Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2012

MATHEMATICS – ORDINARY LEVEL

PAPER 1 (300 marks)

FRIDAY, 8 JUNE – AFTERNOON, 2:00 to 4:30

Attempt **SIX QUESTIONS** (50 marks each).

WARNING: Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

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1. **(a)** When Katie had travelled 140 km, she had completed $\frac{4}{9}$ of her journey.

Find the length of her journey.

(b) Robert's electricity bill gave the following data:

Unit type	*Present reading*	*Previous reading*	*Unit price*
Day rate	35 087	34 537	€0·1506
Night rate	17 213	16 853	€0·0745

(i) Calculate the total cost of the units used.

Robert also pays a standing charge of €24·89 and a levy of €5·46.
VAT at the rate of 13·5% is charged on all amounts.

(ii) Calculate the total amount of Robert's electricity bill.

(c) A retailer bought 40 toys at €24·75 each.
He sold 10 of the toys at €33·88 each and sold the remaining 30 toys at a reduced price.
His total sales amounted to €1270.

(i) Write his total profit on the transaction as a percentage of his cost.
Give your answer correct to one decimal place.

(ii) Find the reduced selling price of each of the remaining 30 toys.

2. **(a)** Solve for x and y

$$x - y = 4$$
$$2x + y = 5.$$

(b) Let $f(x) = x^3 + 2x^2 - x - 2$.

(i) Show, by division, that $x - 1$ is a factor of $f(x)$.

(ii) Hence, or otherwise, find the other factors of $f(x)$.

(c) Let $g(x) = \frac{1}{x^2} - \frac{1}{2x}$ and $h(x) = 1 - \frac{2}{x}$, where $x \neq 0$ and $x \in \mathbb{R}$.

(i) Show that $h(x) = -2x\left[g(x)\right]$.

(ii) Find the values of x for which $g(x) = h(x)$.

3. **(a)** Given that $(t-1)x = 2-5t$, find the value of x when $t = 7$.

(b) **(i)** Solve for x and y

$$x - y + 5 = 0$$
$$x^2 + y^2 = 17.$$

(ii) Which solution gives the lesser value of $x - 2y$?
Write down this value.

(c) **(i)** Simplify $\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)\left(\sqrt{x} + \frac{2}{\sqrt{x}}\right)$, where $x > 0$ and $x \in \mathbb{R}$.

(ii) Hence, solve $\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)\left(\sqrt{x} + \frac{2}{\sqrt{x}}\right) = 3$, where $x > 0$.

(iii) Verify your solution.

4. **(a)** Given that $6 - 4i + 3u = 5i$, where $i^2 = -1$,

(i) find u,

(ii) plot u on an Argand diagram.

(b) Let $z = 1 + i$.

(i) Find $|z|$.

(ii) Show that $z^2 + \bar{z}^2 = 0$, where $\bar{z}$ is the complex conjugate of z.

(iii) Verify that $\frac{1+5i}{3+2i} = z$.

(c) Let $w = 3 + 4i$.
Find the real numbers k and t such that

$$w^2 - (k+t)w + t = 0.$$

5. **(a)** The n^{th} term of a sequence is $T_n = \dfrac{2n-1}{n+1}$.

Find the sum of the second and third terms of the sequence.

(b) The first term of an arithmetic series is 2 and the eighth term is 30.

(i) Find T_3, the third term of the series.

(ii) Find S_{10}, the sum of the first ten terms of the series.

(c) The n^{th} term of a series is $T_n = \dfrac{2}{3^{n+1}}$.

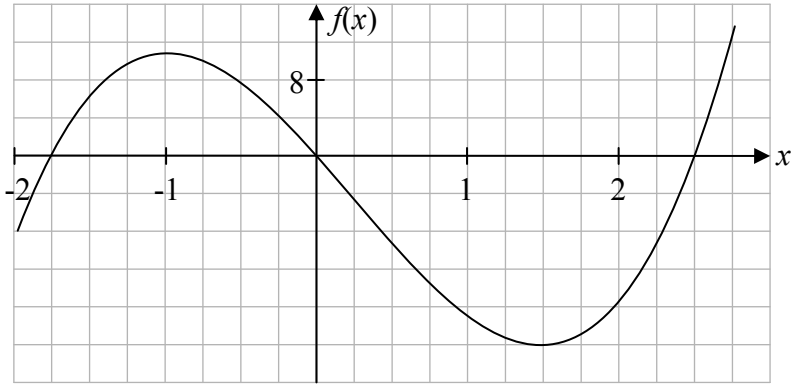
(i) Write, in terms of n, an expression for T_{n-1}, the $(n-1)^{\text{st}}$ term.

(ii) Prove that the series is geometric.

(iii) Show that $S_9 = \dfrac{1}{3} - \dfrac{1}{3^{10}}$, where S_9 is the sum of the first nine terms of the series.

6. **(a)** Let $h(x) = ax + b$, where $x \in \mathbb{R}$.
Given that $h(0) = 3$ and $h(2) = -5$, find the value of a and the value of b.

(b) The diagram shows part of the graph of a function f.



Use the graph to estimate

(i) the values of x for which $f(x) = 0$,

(ii) the values of x for which $f'(x) = 0$, where $f'(x)$ is the derivative of $f(x)$,

(iii) the range of values of x for which $f'(x) < 0$.

(c) Let $g(x) = x(3x^2 - 9)$, where $x \in \mathbb{R}$.

(i) Find $g'(x)$, the derivative of $g(x)$.

(ii) Find the co-ordinates of the local maximum point and of the local minimum point of the curve $y = g(x)$.

(iii) Draw the graph of the function $g'(x)$, the derivative of $g(x)$, in the domain $-2 \le x \le 2$.

7. **(a)** Differentiate $y = 6x - x^2 - 5x^4$ with respect to x.

(b) **(i)** Differentiate $y = (3x^2 + 2)(x^3 - x)$ with respect to x.

(ii) Given that $y = (x^3 - 2x^2 + 4)^5$, find the value of $\frac{dy}{dx}$ when $x = -1$.

(c) A ball is thrown vertically down from the top of a high building.
The distance, s metres, the ball falls is given by

$$s = 3t + 5t^2$$

where t is the time in seconds from the instant the ball is thrown.

(i) Find the speed of the ball after 3 seconds.

(ii) Find the time t when the ball is falling at a speed of $23\ \text{ms}^{-1}$.

(iii) The ball hits the ground at a speed of $38\ \text{ms}^{-1}$.
How high is the building?

8. **(a)** Let $g(x) = k(1 - x)$, where $x \in \mathbb{R}$.

Given that $g(-5) = 20$, find the value of k.

(b) Let $f(x) = \frac{5 + x^2}{2 - x}$, where $x \in \mathbb{R}$ and $x \neq 2$.

(i) Find $f(5)$.

(ii) Find $f'(x)$, the derivative of $f(x)$.

(iii) Show that $f'(x) = 0$ at $x = -1$.

(c) Let $h(x) = 5 + 3x - x^2$, where $x \in \mathbb{R}$.

(i) Find the co-ordinates of the point P at which the curve $y = h(x)$ cuts the y-axis.

(ii) Find the equation of the tangent to the curve $y = h(x)$ at P.

(iii) The tangent to the curve $y = h(x)$ at $x = t$ is perpendicular to the tangent at P.
Find the value of t.

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