महाराष्ट्र वनसेवा मुख्य परीक्षा - २०१२

2012
STATISTICS (Optional)
सांख्यिकी (वैकल्पिक)

100126

Time : 3 hours   Maximum Marks : 200

Note :
(i)   *Answers must be written in English.*
(ii) *Question **No. 1** is **compulsory**. Of the remaining questions, attempt **any Four** selecting one question from **each** section.*
(iii) *Figure to the **RIGHT** indicates marks of the respective question.*
(iv) *Number of optional questions upto the prescribed number in the order in which they have been solved will only be assessed. Excess answers will not be assessed.*
(v)   *Statistical and logarithmic tables will be supplied on request.*
(vi) *Use of your own simple electronic calculator is allowed.*
(vii) *Candidates should not write roll number, any name (including their own), signature, address or any indication of their identity anywhere inside the answer book otherwise they will be penalised.*

1.   Answer *any four* of the following :
  (a) Let X be a gamma random variable (r.v.) with shape parameter α and scale   **10**
       parameter λ.
       (i)   Obtain the moment generating function (m.g.f.) of X.
       (ii) Let Y be another gamma r.v. with shape parameter β and scale
             parameter λ and which is independent of X. Using the m.g.f. obtain the
             distribution of $X + Y$.
  (b) In a CRD with 4 treatments, each having 5 replicates, obtain an expression for   **10**
       the residual sum of squares (R_0^2) and state, giving reasons, its distribution.
  (c) Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with the probability   **10**
       density function (p.d.f.) given by :

$$f(x) = \begin{cases} \theta(1-x)^{\theta-1}, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

       and $\theta > 1$. Find the maximum likelihood estimator (MLE) of θ.
  (d) Construct the dual of the following Linear Programming (L.P.) primal problem.   **10**
            Minimize $z = 5x_1 + 2x_2$
            subject to $x_1 - x_2 \geq 3$
                         $2x_1 + 3x_2 \geq 5$
                         $x_1, x_2 \geq 0$
       State one feasible solution for each problem. Are the solutions you have stated
       optimal for both the problems ?
  (e) Let X_1, X_2, X_3, X_4, X_5 be independent normal r.v.s. with mean μ and variance σ^2.   **10**

       Let $U = \dfrac{1}{9}\{(2X_1 - X_2 - X_3)^2 + (2X_2 - X_1 - X_3)^2 + (2X_3 - X_1 - X_2)^2\}$

       State, giving reasons, the distribution of $W = U/(X_4 - X_5)^2$. Obtain 'a' such that
       $P[W > a] = 0.05$

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SECTION - A

2. Answer the following sub-questions :

(a) Let $\{X_n, n \geqslant 1\}$ be a sequence of i.i.d. r.v.s. with the common m.g.f. M(t). Let N be **15** a Poisson r.v. with mean λ. Let $S_N = X_1 + \ldots + X_N$ and $S_o = 0$. Obtain the m.g.f. of S_N, when N is independent of $\{X_n\}$.

(b) Suppose X is a Poisson r.v. with mean λ and Y is Binomial $(m, \lambda/m)$, and X and Y **15** are independent. Derive a formula for $P[X = Y]$.

(c) Let X be an exponential r.v. such that $P[X \leq 2] = 0.2$. **10**

   (i) Obtain the mean of X.

   (ii) Obtain $P[X > 2 + S / X > S]$. Justify your answer.

3. Answer the following sub-questions :

(a) Let the joint density of (X, Y) be **15**

$$f(x, y) \left\} \begin{array}{l} = 2/(1 + x + y)^3, \ x > 0, \ y > 0 \\ = 0 \qquad\qquad\qquad , \text{otherwise} \end{array} \right.$$

   (i) Obtain the marginal density of X.

   (ii) Determine whether X and Y are independent.

(b) Let $X_1, X_2, \ldots, X_{100}$ be independent Poisson r.v.s. with mean 4. Using the **15** appropriate approximation and tables obtain :

   $P[X_1 + X_2 + \ldots + X_{100} \leq 450]$

(c) Let X be a r.v. with p.d.f. $f(x) = \dfrac{e^{-x} x^n}{n!}$, $x > 0$, $f(x) = 0$, $x \leq 0$, and n a positive **10** integer. Let Y be a Poisson r.v. with mean λ. Show that $P[X \geqslant \lambda] = P[Y \leq n]$.

SECTION - B

4. Answer the following sub-questions :

(a) For a simple random sampling without replacement from a population of size N, **15** let $\bar{y}$ denote the sample mean. Show that $N\bar{y}$ is an unbiased estimator of the population total.

(b) In a RBD with 4 treatments and 5 blocks, derive a test of hypothesis of equality of **10** any two treatment effects.

(c) Let X_1 and X_2 be two independent exponential r.v.s. with mean $1/\lambda$ and let **15** $U_1 = \min(X_1, X_2)$, $U_2 = \max(X_1, X_2)$. Show that the r.v.s. $V_1 = 2U_1$ and $V_2 = U_2 - U_1$ are independent and obtain their distribution.

5. Answer the following sub-questions :

(a) In a stratified sampling with proportional allocation, consider a separate regression estimator (with known regression coefficient) of the population mean. Obtain the expected value and the variance of this estimator.

(b) Suppose a 2^3 factorial experiments with factors A, B, C, are to be run in two **10** blocks with (ABC) confounded and the design is to be replicated four times.
(i) Write down the two blocks.
(ii) Write down the corresponding analysis of variance table.

(c) Let X_1, X_2, X_3 be independent r.v.s. with mean zero and variance 1, and let **15** $S_n = X_1 + \ldots + X_n$. Obtain the multiple correlation coefficient between S_3 and (S_1, S_2). Examine whether it is the same as the correlation between S_2 and S_3.

SECTION - C

6. Answer the following sub-questions :

(a) Let X_1, X_2, $\ldots$, X_n be a random sample from a Bernoulli distribution with **15** $P[X_1 = 1] = P = 1 - P[X_1 = 0]$, $0 < P < 1$.
(i) Show that $X_1 + X_2 + \ldots + X_n$ is a sufficient statistic for P.
(ii) Obtain an unbiased estimator for $P(1-P)$.
(iii) Is the estimator you have given in (ii) the only unbiased estimator for $P(1-P)$? Give reasons for your answer.

(b) Measurements of the diameters of a random sample of 400 ball bearings made by **10** a certain machine showed a mean of 10.5 mm and a standard deviation of 0.8 mm. Find the 95% confidence limit for the mean diameter of the ball bearings. What should be the size of the sample if one wants to be 95% confident that the error of the estimate will not exceed 0.01 ?

(c) A sample of size 1 is taken from a distribution with p.d.f. **15**

$$f(x) = \begin{cases} \frac{2}{\theta^2} \bullet (\theta - x) , & \text{if } 0 < x < \theta \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find the most powerful test of size $\alpha = 0.05$ of $H_0 : \theta = 1$ against $H_1 : \theta = 2$. State any major result you might use.

7. Answer the following sub-questions :

(a) Let X be an observation from a distribution with p.d.f. **15**

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} , & \text{if } x > 0 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Where $\theta > 0$.
(i) Obtain the Cramér - Rao lower bounds corresponding to θ and θ^2.
(ii) Obtain an unbiased estimator of θ^2 and determine whether its variance attains the Cramér - Rao lower bound.

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from uniform $(0, \theta)$ distribution. Obtain the maximum likelihood estimator of θ. Is the estimator unbiased for θ ? Give reasons for your answer.

(c) A company claims that the lifetime of a type of battery it manufactures is more than 250 hours. Measurement on lifetimes of 20 batteries of the company, selected randomly, are given below :   **15**

$$271 \quad 230 \quad 190 \quad 280 \quad 226$$
$$286 \quad 275 \quad 264 \quad 286 \quad 282$$
$$274 \quad 295 \quad 242 \quad 225 \quad 268$$
$$253 \quad 216 \quad 291 \quad 241 \quad 252$$

Using a distribution free test, determine whether the company's claim is justified at 0.05 significance level.

SECTION - D

8. Answer the following sub-questions :

(a) The following table gives the activities and the time required in a construction project.   **10**

Activity	Preceding activity	Time (days)
1 - 2	-	20
1 - 3	-	25
2 - 3	1 - 2	10
2 - 4	1 - 2	12
3 - 4	1 - 3, 2 - 3	5
4 - 5	2 - 4, 3 - 4	10

(i) Draw the activity network of the project.

(ii) Find the total float and the free-float for each activity. What do these indicate ?

(b) (i) Define : Laspeyres' Index number, Paasche's Index number and Fisher's Ideal Index.   **15**

(ii) Prove that Fisher's ideal index lies between Laspeyres' and Paasche's index numbers. What advantage does the Fisher ideal index have over the other two index numbers ?

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(c) Samples of $n = 6$ items are taken from a manufacturing process at regular intervals. A normally distributed quality characteristic is measured and $\bar{x}$ and S values are calculated for each sample. After 50 subgroups have been analyzed we have

$$\sum_{i=1}^{50} \bar{x}_i = 1000 \quad \text{and} \quad \sum_{i=1}^{50} S_i = 75$$

(i) Compute the control limits for the $\bar{x}$ and S control charts.

(ii) Assume that all points on both charts plot within the control limits. What are the natural tolerance limits of the process ?

9. Answer the following sub-questions :

(a) The following table gives the cost in Rs. (thousands) of transportation of one item from a plant to a ware-house.   **10**

Plant	Ware - House				Supply
	1	2	3	4	
P_1	3	4	9	2	23
P_2	6	5	8	8	27
Demand	12	13	15	10	

Obtain the initial feasible solutions using two different methods. Which method results in a lesser cost ?

(b) The following data gives the number (in lakhs) of cars produced for the years 1970 to 1979.   **15**

Year	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979
Cars	9.47	9.36	8.65	8.25	7.80	7.45	7.34	7.0	6.86	7.04

Obtain the trend values using :

(i) the method of semiaverages by taking the average as the mean;

(ii) the 5 - year moving average method.

State one disadvantage of the moving average method.

(c) Explain :   **15**

(i) Single sampling plan,

(ii) Double sampling plan,

(iii) Operating characteristic (OC) curve.

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