binom{15}{3}} = 0{,}0220$ (8)

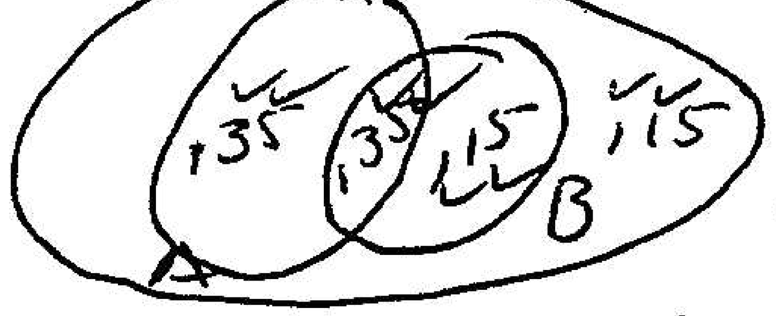
24.1.2 $\dfrac{\binom{5}{1}\binom{7}{1}\binom{3}{1}}{\binom{15}{3}} = 0{,}2308$ (8)

24.2.1 $\dfrac{13!}{3!\cdot 8!\cdot 2!} = 86486400$ (6)

24.2.2 $\dfrac{\dfrac{11!}{3!\,3!} + \dfrac{11!}{3!\cdot 2!} + \dfrac{11!}{3!\,2!}}{86486400} = 0{,}0897$ (12)

Question 25

25.1. $P(A \cap B) = 0.7 \times 0{,}5 = 0{,}35$



(8)

25.2.1 $0{,}35$ (2)

25.2.2. $0{,}15$ (2)

25.3. $1 - P(B \neq 0) > 0{,}9$

$1 - \binom{n}{0}\cdot 0{,}5^n \cdot 0{,}5^0 > 0{,}9$

$0{,}5^n < 0{,}1$ (10)

$n > \dfrac{\log 0{,}1}{\log 0{,}5} = 3{,}3 \quad \therefore n = 4.$

Question 26

$P(X > 2) = 1 - P(X = 0, 1, 2)$

$= 1 - \binom{15}{0}\cdot 0{,}95^{15} - \binom{15}{1} 0{,}05\cdot 0{,}95^{14} - \binom{15}{2} 0{,}05^2\cdot 0{,}95^{13}$ (14)

$= 0{,}0362$

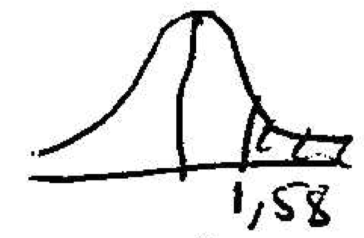
Question 27

27.1. $P(X > 50) = P\left(Z > \dfrac{50 - 38{,}5}{7{,}3}\right)$

$= P(Z > 1{,}58)$

$= 0{,}5 - 0{,}4429$

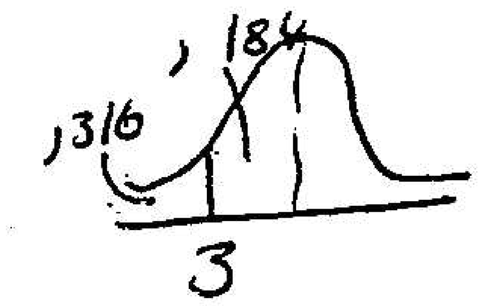
$= 0{,}0571$



(8)

27.2 enige aanvaarbare (2)

27.3.



$z = -0{,}48$

$\therefore \dfrac{X - 38{,}5}{7{,}3} = -0{,}48$

$X = 34{,}996 = 35\text{cm}.$

Question 28

$1{,}96\sqrt{\dfrac{,47 \cdot ,53}{n}} < 0{,}01 \quad p = 0{,}47$

$n > 9569{,}4$ (10)

$\therefore 9570$ persone