

## Mathematical Science

### Paper II

**Time Allowed : 75 Minutes]**

**[Maximum Marks : 100**

**Note :** This Paper contains **Fifty (50)** multiple choice questions. Each question carries **Two (2)** marks. Attempt *All* questions.

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| <p>1. Let <math>G_n = \left(0, 1 + \frac{1}{n}\right)</math> for <math>n \in \mathbb{N}</math>. Then <math>\bigcap G_n</math> is :</p> <p>(A) closed</p> <p>(B) open</p> <p>(C) both open and closed</p> <p>(D) neither open nor closed</p> <p>2. Let <math>A_n = \left[\frac{1}{n}, 1\right]</math>. Then <math>\bigcup_{n=1}^{\infty} A_n</math> is :</p> <p>(A) <math>(0, 1)</math></p> <p>(B) <math>(0, 1]</math></p> <p>(C) <math>[0, 1]</math></p> <p>(D) <math>[0, 1)</math></p> <p>3. The series <math>\sum a_n z^n</math> represents an entire function, if :</p> <p>(A) <math>\lim_{n \rightarrow \infty}  a_n ^{1/n} = 0</math></p> <p>(B) <math>\lim_{n \rightarrow \infty}  a_n ^{1/n}</math> is a positive real number</p> <p>(C) <math>\overline{\lim}_{n \rightarrow \infty}  a_n ^{1/n} = \infty</math></p> <p>(D) <math>\overline{\lim}_{n \rightarrow \infty}  a_n ^{1/n} = 0</math></p> | <p>4. Let <math>f</math> be analytic in a bounded domain <math>D</math> with <math>f(z) = f(2z)</math> for every <math>z \in D</math>. Then</p> <p>(A) <math>f(z)</math> is a non-zero constant</p> <p>(B) <math>f(z)</math> is the identity function of <math>D</math></p> <p>(C) <math>f(z) \not\equiv 0</math> in <math>D</math></p> <p>(D) <math>f(z) \equiv 0</math> in <math>D</math></p> <p>5. A monotone function :</p> <p>(A) has discontinuities everywhere</p> <p>(B) is continuous everywhere</p> <p>(C) has countably many discontinuities</p> <p>(D) has countably many points of continuity</p> |
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[P.T.O.]

6. Let  $S = \{u_1, u_2, \dots, u_p\}$  be a linearly independent subset of a vector space  $V = \langle v_1, \dots, v_q \rangle$ . Then :
- (A)  $p < q$   
 (B)  $p = q$   
 (C)  $p < q$   
 (D)  $p > q$
7. If  $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$  is the identity map, then nullity  $T = ?$
- (A) 0  
 (B) 1  
 (C) 2  
 (D) 3
8. In which of the following alternatives a subset  $T$  of the set :  
 $S = \{(2, 0, 0), (2, 2, 2), (2, 2, 0), (0, 2, 0)\}$   
 is not a basis of  $\mathbf{R}^3(\mathbf{R})$  ?
- (A)  $T = \{(2, 0, 0), (2, 2, 0), (2, 2, 2)\}$   
 (B)  $T = \{(2, 0, 0), (2, 2, 2), (0, 2, 0)\}$   
 (C)  $T = \{(2, 0, 0), (2, 2, 0), (0, 2, 0)\}$   
 (D)  $T = \{(2, 2, 0), (2, 2, 2), (0, 2, 0)\}$
9. The dimension of the space of diagonal  $n \times n$  matrices is :
- (A)  $n$   
 (B)  $n^2$   
 (C)  $n(n - 1)/2$   
 (D)  $n(n + 1)/2$
10. Let  $A_1, \dots, A_n$  be column vectors of size  $m$ . Assume that they have coefficients in  $\mathbf{R}$ , and they are linearly independent over  $\mathbf{R}$ . Then :
- (A) They are linearly independent over  $\mathbb{C}$   
 (B) They are linearly dependent over  
 (C) They form a basis for  $\mathbf{R}$   
 (D) They form a subspace for  $\mathbf{R}$

11. Let E, F and G be mutually exhaustive events such that E and F are mutually exclusive and F and G are independent events. Then a feasible assignment of probabilities is :

- (A)  $P(E) = 0.5, P(F \cup G) = 0.5, P(F) = 0.2$
- (B)  $P(E) = 0.2, P(F \cup G) = 0.65, P(F) = 0.5$
- (C)  $P(E) = 0.2, P(F \cup G) = 0.8, P(F) = 0.4$
- (D)  $P(E) = 0.5, P(F \cup G) = 0.36, P(F) = 0.2$

12. Let N be a random variable with

$$P(N = n) = \frac{1}{n(n+1)}, n = 1, 2, \dots$$

Then  $E[N]$  is :

- (A)  $\frac{1}{10}$
- (B)  $\frac{3}{10}$
- (C) 1
- (D) infinity

13. The probability that a certain machine will produce a defective item is  $\frac{1}{4}$ . If a random sample of 8 items is taken from the output of the machine, what is the probability that there will be 7 or more defectives in the sample ?

- (A)  $\frac{25}{(256)^2}$
- (B)  $\frac{4}{(256)^2}$
- (C)  $\frac{24}{(256)^2}$
- (D)  $\frac{5}{(256)^2}$

14. Let X follow a Poisson (2) distribution. Then :

- (A) The r.v.  $2X$  follows Poisson (4) and  $\frac{X}{2}$  follows Poisson (1)
- (B) Both  $2X$  and  $\frac{X}{2}$  are not Poisson r.v.s.
- (C) The r.v.  $2X$  follows Poisson (4) but  $\frac{X}{2}$  is not a Poisson r.v.
- (D) The r.v.  $2X$  is not a Poisson r.v. but  $\frac{X}{2}$  is Poisson (1)

15. Consider the Linear Programming Problem

$$\text{Maximize } Z = x_1 + x_2$$

$$\text{Subject to } x_1 + 3x_2 < 4$$

$$3x_1 + x_2 < 4$$

$$x_1, x_2 > 0$$

For this problem the value of the objective function at the optimal solution is :

- (A) 2
- (B) 4
- (C) 1
- (D) 3

16. Maximization assignment problem is transformed into a minimization problem by :

- (A) Subtracting all the elements of a column from the highest element of that column
- (B) Subtracting each element of the profit matrix from the highest element of the matrix
- (C) Subtracting all the elements in a row from the highest element of that row
- (D) Any of the above

17. Consider the function  $f(x) = \frac{1}{x}$  on  $[1, \infty]$  and  $g(x) = \frac{1}{x}$  on  $x > 0$ .

Then :

- (A) both  $f$  and  $g$  are uniformly continuous
- (B)  $g$  is uniformly continuous but  $f$  is not
- (C) neither  $f$  nor  $g$  is uniformly continuous
- (D)  $f$  is uniformly continuous but  $g$  is not

Or

Let X and Y be two random variables with  $E[Y | X] = X$  with probability 1.

Then :

- (A)  $\text{Cov}(X, Y) = 0$
- (B)  $\text{Cov}(X, Y) = E[Y | X]$
- (C)  $\text{Cov}(X, Y) = \text{Var}(Y)$
- (D)  $\text{Cov}(X, Y) = \text{Var}(X)$

18. If  $f(x)$  is monotonic increasing on  $(a, b)$  and  $a < c < b$ , then  $\lim_{x \rightarrow c^-} f(x) =$

- (A)  $\inf\{f(x) \mid x < c\}$
- (B)  $\sup\{f(x) \mid x > c\}$
- (C)  $\sup\{f(x) \mid x < c\}$
- (D)  $\inf\{f(x) \mid x > c\}$

Or

Let  $X$  and  $Y$  be two independent r.v.s such that  $X$  follows the exponential distribution with mean 2 and  $Y$  follows Binomial  $\left(8, \frac{1}{2}\right)$ . Then the variance of  $X + 2Y$  :

- (A) is 10.
- (B) can not be computed from the given information.
- (C) is 12.
- (D) is 8.

19. In  $\mathbf{R}$  let  $F_n = \left(-\frac{1}{n}, \frac{1}{n}\right), \forall n \in \mathbf{N}$ .

Then,  $\bigcap F_n$  is :

- (A)  $\{0\}$
- (B)  $\phi$
- (C) both open and closed
- (D) neither open nor closed

Or

Let  $X_1, X_2, \dots, X_n$  be a random sample from  $N(\mu, \mu)$ , where mean = variance =  $\mu$  is unknown. Then which of the following statements is *not* true ?

- (A)  $(\sum X_i^2)$  is sufficient for  $\mu$
- (B)  $(\sum X_i)$  is sufficient for  $\mu$
- (C)  $(\sum X_i, \sum X_i^2)$  is jointly sufficient for  $\mu$
- (D) Sufficient statistics do not exist

20. Consider the sequence :

$$u_n = \frac{(-1)^n 10^7}{n} \text{ and } v_n = \frac{5^n}{\lfloor n \rfloor}$$

Then,

- (A) only  $\{u_n\}$  is convergent
- (B) only  $\{v_n\}$  is convergent
- (C) both of these have the same limit
- (D)  $\{u_n\}$  is oscillating and  $\{v_n\}$  is convergent

Or

Let the random variables  $X_1, X_2$  be distributed as Poisson variates with mean  $\lambda$ . Then number of unbiased estimators of  $\lambda$  is :

- (A) 3
- (B) 2
- (C) infinity
- (D) 4

21.  $\int_0^1 \left( 1 - \frac{x}{1!} + \frac{x^2}{2!} - \dots \right) e^{2x} dx =$

- (A)  $e - 1$
- (B)  $e$
- (C)  $e^2$
- (D)  $e + 1$

Or

Let the random variable X follow  $U(\theta, \theta + 1)$ . Then which of the following statements is *not* correct ?

- (A)  $\text{Min}_i X_i = X_{(1)}$  is an mle which is sufficient for  $\theta$
- (B)  $\left( \bar{X} - \frac{1}{2} \right)$  is an unbiased estimate of  $\theta$
- (C) UMVUE will not exist for  $\theta$
- (D) Any value of  $\theta$  in the interval  $[X_{(n)} - 1, X_{(1)}]$  is an m/e

22. Let  $\mathbf{R}$  be the set of real numbers.

Let  $f: \mathbf{R} \rightarrow \mathbf{R}$  be such that

$$|f(x) - f(y)| < |x - y|^3, \text{ for all } x, y \in \mathbf{R}.$$

Then the value of the function  $f(x)$  is :

- (A)  $x$
- (B)  $x^2$
- (C) Zero
- (D) a constant

Or

Let  $z_1$  and  $z_2$  be independent standard normal variables and let  $y_1 = z_1 z_2^2$  then the correlation between  $z_1$  and  $y_1$  equals :

- (A)  $\frac{\sqrt{3}}{2}$
- (B) 0
- (C)  $5^{-1/2}$
- (D)  $\frac{1}{\sqrt{3}}$

$$f(x) = x^{-2} I_{(1, \infty)}(x)$$

23. Consider the following two statements :

- (a)  $e^z, z \in \mathbb{C}$  is a one-one function
  - (b)  $\sin z, z \in \mathbb{C}$  is a bounded entire function
- (A) both (a) and (b) are false
  - (B) both (a) and (b) are true
  - (C) only (b) is true
  - (D) only (a) is true

Or

Let  $X_1, X_2, \dots, X_n$  be iid r.v.s with pdf  $f(x)$ , where

Then :

- (A)  $EX_{(1)} = \frac{n}{n-1}$ , where  $X_{(1)} = \text{Min}_i X_i$
- (B)  $EX_1$  is finite
- (C)  $EX_{(n)} = \frac{n}{n+1}$ ,  $X_{(n)} = \text{Max}_i X_i$
- (D)  $EX_{(1)}$  and  $EX_{(n)}$  do not exist

24. The non-zero roots of the equation

$$(1+z)^5 = (1-z)^5 :$$

- (A) are real
- (B) some are real and some are purely imaginary
- (C) are purely imaginary
- (D) are complex numbers not lying on  $x$ -axis or  $y$ -axis

Or

To obtain a critical (region | value (or cut-off point) in testing a statistical hypothesis, we need the distribution of a test statistic :

- (A) without any assumption
- (B) under  $H_1$
- (C) under  $H_0$
- (D) all of the above

25. Let  $z \in \mathbb{C}$ . The inequality  $|z + 1| > |z - 1|$  is :

- (A) true iff  $\text{Re } z > 0$
- (B) Always true
- (C) Never true
- (D) True iff  $\text{Im } z > 0$

Or

In the usual two way classification model with one observation per cell

$$E(y_{ij}) = \alpha_i + \beta_j, \quad V(y_{ij}) = \sigma^2,$$

$$i = 1, \dots, a;$$

$$j = 1, \dots, b$$

Which parametric function is not estimable ?

- (A)  $\alpha_1 - \beta_1$
- (B)  $\alpha_1 + \beta_1$
- (C)  $\alpha_1 - \alpha_2$
- (D)  $\beta_1 - \beta_2$

26.  $f(z) = \text{cosec } z$  has :

- (A) infinitely many simple poles and  $z = \infty$  a double pole
- (B)  $z = 0$  an essential singularity,  $z = \infty$  a double pole
- (C) infinitely many simple poles and double pole at  $z = \infty$  and essential singularity at  $z = 0$
- (D) infinitely many simple poles and  $z = 0$  essential singularity



Or

Which of the following pairs represents linear regression line ?

- (A)  $\hat{y} = 3x + 5; \hat{x} = 2y - 3$
- (B)  $\hat{y} = 2x + 4; \hat{x} = 0.5y - 2$
- (C)  $\hat{y} = 1.5x + 1; \hat{x} = -0.5y + 0.8$
- (D)  $\hat{y} = 3x^2 + 4; \hat{x} = 0.2y^2 + 1$

27. If C is the circle  $|z - 2| = 2$ , then

$$\int_C \frac{dz}{z - 5} =$$

- (A) 0
- (B)  $2\pi i$
- (C)
- (D)

Or

In a linear model  $E(y_i) = \alpha + \beta x_i$ ;  $i = 1, \dots, n$  and  $V(y_i) = \sigma^2$ . and  $\hat{\alpha}$  and  $\hat{\beta}$  are least squares estimators of  $\alpha$  and  $\beta$  respectively.

If  $s^2 = \sum_{i=1}^n (y_i - \hat{\alpha} - \hat{\beta}x_i)^2$  then

unbiased estimator of  $\sigma^2$  is :

- (A)  $\frac{s^2}{(n - 2)}$
- (B)  $\frac{s^2}{(n - 1)}$
- (C)  $\frac{s^2}{n}$
- (D)  $\frac{ns^2}{(n - 1)}$

28. Let  $f(z)$  be analytic in  $D = \{z \mid |z| < 1\}$  and  $f\left(\frac{1}{n}\right) = 0$   $n = 2, 3, \dots$ .

Then :

- (A)  $f$  may not have any zero other than ones given
- (B)  $f \equiv 0$  in  $D$
- (C) in addition to  $\{\frac{1}{n} \mid n = 2, 3, \dots\}$ , the only zero of  $f$  in  $D$  is 0
- (D)  $f(z) = 0$  for  $z \in (-1, 1)$  but  $f$  may not be zero at any other point

Or

In a RBD with  $b$  blocks and  $v$  treatments a single yield is missing.

If  $B$  and  $T$  respectively denote the total of block and treatment which contain the missing value and  $G$  is the grand total, then estimate of the missing value is given by :

- (A)  $(vb + bT - G)/(b - 1)(v - 1)$
- (B)
- (C)
- (D)

29. The value of \_\_\_\_\_ is :

- (A)  $2^{99}$
- (B)  $3^{99}$
- (C)  $-2^{99}$
- (D)  $-3^{99}$

Or

Given below is a  $2^3$  design with 3 factors A,B,C in two blocks  $B_1$  and  $B_2$ . Identify  $x$  and  $y$  so that the block contrast  $B_1 - B_2$  represents main effect A :

$$B_1 : (1) \quad x \quad c \quad bc$$

$$B_2 : a \quad ab \quad y \quad abc$$

- (A)  $x = ac, y = b$
- (B)  $x = a, y = ac$
- (C)  $x = b, y = ac$
- (D)  $x = ac, y = a$

30. The value of the integral

$$\int_0^{1+i} (x - y + ix^2) dz$$

along the straight line from  $z = 0$  to  $z = 1 + i$  is :

- (A)  $\frac{i}{3}$
- (B)  $i(1 + i)$
- (C)  $\frac{1}{3}(i - 1)$
- (D)  $\frac{1}{6}(i - 3)$

Or

Or

The degrees of freedom for the error in a Latin Square design with 5 rows, 5 columns and 5 treatments with two missing observations is :

When data are collected in a statistical study for only a portion or subset of all elements. If interest we are using a :

- (A) 22
- (B) 10
- (C) 12
- (D) 14

- (A) Census
- (B) Sampling frame
- (C) Population
- (D) Sample

31. Let  $G = \{1, -1\}$ . Then the group  $(G, \bullet)$  is :

32. In  $S_n$ , the number of distinct cycles of length  $r < n$  is :

- (A) isomorphic to  $(\mathbf{Z}, +)$
- (B) homomorphic image of  $(\mathbf{Z}_5, +)$
- (C) isomorphic to  $(\mathbf{Z}_5, +)$
- (D) a homomorphic image of  $(\mathbf{Z}, +)$

- (A)  $r!$
- (B)  $(n - r)!$
- (C)  $n!/(n - r)!$
- (D)  $\frac{1}{r} \cdot \frac{n!}{(n - r)!}$

Or

Which of the following is *not* the goal of descriptive statistics ?

- (A) Summarizing data
- (B) Displaying aspects of the collected data
- (C) Reporting numerical findings
- (D) Estimating characteristics of the population

33. If  $p$  is a group of order  $p^n$ , ( $n > 1$ ),

- (A)  $O(Z(G))$  need not be a power of  $p$
- (B)  $Z(G)$  is not singleton
- (C)  $Z(G)$  is not commutative
- (D)  $Z(G)$  need not be a normal subgroup of  $G$

Or

Which of the following statements is *correct* ?

- (A) In a statistics problem, characteristics of a sample are assumed to be unknown
- (B) Probability reasons from the population to the sample (deductive reasoning) whereas inferential statistics reasons from the sample to the population (inductive reasoning)
- (C) Hypothesis testing and estimation by confidence intervals are the least important types of inferential statistical procedures
- (D) In a probability problem, characteristics of sample are assumed to be unknown

34. If  $G$  is a group of order 108 then :

- (A)  $G$  has a unique normal subgroup
- (B)  $G$  is cyclic
- (C)  $G$  is non-communicative
- (D)  $G$  is not simple

*Or*

Which of the following statements is *correct* ?

- (A) Color of ten automobiles recently purchased at a certain dealership is an example of a univariate data set
- (B) Height and weight for each basketball player on Pune University team is an example of bivariate data set
- (C) The systolic blood pressure, diastolic blood pressure, and serum cholesterol level for each patient participating in a research study is an example of multivariate data set
- (D) All of the above statements are correct

35.  $G$  is a group of order  $pq$  where  $p$  and  $q$  are primes.

Then :

- (A)  $G$  is solvable only if  $G$  is abelian
- (B)  $G$  is solvable only if  $p > q$  and  $q \nmid p - 1$
- (C)  $G$  is solvable only if  $p = q$
- (D)  $G$  is always solvable

*Or*

The expected number of heads in 300 tosses of a fair coin is :

- (A) 300
- (B) 250
- (C) 200
- (D) 150

36. Let  $R$  be a commutative ring.

Then :

- (A) Every ideal of  $R$  is maximal
- (B)  $R$  is a field if and only if  $R$  does not have a proper non-zero ideal
- (C) Every proper ideal of  $R$  is prime
- (D) A proper ideal of  $R$  is maximal if and only if it is prime

Or

Which of the following is *not* a measure of center ?

- (A) The mean
- (B) The variance
- (C) The median
- (D) The trimmed mean

37. In which of the alternatives a subset  $W$  of a vector space  $\mathbf{R}^3(\mathbf{R})$  is not a subspace ?

- (A)  $W = \{(a, b, 0) \mid a, b \in \mathbf{R}\}$
- (B)  $W = \{(a, b, c) \mid a + b + c = 0\}$
- (C)  $W = \{(a, b, c) \mid a^2 + b^2 + c^2 < 1\}$
- (D)  $W = \{(a, a, 0) \mid a \in \mathbf{R}\}$

Or

Economic Order Quantity (EOQ) in Inventory problem, results in :

- (A) reduced chances of stock outs
- (B) miximization of set-up cost
- (C) equalization of carrying cost and procurement cost
- (D) favourable procurement price

38. Let  $D$  denote the derivative which we view as a linear map on the space of differential functions and  $k$  be a non-zero integer. Then the eigenvectors of  $D^2$  are :

- (A)  $\sin x$  and  $\sin kx$
- (B)  $\cos x$  and  $\cos kx$
- (C)  $k \sin x$  and  $k \cos x$
- (D)  $\sin kx$  and  $\cos kx$

Or

For a two person game, in game theory with A and B, the minimizing and the maximizing players, the optimal strategies are :

- (A) maximax for A and minimax for B
- (B) minimin for A and maximin for B
- (C) maximin for A and minimax for B
- (D) minimax for A and maximin for B
39. If  $T : \mathbf{R}^3(\mathbf{R}) \rightarrow \mathbf{R}^2(\mathbf{R})$  is defined by  $T(x_1, x_2, x_3) = (x_1, x_2)$ , then which of the following alternatives is not true for T.
- (A) T is a linear transformation
- (B) T is an isomorphism
- (C) Range of T =  $\mathbf{R}^2$
- (D)  $\text{Ker } T = \{(0, 0, x_3) | x_3 \in \mathbf{R}\}$

Or

Sequencing problem involving processing of two jobs on 'n' machines :

- (A) cannot be solved graphically
- (B) can be solved graphically
- (C) has a condition that the processing of two jobs must be in the same order
- (D) none of the above
40. Let V be a finite dimensional space over  $\mathbb{C}$  and let  $T : V \rightarrow V$  be a linear map. Assume that all the eigenvalues of T are equal to 0. Then :
- (A) T is not nilpotent
- (B) T is diagonalizable
- (C) There is an integer  $r > 1$  such that  $T^r = 0$  (zero map)
- (D) T is a zero map

Or

For a "Poisson exponential single server and infinite population" queuing model, which of the following is *not* correct :

- (A)  $E(m) = \lambda E(w)$
- (B)  $E(n) = \lambda E(v)$
- (C)  $E(n) = E(m) -$
- (D)  $E(v) = E(w) + \frac{1}{\mu}$

41. Let  $\dim V > \dim W$  and let  $T : V \rightarrow W$  be a linear map. Then :

- (A)  $\text{Ker } T = \{0\}$
- (B)  $\dim \text{Ker } T = \dim \text{Im } T$
- (C)  $\text{Ker } T$  is not  $\{0\}$
- (D)  $T$  is invertible

Or

The types of probability sampling are :

- (A) random sampling, snowball sampling and lottery method
- (B) computer methods, lottery methods and snowball sampling
- (C) simple random and systematic sampling
- (D) random numbers, random sampling and computer methods

42. The dimension of the subspace of  $\mathbf{K}^n$  consisting of those vectors  $A = (a_1, \dots, a_n)$  such that  $a_1 + \dots + a_n = 0$  is :

- (A)  $n$
- (B)  $n - 1$
- (C)  $n/2$
- (D)  $\frac{n - 1}{2}$

Or

A cluster sampling is when :

- (A) units are clustered together after the study to enhance data analysis
- (B) in the first instance groups of people are chosen for the study
- (C) a quota of people is chosen for the study
- (D) units are clustered together after sample selection for data analysis



43. Let  $a \in \mathbf{K}$  and  $a \neq 0$ . For the matrix

$$\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} :$$

- (A) the eigen-vectors of the matrix generate 2-dimensional space
- (B) the eigen-vectors of the matrix generate 1-dimensional space
- (C) if  $\mathbf{K} = \mathbf{R}$ , then the characteristic polynomial and minimal polynomial are same
- (D) the eigen-vectors are orthogonal

Or

In a study of attitudes to university policies, a researcher interviewed 150 first-year students, 130 second-year students and 100 third-year students. The sampling procedure used in this study was :

- (A) probability sampling
- (B) stratified sampling
- (C) quota sampling
- (D) temporal sampling

44. Consider a homogeneous equation  $y'' + ay' + by = 0$ . Its characteristic equation has a root  $r$  of multiplicity two. Then the Wronskian  $W$  of the solutions of the equation is :

- (A)  $W = e^{2rx}$
- (B)  $W = xe^{2rx}$
- (C)  $W = xe^{r^2x}$
- (D)  $W = e^{r^2x}$

Or

A researcher decides to increase the size of his random sample from 1500 to 4000. The effect of this increase is to :

- (A) reduce the variability of the estimate
- (B) reduce the bias of the estimate
- (C) increase the standard error of the estimate
- (D) have no effect because the population size is the same

45. The partial differential equation which represents all surfaces of revolution about  $z$ -axis is represented by :

(A)  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0$

(B)  $x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 0$

(C)  $y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = 0$

(D)  $y \frac{\partial z}{\partial x} - x \frac{\partial z}{\partial y} = 0$

Or

Which of the following statements is *incorrect* about the sampling distribution of the sample mean ?

- (A) The standard error of the sample mean will decrease as the sample size increases
- (B) The sample mean is unbiased for the true (unknown) population mean
- (C) The sampling distribution shows how the sample mean will vary among repeated samples
- (D) The sampling distribution shows how the sample is distributed around the sample mean

46. The order of the differential equation of the family of circles of variable radius  $r$  with centres on the  $x$ -axis is :

(A) 2

(B) 3

(C) 4

(D) 5

Or

Multiple correlation coefficient *cannot* be negative because :

- (A) it is the maximum among all possible correlation coefficients between the dependent variable and a linear combination of the independent variables
- (B) There are enough independent variables having positive correlation with the dependent variable
- (C) We take the positive square root
- (D) We reject the negative value

47. The solution of homogeneous initial value problem is

$$y = 2e^{10x} + \sin 3x,$$

then the least possible order of the differential equation is :

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Or

Hotelling  $T^2$  statistic is a multivariate generalization of :

- (A) Chi-square statistic
  - (B) Student  $t$ -test
  - (C) Snedecor's F-statistic
  - (D) Mahalanobis'  $D^2$ -statistic
48. The set of all spheres with centres on the  $z$ -axis and of radius  $a$  is represented by the :
- (A) first order ordinary differential equation
  - (B) first order partial differential equation
  - (C) second order ordinary differential equation
  - (D) second order partial differential equation

Or

Which of the following statements is *false* ?

- (A) A physical interpretation of the sample mean  $\bar{x}$  demonstrates how it measures the centre of a sample
- (B) The sample median is very sensitive to extremely small or extremely large data values (outliers)
- (C) The sample median is the middle value when the observations are ordered from smallest to largest
- (D) The sample mean is very sensitive to extremely small or extremely large data values (outliers)

49. Consider the equation  $L(y) = y'' + a_1y' + a_2y = 0$ , where  $a_1$  and  $a_2$  are real constants. Then every solution of  $L(y) = 0$  tends to zero as  $x \rightarrow \infty$  if :

- (A)  $a_1 > 0$
- (B)  $a_1 < 0$
- (C)  $a_1 = 0$
- (D)  $a_1 \neq 0, a_2 > 0$

Or

Let  $\{y_n, n > 1\}$  be a sequence of independent standard normal variables.

Let  $X_n = Y_n^3 - 1, n > 1$ . Then

is :

- (A) 1
- (B)  $\int_{-\infty}^0 \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx$
- (C)  $\int_{-\infty}^{-1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx$
- (D)  $\frac{1}{2}$

50. Let  $\phi_1$  and  $\phi_2$  be differentiable functions on an interval I and  $W(\phi_1, \phi_2)$  be the Wronskian of  $\phi_1, \phi_2$ . Consider the following two statements

- (I)  $W(\phi_1, \phi_2)(x_0) \neq 0$  for some  $x_0 \in I \Rightarrow \phi_1, \phi_2$  are linearly independent
- (II)  $\phi_1, \phi_2$  are linearly independent functions on I  $\Rightarrow W(\phi_1, \phi_2)(x) \neq 0$

then :

- (A) both (I) and (II) are false
- (B) both (I) and (II) are true
- (C) only (I) is true
- (D) only (II) is true

Or

In almost all non-parametric tests, which of the following assumptions is always true ?

- (A) The form of the distribution function ( $df$ ) is known
- (B) The  $df$  is discrete
- (C) The distribution is normal
- (D) The  $df$  is continuous

**ROUGH WORK**

**FEB - 30213/II**

**FEB - 30213/11**

**ROUGH WORK**

Test Booklet No.

प्रश्नपत्रिका क्र.

**F**

**Paper-II**

**MATHEMATICAL SCIENCE**

Seat No.

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(In figures as in Admit Card)

Seat No.

(In words)

OMR Sheet No.

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(To be filled by the Candidate)

**Signature and Name of Invigilator**

1. (Signature) .....

(Name) .....

2. (Signature) .....

(Name) .....

**FEB - 30213**

**Time Allowed : 1¼ Hours]**

**[Maximum Marks : 100**

**Number of Pages in this Booklet : 24**

**Number of Questions in this Booklet : 50**

**Instructions for the Candidates**

- Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
- This paper consists of 50 objective type questions. Each question will carry two marks. All questions of Paper-II will be compulsory, covering entire syllabus (including all electives, without options).
- At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
  - To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
  - Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.**
  - After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
- Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.  
**Example :** where (C) is the correct response.  

(A)	(B)	(C)	(D)
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- Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only**. If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
- Read instructions given inside carefully.
- Rough Work is to be done at the end of this booklet.
- If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
- You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
- Use only Blue/Black Ball point pen.**
- Use of any calculator or log table, etc., is prohibited.**
- There is no negative marking for incorrect answers.**

**विद्यार्थ्यांसाठी महत्त्वाच्या सूचना**

- परिक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
- सदर प्रश्नपत्रिकेत 50 बहुपर्याय प्रश्न आहेत. प्रत्येक प्रश्नास दोन गुण आहेत. या प्रश्नपत्रिकेतील सर्व प्रश्न सोडविणे अनिवार्य आहे. सदरचे प्रश्न हे या विषयाच्या संपूर्ण अभ्यासक्रमावर आधारित आहेत.
- परीक्षा सुरु झाल्यावर विद्यार्थ्यांला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनीटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी आवश्यक तपासून पहाव्यात.
  - प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्विकारू नये.
  - पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चुकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळी वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.**
  - वरीलप्रमाणे सर्व पडताळून पहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
- प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळ/निळ्या करावा.  
**उदा. :** जर (C) हे योग्य उत्तर असेल तर.  

(A)	(B)	(C)	(D)
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- या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत. इतर ठिकाणी लिहिलेली उत्तरे तपासली जाणार नाहीत.
- आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
- प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
- जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणा व्यतिरीक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूप केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गाचा अवलंब केल्यास विद्यार्थ्यांला परीक्षेस अपात्र ठरविण्यात येईल.
- परीक्षा संपल्यानंतर विद्यार्थ्यांनी मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापी, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
- फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.**
- कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.**
- चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.**