# KCSE MATHEMATICS PAPER 121/2 2009

## SECTION I (50 Marks)

*Answer all the questions in this section in the spaces provided.*

1. A farmer feeds every two cows on 480kg of hay for four days. The farmer has 20 160 Kg of hay which is just enough to feed his cows for 6 weeks. Find the number of cows in the farm
(3 marks)

2. Find a quadratic equation whose roots are $1.5 + \sqrt{2}$ and $1.5 - \sqrt{2}$, expressing it in the form $ax^2 + bx + c = 0$, where a, b and c are integers. (3 marks)

3. The mass of a wire $m$ grams(g) is partly a constant and partly varies as the square of its thickness $t$ mm. When $t = 2$mm, $m = 40$g and when $t = 3$mm, $m = 65$g. Determine the value of $m$ when $t = 4$ mm. (4 marks)

4. In the figure below, O is the centre of the circle and radius ON is perpendicular to the line TS at N.
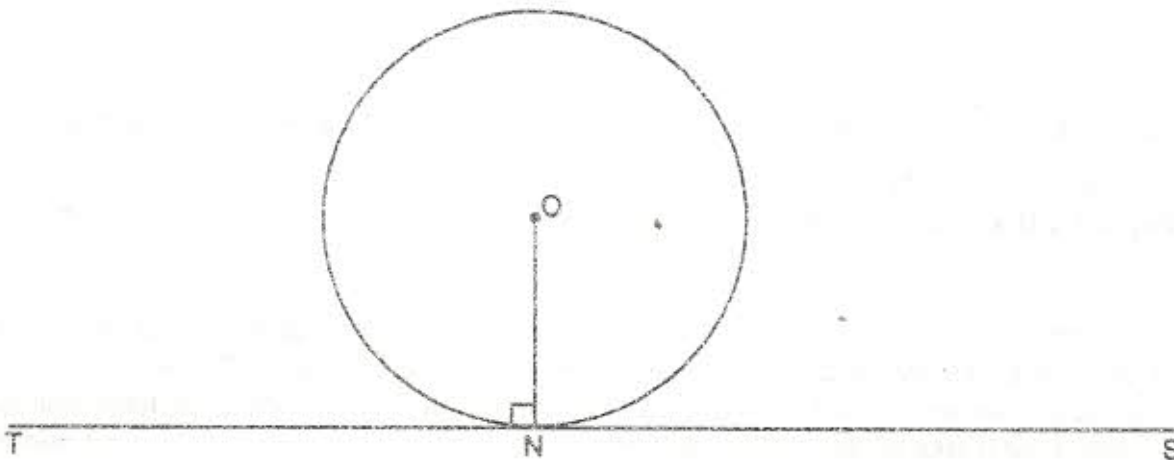


Using a ruler and a pair of compasses only, construct a triangle ABC to inscribe the circle, given that angle ABC = 60°, BC = 12cm and points B and C are on the line TS. (4 marks)

5. A solution was gently heated, its temperature readings taken at intervals of 1 minute and recorded as shown in the table below.

| Time (Min) | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Temperature (°c) | 4 | 5.2 | 8.4 | 14.3 | 16.8 | 17.5 |

(a) Draw the time temperature graph on the grid provided. (2 marks)
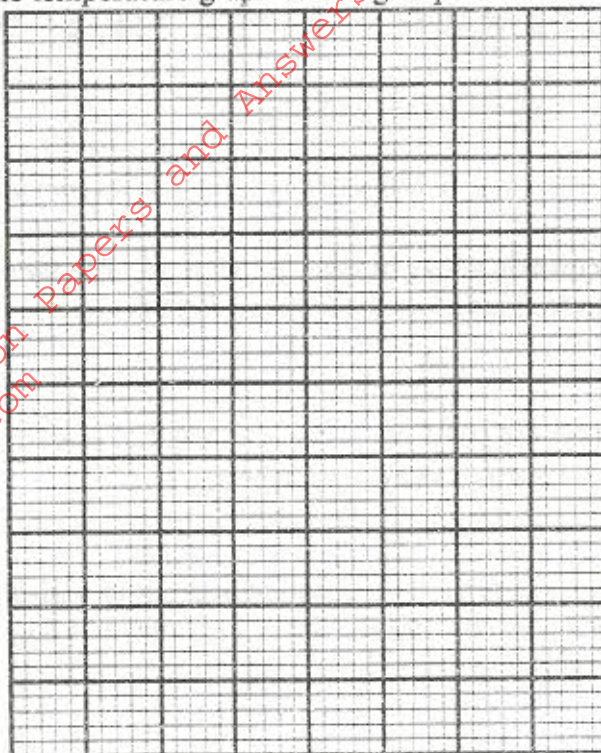


(b) Use the graph to find the average rate of change in temperature between $t = 1.8$ and $t = 3.4$ (2 marks)

6. Vector $OA = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $OB = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$. Point C is on OB such that $CB = 2OC$ and point D is on AB such that $AD = 3DB$.
Express **CD** as a column vector. (3 marks)

7. In a certain commercial bank, customers may withdraw cash through one of the two tellers at the counter. On average, one teller takes 3 minutes while the other teller takes 5 minutes to serve a customer. If the two tellers start to serve the customers at the same time, find the shortest time it take to serve 200 customers. (3 marks)

8. (a) Expand and simplify the binomial expression $(2 - x)^7$ in ascending powers of $x$. (2 marks)

(b) Use the expansion up to the fourth term to evaluate $(1.97)^7$ correct to 4 decimal places. (2 marks)

9. The area of triangle FGH is 21cm². The triangle FGH is transformed using the matrix $\begin{pmatrix} 4 & 5 \\ 1 & 2 \end{pmatrix}$. Calculate the area of the image of triangle FGH. (2 marks)

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10. Simlify $\dfrac{\sqrt{3}}{\sqrt{3} - \sqrt{2}}$

11. A circle whose equation is $(x - 1)^2 + (y - k)^2 = 10$ passes through the point $(2, 5)$. Find the coordinates of the two possible centres of the circle. (3 marks)

12. On a certain day, the probability that it rains is $\frac{1}{7}$. When it rains the probability that Omondi carries an umbrella is $\frac{2}{5}$. When it does not rain the probability that Omondi carries an umbrella is $\frac{1}{6}$. Find the probability that Omondi carried an umbrella that day. (2 marks)

13. Point P($40^0$S, $45^0$E) and point Q($40^0$S, $60^0$W) are on the surface of the Earth. Calculate the shortest distance along a circle of latitude between the two points. (4 marks)

14. Solve $4 - 4\cos^2\alpha = 4\sin\alpha - 1$ for $0^0 \le \alpha \le 360^0$. (4 marks)
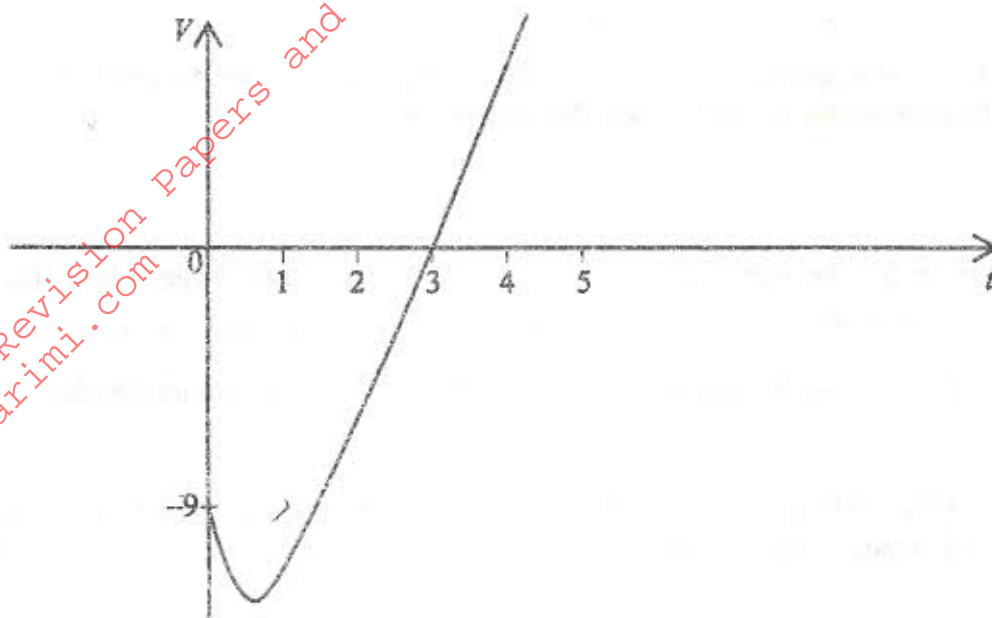
15. In the figure below, AT is a tangent to the circle at A. Angle ATB = $48^0$, BC = 5cm and CT= 4cm.
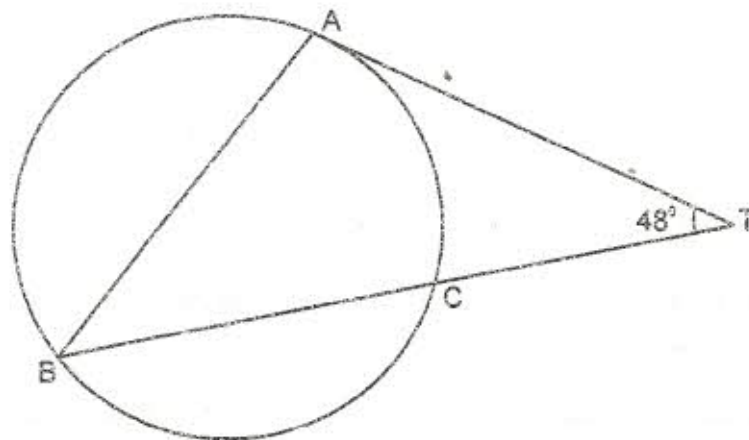


Calculate the length AT. (2 marks)

16. A particle moves in a straight line with a velocity $V$ms$^{-1}$. Its velocity after $t$ seconds is given by $V = 3t^2 - 6t - 9$.
The figure below is a sketch of the velocity-time graph of the particle.



Calculate the distance the particle moves between $t = 1$ and $t = 4$. (4 marks)

## SECTION II (50 Marks)

*Answer only five questions in this section*

17. A water vendor has a tank of capacity 18900 litres. The tank is being filled with water from two pipes A and B which are closed immediately when the tank is full. Water flows at the rate of 150 000cm³/minute through pipe A and 120 000cm³/minute through pipe B.

(a) If the tank is empty and the two pipes are opened at the same time, calculate the time it takes to fill the tank. (3 marks)

(b) On a certain day the vendor opened the two pipes A and B to fill the empty tank. After 25 minutes he opened the outlet tap to supply water to his customers at an average rate of 20 litres per minute.

   (i) Calculate the time it took to fill the tank on that day (4 marks)

   (ii) The vendor supplied a total of 542 jerricans, each containing 25 litres of water, on the day. If the water that remained in the tank was 6,300 litres, calculate, in litres, the amount of water that was wasted. (3 marks)

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18. At the beginnig of the year 1998, Kanyingi bought two houses, one in Thika and the other one in Nairobi, each at Ksh 1 240 000. The value of the house in Thika appreciated at the rate of 12% p.a.

    (a) Calculate the value of the house in Thika after 9 years, to the nearest shilling.

    (2 marks)

    (b) After $n$ years, the value of the house in Thika was Ksh 2 741 245 while the value of the house in Nairobi was Ksh 2 917 231.
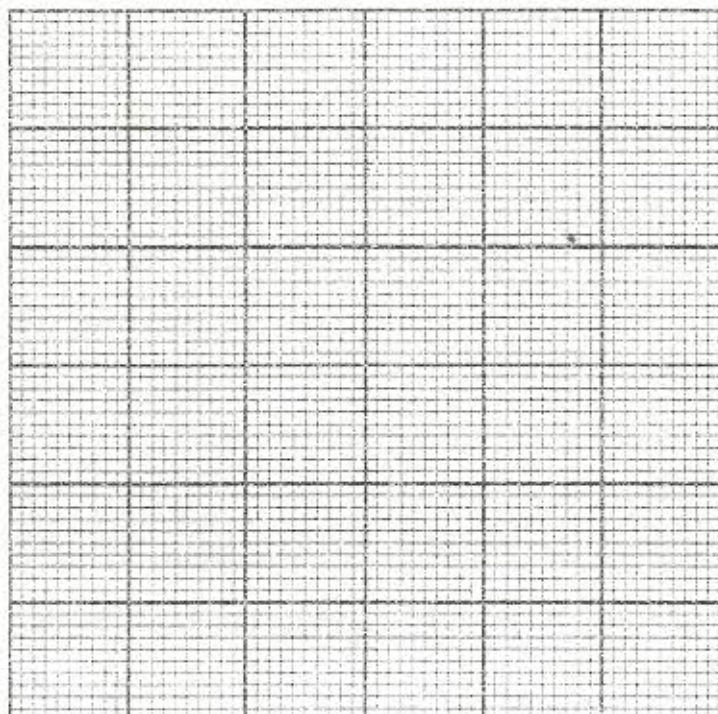
    (i) Find $n$. (4 marks)

    (ii) Find the annual rate of appreciation of the house in Nairobi (4 marks)

19. The table below shows the number of goals scored in handball matches during a tournament.

| Number of goals | 0 - 9 | 10 - 19 | 20 - 29 | 30 - 39 | 40 - 49 |
|---|---|---|---|---|---|
| Number of matches | 2 | 14 | 24 | 12 | 8 |

    (a) Draw a cumulative frequency curve on the grid provided. (5 marks)
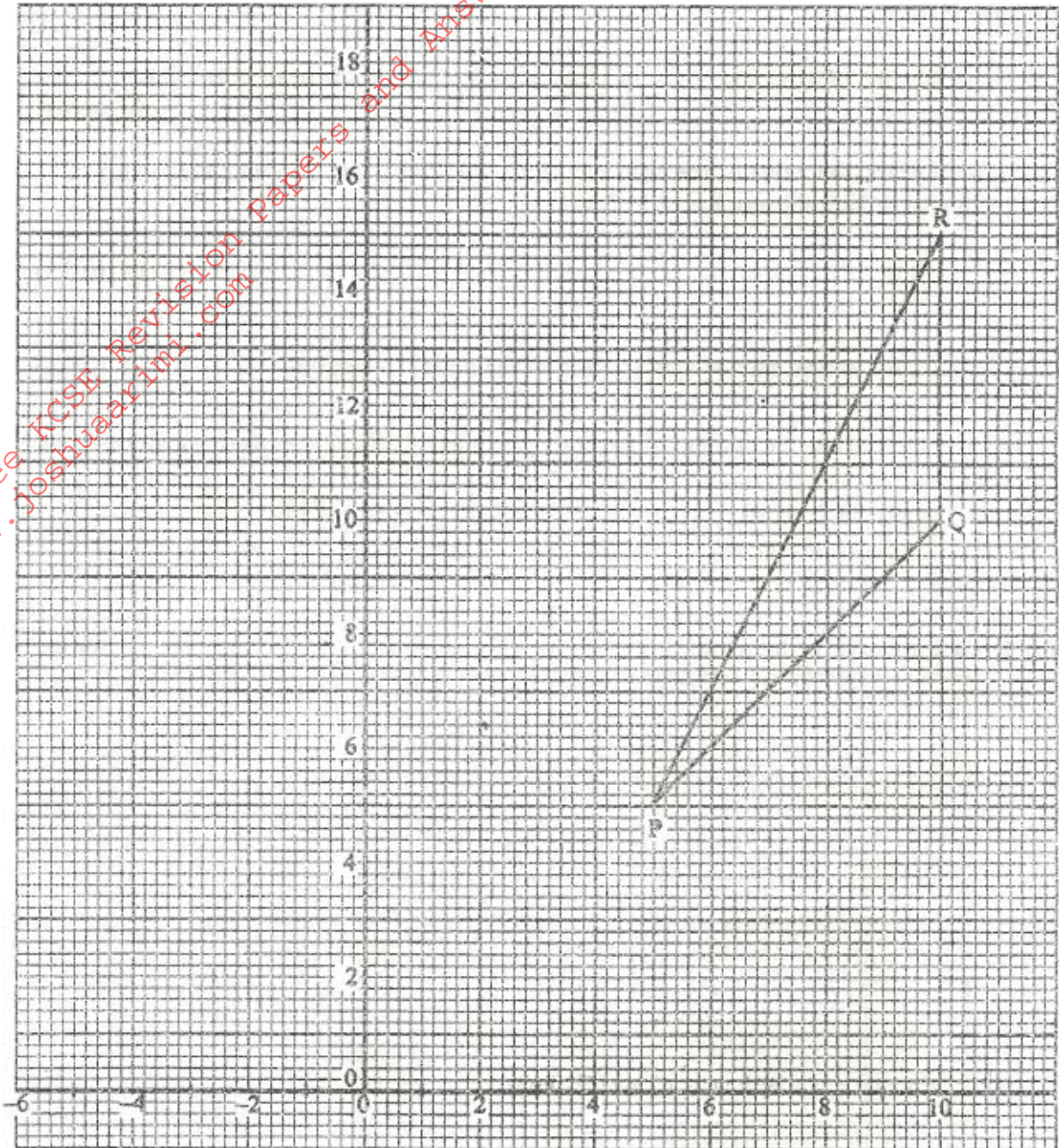


    (b) Using the curve drawn in (a) above determine:

    (i) the median; (1 mark)

    (ii) the number of matches in which goals scored were not more than 37; (1 mark)

    (iii) the inter-quartile range. (3 marks)

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20. Triangle PQR shown on the grid has vertices P(5,5), Q(10,10) and R(10,15)



(a) Find the coordinates of the points P' Q' and R', the images of P,Q,R respectively under transformation M whose matrix is

(2 marks)

$$\begin{pmatrix} -0.6 & 0.8 \\ 0.8 & 0.6 \end{pmatrix}$$

(b) Given that M is a reflection:

    (i) draw triangle P'Q'R' and the mirror line of the reflection;    (2 marks)

    (ii) determine the equation of the mirror line of the reflection.    (1 mark)

(c) Triangle P"Q"R" is the image of triangle P'Q'R' under reflection N where N is a reflection in the y-axis.

    (i)   Draw triangle P"Q"R".                       (1 mark)

    (ii)  Determine a 2 × 2 matrix equivalent to the transformation NM.    (2 marks)

    (iii) Describe fully a single transformation that maps triangle PQR onto triangle P"Q"R".                                       (2 marks)

21.    The table below shows income tax rates.

| Monthly income in Kenya shillings (Ksh) | Tax rate percentage (%) in each shilling |
|---|---|
| Up to 9 680 | 10 |
| from 9 681 to 18 800 | 15 |
| From 18 801 to 27 920 | 20 |
| From 27 921 to 37 040 | 25 |
| From 37 041 and above | 30 |

In a certain year, Robi's monthly taxable earnings amounted to Ksh 24200.

(a)  Calculate the tax charged on Robi's monthly earnings.           (4 marks)

(b)  Robi was entitled to the following tax reliefs:
    I:    monthly personal relief of Ksh 1, 056;
    II   monthly insurance relief at the rate of 15% of the premium paid.

    Calculate the tax paid by Robi each month, if she paid a monthly premium of Ksh 2400 towards her life insurance policy.               (2 marks)

(c)  During a certain month, Robi received additional earnings which were taxed at 20% in each shilling. Given that she paid 36.3% more tax that month, calculate the percentage increase in her earnings.               (4 marks)

**22.** The figure below shows a right pyramid mounted onto a cuboid. $AB = BC = 15\sqrt{2}$ cm $CG = 8$ cm and $VG = 17\sqrt{2}$ cm.
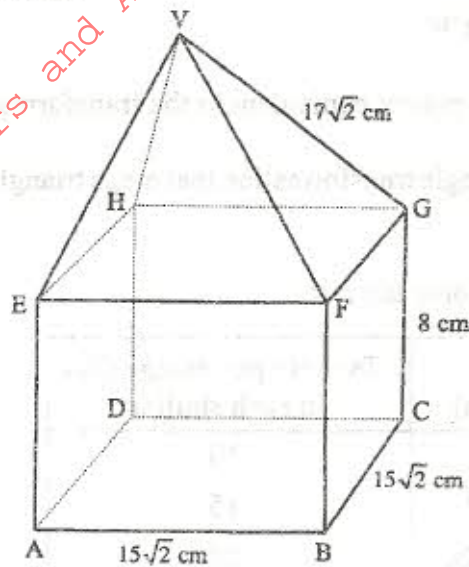


Calculate

(a) the length of AC; (1 mark)

(b) the angle between the line AG and the plane ABCD;

(c) the vertical height of point V from the plane ABCD; (3 marks)

(d) the angle between the planes EFV and ABCD (3 marks)

**23.** (a) The first term of an Arithmetic Progression (AP) is 2. The sum of the first 8 terms of the AP is 156.
  (i)  Find the common difference of the AP. (2 marks)

  (ii) Given that the sum of the first $n$ terms of the AP is 416, find $n$. (2 marks)

(b) The 3$^{rd}$, 5$^{th}$ and 8$^{th}$ terms of another AP form the first three terms of a Geometric Progression (GP). If the common difference of the AP is 3, find:
  (i)  the first term of the GP; (4 marks)

  (ii) the sum of the first 9 terms of the GP, to 4 significant figures. (2 marks)

**24.** Amina carried out an experiment to determine the average volume of a ball bearing. she started by submerging three ball bearings in water contained in a measuring cylinder. She then added one ball at a time into the cylinder until the balls were nine.

The corresponding readings were recorded as shown in the table below.

| Number of ball bearings ($x$) | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|
| Measuring cylinder reading ($y$) | 98.0 | 105.0 | 123.0 | 130.5 | 145.6 | 156.9 | 170.0 |

(a) (i) On the grid provided, plot $(x, y)$ where $x$ is the number of ball bearings and $y$ is the corresponding measuring cylinder reading. (3 marks)
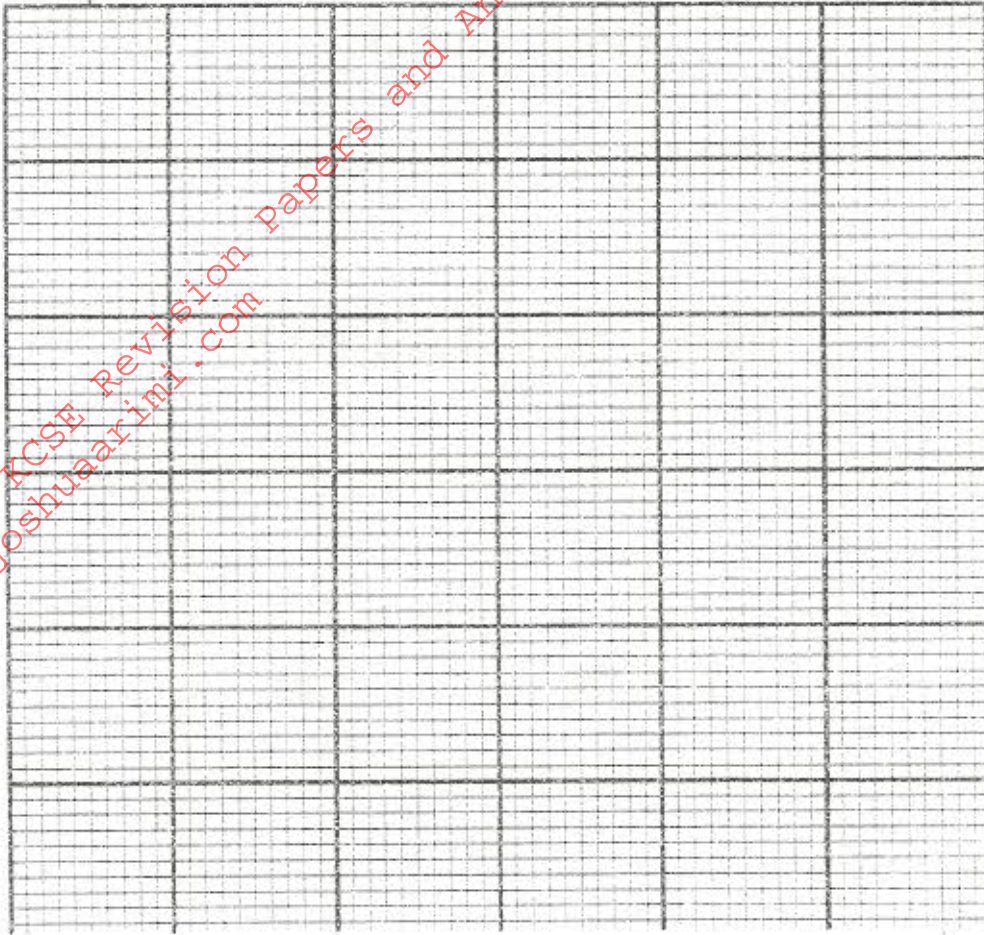


(ii) Use the plotted points to draw the line of best fit. (1 mark)

(b) Use the line of best fit to determine:
   (i) the average volume of a ball bearing; (2 marks)

   (ii) the equation of the line. (2 marks)

(c) Using the equation of the line in b(ii) above, determine the volume of the water in the cylinder. (2 marks)