# Mathematical Science

## Paper III

**Time Allowed : 2½ Hours]** **[Maximum Marks : 150**

**Note :** This Paper contains **Seventy Five (75)** multiple choice questions, each question carrying **Two (2)** marks. Attempt *All* questions.

1. Which of the following sequences has a convergent subsequence ?

(A) $a_n = (-1)^n n$

(B) $a_n = \sqrt{n} + \cos\sqrt{n}$

(C) $a_n = \cos n^2 + i \sin n^2$

(D) $a_n = \dfrac{e^{\sqrt{n}}}{n^5}$

*Or*

Gauss-Seidel method converges as fast as the Jacobi method :

(A) Fourth

(B) Thrice

(C) Twice

(D) Fifth

2. Let A be a linear transformation from $\mathbf{R}^n$ to $\mathbf{R}^n$. If $\bar{x} \in \mathbf{R}^n$, then the derivative of A at $\bar{x}$ is given by :

(A) $A'(\bar{x}) = 0$, the zero transformation

(B) $A'(\bar{x}) = I$, the identity transformation

(C) $A'(\bar{x}) = A$

(D) $A'(\bar{x}) = A.A$

*Or*

The value of :

$$\int_0^1 \frac{1}{1+x}\,dx,$$

correct to three decimal places, by the Simpson's rule with $h = 0.5$ and

| | | | |
|---|---|---|---|
| $x$ : | 0.0 | 0.5 | 1.0 |
| $\frac{1}{1+x} = y$ : | 1.000 | 0.6667 | 0.5000 |

(A) 0.894

(B) 0.794

(C) 0.694

(D) 0.594

3. The Taylor series for sin $x$ about O is :

(A) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - .....$

(B) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - .....$

(C) $x - \frac{x^3}{3} + \frac{x^5}{5} - .....$

(D) $x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + .....$

*Or*

The Lagrange interpolating polynomial of degree two approximating the function $y = l_n x$, defined by the following table values :

| $x$ | $y = l_n x$ |
|---|---|
| 2 | 0.69315 |
| 2.5 | 0.91629 |
| 3.0 | 1.09861 |

(A) $l_2(x) = -0.81366x^2 + 0.8164x - 0.60761$

(B) $l_2(x) = -0.8164x^2 + 0.81366x - 0.60761$

(C) $l_2(x) = 0.8164x^2 + 0.81366x - 0.60761$

(D) $l_2(x) = 0.81366x^2 + 0.8164x - 0.60761$

4. A function $f : [a, b] \to \mathbf{R}$ is Riemann integrable if and only if it is bounded and :

(A) it is discontinuous only at a finite number of points

(B) it is either monotone or it has only a finite number of discontinuities

(C) the set of its discontinuities is of Lebesgue measure zero

(D) the set of its discontinuities is countable

*Or*

If $f(x)$ is continuous function in $x_0 \le x \le x_n$ then given any $\in > 0$, there exists a polynomial $p(x)$ such that :

(A) $|f(x) - p(x)| < \in$; for some $x$ in $(x_0, x_n)$

(B) $|f(x) - p(x)| \le \in$; for some $x$ in $(x_0, x_n)$

(C) $|f(x) - p(x)| < \in$; for all $x$ in $(x_0, x_n)$

(D) $|f(x) - p(x)| \le \in$; for all $x$ in $(x_0, x_n)$

5. The solution of the following linear programming problem is :

Max. $z = 3x_1 + 2x_2$

Subject to constraints

$$x_1 + x_2 \geq 3$$

$$x_1 - x_2 \leq 1$$

$$\text{and } x_1, x_2 \geq 0$$

(A) unbounded

(B) $x_1 = 2,\ x_2 = 1$

(C) $x_1 = 1,\ x_2 = 2$

(D) $x_1 = 0,\ x_2 = 3$

6. If the $p$th variable of the primal is unrestricted in sign, then the $p$th constraint of the dual is :

(A) Less than type

(B) More than type

(C) Equality type

(D) Not necessary equality

7. Inclusion of an additional constraint in the existing set of constraints will cause a(n) :

(A) change in objective function coefficients

(B) change in constraint coefficients

(C) increase in the optimal objective function value

(D) decrease in the optimal objective function value

8. The optimal solution of the quadratic programming problem :

Max. $z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2$

Subject to constraints

$$x_1 + 2x_2 + x_3 = 10$$

$$x_1 + x_2 + x_4 = 9$$

$$\text{and } x_1, x_2, x_3\ x_4 \geq 0$$

(A) $x_1 = 0,\ x_2 = 5,\ x_3 = 0,\ x_4 = 4$

(B) $x_1 = 4,\ x_2 = 0,\ x_3 = 6,\ x_4 = 5$

(C) $x_1 = 2,\ x_2 = 4,\ x_3 = 0,\ x_4 = 4$

(D) $x_1 = 9,\ x_2 = 0,\ x_3 = 1,\ x_4 = 0$

9. Let $f(z)$ be an analytic function within and on $|z - 2| = 3$. Suppose $|f(z)|$ has maximum value 2 on $|z - 2| = 3$. Then the value of $|f''(2)|$ is :

(A) $> \frac{5}{9}$

(B) $\leq \frac{4}{9}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

*Or*

In an experimental research study, the primary goal is to isolate and identify the effect produced by the ....................... .

(A) dependent variable

(B) independent variable

(C) interaction effect

(D) confounding variable

10. The value of the integral :

$$\int_c \left(x^2 + y^2 - 2ixy\right) dz$$

along $c$, where $c$ is a line from $z = 0$ to $z = 1 + i$ is :

(A) $\frac{4}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{(1 + i)(2 - i)}{3}$

(D) $\frac{(1 - i)(2 + i)}{3}$

*Or*

A researcher studies achievement by children in poorly funded elementary schools. She believes that parent involvement has an impact on children by increasing their motivation to do school work. Thus, in her model, greater parent involvement leads to higher student motivation, which in turn creates higher student achievement. Student motivation is what kind of variable in this study ?

(A) Mediating or intervening variable

(B) Confounding variable

(C) Control variable

(D) Independent variable

11. The degree of the extension field $Q\left(\sqrt{5}, \sqrt{3}\right)$ over Q is :

(A) 4

(B) 3

(C) 2

(D) 1

*Or*

Select the most suitable words to complete the statement : "The entire group of objects or people about which information is wanted is called the ...................... . Individual members are called..................... . The ..................... is the part that is actually examined in order to gather information."

(A) population, units, sample

(B) sample, units, target population

(C) whole, items of interest, response group

(D) population, explanatory variable, subgroup

12. Let $M_2(z)$ be the ring of $2 \times 2$ matrices over $z$.
Consider the following statements :
(1) $M_2(z)$ is a field
(2) $M_2(z)$ is an integral domain
(3) $M_2(z)$ is not an integral domain
(4) $M_2(z)$ is a division ring
Then :

(A) Only statement (3) is true

(B) Only statements (1) and (2) are true

(C) Only statement (4) is true

(D) All the statements are true

*Or*

If most of the measurements in a large data set are of approximately the same magnitude except for a few measurements that are quite a bit larger, how would the mean and median of the data set compare and what shape the histogram of the data set have ?

(A) The mean would be larger than the median and the histogram would be skewed with a long right tail

(B) The mean would be larger than the median and the histogram would be skewed with a long left tail

(C) The mean would be equal to the median and the histogram would be symmetrical

(D) The mean would be smaller than the median and the histogram would be skewed with a long right tail

13. Consider the ring $z_{36}$. This ring has :

(A) only one prime ideal

(B) exactly two prime ideals

(C) only one maximal ideal

(D) no proper ideals

*Or*

Let $X_n = X$ and $Y_n = -X$, where $X \sim N(0, 1)$. Which of the following statements are correct ?

(*i*) $X_n \xrightarrow{d} \tilde{X}$, where $\tilde{X}$ is an iid copy of X

(*ii*) $Y_n \xrightarrow{d} X$

(*iii*) $X_n + Y_n \xrightarrow{d} \tilde{X} + X$

(*iv*) $X_n + Y_n \xrightarrow{p} 0$

(A) (*i*), (*ii*) and (*iii*)

(B) (*i*), (*ii*) and (*iv*)

(C) (*i*) and (*ii*)

(D) (*ii*), (*iii*) and (*iv*)

14. Which of the following statements is true ?

(A) There exists a field of order 24

(B) There exists a field of order $pq$ with $pq$ distinct primes.

(C) There exists a field of order $2k$, $(k > 2)$

(D) There exists a field of order 25

*Or*

Let $\Omega = R$, $F = \mathbf{B}(R)$ = Borel σ-field :

$$A_n = \begin{cases} \left[0, \frac{1}{n}\right] & n \text{ odd} \\ \left[1 - \frac{1}{n}, 1\right] & n \text{ even} \end{cases}$$

Then which of the following is true ?

(A) $\underline{\lim}\ A_n = \overline{\lim}\ A_n = \phi$

(B) $\underline{\lim}\ A_n = \{1\}$

(C) $\underline{\lim}\ A_n = \overline{\lim}\ A_n = R$

(D) $\overline{\lim}\ A_n = \{0, 1\}$

15. The matrix of the linear mapping $f : \mathrm{R}^2 \rightarrow \mathrm{R}^3$, given by :

$$f(x, y) = (2x, y, x - y)$$

with respect to the standard bases is :

(A) $\begin{bmatrix} 2 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & 1 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$

*Or*

Let $X_1, X_2, \ldots\ldots\ldots\ldots X_n$ be iid realization from F. Consider the following sequence :

$$T_n(x) = \frac{1}{n} \sum_{i=1}^{n} I_{[X_i \le x]},$$

where $I_A$ denote the indicator function of the set A and $x \in \mathbf{R}$ fixed. Which of the following statements are correct ?

(*i*) Using WLLN, $T_n(x) \xrightarrow{p} F(x)$

(*ii*) Using WLLN, $T_n(x) \xrightarrow{d} F(x)$

(*iii*) Using SLLN, $T_n(x) \xrightarrow{a.s.} F(x)$

(*iv*) $E(T_n(x)) = F(x)$, $\mathrm{Var}(T_n(x)) = \dfrac{F(x)(1 - F(x))}{n}$

(A) (*i*), (*iii*) and (*iv*)

(B) (*i*), (*ii*) and (*iii*)

(C) (*ii*), (*iii*) and (*iv*)

(D) (*i*), (*ii*) and (*iv*)

16. Which of the following subspaces of $\mathbf{R}$ are not homeomorphic ?

(A) $[0, 1)$ and $[0, \infty)$

(B) $(0, 1)$ and $[0, 1]$

(C) The set of all integers and the set of all even integers

(D) $[1, 2]$ and $[2, 4]$

*Or*

Let X be a random variable with probability distribution :

$P[X = k] = p_k$, $k = 1, 2, ..........$ .

Then :

(A) $E(X) \leq \sum_{k=1}^{\infty} P[X \geq k]$

(B) $E(X) = \sum_{k=1}^{\infty} P[X \geq k]$

(C) $E(X) \geq \sum_{k=1}^{\infty} P[X \geq k]$

(D) E(X) does not exist

17. Which of the following is false ?

(A) If $f : \mathbf{R} \to \mathbf{R}$ is a Lipschitz map, then $f$ is uniformly continuous

(B) Any map $f : \mathbf{N} \to \mathbf{R}$ is uniformly continuous

(C) If $f : \mathbf{R} \to \mathbf{R}$ is continuous and $f(x) \to 0$ as $x \to \pm \infty$, then $f$ is uniformly continuous.

(D) If $f : (0, 1) \to \mathbf{R}$ is continuous, then $f$ is uniformly continuous

*Or*

A card is drawn randomly from a deck of ordinary playing cards. You win Rs. 10 if the card is a spade or an ace. What is the probability that you will win the game ?

(A) $\frac{1}{13}$

(B) $\frac{13}{52}$

(C) $\frac{17}{52}$

(D) $\frac{4}{13}$

18. Let $\{E_n\}$ be a sequence of measurable sets. Which of the following is false ?

(A) If $E_n \supset E_{n+1}$ for each $n$, then $m\left(\bigcap_{n=1}^{\infty} E_n\right) = \lim_{n\to\infty} m(E_n)$

(B) If $E_n \subset E_{n+1}$ for each $n$, then $m\left(\bigcup_{n=1}^{\infty} E_n\right) = \lim_{n\to\infty} m(E_n)$

(C) $m(\lim\inf_{n \to \infty} E_n) \leq \lim\inf_{n \to \infty} m(E_n)$

(D) $m(\lim\sup_{n \to \infty} E_n) \geq \lim\sup_{n \to \infty} m(E_n)$, provided that $m\left(\bigcup_{n=k}^{\infty} E_n\right) < \infty$ for some $k \geq 1$.

*Or*

Suppose a die is tossed 5 times. What is the probability of getting exactly 2 fours ?

(A) 0.161

(B) 0.171

(C) 0.250

(D) 0.333

19. Which of the following is false ?

(A) If $f$ is measurable then so is $|f|$

(B) If $|f|$ is measurable so is $f$

(C) If $f$ is integrable so is $|f|$

(D) If $f$ is measurable and $|f|$ is integrable, so is $f$

*Or*

A restaurant manager is considering a new location for his restaurant. The projected annual cash flow for the new location is (in Rs.)

| **Cash flow** | **Probability** |
|---|---|
| 10,000 | 0.10 |
| 30,000 | 0.15 |
| 70,000 | 0.50 |
| 90,000 | 0.15 |
| 1,00,000 | ? |

The expected cash flow for the new location is :

(A) Rs. 12,800

(B) Rs. 64,000

(C) Rs. 70,000

(D) Rs. 60,000

20. A group of order $p^e$, where $p$ is prime and $e > 1$, is never :

(A) Simple

(B) Abelian

(C) Cyclic

(D) Solvable

*Or*

The chances that you will ticketed for illegal parking on campus are about $\frac{1}{3}$. During the last nine days, you have illegally parked every day and have *not* been ticketed. Today, on the 10th day, you again decide to park illegally. The chances that you will be caught are :

(A) still equal to $\frac{1}{3}$

(B) greater than $\frac{1}{3}$

(C) less than $\frac{1}{3}$

(D) equal to $\frac{1}{10}$

21. The field with 16 elements contains a subfield with $k$ elements if :

(A) $k = 8$

(B) $k = 4$

(C) $k = 9$

(D) $k = 11$

*Or*

Suppose a die is tossed 5 times. What is the probability of getting exactly 2 fours ?

(A) 0.028

(B) 0.161

(C) 0.177

(D) 0.333

22. The multiplicative group of non-zero elements of a field F is cyclic if :

(A) $F = \mathbf{R}$

(B) $F = \mathbf{C}$

(C) $F = \mathbf{F}_{625}$

(D) $F = \mathbf{Q}$

*Or*

A public opinion poll surveyed a simple random sample of voters. Respondents were classified by gender (male and female) and by voting preference Republican, Democrat or Independent. Results are shown below :

| | Voting preferences | | | |
|---|---|---|---|---|
| | Republican | Democrat | Independent | Total |
| Male | 200 | 150 | 50 | 400 |
| Female | 250 | 300 | 50 | 600 |
| Total | 450 | 450 | 100 | 1000 |

If you conduct a chi-square test of independence, what is the expected frequency count of male independents ?

(A) 10

(B) 25

(C) 40

(D) 60

23. The roots of which polynomial cannot be expressed by radicals over Q ?

(A) $x(x^4 + x^3 + 3) + 3$

(B) $x^3(x^3 + 2) + 1$

(C) $x(x^4 + x^2 + x) + 1$

(D) $x(x^2 - 4)(x^2 + 4) + 2$

*Or*

It has been estimated that about 30% of frozen chicken contain enough salmonella bacteria to cause illness if improperly cooked. A consumer purchases 5 frozen chickens. What is the probability that the consumer will have less than 2 contaminated chickens ?

(A) 0.4206

(B) 0.3233

(C) 0.1181

(D) 0.5282

24. Let F be a linear operator on a normed space X. Then which of the following statements is *not* true ?

(A) F is bounded whenever X is finite dimensions

(B) F is bounded whenever it is continuous at a point of X

(C) F is bounded whenever X is a Banach space

(D) F is bounded whenever it is uniformly continuous

*Or*

Which of the following statements is *not* true about Poisson probability distribution with parameter $\lambda$.

(A) The mean of the distribution is $\lambda$

(B) The variance of the distribution is $\lambda$

(C) The coefficient of variation is 1

(D) The parameter $\lambda$ must be greater than zero

25. Let X be a normed linear space and $x, y \in X$. Then there may not be a bounded linear functional $f$ on X such that :

(A) $f(x - y) = \|x - y\| + 1$

(B) $f(x) = f(y)$

(C) $f(x - y) = \|x - y\|$

(D) $f(x - y) = \dfrac{\|x - y\|}{2}$

*Or*

A study was conducted to estimate the effectiveness of assignments in an introductory statistics course. Students of teacher A received no assignments, while students of teacher B received assignments. The final grade of each student was recorded and it was found that the 95% confidence interval for the difference in the mean grades (Group A—Group B) was computed to be $-3.5 \pm 1.8$. This means :

(A) there is evidence that assignments caused better grades because the 95% confidence interval does not cover 0

(B) there is evidence that assignments caused better grades because the difference in the population means is less than zero

(C) there is evidence that assignments do not cause better grades because the 95% confidence interval does not cover zero

(D) there is little evidence that assignments caused better grades because the 95% confidence interval does not cover zero

26. Let A be a bounded linear transformation from a Banach space X to a Banach space Y. Suppose $\frac{X}{\ker A}$ is isomorphic to image of A. Then :

(A) ker A is closed and $I_m$ A is open

(B) ker A is closed and $I_m$ A is also closed

(C) ker A is open

(D) $I_m$ A is open

*Or*

With regard to the chi-squared test :

(A) it is used as an alternative to the $t$-test to determine the difference between two means

(B) the number of degrees of freedom is the number of independent comparisons

(C) the larger the value of the chi-squared test, the less likely it is to be significant

(D) the null hypothesis is not required

27. Let $<x_n>$ be a bounded sequence of elements in a separable Hilbert space. Then :

(A) there is a subsequence of $<x_n>$ which converges weakly

(B) every subsequence of $<x_n>$ converges weakly

(C) there is a subsequence of $<x_n>$ which converges strongly

(D) every subsequence of $<x_n>$ converges strongly

*Or*

The correlation coefficient :

(A) describes the association between two variables

(B) is measured on a scale of 0 to 1

(C) describes the degree of agreement between two variables

(D) is positive when a positive value of one variable implies that the other variable also takes a positive value

28. The space [0, 1] × [0, 1] in the dictionary order topology is not :

(A) connected

(B) locally connected

(C) locally path connected

(D) Hausdorff

*Or*

In a normal distribution :

(A) the coefficient of variation is the same as the standard deviation

(B) the mean is higher than the median

(C) 95% of observations lie within one standard deviation of the mean

(D) Mann-Whitney test is suitable for analysis

29. Suppose X is Lindelöf and Y is compact. Then X × Y is :

(A) metrizable

(B) second countable

(C) compact

(D) Lindelöf

*Or*

Let $x_1, x_2, \ldots\ldots x_n$ be a random sample from uniform distribution $v(0, \theta)$. Let $\delta(x) = 2\bar{x}$. Which of the following statements is *true* :

(A) $\delta(x)$ is biased and $v(\delta(x)) > \frac{\theta^2}{n}$

(B) $\delta(x)$ is biased and $v(\delta(x)) < \frac{\theta^2}{n}$

(C) $\delta(x)$ is unbiased and $v(\delta(x)) > \frac{\theta^2}{n}$

(D) $\delta(x)$ is unbiased and $v(\delta(x)) < \frac{\theta^2}{n}$

30. Which of the following spaces is normal ?

(A) Product of two normal spaces

(B) Closed subspace of a normal space

(C) The product space $\mathbf{R}^J$, where J is uncountable

(D) Subspace of a normal space

*Or*

Let $x_1$, $x_2$, ...... $x_n$ be a random sample from exponential distribution with mean $\lambda$. Let $\bar{x}$ be an estimator of $\lambda$. Then $\bar{x}$ is :

(A) biased and efficient

(B) unbiased, MLE and efficient

(C) biased, MLE but not efficient

(D) unbiased but not efficient

31. The fundamental group of a topological space X is trivial if X is :

(A) $\{(x_1, x_2) \in \mathbf{R}^2 / 0 < x_1^2 + x_2^2 \leq 1\}$

(B) convex subset of $\mathbf{R}^2$

(C) $\mathbf{R}^2 - \{0\}$

(D) Circle in $\mathbf{R}^2$

*Or*

Let $X \sim N(0, \sigma^2)$ then :

(A) X is complete but $X^2$ is not complete

(B) X is not complete but $X^2$ is complete

(C) X and $X^2$ are both complete

(D) X and $X^2$ are both not complete

32. Which of the following numbers can not be expressed as a sum of two squares ?

(A) 23

(B) 24

(C) 25

(D) 26

*Or*

Let $x_1, x_2, .... x_n$ be a random sample from $N(\theta, 1)$. Then the critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$ is of the form :

(A) $\bar{x} > k$

(B) $\bar{x} < k$

(C) $k_1 < \bar{x} < k_2$

(D) $\Sigma(x_i - \bar{x})^2 > k$

33. For which value of $n$, the statement "In a party of $n$ persons, there are three persons, who know each other or there are three persons who are unknown to each other" is always true ?

(A) 3

(B) 4

(C) 5

(D) 7

*Or*

The degrees of freedom of chi-square for testing goodness of fit with $n$ observations and $k$ classes are :

(A) $n - 1$

(B) $n - k$

(C) $k$

(D) $k - 1$

34. Which of the following statements is *not true* ?

(A) Every connected graph has a spanning tree

(B) There is no odd degree vertex in a Eulerian graph

(C) Complement of a disconnected graph is connected

(D) In a Hamiltonian graph, there is a cycle passing through every edge

*Or*

$X_1, X_2, \ldots X_n$ are i.i.d. r.v.s. with $E(X_i) = \theta$ and $V(X_i) = \theta$. Assymptotic distribution of $\sum_{i=1}^{n} X_i$ is :

(A) $N(n\theta, \theta)$

(B) $N(\theta, \frac{\theta}{n})$

(C) $N(\theta, n\theta)$

(D) $N(n\theta, n\theta)$

35. Let P be a poset and $f : P \to P$ be order preserving map. There exists an element $x \in P$ such that $f(x) = x$ if :

(A) P is a lattice

(B) P is a finite poset

(C) P is a chain

(D) P is a bounded poset

*Or*

$x_1, x_2, \ldots\ldots x_n$ is a random sample of size $n$ from exponential distribution with mean $\frac{1}{\theta}$. Let $x_{(1)}$ be the first ordered statistic. $E(x_{(1)})$ is :

(A) $n\theta$

(B) $\frac{1}{n\theta}$

(C) $\theta$

(D) $\frac{1}{\theta}$

36. The solution of the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ in a rectangle R : $|x - x_0| \le a$, $|y - y_0| \le b$ exists and unique if :

(A) $f(x, y)$ is continuous in R and bounded

(B) $f(x, y)$ is continuous, bounded and satisfies Lipschitz condition on R

(C) $f(x, y)$ is continuous and satisfies Lipschitz condition in R

(D) $f(x, y)$ is bounded and satisfies Lipschitz condition in R

*Or*

$X_1, X_2, \ldots\ldots X_n$ are i.i.d. r.v.s. from U(0, 1) approximate distribution of $\sum_{i=1}^{12} X_i - 6$ is :

(A) $N\left(0, \frac{n}{12}\right)$

(B) N(0, 1)

(C) N(6, 1)

(D) N(0, $n$)

37. The eigen values of the one-dimensional wave equation $y_{tt} = c^2 y_{xx}$, $0 < x < 1$ are :

(A) $\pi, -\pi$

(B) $\pi, \frac{\pi}{2}$

(C) $\pi, 2\pi, 3\pi, .....$

(D) $2\pi, 3\pi$

*Or*

If

$$\rho = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix},$$

then $\rho^2_{1.23} =$

(A) $\frac{1}{4}$

(B) $\frac{2}{3}$

(C) $\frac{1}{3}$

(D) $\frac{3}{4}$

38. The integral surface of the equation

$$yp - xq = 0$$

passing through the equations $x = 0$, $z = y^2$ is :

(A) $x^2 - y^2 = z^2$

(B) $x^2 + y^2 + z^2 = 5$

(C) $x^2 + y^2 = z$

(D) $x^2 + y^2 = 4z$

*Or*

To test the significance of correlation coefficient, the appropriate test statistic is :

(A) $\frac{r\sqrt{n-3}}{\sqrt{1-r^2}}$

(B) $\frac{r\sqrt{n-1}}{\sqrt{1-r^2}}$

(C) $\frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$

(D) $\frac{r\sqrt{n}}{\sqrt{1-r^2}}$

39. Let $J_n(x)$ be a Bessel function of order $n$. Consider :

(1) $J_0'(x) = -J_1(x)$,

(2) $J_n'(x) = \frac{n}{x} J_n(x) - J_{n+1}(x)$,

(3) $J_n'(x) = J_{n-1}(x) - \frac{n}{x} J_n(x)$.

Then :

(A) only (1) is true

(B) only (2) and (3) are true

(C) all (1), (2) and (3) are true

(D) none of (1), (2) and (3) is true

*Or*

If $(X_1, X_2, X_3)'$ follow $N(\mu, \Sigma)$ where $\mu' = [1, 2, 3]$,

$$\Sigma = \begin{bmatrix} 1 & +\frac{1}{2} & 0 \\ +\frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

then $X_1 - X_2 + X_3$ follows :

(A) $N\left(2, \frac{5}{2}\right)$

(B) $N(0, 2)$

(C) $N(2, 2)$

(D) $N(2, 3)$

40. The number of non-negative integer solutions to the equation $5x + 3y = 42$ is :

(A) 1

(B) 3

(C) 4

(D) 2

*Or*

Let $X = (X_1, X_2, X_3)'$ with $V(X) = \Sigma$. Eigen values of $\Sigma$ are .6, .4 and 2. Then the proportion of variance explained by the first principal component is :

(A) .2

(B) .6666

(C) .5

(D) .3333

41. The unit digit in the decimal expansion of $7^{100}$ is :

(A) 5

(B) 3

(C) 2

(D) 1

*Or*

The least square regression line is the line :

(A) which is determined by use of a function of the distance between the observed Y's and the predicted Y's

(B) for which the sum of residuals about the line is zero

(C) for which the sum of squares of the residuals about the line is zero

(D) which has the smallest sum of the squared residuals of any line through the data values

42. Consider the following congruences :

(1) $x^2 \equiv 31 (\text{mod } 7)$

(2) $x^2 \equiv 7 (\text{mod } 31)$

Then :

(A) (1) has a solution but not (2)

(B) Both the congruences have a solution

(C) None of (1) and (2) has a solution

(D) (2) has a solution but not (1)

*Or*

There is an approximate linear relationship between the height of females and their age (from 5 to 18 years) described by :

height = 50.3 + 6.01 (age)

where height is measured in cm and age is in years

Which of the following is *not* correct ?

(A) The estimated slope is 6.01 which implies that children increase by about 6 cm for each year they grow older

(B) The estimated height of a child who is 10 years old is about 110 cm

(C) The average height of children when they are 5 years old is about 50% of the average height when they are 18 years old

(D) The estimated intercept is 50.3 cm which implies that children reach this height when they are $\frac{50.3}{6.01} = 8.4$ years old

43. The number of solutions to the congruence $x^3 \equiv 3 \pmod 7$ is :

(A) 3

(B) 2

(C) 1

(D) No solution

*Or*

Which of the following statements is *not* true ?

(A) The ANOVA problem is referred to non-parametric hypothesis testing

(B) The ANOVA refers to a collection of experimental situations and statistical procedures for the analysis of quantitative responses from experimental units

(C) The simplest ANOVA problem is referred to as one-way ANOVA

(D) Single-factor ANOVA focusses on a comparison of more than two populations or treatment means

44. Let a one-dimensional harmonic oscillator has Lagrangian

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2,$$

then the path of the oscillator in the phase space is :

(A) a circle

(B) a hyperbola

(C) an ellipse

(D) a straight line

*Or*

In a single factor ANOVA problem involving five populations, with a random sample of four observations from each one, it is found that

SST$r$ = 16.1408 and SSE = 37.3801.

Then the value of the test statistic is :

(A) 0.432

(B) 0.812

(C) 1.619

(D) 2.316

45. A bead sliding on a uniformly rotating wire in a force-free space. Then the Hamiltonian of the bead :

(A) represents total energy and constant of motion

(B) represents total energy but not a constant of motion

(C) does not represent total energy but represents constant of motion

(D) neither represents total energy nor a constant of motion

*Or*

Stratified sampling will achieve the maximum gain in precision if :

(A) Sample is allocated to strata according to proportional allocation

(B) Sample is allocated to strata according to Neyman allocation

(C) Stratum mean squares are minimized

(D) Stratum sizes are proportional to stratum means

46. A Lagrangian of a particle of mass $m$ constrained to move on the plane curve $xy = c$, where $c$ is constant, is :

(A) $L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) - mgy$

(B) $L = \frac{1}{2}m\left(1 - \frac{c^2}{x^4}\right)\dot{x}^2 + mg\frac{c}{x}$

(C) $L = \frac{1}{2}m\left(1 + \frac{c^2}{x^4}\right)\dot{x}^2 - mg\frac{c}{x}$

(D) $L = \frac{1}{2}m\dot{x}^2 - \frac{mgc}{x}$

*Or*

Cluster sampling provides an unbiased estimator of the population mean only if :

(A) all clusters are of equal size

(B) all clusters are homogeneous

(C) cluster sizes are known

(D) all clusters are included in the sample

47. Which one of the following Lagrangians does not produce the equation of motion $\ddot{q} - q = 0$.

(A) $L = (q + \dot{q})^2$

(B) $L = q^2 + \dot{q}^2$

(C) $L = q^2 - \dot{q}^2$

(D) $L = (q - \dot{q})^2$

*Or*

The ratio estimator can be used in stratified sampling only if :

(A) Sample allocation is proportional allocation

(B) The concomitant variable is observed on all selected sampling units

(C) Stratum means for the concomitant variable are known

(D) Strata are homogeneous with respect to the concomitant variable

48. The tangent vector field(s) to the sphere $x^2 + y^2 + z^2 = r^2$ is (are) :

(1) $(x_2, x_1, 0)$

(2) $(x_2, -x_1, 0)$

(3) $(x_3, 0, -x_1)$

(4) $(x_1, x_2, x_3)$

Then :

(A) only (1) is true

(B) both (1) and (2) are true

(C) only (2) and (3) are true

(D) only (4) is true

*Or*

Horvitz-Thompson estimator is unbiased for population mean because :

(A) it assigns unequal weights to different sampling units

(B) it assumes sample selection by simple random sampling

(C) it compensates for the unequal probabilities of selection in construction of the estimator

(D) it takes the sum over all population units

49. Consider a curve $\alpha : \mathbf{R} \to E^2$, defined by

$$\alpha(t) = (x(t),\ y(t))$$

Then the curvature of the curve is given by :

(A) $\dfrac{x\dot{y} - y\dot{x}}{\left(\dot{x}^2 + \dot{y}^2\right)^{\frac{3}{2}}}$

(B) $\dfrac{x\dot{y}^2 - y\dot{x}^2}{\left(\dot{x}^2 + \dot{y}^2\right)^{\frac{3}{2}}}$

(C) $\dfrac{x\ddot{y} - y\ddot{x}}{\left(\dot{x}^2 + \dot{y}^2\right)^{\frac{3}{2}}}$

(D) $\dfrac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{\left(\dot{x}^2 + \dot{y}^2\right)^{\frac{3}{2}}}$

*Or*

In a connected design, the diagonal elements of C-matrix are all may be :

(A) Negative

(B) Non-negative

(C) Zero

(D) Positive

50. The values of $c$ for which

$$M : z(z - 2) + xy = c$$

represents a surface are :

(A) different from zero

(B) different from 1

(C) different from 2

(D) different from –1

*Or*

If a $3^4$-factorial experiment is conducted in a block of size 9, then the number of generalized interactions confounded will be :

(A) 3

(B) 4

(C) 1

(D) 2

51. The Gaussian curvature at any point of a surface $\bar{r} = a(\sin u \cos v, \sin u \sin v, \cos u)$ is :

(A) $a$

(B) $\frac{1}{a}$

(C) $\frac{1}{a^2}$

(D) $\frac{a^2}{2}$

*Or*

Consider the following statements :

(1) If at least one element of incidence matrix of a block design is zero, then the design is incomplete block design.

(2) If at least one element of incidence matrix of a block design is zero, then the design is non-orthogonal design.

Of these statements :

(A) Only (1) is true

(B) Only (2) is true

(C) Both (1) and (2) are true

(D) Neither (1) nor (2) is true

52. The extremal of the functional

$$I(y(x)) = \int_1^2 \left(\frac{x^3}{y'^2}\right) dx$$

satisfying $y(1) = 0$, $y(2) = 3$ is :

(A) $y = x - 1$

(B) $y = \frac{x^2}{2} - 1$

(C) $y = x^2 - 1$

(D) $y = \frac{x^2}{2} + x - 1$

*Or*

With $n$-variables the number of linearly independent contrast are :

(A) $\begin{pmatrix} n \\ 2 \end{pmatrix}$

(B) $n + 1$

(C) $n - 1$

(D) $n$

53. The functional

$$I(y(x)) = \int_0^{\frac{\pi}{4}} \left(y^2 - y'^2\right) dx$$

satisfying $y(0) = 0$ attained its extremal on :

(A) $y = x$

(B) $y = 0$

(C) $y = \sin x$

(D) $y = \cos x$

*Or*

Consider a stationary time series $X_t = \mu + \phi X_{t-1} + Z_t$, $t = 0, \pm 1, \pm 2, \ldots\ldots\ldots$, $Z_t \sim$ iid Normal $(0, \sigma^2)$. Which of the following statements are *correct* ?

(*i*) Best linear forecast $P_t X_{t+k}$ is a weighted average of $E(X_t)$ and latest observation $X_t$

(*ii*) $P_t X_{t+k} \to E(X_t)$ as $k \to \infty$

(*iii*) Forecast mean square error of $P_t X_{t+k}$ approaches to $\sigma^2/(1 - \phi^2)$

(*iv*) Best linear forecast $P_t X_{t+k}$ will be $X_t$ itself for all $k$

(A) (*i*) and (*ii*)

(B) (*i*), (*ii*) and (*iii*)

(C) (*ii*), (*iii*) and (*iv*)

(D) (*i*) and (*iv*)

54. A curve of fixed perimeter $l$ that encloses maximum area is :

(A) a circle centred at (0, 0) and of radius $l$

(B) a square with diagonal $\frac{l}{2\pi}$

(C) a circle centred at arbitrary point $(a, b)$ and of radius $\frac{l}{2\pi}$

(D) a rectangle with diagonal $2\pi l$

*Or*

Let $\{X_t\}$ and $\{Y_t\}$ be a stationary AR(1) and AR(2) process respectively. Suppose $X_t$ and $Y_s$ are uncorrelated at all leads and lags. Then, which of the following statements are *correct* ?

(*i*) $X_t + Y_t$ is a stationary process

(*ii*) $X_t + Y_t$ is AR(3)

(*iii*) $X_t - Y_t$ is MA(1)

(*iv*) $X_t + Y_t$ is ARMA(3, 2)

(A) (*i*) and (*ii*)

(B) (*i*) and (*iii*)

(C) (*i*) and (*iv*)

(D) (*iii*) and (*iv*)

55. The extremal of the functional

$$I\big(y(x)\big) = \int_{x_1}^{x_2} \frac{\sqrt{1+y'^2}}{\sqrt{x}}\, dx$$

is a :

(A) cycloid

(B) circle

(C) catenary

(D) parabola

*Or*

Given the following functions, which of them can be auto-correlation functions of a weak stationary process ?

(*i*) $\rho(h) = 1 + |h|$, $h = 0, \pm 1, \pm 2, \ldots$

(*ii*) $\rho(h) = \phi^{|h|}$, $|\phi| < 1$, $h = 0, \pm 1, \pm 2, \ldots$

(*iii*) $\rho(h) = \begin{cases} 1 & \text{if } \quad h = 0 \\ 0 & \text{otherwise} \end{cases}$

(*iv*) $\rho(h) = \begin{cases} \dfrac{\theta}{1+\theta^2} & \text{if } \quad |h| = 1 \\ 0 & \quad |h| > 1 \\ 1 & \quad h = 0 \end{cases}$

(A) (*ii*), (*iii*) and (*iv*)

(B) (*i*), (*ii*) and (*iii*)

(C) (*i*), (*iii*) and (*iv*)

(D) (*i*), (*ii*) and (*iv*)

56. Consider the following statements :

(*i*) If a kernel is symmetric then all its iterated kernels are symmetric

(*ii*) The eigen functions of a symmetric kernel corresponding to different eigen values need not be orthogonal.

Then :

(A) both (*i*) and (*ii*) are true

(B) both (*i*) and (*ii*) are false

(C) only (*i*) is true

(D) only (*ii*) is true

*Or*

Consider two MA(1) processes $X_t = Z_t - \frac{1}{\theta} Z_{t-1}$ and $Y_t = Z_t - \theta Z_{t-1}$, where $Z_t \sim$ white noise $(0, \sigma^2)$. Which of the following statements are *true* ?

(*i*) Both $\{X_t\}$ and $\{Y_t\}$ processes have the same covariance function

(*ii*) When $|\theta| < 1$, $\{Y_t\}$ process is preferred

(*iii*) Both $\{X_t\}$ and $\{Y_t\}$ processes are causal

(*iv*) Both $\{X_t\}$ and $\{Y_t\}$ processes have exponentially decaying ACF function

(A) (*i*), (*iii*) and (*iv*)

(B) (*ii*), (*iii*) and (*iv*)

(C) (*i*), (*ii*) and (*iii*)

(D) (*i*), (*ii*) and (*iv*)

57. The initial value problem corresponding to the integral equation

$$x(t) = t^3 + \int_0^t (t-s)^2 x(s)\, ds$$

is :

(A) $x'''(t) + 2x(t) = 0,\ x(0) = x'(0) = x''(0) = 0$

(B) $x'''(t) - 2x(t) = 6,\ x(0) = x'(0) = x''(0) = 0$

(C) $x'''(t) - 2x(t) = 0,\ x(0) = x'(0) = x''(0) = 0$

(D) $x'''(t) + 2x(t) = 6,\ x(0) = x'(0) = x''(0) = 0$

*Or*

Patients arrive at doctor's clinic according to Poisson process with rate $\lambda = \frac{1}{10}$ minute. The doctor will not examine the patient until at least 3 patients are in the waiting room. Then the expected waiting time until the first patient is admitted to see the doctor is :

(A) 10 minutes

(B) 30 minutes

(C) 20 minutes

(D) $\frac{10}{3}$ minutes

58. The resolvent kernel of the Volterra integral equation with kernel $k(t, s) = 1$ is :

(A) $\lambda(t - s)^2$

(B) $e^{\lambda(t - s)^2}$

(C) $e^{\lambda(t - s)}$

(D) $\lambda(t - s)$

*Or*

Let $\{X_n, n \geq 0\}$ be a branching process with $E(X_1) = m$. If $W_n = \frac{X_n}{m^n}$, then $E(W_n)$ is :

(A) $m$

(B) $m^n$

(C) 1

(D) $m^{n-1}$

59. Consider the integral equation

$$x(t) = 1 + \lambda \int_0^1 (1 - 3ts)x(s)\, ds,$$

where $\lambda$ is a real parameter. Then the Neumann series for the integral equation converges for :

(A) $-2 < \lambda < 2$

(B) $0 < \lambda < 3$

(C) $-3 < \lambda < 0$

(D) $-3 < \lambda < 3$

*Or*

Let a random process be defined as $x(t) = y \cos \omega t,\ t \geq 0$

where $\omega$ is a constant and $y$ is a uniform random variable over (0, 1), then the covariance function $k_x(t, s)$ of $x(t)$ is :

(A) $\frac{1}{12} \cos \omega t . \cos \omega s$

(B) $\frac{1}{4} \cos \omega t . \cos \omega s$

(C) $\frac{1}{3} \cos \omega t . \cos \omega s$

(D) $\frac{1}{2} \cos \omega t$

60. The order of convergence in Newton-Raphson method is :

(A) 0

(B) 1

(C) 2

(D) 3

*Or*

Consider a state Markov chain with states 0 and 1, and transition probability matrix

$$P = \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Then the state is :

(A) Ergodic

(B) Absorbing

(C) Transient

(D) Recurrent

61. The $n$th order divided difference $[x_1, x_2, ......, x_{n+1}]$ of the function $\frac{1}{x}$ is ......................... .

(A) $\frac{-1}{x_1 x_2 \ldots\ldots x_{n+1}}$

(B) $\frac{(-1)^n}{x_1 x_2 \ldots\ldots x_{n+1}}$

(C) $\frac{-1}{x_1 x_2 \ldots\ldots x_n}$

(D) $\frac{(-1)^n}{x_1 x_2 \ldots\ldots x_n}$

*Or*

Pearle's vital Index in the context of vital statistics is :

(A) Ratio of the number of births in the given period and the number of deaths in the same given period, multiplied by 100

(B) Ratio of the number of deaths in the given period and the number of births in the same given period, multiplied by 100

(C) Ratio of the number of births in the given period and the number of deaths in the same given period, multiplied by 1000

(D) Ratio of the number of deaths in the given period and the number of births in the same given period, multiplied by 1000

62. The area of a circle of diameter $d$ is given for the following values :

| ***d*** | **area** |
|---|---|
| 80 | 5025 |
| 85 | 5674 |
| 90 | 6362 |
| 95 | 7088 |
| 100 | 7854 |

The area of a circle of diameter 105 by interpolation formula is :

(A) 8666

(B) 8546

(C) 8425

(D) 8353

*Or*

The crude and standardised death rate from the following data is :

| Age-group (years) | Population | Deaths |
|---|---|---|
| under 10 | 20,000 | 600 |
| 10—20 | 12,000 | 240 |
| 20—40 | 50,000 | 1250 |
| 40—60 | 30,000 | 1050 |
| Above 60 | 10,000 | 500 |

(A) 29.80 and 29.83

(B) 27.40 and 31.40

(C) 31.40 and 27.40

(D) 30.50 and 34.50

63. An approximate value of $y$ when $x = 0.2$ by Runge-Kutta fourth order method is .......................;

given that $y' = x + y$ and $y(0) = 1$.

(A) 1.5418

(B) 1.2428

(C) 1.2534

(D) 1.1327

*Or*

The number of persons dying at age 75 is 476 and the complete expectation of life at 75 and 76 years are 3.92 and 3.66 years. Then what are the numbers living at ages 75 and 76 ?

(A) 2660 and 2181

(B) 2675 and 2199

(C) 2575 and 2099

(D) 2565 and 2081

64. The Laplace transform of the periodic function F($t$) with period T > 0 is :

(A) $\dfrac{\int_0^T e^{-pt} F(t)\, dt}{T}$

(B) $\dfrac{\left(\int_0^T e^{-pt} F(t)\, dt\right)}{\left(1 - e^{-pT}\right)}$

(C) $\dfrac{\left(\int_0^T e^{pt} F(t)\, dt\right)}{T}$

(D) $\dfrac{\left(\int_0^T e^{pt} F(t)\, dt\right)}{\left(1 - e^{-pT}\right)}$

*Or*

Let $P_n$ is the probability of the mean of a sample of size $n$ falling outside the control limits. Then what is the probability that at most X-sample are to be taken for at least $s$-points to go out of control :

(A) $\sum_{s=0}^{r-1}\left[\binom{X}{s} P_n^s (1-P_n)^{X-s}\right]$

(B) $1-\sum_{s=0}^{r-1}\left[\binom{X}{s} P_n^s (1-P_n)^{X-s}\right]$

(C) $\dfrac{1}{\sum_{s=0}^{r-1}\left[\binom{X}{s} P_n^s (1-P_n)^{X-s}\right]}$

(D) $\dfrac{1}{\left[1-\sum_{s=0}^{r-1}\left\{\binom{X}{s} P_n^s (1-P_n)^{X-s}\right\}\right]}$

65. Which of the following is the inverse Laplace transform of $\dfrac{3p+6}{p^2+8p+25}$ ?

(A) $e^{-4t}(3\cos(3t) + 9\sin(3t))$

(B) $e^{-4t}(3\cos(3t) + 6\sin(3t))$

(C) $e^{-4t}(3\cos(3t) - 3\sin(3t))$

(D) $e^{-4t}(3\cos(3t) - 2\sin(3t))$

*Or*

If $R_s(t)$, $R_p(t)$ and $R_k^n(t)$ are the reliabilities of a series, parellel and $k$-out-of-$n$ systems respectively then for a system having identical components, which of the following is *true* ?

(A) $R_s(t)/R_p(t) \geq 1$, $R_p(t) - R_s(t) \geq 0$

(B) $R_k^n(t)/R_p(t) \geq 1$, $R_p(t) - R_k^n(t) \geq 0$

(C) $R_s(t)/R_k^n(t) \geq 1$, $R_s(t) - R_p(t) \geq 0$

(D) $R_p(t)/R_k^n(t) \geq 1$, $R_k^n(t) - R_s(t) \geq 0$

66. The Fourier cosine transform of $\dfrac{1}{1+t^2}$ is :

(A) $\sqrt{\dfrac{\pi}{2}}\, e^{-p^3}$

(B) $\sqrt{\dfrac{\pi}{2}}\, e^{-p^2}$

(C) $\sqrt{\dfrac{\pi}{2}}.\, e^{p^2}$

(D) $\sqrt{\dfrac{\pi}{2}}.\, e^{-p}$

*Or*

Three components with failure rates $5 \times 10^{-4}$, $3 \times 10^{-3}$ and $4 \times 10^{-3}$ respectively are arranged in series, the MTSF will be :

(A) $\frac{31}{12} \times 10^3$

(B) $\frac{31}{12} \times 10^4$

(C) $\frac{12}{31} \times 10^4$

(D) $\frac{2}{15} \times 10^3$

67. The inverse Fourier transform of $e^{-|p|}$ is :

(A) $\frac{\sqrt{2}}{\sqrt{\pi}\,(1-t)}$

(B) $\frac{\sqrt{2}}{\left(\sqrt{\pi}\,(1-t^2)\right)}$

(C) $\frac{\sqrt{2}}{\sqrt{\pi}\,(1+t^2)}$

(D) $\frac{\sqrt{2}}{\sqrt{\pi}\,(1+t)}$

*Or*

Three components with failure rates as $2 \times 10^{-3}$, $3 \times 10^{-3}$ and $5 \times 10^{-3}$ are arranged in series, then the system failure rate will be :

(A) $3 \times 10^{-8}$

(B) $5 \times 10^{-3}$

(C) $10^{-2}$

(D) $2 \times 10^{-2}$

68. Which one of the following is *not* an entire function ?

(A) $f(z) = e^z$

(B) $f(z) = e^{-z}$

(C) $f(z) = e^{iz}$

(D) $f(z) = e^{\bar{z}}$

*Or*

The ASN of a double sampling plan reduces to that of single sampling plan if the probability of making a decision on the basis of first sample is :

(A) $\frac{3}{4}$

(B) $\frac{1}{2}$

(C) 0

(D) 1

69. Consider the following two statements :

(*i*) Every function analytic in the extended plane is constant

(*ii*) If $f(z)$ is entire function such that $f(z) = u + iv$ and $u^2 \leq v^2 + 2012$, then $f(z)$ is constant

Then :

(A) both (*i*) and (*ii*) are false

(B) both (*i*) and (*ii*) are true

(C) only (*i*) is true

(D) only (*ii*) is true

*Or*

In the context of queueing theory which of the following statements are *correct* ?

(*i*) queueing theory deals with situations where customers arrive, wait for the service, get the service and leave the system

(*ii*) customers in queueing theory might include humans machines, ships, letters etc.

(*iii*) A queue refers to physical presence of the customers waiting to be served

(*iv*) A study of queueing theory helps the manager to establish an optimum level of service

(A) (*i*), (*ii*) and (*iii*)

(B) (*i*), (*ii*) and (*iv*)

(C) (*i*) and (*iv*)

(D) (*i*) and (*iii*)

70. Which one of the following is *true* ?

(A) $f(z)$ and $f(\bar{z})$ are simultaneously analytic

(B) $f(z)$ and $\overline{f(z)}$ are simultaneously analytic

(C) $f(z)$ and $\overline{f(\bar{z})}$ are simultaneously analytic

(D) $f(\bar{z})$ and $\overline{f(\bar{z})}$ are simultaneously analytic

*Or*

Average number of customers in the system for (M|G|1) queueing model, where $\rho = \frac{\lambda}{\mu}$ is :

(A) $\frac{\lambda^2\sigma^2 + \rho^2}{2\lambda(1-\rho)} + \frac{1}{\mu}$

(B) $\frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)}$

(C) $\frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)} + \rho$

(D) $\frac{\lambda^2\sigma^2 + \rho^2}{2\lambda(1-\rho)}$

71. Consider the following two statements :

(*i*) If a function $f : \mathbf{C} \to \mathbf{C}$ is entire and bounded then $f(z)$ is constant.

(*ii*) If a function $f : \mathbf{C} \to \mathbf{C}$ is entire and real part of $f(z)$ is bounded, then $f(z)$ is constant.

Then :

(A) both the statements (*i*) and (*ii*) are false

(B) only (*i*) is true

(C) only (*ii*) is true

(D) both the statements (*i*) and (*ii*) are true

*Or*

In the context of Inventory models, which of the following statements is *correct* ?

(A) Re-order quantity in a "fixed-order interval system" equals economic order quantity

(B) Review period of the items is always kept higher than its lead time

(C) In periodic review system of inventory control, the stock is usually replenished at unequal time interval

(D) The (S, s) system requires high safety stock volumes.

72. Let S be the set of real numbers which can be represented in the form

$$\frac{a_1}{5} + \frac{a_2}{5^2} + \ldots\ldots + \frac{a_n}{5^n} + \ldots\ldots,$$

where $a_n = 0$ or 2 for each $n$.

Then $m(\mathrm{S})$ is :

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) $\frac{2}{5}$

*Or*

The probability distribution of monthly sales of a certain item is as follows :

| **Monthly Sale** | **Probability** |
|---|---|
| 0 | 0.02 |
| 1 | 0.05 |
| 2 | 0.30 |
| 4 | 0.20 |
| 5 | 0.10 |
| 6 | 0.06 |

The cost of carrying inventory is Rs. 10 per unit per month. The current policy is to maintain a stock of 4 items at the beginning of each month. Assuming that the cost of shortage is proportional to both time and quantity short, then what is the imputed cost of shortage of one item for one unit of time.

(A) Rs. 115 < $C_2$ < Rs. 390

(B) Rs. 100 < $C_2$ < Rs. 290

(C) Rs. 120 < $C_2$ < Rs. 410

(D) Rs. 110 < $C_2$ < Rs. 290

73. For $f(x) = |x|$, the four Dini derivatives are given by :

(A) $D^+ = 1$, $D^- = 1$, $D_+ = -1$, $D_- = -1$

(B) $D^+ = D_+ = 1$, $D^- = D_- = -1$

(C) $D^+ = 1$, $D^- = 1$, $D_+ = 1$, $D_- = -1$

(D) $D^+ = 1$, $D^- = -1$, $D_+ = 1$, $D_- = 1$

*Or*

For any multi-stage problem, the solution by dynamic programming involves :

(*i*) The recurrence relation connecting optimum decision function for the $(n - 1)$ stage process

(*ii*) The relation giving the optimum decision function for a one-stage process

(*iii*) The optimum decision function for $n$-stage process

(*iv*) If the number of stage is large, take a limiting process and solve the resulting functional equation

(A) (*i*) and (*ii*)

(B) (*i*), (*ii*), (*iii*) and (*iv*)

(C) (*i*) and (*iv*)

(D) (*i*) and (*iii*)

74. Let $\{f_n\}$ be a sequence of non-negative measurable functions and $\lim f_n = f$ a.e., then :

(A) $\int f dx = \lim\inf \int f_n \, dx$

(B) $\int f dx \le \lim\inf \int f_n \, dx$

(C) $\int f dx \ge \lim\inf \int f_n \, dx$

(D) $\int f dx = \lim\sup \int f_n \, dx$

*Or*

Using dynamic programming, the maximum value of $x_1^2 + 2x_2^2 + 4x_3$ subject to $x_1 + 2x_2 + x_3 \le 8$ and $x_1, x_2, x_3 \ge 0$ is obtained as :

(A) 46

(B) 64

(C) 32

(D) 81

75. Let $\upsilon$ be a signed measure on [[X, S]]. Consider the following statements :

(*i*) Hahn decomposition of the space X is unique

(*ii*) Jordan decomposition of the signed measure $\upsilon$ is unique.

Then :

(A) Only (*i*) is true

(B) Only (*ii*) is true

(C) Both (*i*) and (*ii*) are true

(D) Both (*i*) and (*ii*) are false

*Or*

In a decision-making situation the :

(A) Total number of courses of action cannot be more than the number of events

(B) Laplace principle is based on the premise of equally-likely occurrence of possible events

(C) Minimax is an optimist's choice while minimum is a pessimist's criterion

(D) For any utility function, the units of measuring utility should always range between zero and 100

**ROUGH WORK**

Test Booklet No.
प्रश्नपत्रिका क्र.

**M**

# Paper-III
# MATHEMATICAL SCIENCE

**Signature and Name of Invigilator**

1. (Signature) ..........................................

(Name) ..................................................

2. (Signature) ..........................................

(Name) ..................................................

Seat No. | | | | | | |

(In figures as in Admit Card)

Seat No. ..............................................................

(In words)

OMR Sheet No. | | | | | | |

(To be filled by the Candidate)

## FEB - 30313

**Time Allowed : 2½ Hours]** **[Maximum Marks : 150**

Number of Pages in this Booklet : **40** Number of Questions in this Booklet : **75**

**Instructions for the Candidates**

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. This paper consists of **75** objective type questions. Each question will carry *two* marks. *All* questions of Paper-III will be compulsory, covering entire syllabus (including all electives, without options).
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (*i*) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   (*ii*) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.**
   (*iii*) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   **Example :** where (C) is the correct response.
   (A) (B) ● (D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table, etc., is prohibited.**
12. **There is no negative marking for incorrect answers.**

**विद्यार्थ्यांसाठी महत्त्वाच्या सूचना**

1. परिक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. सदर प्रश्नपत्रिकेत 75 बहुपर्यायी प्रश्न आहेत. प्रत्येक प्रश्नास **दोन** गुण आहेत. या प्रश्नपत्रिकेतील **सर्व** प्रश्न सोडविणे अनिवार्य आहे. सदरचे प्रश्न हे या विषयाच्या संपूर्ण अभ्यासक्रमावर आधारित आहेत.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनीटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी आवश्य तपासून पहाव्यात.
   (*i*) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्विकारू नये.
   (*ii*) **पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चूकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.**
   (*iii*) वरीलप्रमाणे सर्व पडताळून पहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   **उदा. :** जर (C) हे योग्य उत्तर असेल तर.
   (A) (B) ● (D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहिलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणा व्यतिरीक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापी, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. **फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.**
11. **कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.**
12. **चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.**