2013. M227

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2013

Mathematics (Project Maths – Phase 2)

Paper 1

Ordinary Level

Friday 7 June   Afternoon 2:00 – 4:30

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Grade

Instructions

There are **three** sections in this examination paper:

Section A	Concepts and Skills	100 marks	4 questions
Section B	Contexts and Applications	100 marks	2 questions
Section C	Functions and Calculus (old syllabus)	100 marks	2 questions

Answer all eight questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

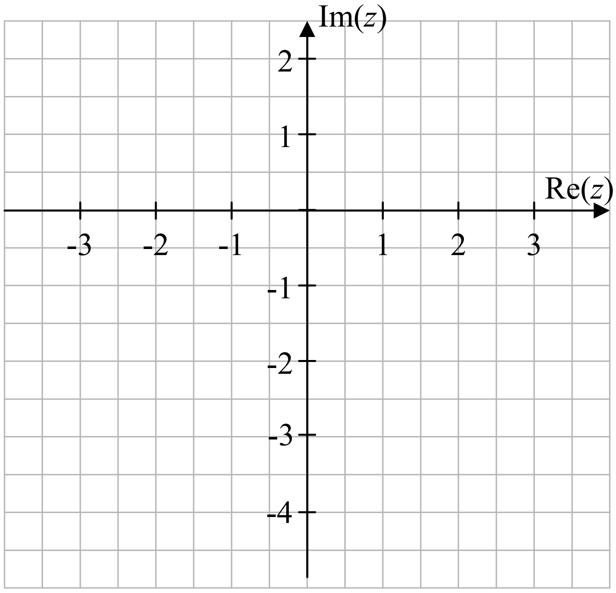
Section A	Concepts and Skills	100 marks

Answer **all four** questions from this section.

Question 1 **(25 marks)**

Let $z_1 = 3 - 4i$ and $z_2 = 1 + 2i$, where $i^2 = -1$.

(a) Plot z_1 and z_2 on the Argand diagram over.



(b) From your diagram, is it possible to say that $|z_1| > |z_2|$?
Give the reason for your answer.

Answer:

Reason:

(c) Verify algebraically that $|z_1| > |z_2|$.

(d) Find $\dfrac{z_1}{z_2}$ in the form $x + yi$, where $x, y \in \mathbb{R}$.

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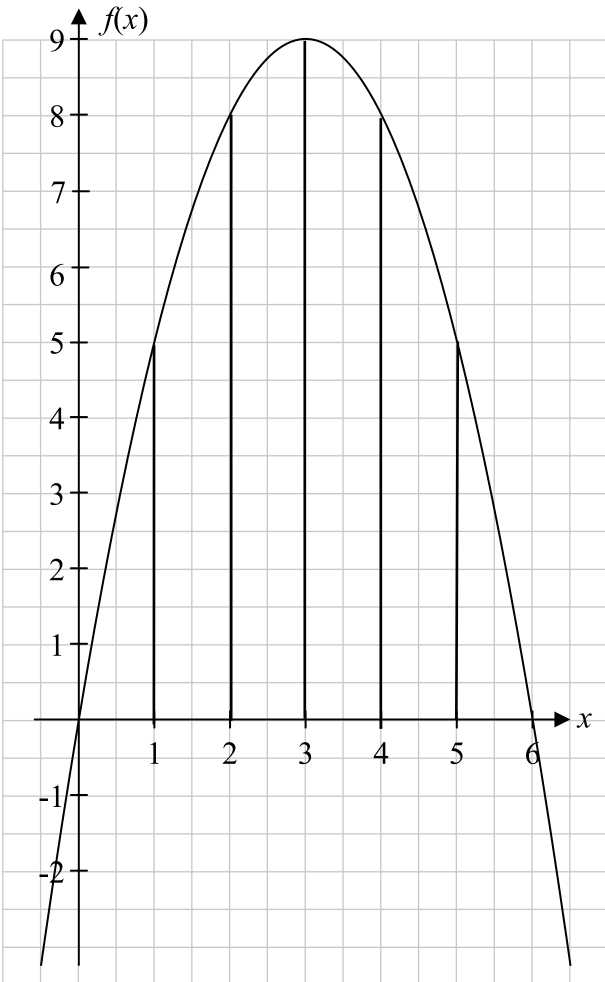
Question 2 **(25 marks)**

The diagram shows the graph of the function $f(x) = 6x - x^2$ in the domain $0 \le x \le 6,\ x \in \mathbb{R}$.

(a) Find $f(0)$, $f(1)$, $f(2)$, $f(3)$, $f(4)$, $f(5)$ and $f(6)$. Hence, complete the table below.

x	0	1	2	3	4	5	6
$f(x)$							



(b) Use the trapezoidal rule to estimate the area of the region enclosed between the curve and the x-axis in the given domain.

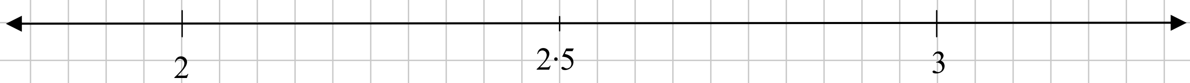
Question 3 **(25 marks)**

(a) The mean distance from the earth to the sun is 149 597 871 km. Write this number in the form $a \times 10^n$, where $1 \le a < 10$ and $n \in \mathbb{Z}$, correct to two significant figures.

(b) **(i)** Write each of the numbers below as a decimal correct to two decimal places.

	A	**B**	**C**	**D**	**E**	**F**	**G**
Number	$2{\cdot}1$	$\sqrt{5}$	$\dfrac{243}{85}$	$\tan 70°$	$\dfrac{3\pi}{4}$	250%	$\left(1+\dfrac{1}{10}\right)^{10}$
Decimal Number	$2{\cdot}10$						

(ii) Mark 5 of the numbers in the table on the number line below and label each number clearly.



(c) Solve the equation $27^{2x} = 3^{x+10}$.

Question 4 **(25 marks)**

(a) Given that $R = (1 + 0\cdot015)^{12}$, find the value of R, correct to 2 decimal places.

(b) Michael has a credit card with a credit limit of €1000. Interest is charged monthly at 1·5% of the amount owed. Michael gets a bill at the end of each month. At the start of January, Michael owes €800 on his credit card. If Michael makes no repayments and no more purchases, show that he will exceed his credit limit after 15 months.

(c) Michael buys an item costing £95 on the internet and pays with his credit card. If the exchange rate is €1 = £0·8473, calculate, correct to the nearest cent, the amount that will be included on Michael's credit card bill.

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Section B	Contexts and Applications	100 marks

Answer **both** Question 5 **and** Question 6 from this section.

Question 5 **(40 marks)**

Two identical cylindrical tanks, A and B, are being filled with water. At a particular time, the water in tank A is 25 cm deep and the depth of the water is increasing at a steady rate of 5 cm every 10 seconds. At the same time the water in tank B is 10 cm deep and the depth of the water is increasing at a steady rate of 7·5 cm every 10 seconds.

(a) Draw up a table showing the depth of water in each tank at 10 second intervals over two minutes, beginning at the time mentioned above.

(b) Each tank is 1 m in height. Find how long it takes to fill each tank.

Tank A:

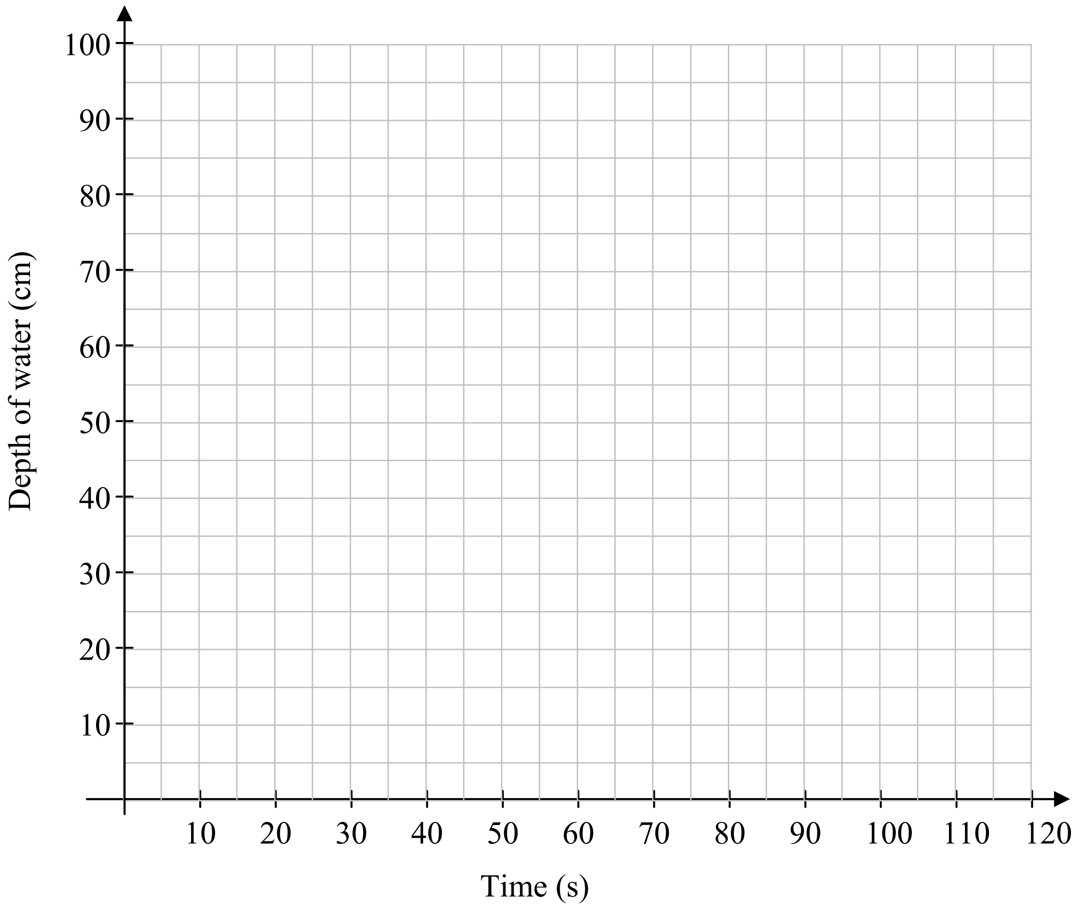
Tank B:

(c) For each tank, write down a formula which gives the depth of water in the tank at any given time. State clearly the meaning of any letters used in your formulas.

Tank A:

Tank B:

(d) For each tank, draw the graph to represent the depth of water in the tank over the 2 minutes.



(e) Find, from your graphs, how much time passes before the depth of water is the same in each tank.

Answer: ______________________________

(f) Verify your answer to part **(e)** using your formulas from part **(c)**.

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Question 6 **(60 marks)**

Two brothers, Eoin and Peter, began work in 2005 on starting salaries of €20 000 and €17 000 per annum, respectively. Eoin's salary increased by €500 per annum and Peter's salary increased by €1250 per annum. This salary pattern will continue.

(a) Complete the table, showing the annual salary of each brother for the years 2005 to 2010.

Year	1	2	3	4	5	6
Eoin's salary (€)	20 000					
Peter's salary (€)	17 000					

(b) In what year will both brothers earn the same amount?

Answer: ______________________

(c) Eoin claims that their salaries over the years can be represented by an arithmetic sequence.

(i) Explain what an arithmetic sequence is.

(ii) Do you agree with Eoin? Explain your answer.

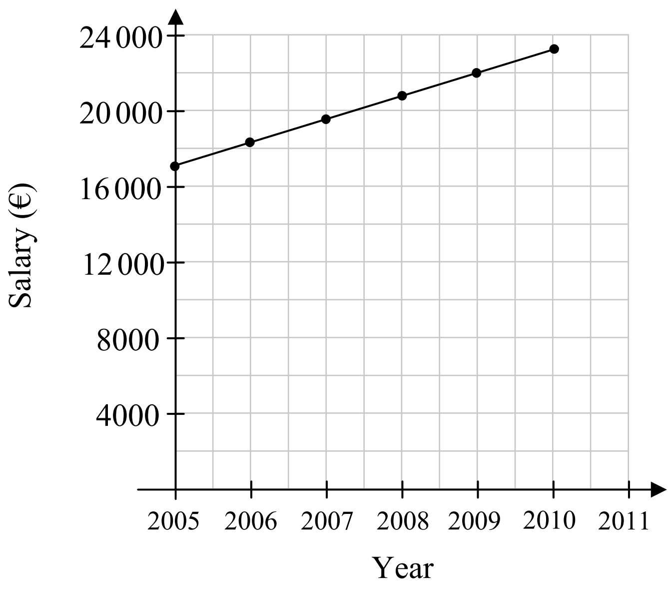
(d) Find, in terms of n, a formula that gives Eoin's salary in the n^{th} year of the pattern.

(e) Using your formula, or otherwise, find Eoin's salary in 2015.

(f) Find, in terms of n, a formula that gives the total amount earned by Peter from the first to the n^{th} year of the pattern.

(g) Using your formula, or otherwise, find the total amount earned by Peter from the start of 2005 up to the end 2015.

(h) Give one reason why the graph below is not an accurate way to represent Peter's salary over the period 2005 to 2011.



Section C	Functions and Calculus (Old Syllabus)	100 marks

Answer **both** Question 7 **and** Question 8 from this section.

Question 7 **(50 marks)**

(a) Let $y = 2x^3 - 3x^2 - 1$. Find $\frac{dy}{dx}$.

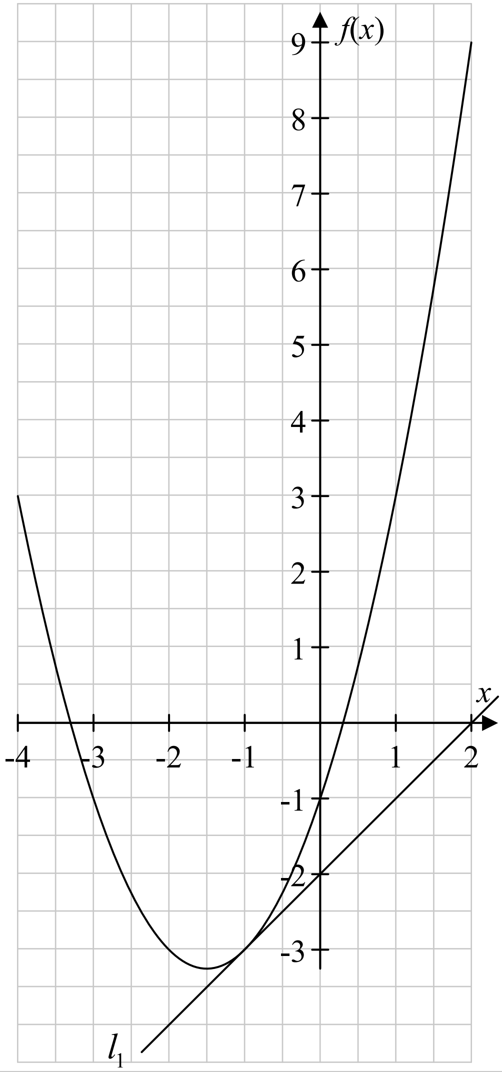
(b) **(i)** Differentiate $(2x^2 + 3x + 1)(x^3 - x + 2)$ with respect to x.

(ii) Let $y = \frac{3x}{2x+5}$, where $2x + 5 \neq 0$. Find the value of $\frac{dy}{dx}$ at $x = 0$.

(c) The diagram opposite shows graphs of the quadratic function $f(x) = x^2 + 3x - 1,\ x \in \mathbb{R}$ and the line l_1.

The line l_1 passes through the point (2, 0) and is a tangent to the curve at the point (−1, −3).

(i) Find the slope of l_1, using a slope formula.



(ii) Find $f'(x)$, the derivative of $f(x)$.

(iii) Verify your answer to **(i)** above by finding the value of $f'(x)$ at $x = -1$.

(iv) The line l_2 is perpendicular to l_1 and is also a tangent to the curve $f(x)$. Find the co-ordinates of the point at which l_2 touches the curve.

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Question 8 **(50 marks)**

(a) Given that $f(x) = 12 - x - x^2$, find the value of x for which $f'(x) = 0$, where $f'(x)$ is the derivative of $f(x)$.

(b) Let $g(x) = x^3 - 9x^2 + 24x - 20$, where $x \in \mathbb{R}$.

(i) Find the co-ordinates of the local maximum point and of the local minimum point of the function g.

(ii) Hence, draw a sketch of the function g.

(c) A stone is thrown vertically upwards. The height s meters, of the stone after t seconds is given by:

$$s = 5(4t - t^2)\ .$$

(i) Find the height of the stone after 1 second.

(ii) Show that the stone momentarily stops two seconds after being thrown, and find its height at that time.

(iii) Show that the acceleration of the stone is constant.

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