SENIOR CERTIFICATE EXAMINATION
SENIORSERTIFIKAAT-EKSAMEN

OCTOBER / NOVEMBER
OKTOBER / NOVEMBER

2004

ADDITIONAL MATHEMATICS

ADDISIONELE WISKUNDE

HG **302-1/0**

18 pages
18 bladsye

302 1 0 HG

GAUTENGSE DEPARTEMENT VAN ONDERWYS

SENIORSERTIFIKAAT-EKSAMEN

ADDISIONELE WISKUNDE HG

TYD: 3 uur

PUNTE: 400

INSTRUKSIES:

- Hierdie vraestel bestaan uit VYF afdelings.
- Afdeling A is VERPLIGTEND.
- Beantwoord ook nog enige TWEE ander afdelings uit Afdelings B, C, D en E.
- Elke afdeling moet in 'n **aparte antwoordboek beantwoord word en die betrokke afdeling moet duidelik op die buiteblad aangetoon word**. Plaas alle antwoordboeke in die antwoordboek vir Afdeling A voordat jy dit inlewer.
- Nie-programmeerbare sakrekenaars mag gebruik word, tensy daar anders aangedui word.
- Hierdie vraestel bestaan uit 15 bladsye. Statistiese tabelle en formuleblaaie kom onderskeidelik voor op bladsye 16, 17 en 18.
- Alle noodsaaklike berekeninge moet duidelik getoon word.
- Alle hoeke is in radiale en antwoorde moet ook in radiale gegee word.
- Skryfwerk moet leesbaar wees.

GAUTENG DEPARTMENT OF EDUCATION

SENIOR CERTIFICATE EXAMINATION

ADDITIONAL MATHEMATICS HG

TIME: 3 hours

MARKS: 400

INSTRUCTIONS:

- This examination paper consists of FIVE sections.
- Section A is COMPULSORY.
- A further TWO sections must be answered from Sections B, C, D and E.
- Each section must be **answered in a separate answer book and the relevant section must be clearly indicated on the cover**. Place all answer books inside the answer book for Section A before handing them in.
- Unless otherwise indicated, non-programmable calculators may be used.
- The examination paper consists of 15 pages. Statistical tables and formula sheets can be found on pages 16, 17 and 18, respectively.
- All essential calculations must be clearly shown.
- All angles are in radians and answers must also be given in radians.
- Please write legibly.

AFDELING A
VERPLIGTEND
CALCULUS

VRAAG 1

Bepaal die volgende limiete as hulle bestaan:

1.1 $\lim\limits_{x\to\infty}\dfrac{3x^2-2x+1}{-4x^2}$ (4)

1.2 $\lim\limits_{x\to 0}\dfrac{x^2+2x}{3\sin 2x}$ (8)

[12]

VRAAG 2

$$f(x)=\begin{cases} px^2 & x<-2 \\ 2q & x=-2 \\ x & -2<x<0 \\ \tan x & 0\le x<1 \\ 2 & x\ge 1 \end{cases}$$

2.1 Bespreek die kontinuïteit van $f(x)$ by die volgende punte en indien nie kontinu nie, noem ook die tipe diskontinuïteite:

2.1.1 $x=1$ (6)

2.1.2 $x=0$ (6)

2.2 Bespreek algebraïes die differensieerbaarheid van $f(x)$ by dieselfde punte. Dit is nie nodig om vanuit eerste beginsels te werk nie:

2.2.1 $x=1$ (2)

2.2.2 $x=0$ (6)

2.3 Bepaal die waardes van p en q waarvoor $f(x)$ kontinu sal wees by $x=-2$. (8)

[28]

SECTION A
COMPULSORY
CALCULUS

QUESTION 1

Determine the following limits if they exist:

1.1 $\lim\limits_{x\to\infty}\dfrac{3x^2-2x+1}{-4x^2}$ (4)

1.2 $\lim\limits_{x\to 0}\dfrac{x^2+2x}{3\sin 2x}$ (8)

[12]

QUESTION 2

$$f(x)=\begin{cases} px^2 & x<-2 \\ 2q & x=-2 \\ x & -2<x<0 \\ \tan x & 0\le x<1 \\ 2 & x\ge 1 \end{cases}$$

2.1 Discuss the continuity of $f(x)$ at the following points and if not continuous, state what kind of discontinuity occurs:

2.1.1 $x=1$ (6)

2.1.2 $x=0$ (6)

2.2 Discuss algebraically the differentiability of $f(x)$ at the same points. It is not necessary to work from first principles:

2.2.1 $x=1$ (2)

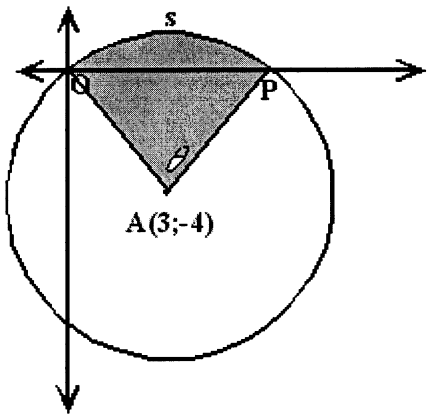
2.2.2 $x=0$ (6)

2.3 Determine the values of p and q which will make $f(x)$ continuous at $x=-2$. (8)

[28]

VRAAG 3

Die sirkel in die skets se middelpunt is (3; -4) en dit gaan deur die oorsprong sowel as die punt P. Die klein boog OP se lengte is **s** eenhede.



3.1 Bereken die radius van die sirkel. (4)

3.2 As s = 6,435 eenhede en r = 5 eenhede, bereken

3.2.1 θ (4)

3.2.2 die oppervlakte van die geskakeerde sektor AOP. (6)

[14]

VRAAG 4

As $f(x) = \cos x$, $g(x) = bg\cos x$ en $h(x) = bg\sin x$ is,
bepaal, sonder 'n sakrekenaar, die waarde van

4.1 $g\left(\frac{-1}{\sqrt{2}}\right)$ (4)

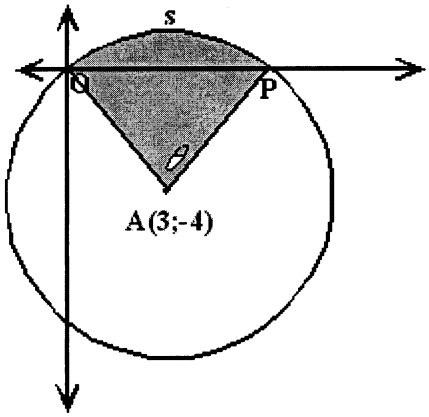
4.2 $h \circ f\left(\frac{4\pi}{3}\right)$ (6)

4.3 $f \circ h\left(\frac{1}{p}\right)$ as $p < 0$ (8)

[18]

QUESTION 3

The circle shown in the diagram has centre (3;-4) and passes through the origin as well as the point P. The small arc OP has length of **s** units.



3.1 Calculate the radius of the circle. (4)

3.2 If s = 6,435 units and r = 5 units, calculate

3.2.1 θ (4)

3.2.2 the shaded area of sector AOP. (6)

[14]

QUESTION 4

f(x) = cos x, g(x) = arccos x and ***h(x) = arcsin x.***
Determine, without using a calculator, the value of

4.1 $g\left(\frac{-1}{\sqrt{2}}\right)$ (4)

4.2 $h \circ f\left(\frac{4\pi}{3}\right)$ (6)

4.3 $f \circ h\left(\frac{1}{p}\right)$ as $p < 0$ (8)

[18]

VRAAG 5

5.1 Skets die grafiek van $f(x) = bg\sin 2x$ (6)

5.2 As $F(x) = x.bg\sin 2x + \frac{1}{2}\sqrt{1-4x^2}$, toon aan dat die afgeleide hiervan is $F'(x) = f(x) = bg\sin 2x$ (12)

5.3 5.3.1 Gebruik 5.2 en bepaal die oppervlakte tussen die grafiek van $f(x) = bg\sin 2x$ en die x-as van $x = 0$ tot $x = \frac{1}{2}$. (Laat jou antwoord in terme van π .) (8)

5.3.2 Kleur hierdie oppervlakte in op die grafiek wat jy geteken het in Vraag 5.1. (2)

[28]

VRAAG 6

Gegee die funksie $f(x) = 4x^2 + 4x + 1$, bepaal die oppervlakte onder $f(x)$ (en bo die x-as) tussen $x = 0$ and $x = 3$. Gebruik die tegniek om die oppervlakte te benader deur n stroke van gelyke breedte en die Riemann-som te bepaal vir die oppervlakte van hierdie reghoeke. Laat dan $n \to \infty$. **[20]**

QUESTION 5

5.1 Sketch the graph of $f(x) = \arcsin 2x$ (6)

5.2 If, $F(x) = x.\arcsin 2x + \frac{1}{2}\sqrt{1-4x^2}$, show that the derivative of this is

$F'(x) = f(x) = \arcsin 2x$ (12)

5.3 5.3.1 Use 5.2 to find the area between the graph of $f(x) = \arcsin 2x$ and the x axis from $x = 0$ to $x = \frac{1}{2}$. (Leave your answer in terms of π.) (8)

5.3.2 Shade in this area on the graph you have drawn in Question 5.1. (2)

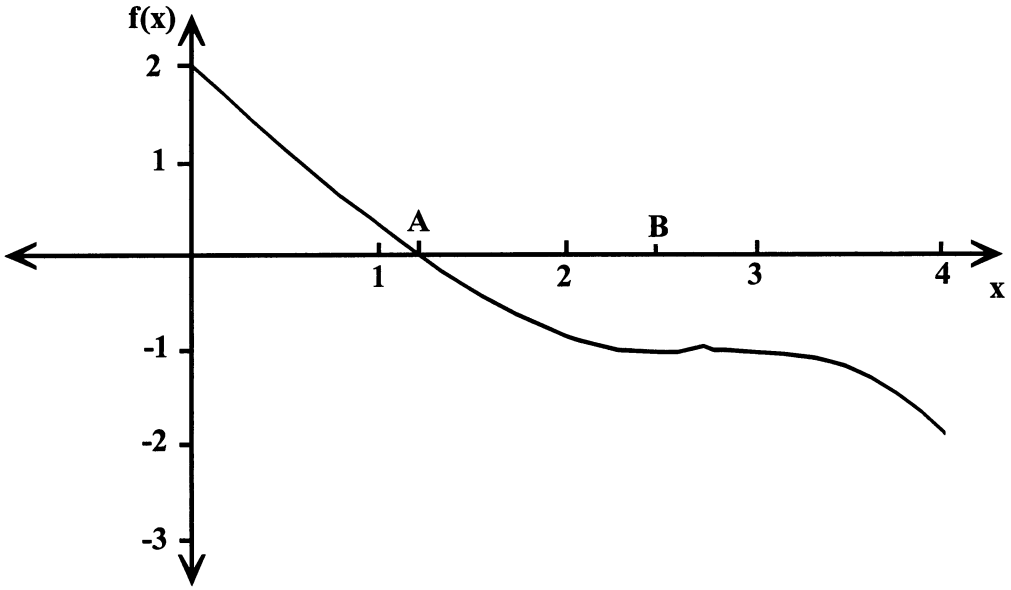
[28]

QUESTION 6

Given the function $f(x) = 4x^2 + 4x + 1$, determine the area below $f(x)$ (and above the x axis) between $x = 0$ and $x = 3$. Use the technique of approximating the area by n rectangles of equal width and finding the Riemann Sum formula for the area of these rectangles. Then let $n \to \infty$. **[20]**

VRAAG 7

Die grafiek toon die kontinue funksie $f(x) = 1 - x + \cos^2 x \qquad x \in [0;4]$



7.1 Toon algebraïes aan dat ***f(x)*** die ***x***-as sny tussen $x = 1$ and $x = 2$ (punt A op die grafiek). (4)

7.2 Gebruik Newton se metode, met `n beginwaarde van $a_0 = 1$, om punt A te bepaal korrek tot twee desimale syfers. (10)

7.3 Bepaal die ***x*** -koördinaat van die stasionêre punt B. Gee die antwoord in terme van π. (10)
[24]

VRAAG 8

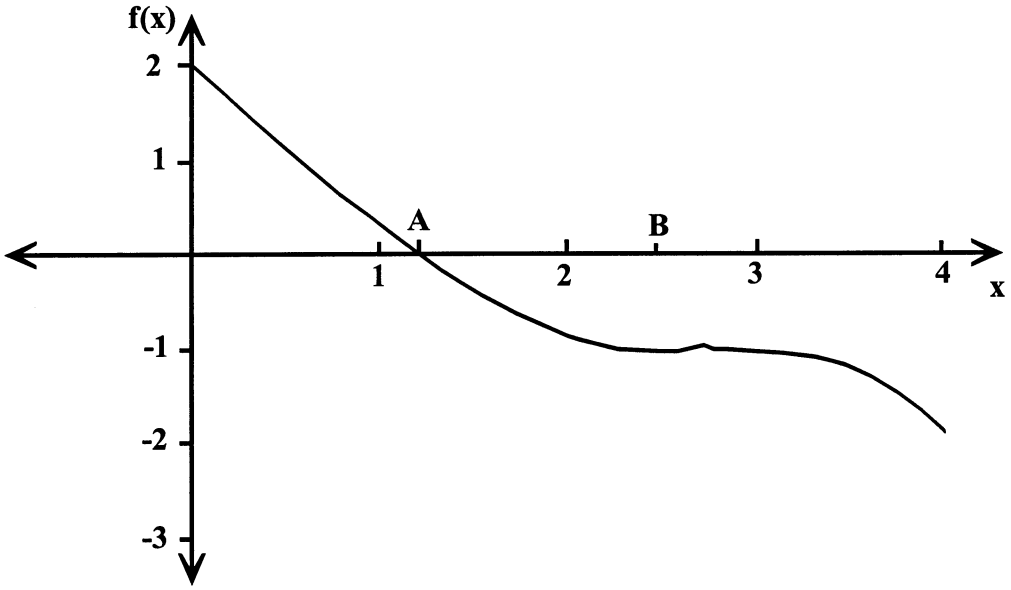
Bepaal die volgende integrale:

8.1 $\int \frac{2x}{\sqrt{1-4x^2}}dx$ (8)

8.2 $\int \frac{2}{\sqrt{1-4x^2}}dx$ (6)
[14]

QUESTION 7

The graph shows the continuous function $f(x) = 1 - x + \cos^2 x \qquad x \in [0;4]$



7.1 Show algebraically that ***f(x)*** has an ***x*** intercept between $x = 1$ and $x = 2$ (point A on the graph). (4)

7.2 Using Newton's Method, with a starting value of $a_0 = 1$ to calculate point A correct to two decimal digits. (10)

7.3 Determine the ***x***-co-ordinate of the stationary point B. Give the answer in terms of π. (10)
[24]

QUESTION 8

Determine the following integrals:

8.1 $\int \frac{2x}{\sqrt{1-4x^2}} dx$ (8)

8.2 $\int \frac{2}{\sqrt{1-4x^2}} dx$ (6)
[14]

VRAAG 9

$$f(x) = \cos x\sqrt{1+\sin x}$$

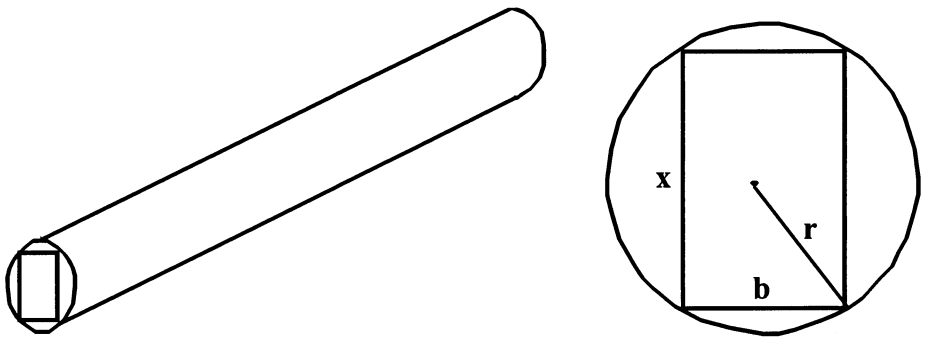
9.1 Bepaal $\int f(x)dx$ (6)

9.2 As die funksie $f(x)$ roteer word om die x-as, ontstaan 'n omwentelingsliggaam. Bereken die volume van hierdie liggaam tussen $x = 0$ en $x = \frac{\pi}{2}$. Gee jou antwoord korrek tot 2 desimale syfers. (16)

[22]

VRAAG 10

'n Reghoekige houtbalk moet gesny word uit 'n eenvormige silindriese stomp met 'n radius van r meter (sien skets). 'n Skets van die dwarsdeursnee word ook gegee.



Die sterkte van die balk (s) word bepaal deur die vergelyking $s = kbx^3$ waar k konstant is en b en x die breedte en lengte van die balk (sien skets).

10.1 Toon aan dat $s = k\sqrt{(4r^2 - x^2)}.x^3$. (8)

10.2 Bereken vervolgens die waarde van x van die sterkste balk wat gesny kan word uit 'n boomstomp met radius 0,2m (maksimeer dus s). Gee die antwoord korrek tot twee desimale syfers. (Dit is nie nodig om aan te toon dat hierdie antwoord wel 'n maksimum sal gee nie). (12)

[20]

TOTAAL VIR AFDELING A: [200]

QUESTION 9

$$f(x) = \cos x\sqrt{1+\sin x}$$

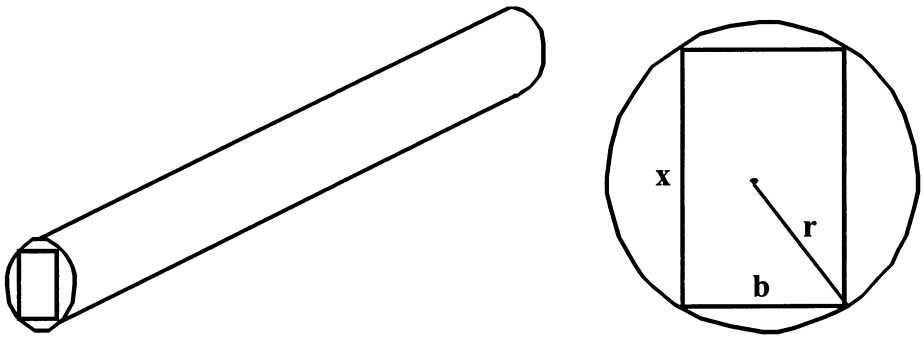
9.1 Determine $\int f(x)\,dx$ (6)

9.2 If the function $f(x)$ is rotated about the x axis, a solid of revolution is generated. Calculate the volume of this solid between $x = 0$ and $x = \frac{\pi}{2}$. Give your answer correct to 2 decimal digits. (16)

[22]

QUESTION 10

A rectangular wooden beam is to be cut from a uniform circular log of radius r metres (see diagram). A detail of the cross-section is also given.



The stiffness of the beam (s) is given by the equation $s = kbx^3$ where k is a constant and b and x is the breadth and length of the beam (see sketch).

10.1 Show that the $s = k\sqrt{(4r^2 - x^2)}.x^3$ (8)

10.2 Hence calculate the value of x of the stiffest beam that can be cut from a log of radius 0.2 m (i.e. maximize s). Give the answer correct to two decimal places. (It is not necessary to show that this answer will give a maximum.) (12)

[20]

TOTAL FOR SECTION A: [200]

Beantwoord enige TWEE van die volgende VIER afdelings.

AFDELING B
WISKUNDE VAN FINANSIES

VRAAG 11

11.1 Mnr. Terrafirma is 'n baie suksesvolle boer in KwaZulu-Natal. Hy verwag 'n baie goeie mielie-oes hierdie jaar en daarom besluit hy om homself met 'n nuwe SSS DVD-speler te bederf. Hy sien in 'n advertensie van een wat beskikbaar is op huurkoop teen 13% per jaar enkelvoudige rente. Geen deposito word vereis nie, maar maandelikse betalings van R400 moet gemaak word aan die einde van elke maand vir 'n periode van twee jaar. Veronderstel dat die DVD gekoop word aan die begin van die maand en dat die betalings begin aan die einde van dieselfde maand. Bereken

11.1.1 hoeveel betaal hy in totaal vir die DVD-speler. (2)

11.1.2 wat die aankoopprys van die DVD-speler is. (4)

11.2 Mnr. Pennypincher, sy buurman, is nie so 'n suksesvolle boer nie, maar hanteer sy finansies baie versigtig. Hy wil dieselfde DVD-speler koop op dieselfde tyd, maar is bewus van die gevare van huurkoopooreenkomste, en hy besluit om eerder 'n banklening aan te gaan sodat hy kontant kan betaal vir die DVD-speler. Sy bank vra rente van 14,5% per jaar maandeliks saamgestel en laat hom toe om die lening terug te betaal oor dieselfde twee jaar tydperk in maandelikse paaiemente. Hierdie betalings moet gemaak word aan die einde van elke maand en dit begin aan die einde van die eerste maand.

11.2.1 Wat sal sy maandelikse paaiement wees gegrond op die veronderstelling dat die DVD-speler presies R7 619.05 kos? (10)

11.2.2 Hoeveel betaal hy uiteindelik vir sy DVD-speler? (2)

11.3 Vergelyk die bedrae wat Mnr. Terrafirma en Mnr. Pennypincher afsonderlik betaal vir sy DVD-speler en verduidelik waarom Mnr. Pennypincher 'n voordeliger ooreenkoms aangegaan het alhoewel die gekwoteerde rentekoers tog meer was. (4)
[22]

Answer any TWO of the following FOUR sections.

SECTION B
FINANCIAL MATHEMATICS

QUESTION 11

11.1 Mr Terrafirma is a very successful farmer in KwaZulu-Natal. He is anticipating a very good maize harvest this year so he decides to treat himself to a new SSS DVD player. He sees one advertised on Hire Purchase at 13% per annum simple interest. No deposit is required but monthly payments of R400 are to be made at the end of each month for a period of two years. Assume that the DVD is purchased at the beginning of the month and that payments begin at the end of the same month. Calculate

11.1.1 how much he pays in total for the DVD. (2)

11.1.2 what the purchase price of the DVD is. (4)

11.2 Mr Pennypincher, his neighbour, is not as successful a farmer, but manages his finances very carefully. He would like to buy the same DVD at the same time but is wary of hire purchase agreements, so he decides to take out a bank loan instead so that he can pay cash for the DVD. His bank charges him interest of 14,5% per annum compounded monthly and allows him the same two-year period to pay back the loan in monthly instalments. These payments are to be made at the end of each month beginning at the end of the first month.

11.2.1 What will his monthly instalments be on the assumption that the DVD costs exactly R7 619.05? (10)

11.2.2 How much does he end up paying in total for his DVD? (2)

11.3 Compare what Mr Terrafirma and Mr Pennypincher each pay for their DVDs and explain why Mr Pennypincher gets a better deal even though the quoted interest rate is higher. (4)

[22]

VRAAG 12

Soos te wagte, het Mnr. Terrafirma 'n rekord-oes behaal en daarom besluit hy om 'n nuwe trekker te koop vir R750 000. Hy kan dit bekostig om R275 000 kontant te betaal en neem 'n lening uit by die bank vir die verskil. Sy bank bied hom 'n rentekoers aan van 12% per jaar kwartaalliks saamgestel. Hy moet maandelikse terugbetalings doen van R10 000 op die lening aan die einde van elke maand, met 'n finale betaling wat minder as R10 000 is, beginnende oor twee maande.

12.1 Toon aan dat die ekwivalente effektiewe rentekoers maandeliks saamgestel $i^{(12)} = 0,99\%$, is (korrek tot 2 desimale syfers). (6)

12.2 Hoe lank sal dit Mnr. Terrafirma neem om sy lening aan die bank terug te betaal? (Gebruik 0,99%) (14)

12.3 Wat sal sy finale paaiement wat minder as R10 000 is, wees? (10)

12.4 Aan die begin van sy derde jaar van terugbetaling, vernietig 'n ernstige droogte Mnr. Terrafirma se hele mielie-oes met die gevolg dat hy nie in staat is om sy lening te betaal vir 12 maande nie, beginnende in die 25ste maand. Wat sal die uitstaande balans wees wat hy skuld aan die begin van die vierde jaar? (10)
[40]

VRAAG 13

Aan die begin van 'n sekere jaar besluit Mnr. Pennypincher dat hy 'n nuwe stroper sal nodig hê oor 5 jaar. Dit kos op die oomblik R750 000. Hy besluit om dadelik met 'n delgingsfonds te begin sodat hy die stroper kan koop oor 5 jaar. Die prys styg met 12,75% per jaar.

13.1 Wat sal die nuwe stroper oor 5 jaar kos? (4)

Hy besluit om aan die einde van elke maand R14 500 in 'n rekening te betaal wat 12% rente per jaar verdien, maandeliks saamgestel. Nadat hy bereken het dat dit hom R1 184 210,21 (tot die naaste sent) sal gee, besef Mnr. Pennypincher dat dit nie genoeg sal wees nie. Hy besluit om die betaling elke jaar te verdubbel aan die einde van Junie. Dit beteken dat hy R29 000 sal inbetaal aan die einde van elke Junie gedurende die 5 jaar. (Hierdie maand stem ooreen met die mielie-oes.)

13.2 Hoeveel sal hierdie **addisionele** jaarlikse belegging (slegs die ekstra R14 500 wat hy telkens in Junie spaar) bydra tot die delgingsfonds? (10)

13.3 Na 5 jaar word Mnr. Pennypincher 'n aanbod gemaak van 2,5% afslag vir kontant! Het hy genoeg geld om te betaal vir sy stroper? (6)
[20]

QUESTION 12

As anticipated, Mr Terrafirma has a bumper harvest so he decides to buy a new tractor for R750 000. He can afford to pay R275 000 cash and takes out a bank loan for the difference. His bank offers him an interest rate of 12% per annum compounded quarterly. He is required to pay back monthly instalments of R10 000 on the loan at the end of each month, with a final payment less than R10 000, starting in two months' time.

12.1 Show that the equivalent effective interest rate compounded monthly is $i^{(12)} = 0,99\%$, (correct to 2 decimal digits). (6)

12.2 How long will it take Mr Terrafirma to repay his bank loan? (Work with 0,99%.) (14)

12.3 What will his final payment, less than R10 000, be? (10)

12.4 At the beginning of his third year of repayment, a serious drought wipes out Mr Terrafirma's entire maize crop with the result that he is unable to pay his loan for 12 months, starting in the 25th month. What will be his outstanding balance owing at the beginning of the fourth year? (10)

[40]

QUESTION 13

At the beginning of a year Mr Pennypincher decides that he will need a new combine harvester in 5 years' time. This cost is R750 000 at present. He decides to set up a sinking fund so that he can purchase it after 5 years. The price escalates at a rate of 12,75% per annum.

13.1 What will the new harvester cost in 5 years' time? (4)

He decides to pay R14 500 at the end of each month into an account which pays 12% interest per annum compounded monthly. Having calculated that this will give him R1 184 210,21 (to the nearest cent), Mr Pennypincher realizes it will not be enough. He decides to double the payment every year at the end of June. This means that he will pay R29 000 at the end of each June during all 5 years. (This month coincides with the maize harvest.)

13.2 How much will this **additional** annual investment (only the extra R14 500 that he saves every June) add to the sinking fund? (10)

13.3 After 5 years Mr Pennypincher is offered a 2,5% discount for cash! Does he have enough money to pay for his harvester? (6)

[20]

VRAAG 14

Die jaarlikse winsgewendheid van mielieboerdery is 'n funksie van q, die oppervlakte waarop daar met mielies geboer word (in honderde hektare). Die inkomste en koste-funksies word onderskeidelik gegee deur:
$R(q) = -0{,}03(q-10)^2 + 3$ en $C(q) = aq^2 + 0{,}08q + 1{,}44$ (R en C in miljoene rand) en waar a 'n konstante is.

14.1 Bereken a as 400 hektaar geplant moet word om gelyk te breek, dit wil sê as q = 4. (6)

14.2 As $a = 0{,}01$, bereken die maksimum wins. (12)
[18]

TOTAAL VIR AFDELING B: [100]

AFDELING C
ANALITIESE MEETKUNDE

VRAAG 15

15.1 $f(x)$ se parametriese vergelyking is $x = 2t^2 \; ; \; y = 4t$

15.1.1 Skryf die vergelyking van $f(x)$ in Cartesiese (kanoniese) vorm neer.

15.1.2 Skets $f(x)$ en toon alle afsnitte. (6)

15.1.3 Gee die vergelyking van die raaklyn aan $f(x)$ met inklinasie $\frac{3\pi}{4}$. (10)

15.2 Gee die vergelyking (in Cartesiese / kanoniese vorm), van die elips waarvan een fokus $(2 \; ; \; 0)$ is en 'n directrieks $x = 18$. (10)

15.3 Raaklyne word getrek vanaf $A(-3 \; ; \; 3)$ na $xy = 16$. Bepaal die koördinate van die raakpunte van hierdie raaklyne met die hiperbool. (16)
[42]

QUESTION 14

The annual profitability of maize farming is a function of q, the area under maize (in hundreds of hectares). The revenue and cost functions are given respectively by:
$R(q) = -0{,}03(q-10)^2 + 3$ and $C(q) = aq^2 + 0{,}08q + 1{,}44$
(R and C in millions of rand) and where a is a constant.

14.1 Calculate a if 400 hectares have to be planted to break even, that is when q = 4. (6)

14.2 If $a = 0{,}01$, calculate the maximum profit. (12)
[18]

TOTAL FOR SECTION B: [100]

SECTION C
ANALYTICAL GEOMETRY 2

QUESTION 15

15.1 $f(x)$ has parametric equation $x = 2t^2$; $y = 4t$

15.1.1 Write the equation of $f(x)$ in Cartesian (canonical) form.

15.1.2 Sketch $f(x)$, labelling any intercepts. (6)

15.1.3 Give the equation of the tangent to $f(x)$ with inclination $\frac{3\pi}{4}$. (10)

15.2 Give the equation (in Cartesian / canonical form), of the ellipse with one focus $(2\ ;\ 0)$ and a directrix $x = 18$. (10)

15.3 Tangents are drawn from $A(-3\ ;\ 3)$ to $xy = 16$. Determine the co-ordinates of the points of contact of these tangents with the hyperbola. (16)
[42]

VRAAG 16

16.1 Bepaal die afstand vanaf $A(3\ ;-2\ ;6)$ tot by die vlak $3x+4y-5z=21$. (10)

16.2 Beskou die vlak P_1, en co-planar lyne L_1 en L_2 wat gegee word deur:

$$P_1 : x+2y-z=5$$
$$L_1 : \frac{x-11}{-4}=\frac{y+2}{2}=\frac{z+8}{5}$$
$$L_2 : \frac{x-1}{1}=\frac{y+2}{-3}\ ;\ z=7$$

16.2.1 Gee die vergelyking van vlak P_2 wat L_1 en L_2 bevat. (12)

16.2.2 Bereken die snylyn van die vlakke P_1 en P_2. (8)
[30]

VRAAG 17

Die lyn met vergelyking $y=mx+c$ is 'n raaklyn aan die ellips met vergelyking

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

17.1 Toon aan dat $c^2=a^2m^2+b^2$. (14)

17.2 Vervolgens, of andersins, bereken die vergelykings van die raaklyne vanaf $A(3\ ;4)$ na die ellips $\frac{x^2}{16}+\frac{y^2}{25}=1$. (14)
[28]

TOTAAL VIR AFDELING C: [100]

QUESTION 16

16.1 Determine the distance from $A(3\ ;-2\ ;6)$ to the plane $3x+4y-5z=21$. (10)

16.2 Consider plane P_1, and co-planar lines L_1 and L_2 given by:

$$P_1: x+2y-z=5$$
$$L_1: \frac{x-11}{-4}=\frac{y+2}{2}=\frac{z+8}{5}$$
$$L_2: \frac{x-1}{1}=\frac{y+2}{-3};\ z=7$$

16.2.1 Give the equation of plane P_2 containing L_1 and L_2. (12)

16.2.2 Calculate the line of intersection of planes P_1 and P_2. (8)

[30]

QUESTION 17

The line with equation $y=mx+c$ is a tangent to the ellipse with equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

17.1 Show that $c^2=a^2m^2+b^2$. (14)

17.2 Hence or otherwise, calculate the equations of the tangents from $A(3\ ;4)$ to the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$. (14)

[28]

TOTAL FOR SECTION C: [100]

AFDELING D
ALGEBRA

VRAAG 18

Bewys deur wiskundige induksie dat

$1^2+2^2+3^2+...+n^2=\frac{n(n+1)(2n+1)}{6}$ vir $n\in\mathbb{N}$ **[16]**

VRAAG 19

$f(x)=x^2+bx+c$ het nulpunte α en β. Bepaal die kwadratiese vergelyking met konstante term gelyk aan $2c-b^2$, wat nulpunte 1 en $-\alpha^2-\beta^2$ het, met koëffisiënte uitgedruk in terme van b en c. **[12]**

VRAAG 20

Bepaal parsiële breuke vir $\frac{2}{x^3+3x^2+2x}$ **[14]**

VRAAG 21

21.1 Stel Eisenstein se kriterium. (6)

21.2 $f(x)=(m+2)x^{12}-140x^7+42x^6+mx^4-210$ is 'n polinoom in $\mathbb{Z}[x]$.
Vir watter waarde(s) van m sal $f(x)$ onontbindbaar wees oor $\mathbb{Z}$ as $|m|<10$? (8)
[14]

VRAAG 22

$f(x)=x^6-4x^4+x^2+6$

22.1 Stel die rasionale wortelstelling. (6)

22.2 Toon aan dat $f(x)$ geen rasionale nulpunte het nie. Gebruik 22.1 en die faktorstelling. (6)

22.3 As $f\left(-\sqrt{2}\right)=0$, faktoriseer $f(x)$ volledig oor :

22.3.1 $\mathbb{Z}$ (12)

22.3.2 $\mathbb{R}$ (2)

22.3.3 $\mathbb{C}$ (2)
[28]

SECTION D
ALGEBRA

QUESTION 18

Prove by mathematical induction that

$1^2+2^2+3^2+...+n^2=\frac{n(n+1)(2n+1)}{6}$ for $n\in\mathbb{N}$ [16]

QUESTION 19

$f(x)=x^2+bx+c$ has zeros α and β. Determine the quadratic equation with constant term equal to $2c-b^2$, which has zeros 1 and $-\alpha^2-\beta^2$, with coefficients expressed in terms of b and c. [12]

QUESTION 20

Determine partial fractions for $\frac{2}{x^3+3x^2+2x}$ [14]

QUESTION 21

21.1 State Eisenstein's criterion. (6)

21.2 $f(x)=(m+2)x^{12}-140x^7+42x^6+mx^4-210$ is a polynomial in $\mathbb{Z}[x]$.
For which value(s) of m will $f(x)$ be irreducible over $\mathbb{Z}$ if $|m|<10$? (8)
[14]

QUESTION 22

$f(x)=x^6-4x^4+x^2+6$

22.1 State the rational roots theorem. (6)

22.2 Show that $f(x)$ has no rational zeros. Use 22.1 and the factor theorem. (6)

22.3 If $f(-\sqrt{2})=0$, factorise $f(x)$ fully over :

22.3.1 $\mathbb{Z}$ (12)

22.3.2 $\mathbb{R}$ (2)

22.3.3 $\mathbb{C}$ (2)
[28]

VRAAG 23

$f(x)=\frac{x^2-1}{x-2}$. $f(x)$ het 'n minimum draaipunt (3,7; 13,4) en 'n maksimum draaipunt by (0,3; 0,7). Skets $f(x)$ en toon alle afsnitte, draaipunte en asimptote aan. **[16]**

TOTAAL VIR AFDELING D: [100]

AFDELING E
STATISTIEK

Rond alle antwoorde af tot vier desimale syfers, tensy anders gevra.

VRAAG 24

24.1 'n Klein pakkie "Jelly tots" het 5 pienk, 7 geel en 3 groen lekkertjies in. Daar is slegs hierdie drie kleure in die pakkie. As ek 3 lekkertjies uithaal, een op 'n keer, sonder terugplasing, bereken

24.1.1 die waarskynlikheid dat al drie pienk sal wees. (8)

24.1.2 die waarskynlikheid dat ek een van elke kleur sal kry. (8)

24.2 Die letters van die woord PARALLELOGRAM word gerangskik om woorde te vorm met dieselfde aantal letters.

24.2.1 Hoeveel **nuwe** woorde kan gemaak word as die woorde nie noodwendig sin moet maak nie? (6)

24.2.2 Wat is die waarskynlikheid dat 'n woord gevorm uit die letters van die woord PARALLELOGRAM sal begin en eindig met dieselfde letter? (12)
[34]

QUESTION 23

$f(x)=\frac{x^2-1}{x-2}$. $f(x)$ has a minimum turning point at (3,7; 13,4) and a maximum turning point at (0,3; 0,7). Sketch $f(x)$ showing all asymptotes, turning points and intercepts. **[16]**

TOTAL FOR SECTION D: [100]

SECTION E
STATISTICS

Round all answers off to four decimal places unless otherwise asked.

QUESTION 24

24.1 A small packet of Jelly tots contains 5 pink, 7 yellow and 3 green sweets. Only these three colours are present in the packet. If I remove 3 jelly tots, one at a time, without replacement, calculate

24.1.1 the probability that all three are pink. (8)

24.1.2 the probability that I will get one of each colour. (8)

24.2 The letters of the word PARALLELOGRAM are arranged to form words with the same number of letters.

24.2.1 How many **new** words can be made if the words don't necessarily have to make sense? (6)

24.2.2 What is the probability that a word formed from the letters of the word PARALLELOGRAM starts and ends with the same letter? (12)
[34]

VRAAG 25

Twee boogskutters, A en B, neem aan 'n kompetisie deel. Die waarskynlikheid dat elkeen op sy eie 'n kolskoot sal skiet, is 0,7 en 0,5. Aanvaar dat die twee waarskynlikhede **onafhanklik** van mekaar is.

25.1 Stel die situasie met behulp 'n Venn-diagram voor. (8)

25.2 Gebruik die Venn-diagram, of andersins, en skryf die waarskynlikheid neer dat

25.2.1 A en B albei 'n kolskoot sal skiet as elkeen een skoot kry. (2)

25.2.2 nie een van die twee 'n kolskoot sal skiet as elkeen een skoot kry. (2)

25.3 Hoeveel skote moet B skiet sodat sy kans op minstens een kolskoot minstens 0,9 is? (10)
[22]

VRAAG 26

'n Elektrisiteitsmaatskappy verkoop 'n groot aantal sonkragpanele (>1000) aan die Oos-Kaapse regering om te installeer in afgeleë gebiede as bron van elektrisiteit. As gevolg van probleme met die vervaardigingsproses, was 5% van hierdie panele onbruikbaar. Voor die besending deur die regering aanvaar word, word 'n steekproef van panele getoets en die aantal onbruikbare panele getel. As meer as 2 van die 15 panele onbruikbaar is, sal die besending teruggestuur word. Wat is die waarskynlikheid dat dit sal gebeur? **[14]**

VRAAG 27

Die galjoen, Suid-Afrika se nasionale vis, is 'n waardevolle vangs vir soutwater-hengelaars. Gebaseer op vangste wat gemaak is, word dit geskat dat die populasie, wat normaal verdeel is, 'n gemiddelde lengte van 38,5 cm het met 'n standaardafwyking van 7,3 cm. Norman, 'n ervare galjoenvisserman wat in die Wes-Kaap woon, beplan vir sy volgende hengelvakansie van die seisoen.

27.1 Op grond van noodsaaklike aannames, wat is die waarskynlikheid dat die volgende galjoen wat Norman vang, langer sal wees as 50 cm? (8)

27.2 Noem een van hierdie aannames. (2)

27.3 Wat is die wetlike perk (lengte van die vis) waarbo 'n galjoen gehou mag word, as 31,6% van (die kleinste) galjoen vangste teruggegooi moet word in die see? Gee jou antwoord noukeurig tot die naaste sentimeter. (10)
[20]

QUESTION 25

Two archers, A and B, take part in a competition. The probability that each one will shoot a bull's eye, is 0,7 and 0,5. Accept that the two events are **independent**.

25.1 Represent the situation with a Venn Diagram. (8)

25.2 Use the Venn Diagram, or otherwise, and write down the probability that

25.2.1 both A and B shoot bull's eyes if each gets one shot only. (2)

25.2.2 not one of them shoots a bull's eye if each gets one shot only. (2)

25.3 How many times must B shoot so that his chance on at least one bull's eye is 0,9? (10)
[22]

QUESTION 26

An electrical company is selling a large number of solar panels (>1000) to the Eastern Cape Government for installation in the rural areas as a source of electricity. Due to difficulties in the manufacturing process, 5% of these panels turn out to be defective. Before the consignment is accepted by the Government, a sample of panels is tested and the number of defective panels counted. If more than 2 of the 15 panels are defective, the consignment is rejected. What is the probability of this occurring? **[14]**

QUESTION 27

The Galjoen, South Africa's national fish, is a valued catch by recreational saltwater anglers. Based on catch returns, it is estimated that the population, which is normally distributed, has an average length of 38,5 cm with a standard deviation of 7,3 cm. Norman, an experienced Galjoen fisherman living in the Western Cape, is planning his next fishing trip for the season.

27.1 Making the necessary assumptions, what is the probability that the next Galjoen Norman catches will be over 50 cm? (8)

27.2 Give one of these assumptions. (2)

27.3 What is the legal limit (length of fish) above which Galjoen may be kept, if 31,6% of (the smallest) Galjoen caught have to be returned to the sea? Give your answer to the nearest centimetre. (10)
[20]

VRAAG 28

'n Meningsopname het bevind dat 'n spesifieke party baie moontlik 47% van die stemme sal kry in die volgende verkiesing. Daar word aanspraak gemaak dat hierdie getal korrek is tot binne 1% met `n 95% vertrouensinterval. Bereken die minimum aantal persone wat ondervra moes word vir hierdie aanspraak om geldig te wees. **[10]**

TOTAAL VIR DIE AFDELING E: [100]

TOTAAL: 400

QUESTION 28

A polling survey found that a particular party is likely to gain 47% of the vote in the upcoming election. This figure is claimed to be correct to within 1% with a 95% confidence interval. Calculate the minimum number of people that would have had to be surveyed for this claim to have been valid. **[10]**

TOTAL FOR SECTION E: [100]

TOTAL: 400

FORMULA SHEET/ FORMULEBLAD

Differential en Integraal Calculus

Differensiaal- en Integraalrekene

$s = r\theta$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$ $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\sin A.\cos B = \frac{1}{2}(\sin(A+B)+\sin(A-B))$

$\sin A.\sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$

$\cos A.\cos B = \frac{1}{2}(\cos(A-B)+\cos(A+B))$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(2n+1)(n+1)}{6}$$

$$a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$$

$$V = \pi \int_a^b [f(x)]^2 dx$$

$$Riemann\ Sum = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$$

F(x)	F '(x)
$a.x^n$	$na.x^{n-1}$
sin x	cos x
cos x	- sin x
tan x	$\sec^2 x$
sec x	sec x.tan x
cot x	$-\operatorname{cosec}^2 x$
cosec x	- cosec x.cot x
arcsin x bgsin x	$\frac{1}{\sqrt{1-x^2}}$
arccos x bgcos x	$-\frac{1}{\sqrt{1-x^2}}$
arctan x bgtan x	$\frac{1}{x^2+1}$
f(x).g(x)	f '(x).g(x)+f(x).g '(x)
$\frac{f(x)}{g(x)}$	$\frac{f'(x).g(x)-f(x).g'(x)}{[g(x)]^2}$
f(g(x))	f '(g(x)).g '(x)

Finance/ Finansies

$F=P(1+i)^n$ $F=P(1-i)^n$

$F=P(1+in)$ $F=P(1-in)$

$$P = x.\frac{1-(1+i)^{-n}}{i} \qquad F = x.\frac{(1+i)^n - 1}{i}$$

Analytical Geometry/ Analitiese Meetkunde

$y=4ax^2$ $yy_1=2a(x+x_1)$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$$

Algebra

$$\alpha+\beta=-\frac{b}{a} \qquad \alpha+\beta+\gamma=-\frac{b}{a}$$

$$\alpha..\beta=\frac{c}{a} \qquad \alpha\beta+\beta\gamma+\alpha\gamma=\frac{c}{a}$$

$$\alpha.\beta.\gamma=-\frac{d}{a}$$

Statistics / Statistiek

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$${}_nP_r = \frac{n!}{(n-r)!} \qquad {}_nC_r = \frac{n!}{(n-r)!\,n!}$$

$$P(R=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

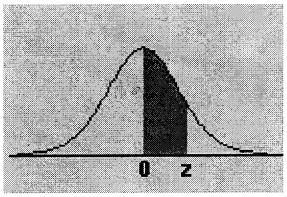
$$P(R=x) = \frac{\binom{p}{x}\binom{N-p}{n-x}}{\binom{N}{n}}$$

$$z = \frac{\bar{X}-\mu}{\sigma}$$

$$P(\bar{X} - 1.96\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + 1.96\frac{\sigma}{\sqrt{n}}) = 0.95$$

$$P\left(p - 1.96\sqrt{\frac{p(1-p)}{n}} < \pi < p + 1.96\sqrt{\frac{p(1-p)}{n}}\right) = 0{,}95$$

Normal Distribution/ Normaalverspreiding



$$P(X \le x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-x^2}{2}} dx$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0		0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Wiskunde Formuleblad/ Mathematics Formula Sheet

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(a+l) \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(1-r^n)}{1-r} \qquad S_n = \frac{a(r^n - 1)}{r-1} \qquad S_\infty = \frac{a}{1-r}$$

$$A = P(1 + \frac{r}{100})^n \qquad A = P(1 - \frac{r}{100})^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2})$$

$$y^2 + x^2 = r^2$$

$$(x-p)^2 + (y-q)^2 = r^2$$

In Δ ABC: $\frac{a}{\sin A} = \frac{b}{\sin B}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \tfrac{1}{2}ab.\sin C$$

$$\cos(A+B) = \cos A.\cos B - \sin A.\sin B$$

$$\sin(A+B) = \sin A.\cos B + \cos A.\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\sin 2A = 2\sin A \cos A$$

END / *EINDE*