PARLIAMENT OF INDIA
(JOINT RECRUITMENT CELL)

MAIN EXAMINATION FOR POSTS OF EXECUTIVE/LEGISLATIVE/COMMITTEE/PROTOCOL OFFI... RESEARCH/REFERENCE OFFICER IN LOK SABHA SECRETARIAT

3rd SEPTEMBER, 2010

MATHEMATICS - Paper-I

INSTRUCTIONS : This question paper consists of 10 questions in two sections A and B. Attempt at least two questions from each section and a total of five questions.
Questions must be answered in English only.

Time: 3 hours | Marks: 300

SECTION - A

Q1- (a) Show that $\lim\limits_{x\to 0} \sin\frac{1}{x}$ does not exist. (15)

(b) Find all asymptotes of the curve :-

$$x^4 + a^4 = (y-a)^2 (x^2 - a^2)$$ (15)

(where $a > 0$ is a real constant).

(c) For what value of p does $\frac{\sin 2x + p\sin x}{x^3}$ tend to a finite limit as $x \to 0$? (15)

(d) Find the area enclosed by the curve $xy^2 = a^2(a-x)$ and y-axis. (15)

Q2- (a) If $u = f(r)$, where $r^2 = x^2+y^2$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r).$$ (12)

(b) Differentiate $x^{\sin x}$ w.r.t $(\sin x)^x$. (12)

(c) If $u = \sec^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$$ (12)

(d) If α and β are distinct real numbers, show that there exists a real number

(e) Evaluate $\lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$.

Q3 - (a) Evaluate

$$\iint_R e^{x^2+y^2}\, dy\, dx$$

where R is the semi circular region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$. (15)

(b) Examine for convergence the improper integral

$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^2}$$ (15)

(c) If Γ denotes the gamma function, prove that

$$\sqrt{\pi}\, \Gamma(2m) = 2^{2m-1}\, \Gamma(m)\, \Gamma\left(m+\tfrac{1}{2}\right)$$

(where $m > 0$). (15)

(d) Find the greatest and smallest values that the function $f(x,y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$. (15)

Q4- (a) Find the value of λ so that the equation

$$x^2 - \lambda xy + 2y^2 + 3x - 5y + 2 = 0$$

may represent a pair of straight lines and find the lines. (15)

(b) Obtain the equation of the sphere which passes through the three points $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ and has its radius as small as possible. (15)

(c) Find the equation of the cone which passes through the common generators of the cones $-2x^2+4y^2+z^2=0$ and $10xy-2yz+5zx=0$ and the line with direction cosines proportional to $1, 2, 3$. (15)

(d) Find the equation of the right circular cylinder whose axis is $x=2y=-z$ and radius is 4. (15)

Q5- Attempt any five parts : $(12\times 5=60)$.

(a) Show that the vectors (a_1, a_2) and (b_1, b_2) in $\mathbb{C}^2$ are linearly dependent if and only if $a_1 b_2 = a_2 b_1$, $\mathbb{C}$ denoting the space of all complex numbers. (12)

(b) Extend the set $S=\{(1,1,0)\}$ to form two different basis of $\mathbb{R}^3(\mathbb{R})$. (12)

(c) Let $A=\{(x,y,0) : x, y \in \mathbb{R}\}$ and of $V = \mathbb{R}^3(\mathbb{R})$. Find the dimension of

(1

(d) Define the rank of a matrix. Find the rank of the matrix

$$\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

12

(e) If A and B are Hermitian matrices such that $A^2 + B^2 = 0$, show that $A = 0$ and $B = 0$. 12

(f) Describe explicitly a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which has its range the subspace spanned by the vectors $(1, 0, -1)$ and $(1, 2, 2)$. 12

SECTION – B

Q6-(a) Find the differential equation correspon[ding to]

$$y = ae^{2x} + be^{-3x} + ce^{x}$$

where a, b, c are arbitrary constants. (15)

(b) Solve :

$$x\frac{dy}{dx} + y = y^2x^3\cos x.$$ (15)

(c) Reduce the differential equation

$$(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$$

into an exact differential equation and hence solve it. (15)

(d) If p denotes $\frac{dy}{dx}$, solve

$$y = 2px + f(xp^2).$$ (15)

Q7 – (a) Use the method of variation of parameters to solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$ (15)

(b) Solve :

$$x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$$ (15)

(c) Solve:

$$x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 2x^5.$$

(d) Solve:

$$e^{3x}(p-1) + p^3 e^{2y} = 0, \quad \text{where } p \equiv \frac{dy}{dx}.$$ (15)

Q8-(a) A ladder leans against a smooth wall, the lower end resting on a rough floor for which the coefficient of friction is $\frac{1}{4}$. Find the inclination of the ladder to the vertical if it is just on the point of slipping. (15)

(b) Consider a system with two degrees of freedom for which the potential energy is given by

$$V = q_1^2 + 3q_1q_2 + 4q_2^2.$$

where q_1 and q_2 are the generalized coordinate of the system. Determine the position of equilibrium and discuss its stability. (15)

(c) Show that, if a rigid body is in equilibrium under the action of four forces, the invariant $(\vec{F}\cdot\vec{G})$ of any two is equal to the invariant $(\vec{F}\cdot\vec{G})$ of [illegible]ther two. Show also that the invariant ([illegible] of any three of the forces is zero. (15)

(d) Find the thrust on a vertical quadrilateral which has one side of length a in the surface, and the opposite side of length b parallel to it at a depth h. Also, if the fluid consists of top layer of density ρ and thickness $\frac{h}{2}$, and the rest of density σ, prove that the thrust on the quadrilateral is

$$\frac{gh^2}{48}\left\{(7a+11b)\rho + (a+5b)\sigma\right\}. \quad (15)$$

Q9 – (a) Two weights w_1 and w_2 rest on a rough plane inclined at an angle α to the horizontal and are connected by a string which lies along the line of greatest slope. If μ_1 and μ_2 are their coefficients of friction with the plane, and $\mu_1 > \tan\alpha > \mu_2$, prove that, if they are both on the point of slipping,

$$\tan\alpha = \frac{\mu_1 w_1 + \mu_2 w_2}{w_1 + w_2} \quad (15)$$

(b) A plane triangular area is immersed liquid of uniform density with its p vertical, one side horizontal and the oppo corner downwards. Its vertical altitude is h, and the horizontal side is at a depth h below the effective surface. Show that its centre of pressure is at a depth $\frac{11}{8}h$ below the surface. 15

(c) Prove that a necessary and sufficient condition for a field $\vec{F}$ to be conservative is that force is gradient of potential energy with its sign reversed. 15

(d) A cable, of weight w per unit length and length $2l$ hangs from two points A and B, at the same height and at a distance $2a$ apart. Show that the maximum tension in the cable is

$$wa\sqrt{\frac{a}{6(l-a)}}$$ 15

Q10- (a) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$ where t is the time. Find the components of its velocity

$\hat{i} - 3\hat{j} + 2\hat{k}$. (12)

(b) If $\vec{A}$ and $\vec{B}$ are irrotational, prove that $\vec{A} \times \vec{B}$ is solenoidal. (12)

(c) Verify Green's theorem in the plane for

$$\int_C (x^2 - xy^3)\,dx + (y^2 - 2xy)\,dy$$

where C is the square with vertices (0, 0), (2, 0), (2, 2), (0, 2). (12)

(d) State Stoke's theorem. Verify Stoke's theorem for $\vec{A} = (2x-y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, where S is the upper surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary. (12)

(e) Calculate the curvature and torsion of the curve $\vec{r} = \vec{r}(u)$ in terms of u, where

$$\vec{r}(u) = (u, u^2, u^3).$$ (12)

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3rd SEPTEMBER, 2010

MATHEMATICS-PAPER-II

INSTRUCTIONS: Answers must be written in English only. Candidates should attempt at least 2 questions from each section and total 5 questions. All questions carry equal marks. All the symbols having their usual meaning.

Time: 3 hours Marks: 300

SECTION A

1. (a) Let P be a Sylow p-subgroup of G. Then any conjugate gpg^{-1} is also a Sylow p- subgroup of G.
   (b) State and prove Cayley's theorem for permutation groups.
   (c) Let R be a commutative ring and A an ideal of R. Then
   $$\sqrt{A} = \{a \in R / a^n \in A, \textit{forsomen} \geq 1\}$$ is an ideal of R.

2. (a) State and prove implicit function theorem.
   (b) Test the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n}$ for uniform convergence on [0,1].
   (c) Let F=2xz**i**-x**j**+y^2**k**, evaluate $\iiint_V F dv$ where V is the region bounded by the surface x=0,y=0,y=6,z=x^2,z=4.

3. (a) Evaluate $\oint_c \frac{e^z dz}{z(1-z)^3}$ where c is (i) $|z| = \frac{1}{2}$, (ii) $|z-1| = \frac{1}{2}$, (iii) $|z| = 2$.
   (b) State and prove Residue theorem. Using this theorem,
   Evaluate $\oint_c \frac{z^2+4}{z^3+2z^2+z} dz$ where c is (i) $|z| = 1$, (ii) $|z+1-i| = 1$, (iii) $|z-1| = 5$.

4. (a) Use duality to solve the following LPP:
   Maximize $Z=2x_1+x_2$
   Subject to the constraints:
   $x_1+2x_2 \leq 10$, $x_1+x_2 \leq 6$
   $x_1-x_2 \leq 2$, $x_1-2x_2 \leq 1$
   $x_1, x_2 \geq 0$

(b) Solve the following transportation problem.
Is the optimal solution obtained by you is unique one? If not, why? What are the alternate optima then?

Source	Destination 1	2	3	4	Supply
1	15	18	22	16	30
2	15	19	20	14	40
3	13	16	23	17	30
Demand	20	20	25	35	100

5. (a) The sub group H of a group G is normal iff $g^{-1}Hg = H \forall g \in G$.

(b) Prove that f(x)=sinx is Reimann integrable on $[0, \pi/2]$.

(c) Examine the convergence and absolute convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n^2+1}$

SECTION-B

6. (a) Find the complete integral of $p^2 x + q^2 y = z$

(b) Consider the problem of heat conduction in a finite rod of length 'l' which is governed by the following initial and boundary value problem:

$u_t = u_{xx}, o \langle x \langle l, t \succ 0$

$u(o,t) = u(l,t) = 0, t \succ 0$

$u(x,0) = x(l-x), 0 \le x \le l$

Find the solution of this problem by the method of separation of variables.

7. (a) Use Gauss-Seidel method to solve the equations correct up to 4 decimal places by performing 7 iterations

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$
$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$
$$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$
$$-x_1 - x_2 - 2x_3 + 10x_4 = -9$$

(b) Find y(0.2), y(0.4),y(0.6) by using the Runge-Kutta fourth order method given that

$$\frac{dy}{dx} = 1 + y^2, y(0) = 0.$$

8. (a) Define holonomic and Non- holonomic systems. Give two examples of each of the system and explain.

(b) Derive the Hamiltonians equations of motion. Discuss the motion of a spherical pendulum using the Hamiltonian equations.

(c) Derive the Lagranges equation of second kind for a holonomic system in independent co-ordinates. Discuss the motion of Harmonic oscillator by using the Legranges equation of second kind.

9. (a) Define a two-dimensional motion. what are its characteristics? What do you mean by a point as well a curve in a two-dimensional motion?

(b) Define a source and its strength of a two-dimensional motion. find the relation between the strength of a source and radial component of the velocity.

(c) Find the complex potential of a source of strength 'm' situated at the origin. Find the streame lines, potential lines and circulation of a two dimensional source flow.

10. (a) Consider the Navier- Stokes equation in the following form:

$$\rho \frac{D\overline{q}}{Dt} = \rho \overline{F} - grad p + \mu \nabla^2 \overline{q} + \frac{\mu}{3} grad(div \overline{q}).$$

Express this equation in x,y,and z directions . How does this equation reduces for an incompressible flow? Consequently deduce Euler's equation of motion by assuming suitable assumptions.

(b) Explain Octal and Hexadecimal systems .Give flow chart for Newton – Raphson method.

(c) Verify $yzdx + (x^2 y - zx)dy + (x^2 z - xy)dz = 0$ is integrable and find its integral.