POSSIBLE ANSWERS FOR :

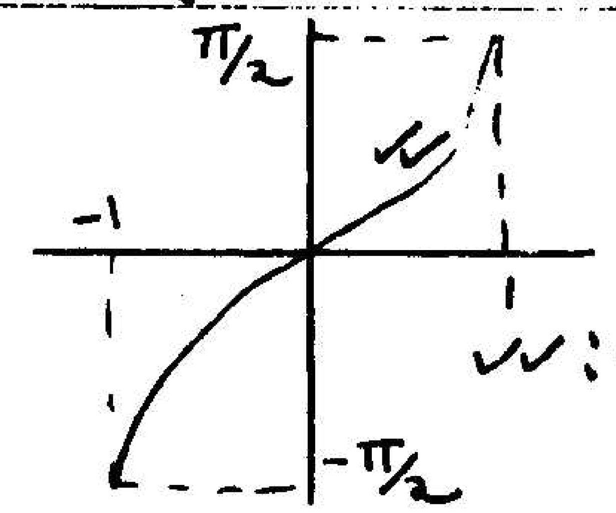
Additional Mathematics Paper Q 2003

Section A

Question 1. [28]

1.1. $D_x: x \in [-1; 1]$
$R: y \in [-\frac{\pi}{2}; \frac{\pi}{2}]$ (4)

1.2



(4)

3.1 $-\pi/3$ (2)

3.2 $\arccos(-\cos \pi/6) = x$
$x = \pi - \pi/6 = \frac{5\pi}{6}$ (6)

3.3 Let $\arctan 1/3 = A$ and $\arctan \frac{1}{7} = B$
$\therefore \tan A = 1/3$ and $\tan B = 1/7$
$x = \tan(A+B)$
$= \frac{\tan A + \tan B}{1 - \tan A . \tan B}$
$= \frac{\frac{1}{3} + \frac{1}{7}}{1 - \frac{1}{3} . \frac{1}{7}} = \frac{1}{2}$ (12)

Question 2 [22]

2.1.1 $\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \cdot \frac{\sqrt{2+h} + \sqrt{2}}{\sqrt{2+h} + \sqrt{2}}$
$= \lim_{h \to 0} \frac{2+h-2}{h(\sqrt{2+h} + \sqrt{2})} = \lim_{h \to 0} \frac{h}{h(\sqrt{2+h} + \sqrt{2})}$
$= \frac{1}{2\sqrt{2}}$ (8)

or

$\left(\frac{0}{0} \therefore L'H\right): \lim_{h \to 0} \frac{\frac{1}{2}(2+h)^{-1/2}}{1}$
$= \frac{1}{2} \cdot 2^{-1/2}$

2.1.2 $\lim_{\theta \to \pi/2} \frac{\pi/2 - \theta}{2 \sin\theta . \cos\theta}$
$= \lim_{\theta \to \pi/2} \frac{\pi/2 - \theta}{2\sin\theta . \sin(\pi/2 - \theta)}$
$= \frac{1}{2}$ (8)

or

$\left(\frac{0}{0} \therefore L'H\right) \lim_{\theta \to \pi/2} \frac{-1}{2 \cos 2\theta} = \frac{-1}{-2} = 1/2$

2.2. $\lim_{x \to -1} \frac{(x-1)(x+1)}{x+1} = -2$
$\therefore f(x) = \begin{cases} \frac{x^2-1}{x+1} & \text{if } x \neq -1 \\ -2 & \text{if } x = -1 \end{cases}$ (6)

Question 3 [32]

3.1.1 $\lim_{x \to 0^-} f(x)$ does not exist.
$\therefore$ jump disc. (4)

3.1.2. $\lim_{x \to 2^-} f(x) = 2$ $\lim_{x \to 2^+} f(x) = 2$
$f(2) = 2$
$\therefore \lim_{x \to 2} f(x) = f(2)$
$\therefore$ Cont. at $x = 2$ (8)

3.2.1. Not diff. because not cont. (2)

3.2.2.
$\lim_{x \to 2^-} f'(x) = \lim_{x \to 2^-} (2x-1) = 3$
$\lim_{x \to 2^+} f'(x) = 0$
$\therefore$ Not differentiable. (6)

3.3.1 $x^2 - x > 0$
$x > 1$ (4)

3.3.2. $2x - 1 > 0$
$\frac{1}{2} < x < 2$ (4)

3.3.3. $f''(x) = 2$ as $x = 1/2$
or: min. at $x = 1/2$ (4)

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Question 4 [28]

4.1. $\frac{1}{2}[\tan(\pi - x)]^{-1/2} . \sec^2(\pi - x).(-1)$ (8)

4.2 $1.\log\sin\frac{x}{2} + x . \frac{1}{\sqrt{1-\frac{x^2}{4}}} . \frac{1}{2}$ or $\left(\frac{x}{2}\right)^2$ (8)

4.3. $y = (1+2x)^{-1}$
$y' = -(1+2x)^{-2} . 2$
$y'' = 2(1+2x)^{-3} . 2.2$
$\frac{d^n y}{dx^n} = n!(-1)^n (1+2x)^{-n-1} . 2^n$ (12)

Question 5 [42]

5.1. $\Delta x_i = \frac{2}{n}$ $\quad -x^2+4=0 \quad \therefore x = \pm 2$

$x_0 = 0$
$x_1 = \frac{2}{n}$
$\vdots$
$x_n = \frac{2i}{n}$

$f(x_i) = -\left(\frac{2i}{n}\right)^2 + 4$
$= -\frac{4i^2}{n^2} + 4$

$\Delta x_i . f(x_i) = -\frac{8i^2}{n^3} + \frac{8}{n}$

$\sum (x_i).f(x_i) = \sum -\frac{8i^2}{n^3} + \sum \frac{8}{n}$

$= \frac{-8}{n^3}\left[\frac{n}{6}(2n+1)(n+1)\right] + \frac{8}{n}.n$

$= \frac{-4}{3n^2}(2n^2+3n+1) + 8$

$= -\frac{8}{3} - \frac{4}{n} - \frac{4}{3n^2} + 8$

As $n \to \infty$: $-\frac{8}{3} + 8 = 5\frac{1}{3}$ (5,3) (20)

If started at $x_0 = -2$: $10\frac{2}{3}$ $\frac{18}{20}$

5.2.1 $\frac{\tan(2\theta+1)}{2} + k$ (6)

5.2.2. $\int \frac{1}{3\sqrt{1-\left(\frac{2x}{3}\right)^2}}\,dx$

$= \frac{\frac{1}{3}\arcsin\left(\frac{2x}{3}\right)}{2/3} + k$ (8)

5.2.3 $-\int(-1-4x)(1-x-2x^2)^{-3}\,dx$

$= -\frac{(1-x-2x^2)^{-2}}{-2} + k$ (8)

Question 6. [48]

6.1. $\pi\int_0^{\pi/2}(1+\cos x)^2\,dx$

$= \pi\int_0^{\pi/2}(1+2\cos x + \cos^2 x)\,dx$

$= \pi\int_0^{\pi/2}\left(1+2\cos x+\frac{1}{2}[1+\cos 2x]\right)dx$

$= \pi\left[x+2\sin x+\frac{1}{2}x+\frac{1}{2}\frac{\sin 2x}{2}\right]_0^{\pi/2}$

$= \pi\left[\frac{\pi}{2}+2\sin\frac{\pi}{2}+\frac{1}{2}.\frac{\pi}{2}+\frac{1}{4}\sin 2.\frac{\pi}{2}\right] - 0$

$= \frac{3\pi^2}{2} + 2\pi$ (14)

6.2.1 $s = 2\theta$ (2)

6.2.2
$V'(\theta) = \frac{1(\theta^2+1) - \theta.2\theta}{(\theta^2+1)^2} = 0$

$\theta^2+1-2\theta^2 = 0$

$\theta^2 = 1 \quad \therefore \theta = +1$ of $\theta = -1$
maks. min. (10)

6.2.3. $s = 2(1) = 2$ (2)

6.3.1 $f'(x) = 3x^2 - 4$

$a_{n+1} = a_n - \frac{a_n^3 - 4a_n + 1}{3a_n^2 - 4}$

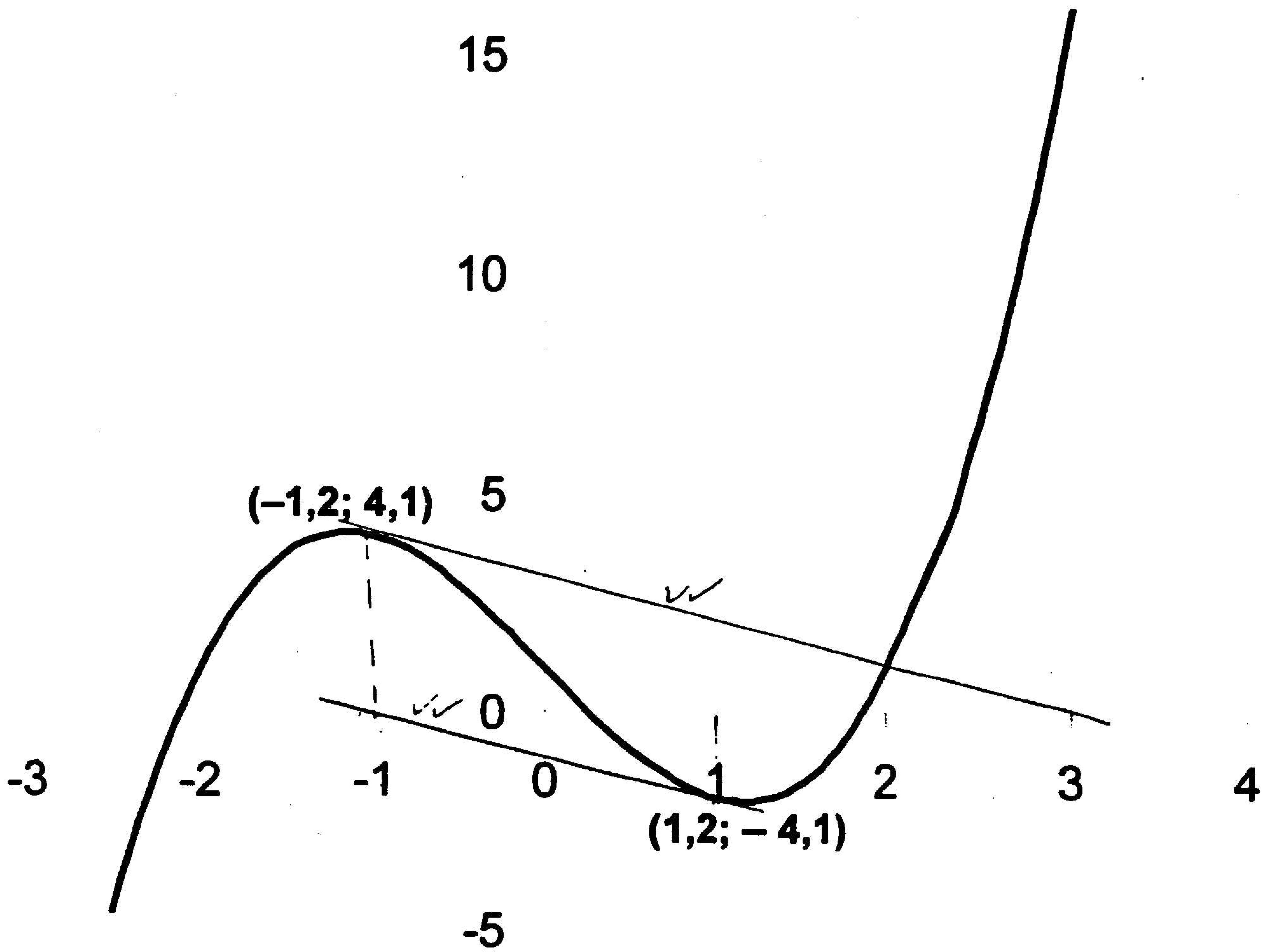
$a_0 = 3$

$a_1 = 2{,}30434$
$a_2 = 1{,}9674$
$a_3 = 1{,}8668\ldots$
$a_4 = 1{,}8608\ldots$
$\therefore x = 1{,}861$ (14)

6.3.2
See graph. (6)

CANDIDATE'S EXAMINATION NUMBER
KANDIDAAT SE EKSAMENNOMMER:

VRAAG 6.3



The ~~tangent~~ tangent drawn at $x=1$, cuts the x-axis at -1. The next tangent drawn at $x=-1$, cuts the x-axis at 3.

Section B

Question 7. [18]

T_4 T_8

x $i = 0{,}0125$ $i = 0{,}04$; $i = 0{,}0125$

$T_4: \; x(1{,}0125)^{48}$

$T_8: \; \frac{x(1{,}0125)^{48}}{2}(1{,}0125)^{48} + \frac{x(1{,}0125)^{48}}{2} \cdot (1{,}04)^{16} =$

R33478.16 (equation)

$x(3{,}34\ldots) = 33478{,}16$

$x = R10000{,}00$ (18)

Question 8 [18]

8.1. $13q^{5/2} + 12q = 14q^{5/2} - 15q$

$q^{5/2} - 27q = 0$

$q(q^{3/2} - 27) = 0$

$q = 0$ or $q^{3/2} = 27$

$q = 9$ (8)

8.2. $W = \int((13q^{5/2} + 12q) - (14q^{5/2} - 15q))\,dq$

$= \int(-q^{5/2} + 27q)\,dq$

$= -\frac{2}{7}q^{7/2} + \frac{27}{2}q^2 - 150(000)$ (6)

8.3 $W(9) = -\frac{2}{7} \cdot 9^{7/2} + \frac{27}{2} \cdot 9^2 - 150$

= R318,6thousand or

R318642.86

Question 9 [32]

9.1. T_0 T_3 T_n

85000 2000 $i = 0{,}0175$ 2000

$85000(1{,}0175)^2 = \frac{2000(1 - 1{,}0175^{-n})}{0{,}0175}$

$1{,}0175^{-n} = 0{,}229\ldots$

$-n = \frac{\log(0{,}229\ldots)}{\log 1{,}0175}$ $n = 84{,}7$

$\therefore$ $85 + 2 \; (86 + 1)$ months. (14)

9.2 $y = 85\,000(1{,}0175)^{87} - \frac{2000(1{,}0175^{84} - 1)}{0{,}0175}(1{,}0175)$

$= R1436.44$ (8)

9.3 $T_{34}: OB = 85\,000(1{,}0175)^{36} - \frac{2000(1{,}0175^{34} - 1)}{0{,}0175}$

$= R66875.39$

$\therefore$ Still owe R6875.39 (10)

Question 10. [32]

10.1 $50\,000(1{,}07)^5 = R70127{,}5.87$ (4)

10.2 $701000 = \frac{x(1{,}01125^{60} - 1)}{0{,}01125}$

$\therefore x = R8243.65$ (8)

10.3 $(1 + i) = 1{,}01125^{12}$

$i = 0{,}14367\ldots$

$r = 14{,}367\ldots\%$ (6)

10.4. correct i (10.3)

$\frac{10000((1+i)^4 - 1)}{i}(1 + i)$

$= R56584{,}23$ (8)

10.5

$50\,000(0{,}85)^5 = R58014.53$

Will be enough (6)

Section C

Question 11 [26]

11.1. $(x+2)^2 + (y-1)^2 = 13$

Centre $(-2;1)$ $r = \sqrt{13}$

$x^2 + (y+2)^2 = 52$

Centre $(0;-2)$ $r = 2\sqrt{13}$

Dist. $= \sqrt{(-2-0)^2+(1+2)^2} = \sqrt{13}$

$\therefore$ $QR = 2\sqrt{13}$ $PR = \sqrt{13}$

$\therefore$ Touch intern.

16

1.2. $\frac{y-1}{x+2} = \frac{-3}{2}$ $\therefore 2y = -3x-4$

Subst. $x^2 + \left(\frac{-3x-4}{2}+2\right)^2 = 52$

$x = \pm 4$ $y = 4$

tangent: $3y = 2x+20$

10

Question 12 [36]

2.1. The locus of all points whose distance from focus and directrix is in a constant ratio, e, is a conic section.

$e=1$: parabola

$e=0$: circle

$0<e<1$: ellipse

$e<0$: hyperbola

12

12.2. $\frac{x^2}{100} + \frac{y^2}{25} = 1$

$\therefore 25 = 100(1-e^2)$

$e^2 = \frac{75}{100} = \frac{3}{4}$ $\therefore e = \frac{\sqrt{3}}{2}$

$\therefore (-5\sqrt{3}; 0)$ and $(5\sqrt{3}; 0)$

Directrix: $x = \frac{-20}{\sqrt{3}}$ $x = \frac{20}{\sqrt{3}}$

14

12.3.1 $a^2 = 36$ $b^2 = 64$

eq.: $\frac{x^2}{36} - \frac{y^2}{64} = 1$

12.3.2. Loci: $(-10;0)$ and $(10;0)$

10

Question 13 [38]

13.1. $\frac{dx}{dt} = 6t$ $\frac{dy}{dt} = 4$ $\therefore \frac{dy}{dx} = \frac{2}{3t}$

tangent: $\frac{y-4t}{x-3t^2} = \frac{2}{3t}$

$\therefore 3ty = 2x + 6t^2$

14

13.2.

At t: $3y = \frac{2x+4p^2}{p} = \frac{2t+6q^2}{q}$

$\therefore x = \frac{6pq(q-p)}{2(q-p)}$ $y = 2(q+p)$

$\therefore T(3pq; 2(q+p))$

12

13.3. $m_{op} = \frac{4p}{3p^2}$ $m_{oq} = \frac{4q}{3q^2}$

$m_{op} \cdot m_{oq} = -1$ $\therefore pq = \frac{-16}{9}$

$T: x = 3pq = \frac{-16}{3}$

$\therefore x = \frac{-16}{3}$

12

Section D

Question 14. [22]

14.1. Let $n=1$: $5^1-2^1=3$

$\therefore$ True for $n=1$ ✓✓

Assume true for $n=k$;

5^k-2^k is div. by 3 ✓✓

Let $n=k+1$:

$5^{k+1}-2^{k+1} = 5.5^k-2.2^k$

$= 5(5^k-2^k)+5.2^k-2.2^k$

$= 5(5^k-2^k)+3.2^k$ ✓✓

which is div. by 3.

$\therefore$ If assumpt. is true for $n=k$, it is also true for $n=k+1$. ✓✓

$\therefore$ True $\forall n\in N$. (12)

14.2

$\alpha+\beta=-2$ and $\alpha\beta=5$ ✓✓

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$ ✓✓

$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{(-2)^2-2.5}{5}$ ✓✓

$=-6/5$ ✓✓ (10)

Question 15 [22]

15.1. Let $a+b\sqrt{c}$ be an irrational zero of $P(x)\in Q[x]$.

Let $a(x)=(x-a-b\sqrt{c})(x-a+b\sqrt{c})$

$=x^2-2xa+a^2-b^2c$. ✓✓

By the Euclidian Property

$P(x)\equiv a(x).q(x)+mx+k$ with ✓✓ m and k rational.

Let $x=a+b\sqrt{c}$: ✓✓

$0=0+m(a+b\sqrt{c})+k$

If $m\neq 0$: $k=-m(a+b\sqrt{c})$ ✓✓

This is impossible since k is rational

$\therefore m=0$

$\therefore k=0$ ✓✓

$\therefore a-b\sqrt{c}$ is ✓✓ a zero of $P(x)$ (12)

15.2 $-2-\sqrt{2}$ ✓✓ a zero ✓✓

$\therefore (x+2+\sqrt{2})(x+2-\sqrt{2})$ a factor

$=x^2+4x+2$ ✓✓

$$\begin{array}{r|rrrrr} & 1 & 0 & 1 & & \\ \hline 1\ 4\ 2 & 1 & 4 & 3 & 4 & 2 \\ & 1 & 4 & 2 & & \\ & & 0 & 1 & 4 & 2 \\ & & & 1 & 4 & 2 \end{array}$$

$\therefore (x^2+4x+2)(x^2+1)$ ✓✓ (10)

Question 16

$\frac{3x^2-x+1}{x^2(x^2+1)}\equiv\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+1}$ ✓✓

$3x^2-x+1\equiv Ax(x^2+1)+B(x^2+1)+(Cx+D)x^2$ ✓✓

$x=0$: $1=B$ ✓✓

x^3: $0=A+C$

x^2: $3=B+D \therefore D=2$ ✓✓

x: $-1=A$ ✓✓ $\therefore C=1$ ✓✓

$\therefore -\frac{1}{x}+\frac{1}{x^2}+\frac{x+2}{x^2+1}$ ✓✓ (14)

Question 17 [18]

17.1 [8]

$$\begin{array}{r|rrrr} & 1 & -2 & & \\ \hline 1\ 2\ 3 & 1 & 0 & 0 & -5 \\ & 1 & 2 & 3 & \\ & & -2 & -3 & -5 \\ & & -2 & -4 & -6 \\ & & & 1 & 1 \end{array}
\qquad
\begin{array}{r|rrr} & 1 & 1 & \\ \hline 1\ 1 & 1 & 2 & 3 \\ & 1 & 1 & \\ & & 1 & 3 \\ & & 1 & 1 \\ & & & 2 \end{array}$$

$\therefore 2$ ✓✓ is the gcd (8)

17.2. $f(x)=g(x).(x-2)+x+1$ ✓✓

$\therefore x+1=f-g.(x-2)$ ✓✓ ...① and

$g(x)=(x+1)(x+1)+2$ ✓✓

$\therefore 2=g-(x+1)(x+1)$ .. ②

① in ②:

$2\equiv g-[f-g.(x-2)](x+1)$ ✓✓

$2\equiv g-f.(x+1)+g.(x-2)(x+1)$

$2\equiv g(1-x^2-x-2)-f.(x-1)$ ✓✓

$\therefore s(x)=(x-1)$ and $t(x)=-x^2-x-1$ (10)

Question 18 [24]

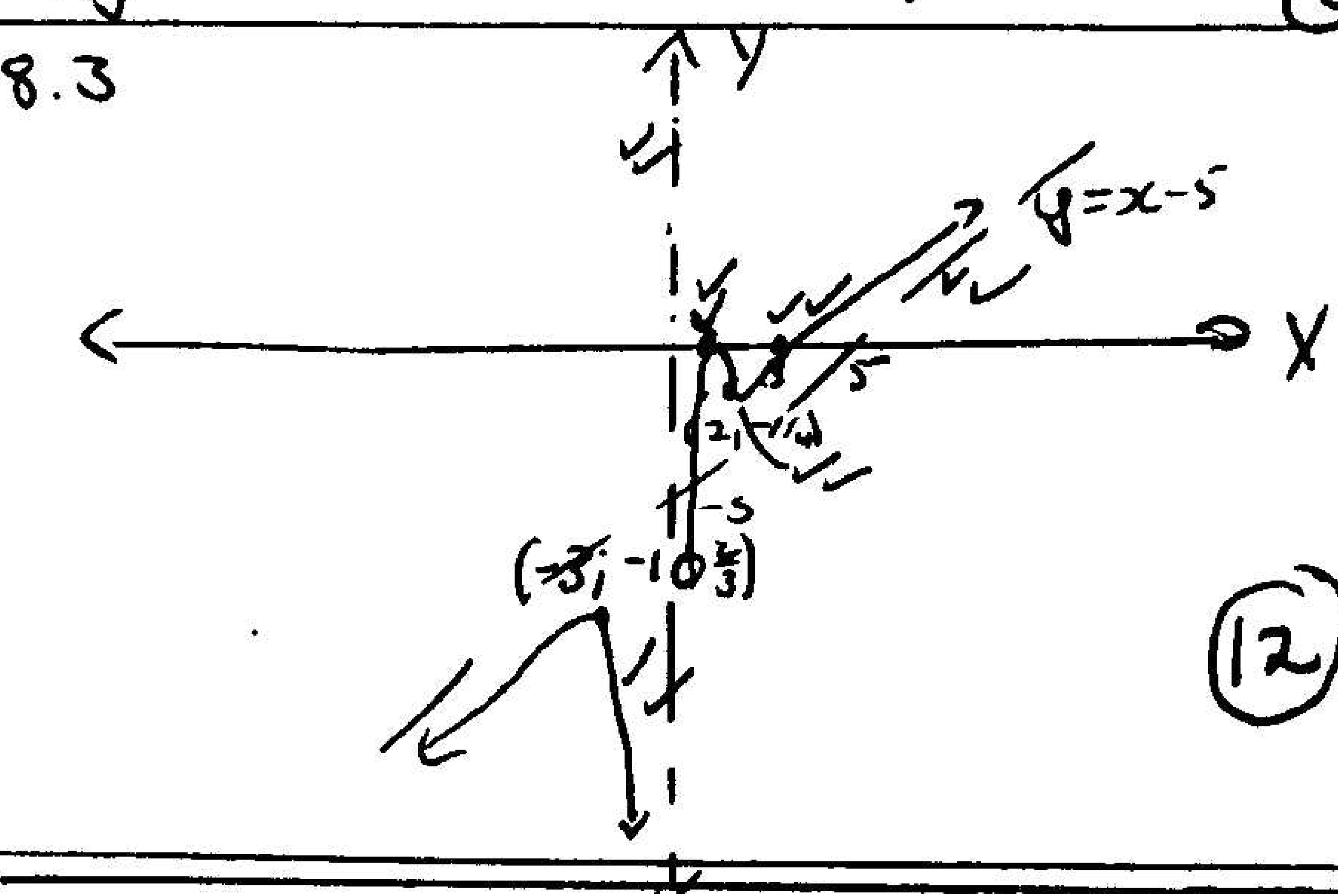
18.1. $x=1$ ✓✓ and $x=3$. No y-int. ✓✓ (4)

18.2. $x=0$ ✓✓ is a vertical

obliqeue: $\frac{(x^2-2x+1)(x-3)}{x^2}=\frac{x^3-5x^2+7x-3}{x^2}$ ✓✓

$\therefore y=x-5$ ✓✓ is an oblique as (8)

18.3



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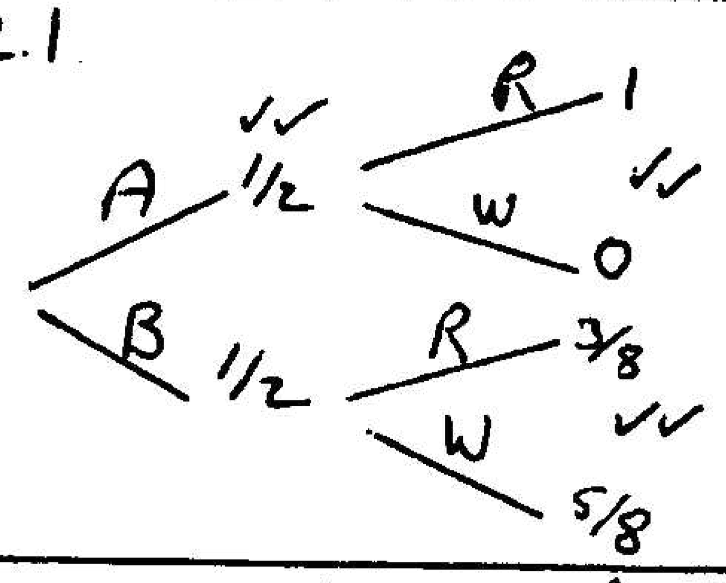
Section E

Question 19 [30]

19.1. $\dfrac{8!}{2!\,2!\,4!} = 420$ (6)

19.2.1

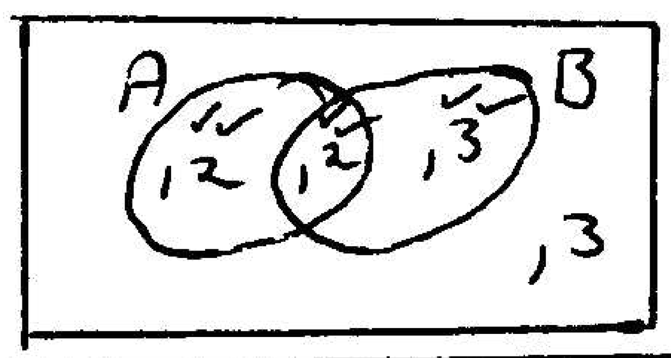


(6)

19.2.2. $\frac{1}{2}\cdot 1 + \frac{1}{2}\cdot\frac{3}{8} = \frac{11}{16}$ (0,6875) (6)

9.3.1. $P(A\cap B) = 0{,}4 \times 0{,}5 = 0{,}2$ (4)

19.3.2



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19.3.3 0,3 (2)

Question 20 [20]

20.1. $\dfrac{\binom{20}{2}\binom{15}{2}}{\binom{35}{4}} = 0{,}38$ (8)

20.2 $P(X\geq 1) = 1 - P(X=0)$

$= 1 - \binom{10}{0}\left(\frac{5}{100}\right)^0\left(\frac{95}{100}\right)^{10} = 0{,}40$ (12)

Question 21 [20]

21.1. $\displaystyle\int_{-\sqrt{3}}^{\sqrt{3}} \frac{c}{1+x^2}\,dx = 1$

$c \text{ bg} \tan x\Big|_{-\sqrt{3}}^{\sqrt{3}} = 1$ subst.

$c\left[\text{bg}\tan\sqrt{3} - \text{bg}\tan(-\sqrt{3})\right] = 1$

$c\left[\pi/3 + \pi/3\right] = 1 \quad \therefore c = \frac{3}{2\pi}$ (12)

21.2. $\displaystyle\int_{-1}^{1} \frac{3}{2\pi}\,\frac{dx}{1+x^2}$ (borders)

$= \arctan\Big|_{-1}^{1}\cdot\frac{3}{2\pi}$

$= \frac{3}{2\pi}\left[\arctan 1 - \arctan(-1)\right]$

$= \frac{3}{2\pi}\cdot\left[\pi/4 + \pi/4\right] = \frac{3}{4}$ (8)

Question 22 [12]

22.1. $p = \frac{267}{823}$ (0,3244...)

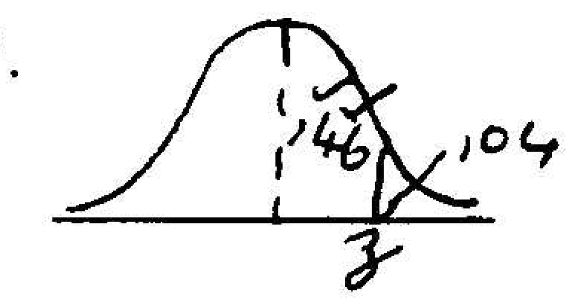
$p \pm 2{,}33\sqrt{\dfrac{p(1-p)}{823}}$

(28,6%; 36,2%) (10)

22.2. Any logic answer (2)

Question 23 [18]

23.1.



$z: 1{,}75$

$\therefore 1{,}75 = \dfrac{100{,}25-95}{\sigma}$

$\therefore \sigma = 3$ (8)

23.2 $P(X<90{,}5) = P\left(Z < \dfrac{90{,}5-95}{3}\right)$

$= P(Z < -1{,}5)$

$= 0{,}5 - 0{,}4332 = 0{,}0668$

$\therefore 668$ soap bars (10)