Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2011

Marking Scheme

MATHEMATICS

Ordinary Level

Contents

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GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

APPLYING THE GUIDELINES

Examples (not exhaustive) of the different types of error:

Blunders (i.e. mathematical errors) (-3)

- Algebraic errors : $8x + 9x = 17x^2$ or $5p \times 4p = 20p$ or $(-3)^2 = 6$
- Sign error $-3(-4) = -12$
- Decimal errors
- Fraction error (incorrect fraction, inversion etc.)
- Cross-multiplication error
- Operation chosen is incorrect. (e.g., multiplication instead of division)
- Transposition error: e.g. $-2x - k + 3 \Rightarrow -2x = 3 + k$ or $-3x = 6 \Rightarrow x = 2$ or $4x = 12 \Rightarrow x = 8$
- Distribution error e.g. $3(2x + 4)$ has $6x + 4$ or $\frac{1}{2}(3 - x) = 5 \Rightarrow 6 - x = 5$
- Omission, if not oversimplified
- Index error
- Factorisation: error in one or both factors of a quadratic: $2x^2 - 2x - 3 = (2x - 1)(x + 3)$
- Root errors from candidate's factors: error in one or both roots
- Error(s) in transcribing formulae from tables (assuming it generates mathematically acceptable answer(s)) Serious errors or over simplifications will merit attempt marks at most (check relevant section of scheme)
- Central sign error in *uv* or *u/v* formulae
- Omission of $\div v^2$ or division not done in *u/v* formula
- Vice-versa substitution in *uv* or *u/v* formulae
- Quadratic formula and its application apply a maximum of two blunders

Slips (-1)

- Numerical slips: 4 + 7 = 10 or 3×6 = 24, but 5 + 3 = 15 is a blunder.
- An omitted round-off or incorrect round-off to a required degree of accuracy, or early rounding-off which effects final answer is penalised as a slip each time.
- However an early round-off which has the effect of simplifying the work is at least a blunder
- Omission of units of measurement or giving the incorrect units of measurement in an answer is treated as a slip, once per section of each question. Only applies where a candidate would otherwise have achieved full marks

Misreadings (-1)

- Writing 2436 for 2346 will not alter the nature of the question so M(-1)
  However, writing 5000 for 5026 will simplify the work and is penalised as at least a blunder.

Note: Correct relevant formula *isolated* and stops: if formula is *not* in Tables, award attempt mark.

QUESTION 1

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (15, 5) marks**	**Att (5, 2)**
Part (c)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

* Incorrect or omitted units: penalise as per guidelines.

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

Aoife and Brian share a prize fund in the ratio 4 : 3. Aoife gets €56.
(i) Find the total prize fund.
(ii) How much does Brian get?

(a) (i)	**5 marks**	**Att 2**
(a) (ii)	**5 marks**	**Att 2**

I		II (Brian First)
(i) $4/7 = 56 \Rightarrow 1/7 = 14$	[2m]	
$7/7 = 14 \times 7 = €98$	[5m]	(ii) $\frac{3}{4} \times 56 = €42$
(ii) $3/7 = 14 \times 3 = €42$	[5m]	(i) Total = 42 + 56 = €98
or		
$€98 - €56 = €42$	[5m]	

* Accept correct answers with or without work
* Accept candidates Total (i) for part (ii), if not over simplified

Blunders (-3)
B1 Mathematical error e.g. incorrect numerator or denominator - once if consistent
B2 Invented total, part (ii)

Slips (-1)
S1 Numerical slips

Attempts (2 marks)
A1 Mentions 7 or $\frac{1}{7}$ and stops - once only

Worthless (0)
W1 Incorrect answer(s) without work
W2 $3 \times 4 = 12$ and stops

Note: Total = 56
$\frac{3}{7} \times 56 = 3 \times 8 = 24$ [2m]
$\frac{4}{7} \times 56 = 4 \times 8 = 32$ [5m]

Part (b) **20 (15, 5) marks** **Att (5, 2)**

The cost of staying for three nights in a hotel in England is £231 sterling.

(i) Find that cost in euro, given that €1 = £0·88 sterling?

(ii) This cost is 5% more than the cost a year ago.
Find, in euro, the cost a year ago.

(b) (i) **15 marks** **Att 5**

I

$$\frac{231}{0.88} = €262.50 \quad [15m]$$

[12m]

or

II

£1 = €1.1363636 [9m]

£231 = €(231×1.1363636) [12m] = €262.4999 [14m] = €262.50 [15m]

* Units not required as in part (a)
* Accept correct answer with or without work

Blunders(-3)

B1 Mathematical error e.g. multiplies instead of dividing by 0.88, giving 203.28, but without work shown merits 0 marks

B2 Invents rate and continues

Slips (-1)

S1 Early rounding-off that affects answer e.g. method II

S2 Fails to round-off, method II

Misreading (-1)

M1 Finds the cost in euro of one night only i.e. gets £77 and continues correctly to €87.50

Note:

$$\frac{0.88}{231} = 0.0038095 \text{ or } 0.0038 \text{ will only merit, at most, attempt marks in b(ii)}$$

[9m] [12m]

(b) (ii) **5 marks** **Att 2**

I	or	II
$105\% = €262.50$ [2m]		$\frac{231}{1.05} = £220$ [2m]
$1\% = \frac{€262.50}{105}$		$\frac{220}{0.88} = €250$ [5m]
$100\% = \frac{262.50}{105} \times 100 = €250$. [5m]		

* Accept candidates answer from (i) if not over simplified but see note on b(i)

Blunders(-3)

B1 Mathematical error e.g. subtracts 5% of €262.50 (€13.125) to get €249.375 (€249.38) or €262.50/0.95 (€276.32)

Slips (-1)

S1 Numerical slips

S2 Incorrect rounding-off, or fails to round-off

Attempts (2 marks)

A1 Mentions 105%

A2 Mentions $5\% = \frac{1}{20}$ or 0.05

A3 Invents answer for (i) and continues

Part (c) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

> The speedometer in a car is faulty. When the car is actually travelling at 57 km/h, the speedometer reads 60 km/h.
>
> **(i)** Calculate the percentage error, correct to one decimal place.
>
> **(ii)** If the percentage error is the same at all speeds, at what speed is the car actually travelling when the speedometer reads 110 km/h?
> Give your answer correct to one decimal place.
>
> **(iii)** The driver is not aware of the fault. He calculates that if he travels at an average speed of 80 km/h as shown on the speedometer, he will reach his destination in four hours.
> How long, correct to the nearest minute, will it actually take him to reach his destination?

(c) (i) **10 marks** **Att 3**

I

Error = 60 – 57 = 3 [4m]

Percentage error = $\frac{3}{57} \times 100 = 5.26\% = 5.3\%$

[7m] [9m] [10m]

II

$\frac{60}{57} \times 100 = 105.263\%$ [4m]

Percentage error = (105.263 – 100) = 5.263% = 5.3%

[7m] [9m] [10m]

Blunders (-3 marks)

B1 Mathematical error e.g. incorrect numerator/denominator - once if consistent

$\frac{3}{60} \times 100 = 5\%$ also incurs S1

Slips (-1)

S1 Incorrect rounding-off, or fails to round-off

Attempts (3 marks)

A1 Correct expression for percentage error and stops

A2 $\frac{60}{57}$ and stops or $\frac{60}{57}$ = 1.05263% = 1.1% (answer)

A3 $\frac{57}{60}$ and stops

A4 Correct answer without work

Note:

$\frac{3}{60} \times 100 = 5\%$ merits [6m] [10m – B1 – S1]

$\frac{57}{60} \times 100 = 95\%$ [3m] continues to $100 - 95 = 5\%$ merits [6m]

(c) (ii) **5 marks** **Att 2**

Speedometer reading = 105.3% of true speed

I $105.3\% = 110$ km/h [2m]

$$100\% = \frac{110}{105.3} \times 100 = 104.46 = 104.5 \text{ km/h}$$

or

II $60 \sim 57$

$$110 \sim \frac{57}{60} \times 110 = 104.5 \text{ km/h}$$

[2m] [5m]

* Accept candidates answer from (i)

Blunders (-3 marks)

B1 Incorrect use of percentage e.g. uses 94.7% of 110 (giving 104.17 = 104.2 km/h)

Slips (-1)

S1 Incorrect rounding-off, or fails to round-off

S2 Incorrect or omitted units

Attempts (2 marks)

A1 Mentions 105.3%

A2 $\frac{57}{60}$ or $\frac{60}{57}$ and stops; at this part

A3 Correct answer without work

Worthless (0)

W1 110 – 3 = 107 km/h

(c) (iii) **5 marks** **Att 2**

I

Distance: $80 \times 4 = 320$ km [2 m]

or

Actual speed: $\dfrac{80}{1.053} = 75.97$ km/h [2 m]

Time: $\dfrac{320}{75.97} = 4.212 = 4$ hours 13 minutes

[4m] [5m]

II

$80 \sim \dfrac{57}{60} \times 80 = 76$ km/h [2m]

$\dfrac{320}{76} = 4.210526316 = 4.211$ [4m]

= 4 hours 13 minutes [5m]

III

$4 \times 1.053 = 4.212$ [4m]

Time = 4 hours 13 minutes [5m]

IV

$\dfrac{320 \times 1.053}{80} = 4.212$ [4m]

Time = 4 hours 13 minutes [5m]

Accept:

Time 4 hours 13 minutes as 4:13 or 4,13 or 4 13

or

253 minutes from $4 \times 60 + 13 = 253$ or ($4.212 \times 60 = 252.72 = 253$) or by calculator function

* Accept candidates answer from (ii)

Blunders (-3 marks)

B1 Error in S/D/T formula

Slips(-1)

S1 Incorrect rounding-off

Attempts (2 marks)

A1 States S/D/T formula and stops

A2 Correct answer without work

QUESTION 2

Part (a)	**15 (10, 5) marks**	**Att (3, 2)**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Part (a) **15 (10, 5) marks** **Att (3, 2)**

Given that $3a(x + 5) = 114$, find the value of x when $a = 4$.

(a) **15 (10, 5) marks** **Att (3, 2)**

Substitution for a: 10 marks
Solving for x: 5 marks

$3a(x + 5) = 114$

$a = 4 \Rightarrow 3(4)(x+5) = 114 \Rightarrow 12(x+5) = 114 \Rightarrow x + 5 = 9.5 \Rightarrow x = 4.5$ or $9/2$ or equivalent

[10m] [5m]

or

$3ax + 15a = 114$ [2m]

$3ax = 114 - 15a$

$$\Rightarrow x = \frac{114-15a}{3a} = \frac{114-15(4)}{3(4)} = \frac{114-60}{12} = \frac{54}{12} = 4.5$$

[10m+2m] [10m+5m]

* Note correct substitution for a at any point merits 10 marks
* Substitution of 4 for x and then solving for a: Att 3 + 5 marks
* Accept correct answer with or without work for full marks

Blunders (-3)
B1 Mathematical error e.g. transposing or incomplete substitution for a

Slips (-1)
S1 Numerical slips

Attempts (3 or 2 marks)
A1 Some effort at multiplying out equation - Att. 2

Part (b) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

(i) Find A, the solution set of $3x - 5 < 7$, $x \in \mathbb{Z}$.

(ii) Find B, the solution set of $\dfrac{-2-3x}{4} \le 1$, $x \in \mathbb{Z}$.

(iii) List the elements of $A \cap B$.

* Do not award multiple attempts for the same piece of work
* No back marking

(b) (i) **10 marks** **Att 3**

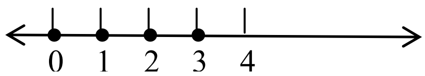
$3x - 5 < 7 \quad \Rightarrow 3x < 12 \quad \Rightarrow x < 4$ [10 m]

or

$A = \{3, 2, ...\}$

or

Shown correctly on number line



* Accept $x < 4$ with or without work
* $x = 4$ with work 7 marks; without work 0 marks
* $x \le 4$ without work 7 marks
* $x \ge 4$ or $x > 4$ without work 0 marks

Blunders (-3)
B1 Mathematical errors

Attempts (3 marks)
A1 Correctly lists/tests one element of A

Worthless (0)
W1 Incorrect graph without work but see A1

(b) (ii) **5 marks** **Att 2**

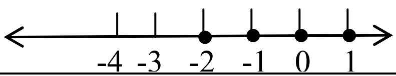
$\dfrac{-2-3x}{4} \le 1 \quad \Rightarrow -2 - 3x \le 4 \quad \Rightarrow -3x \le 6 \quad \Rightarrow x \ge -2$ [5m]

or

$B = \{-2, -1, ...\}$

or

Shown correctly on number line



* Incorrect inequalities treat as above (i)

Blunders (-3)
B1 Mathematical errors e.g. transposing

Attempts (2 marks)
A1 Correctly lists/tests one element of B
A2 Correct answer without work

(b) (iii) **5 marks** **Att 2**

$A = \{3, 2, 1, 0, -1 \ldots\}$ [2m]
or
$B = \{-2, -1, 0, 1, 2, 3 \ldots\}$ [2m]
$A \cap B = \{-2, -1, 0, 1, 2, 3\}$ [5m]

* May plot on a number line
* If marks at (i) and (ii) 0 + 0 award at most Att marks, otherwise accept candidates work
* Answer must be consistent with candidate's inequalities in b(i) and b(ii)

Blunders (-3)
B1 Mathematical error e.g. incorrect set operation

Slips (-1)
S1 Each missing or incorrect element to a maximum of 3

Attempts (2 marks)
A1 Some relevant work at this point e.g. $-2 \le x \le 3$

Part (c) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

Let $f(x) = x^3 - 2x^2 + cx + d$. **(i)** Given that $f(0) = 6$, find the value of d. **(ii)** Given that $f(3) = 0$, find the value of c. **(iii)** Hence, solve the equation $f(x) = 0$.

(c) (i) **5 marks** **Att 2**

$f(x) = x^3 - 2x^2 + cx + d$ $f(0) = 0^3 - 2(0)^2 + c(0) + d = 6 \quad \Rightarrow d = 6$

Blunders (-3)

B1 Mathematical error e.g. 2(0) ≠ 0 or $f(6)$ and continues with some correct work

Attempts (2 marks)

A1 Some effort at substituting 0 for x

A2 Correct answer without work

(c) (ii) **5 marks** **Att 2**

$f(x) = x^3 - 2x^2 + cx + 6$ [2m] $f(3) = 3^3 - 2(3)^2 + c(3) + 6 = 0 \quad \Rightarrow 27 - 18 + 3c + 6 = 0 \quad \Rightarrow 3c = -15 \quad \Rightarrow c = -5$

* Accept candidates d from part(i)

Blunders (-3)

B1 Mathematical error

Attempts (2 marks)

A1 Some effort at substituting 3 for x

A2 Invents value of d and continues with some correct work

A3 $f(0) = 3$ and continues with some correct work

(c) (iii) **5 marks** **Att 2**

$$\begin{array}{r|l} & x^2 + x - 2 \\ \hline x-3 & x^3 - 2x^2 - 5x + 6 \\ & \underline{x^3 - 3x^2} \\ & \quad x^2 - 5x \\ & \quad \underline{x^2 - 3x} \\ & \qquad -2x + 6 \\ & \qquad \underline{-2x + 6} \\ & \qquad\ 0 \quad 0 \end{array}$$

$$x^3 - 2x^2 - 5x + 6 = 0$$

$$\Rightarrow (x-3)(x^2 + x - 2) = 0$$

$$\Rightarrow (x-3)(x-1)(x+2) = 0 \quad [2m]$$

$$\Rightarrow x = 3, \ \ x = 1, \ \ x = -2$$

* Accept candidates answers from parts (i) and (ii) – must solve their equation

* Solutions 1 and –2 found by trial and error (calculator) or inspection must be fully verified otherwise att 2

* Candidates may offer other correct versions e.g. finds quadratic by equating coefficients

Blunders (-3)

B1 Mathematical errors e.g. division, incorrect factors or roots from factors

Slips (-1)

S1 Omits given root, 3.

Attempt (2 marks)

A1 Some effort at division or use of remainder theorem

A2 Substitution for their values of c and/or d

QUESTION 3

Part (a)	**15 marks**	**Att 5**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Part (a) **15 marks** **Att 5**

Multiply $(3x - 1)(2x^2 + 5x - 4)$ and simplify your answer.

(a) **15 marks** **Att 5**

$(3x - 1)(2x^2 + 5x - 4)$

$= 3x(2x^2 + 5x - 4) - 1(2x^2 + 5x - 4)$ [9m]

$= 6x^3 + 15x^2 - 12x - 2x^2 - 5x + 4 = 6x^3 + 13x^2 - 17x + 4$

[12m] [15m]

Blunders (-3)

B1 Mathematical error – each time if different

Attempts (5 marks)

A1 Some correct multiplication

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

> **(i)** Solve for x and y
> $$2x = 13 + 3y$$
> $$\frac{x}{2} = \frac{2-y}{5}.$$
> **(ii)** Hence, find the value of $4\left(x - y^2\right)$.

(b) (i) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

Step 1 Simplification of equation(s) to usable form: 5 marks
Step 2 Solutions for first variable: 5 marks
Step 3 Second variable: 5 marks

$$\begin{aligned} 2x &= 13+3y \\ \frac{x}{2} &= \frac{2-y}{5} \end{aligned} \Rightarrow \begin{aligned} 2x &= 13+3y \\ 5x &= 4-2y \quad [5\text{m}] \end{aligned} \Rightarrow \begin{aligned} 4x &= 26+6y \\ 15x &= 12-6y \\ \hline 19x &= 38 \end{aligned} \Rightarrow x = 2 \quad [5\text{ m}]$$

$2x = 13+3y \Rightarrow 4 = 13+3y \Rightarrow 3y = -9 \Rightarrow y = -3$ [5m]

or

$$\begin{aligned} 2x &= 13+3y \\ \frac{x}{2} &= \frac{2-y}{5} \end{aligned} \Rightarrow \begin{aligned} 2x &= 13+3y \\ 5x &= 4-2y \quad [5\text{m}] \end{aligned}$$

$$\begin{aligned} 2x-3y &= 13 \\ 5x+2y &= 4 \end{aligned} \Rightarrow \begin{aligned} 2x-3y &= 13\times 5 \\ 5x+2y &= 4\times -2 \end{aligned} \Rightarrow \begin{aligned} 10x-15y &= 65 \\ -10x-4y &= -8 \\ \hline -19y &= 57 \end{aligned} \Rightarrow y = -3 \quad [5\text{m}]$$

$2x-3y = 13 \Rightarrow 2x-3(-3) = 13 \Rightarrow 2x = 13-9 \Rightarrow x = 2$ [5m]

* Candidates may use other acceptable approaches
* Correct isolation of x or y merits first 5 marks (Step 1)

Blunders (-3)
B1 Mathematical error

Attempts (2 marks)
A1 Correct answer(s) by trial and error or without relevant work, verified in:
- One equation – one att 2 (final step)
- Both equations – three(3) att 2 (6 marks)

A2 Some effort at a graphical solution – one att only
A3 Fully correct graphical solutions merits three(3) Att 2s, at most

(b) (ii) **5 marks** **Att 2**

$$4\left(x-y^2\right)=4\left(2-(-3)^2\right)=4(2-9)=-28$$

* Accept candidates answers from (i) even if both positive or only merited attempt marks at part(i)

Blunders (-3)
B1 Mathematical error e.g. sign
B2 Incorrect substitution of answers from (i)

Attempt (2 marks)
A1 Correct substitution for x and/or y and stops
A2 $4x-4y^2$ and stops

Worthless (0)
W1 Invented values for x and y, no values for part(i)

Part (c) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

(i)	Solve for x $$\frac{x-1}{x}+\frac{x}{x+1}=\frac{1}{2}, \quad x \neq 0,\ x \neq -1.$$
(ii)	Verify **one** of your solutions.

(c) (i) **10 (5, 5) marks** **Att (2, 2)**

Step 1 – Formation of quadratic equation: 5 marks
Step 2 – Solution of quadratic equation: 5 marks

I $\quad \frac{x-1}{x}+\frac{x}{x+1}=\frac{1}{2} \Rightarrow \frac{(x-1)(x+1)+x^2}{x(x+1)}=\frac{1}{2} \Rightarrow \frac{2x^2-1}{x^2+x}=\frac{1}{2}$

$4x^2-2=x^2+x \Rightarrow 3x^2-x-2=0 \Rightarrow (3x+2)(x-1)=0 \Rightarrow x=-\frac{2}{3},\ x=1$

[5m] [5m]

or

II

$\frac{x-1}{x}+\frac{x}{x+1}=\frac{1}{2}$

LCM = $(x)(x+1)2$

$\frac{(x-1)(x)(x+1)2}{x}+\frac{x(x)(x+1)2}{x+1}=\frac{1(x)(x+1)2}{2}$ [2m]

$(x-1)(x+1)2+(x)(x)2=(x)(x+1)$

$\Rightarrow 2x^2-2+2x^2=x^2+x \Rightarrow 3x^2-x-2=0$ [5m]

$\Rightarrow (3x+2)(x-1)=0 \Rightarrow x=-\frac{2}{3},\ x=1$ [5m]

* Note if equation at Step 1 is linear, maximum marks at Step 2 is Att 2

Blunders(-3)

B1 Mathematical error

Worthless (0)

W1 Trying to find answers by trial and error at this point

(c) (ii)	**5 marks**	**Att 2**

$$\frac{x-1}{x}+\frac{x}{x+1}=\frac{1}{2}$$

$$x=1\Rightarrow\frac{1-1}{1}+\frac{1}{1+1}=0+\frac{1}{2}=\frac{1}{2}$$

Accept

or

$$\frac{x-1}{x}+\frac{x}{x+1}=\frac{1}{2}$$

$$x=\frac{-2}{3}\Rightarrow\frac{\frac{-2}{3}-1}{\frac{-2}{3}}+\frac{\frac{-2}{3}}{\frac{-2}{3}+1}=\frac{5}{2}-2=\frac{1}{2}$$

Accept

* Accept candidates values from (i) with correct conclusion – see B2

* Must verify in given equation i.e. $\frac{x-1}{x}+\frac{x}{x+1}=\frac{1}{2}$

* If linear equation in c(i) award at most att 2 in this part

Blunders (-3)

B1 Mathematical error e.g. fraction error

B2 Incorrect or no conclusion if using incorrect solutions

Attempt (2 marks)

A1 Correct substitution and stops

Worthless (0)

W1 Invented value(s) or uses values not from c(i)

QUESTION 4

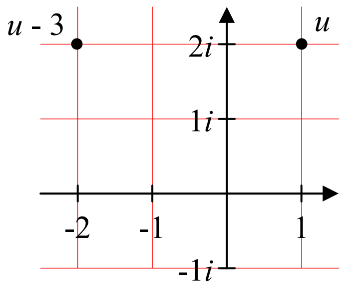
Part (a)	**15 (10, 5) marks**	**Att (3, 2)**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Part (a) **15 (10, 5) marks** **Att (3, 2)**

Let $u = 1 + 2i$, where $i^2 = -1$.
Plot on an Argand diagram
(i) u
(ii) $u - 3$.

(a) (i) **10 marks** **Att 3**
(a) (ii) **5 marks** **Att 2**

(ii) $u - 3 = 1 + 2i - 3 = -2 + 2i$



* No penalty for interchange of real and imaginary axes if consistent

Blunders(-3)
B1 Mathematical error e.g. sign error
B2 Mixes up axes but check * above.

Attempts (3 or 2 marks)
A1 Construction of grid and stops – once only Att 3
A2 Calculates $u - 3$ and stops

Slips (-1)
S1 Numerical slips

Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

Let $z = 2 + 3i$.

(i) Find z^2 in the form $x + yi$, where $x, y \in \mathbb{R}$.

(ii) Show that $z^2 = 4z - 13$.

(iii) Show that $\bar{z}^2 + 13 = 4\bar{z}$, where $\bar{z}$ is the complex conjugate of z.

(b) (i)	**10 marks**	**Att 3**

$$(2+3i)(2+3i) = 2(2+3i) + 3i(2+3i) = 4 + 6i + 6i + 9i^2 = -5 + 12i$$

[4m] [7m] [10m]

Blunders (-3)

B1 Mathematical error e.g. $i^2 \neq -1$

Attempt (3marks)

A1 $z^2 = 4 + 9i^2$ even if continues

A2 Substitution for z

(b) (ii)	**5 marks**	**Att 2**

$$4z - 13 = 4(2+3i) - 13 = 8 + 12i - 13 = -5 + 12i \quad (z^2)$$

* Accept solution of $z^2 - 4z + 13 = 0$
* Accept calculation of $4z - 13$ as same answer in part (i)

Blunders (-3)

B1 Mathematical error e.g. distribution error

Slips (-1)

S1 Incorrect or no conclusion if (i) incorrect

Attempts (2 marks)

A1 Substitution for z

(b) (iii) **5 marks** **Att 2**

I		II
$\bar{z} = \overline{2+3i} = 2-3i$		
$(2-3i)^2 + 13 = 4(2-3i)$	or	$\bar{z}^2 + 13 = 4\bar{z}$
$\Rightarrow 4-6i-6i+9i^2+13 = 8-12i$		$\Rightarrow -5-12i+13 = 4(2-3i)$
$\Rightarrow 8-12i = 8-12i$		$\Rightarrow 8-12i = 8-12i$

* Proof of $\bar{z}^2 = \overline{z^2}$ may be assumed

* If solution of $z^2 - 4z + 13 = 0$ offered at (ii) connection to this part of question must be made

Blunders(-3)

B1 Mathematical error e.g. $\bar{z} \neq 2-3i$

B2 Incorrect or no conclusion if (i) incorrect

Attempts (2 marks)

A1 Conjugate found and stops

Part (c) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

(i) Express $\dfrac{4+2i}{3-i}$ in the form $x+yi$, where $x, y \in \mathbb{R}$.

(ii) Hence, or otherwise, find the real numbers k and t such that

$$\left|\frac{4+2i}{3-i}\right|(k+5i) = \frac{1}{\sqrt{2}}\left(7+(t-1)i\right).$$

(c) (i) **5 marks** **Att 2**

$$\frac{4+2i}{3-i} = \frac{4+2i}{3-i} \times \frac{3+i}{3+i} = \frac{12+4i+6i+2i^2}{9+1} = \frac{10+10i}{10} = 1+i$$

[2m] [4m]

* Must simplify $\dfrac{10+10i}{10}$ to 1 + i for 5 marks, accept $\dfrac{10}{10} + \dfrac{10i}{10}$

Blunders (-3)

B1 Mathematical error e.g. distribution or $i^2 \neq -1$

B2 Incorrect conjugate

Attempts (2 marks)

A1 Finds conjugate and stops

(c) (ii) **10 (5, 5) marks** **Att (2, 2)**

Step 1 – Substitution of $\sqrt{2}$ or candidates modulus: 5 marks
Step 2 – Solution for k and t: 5 marks

$\left|\frac{4+2i}{3-i}\right| = |1+i| = \sqrt{1+1} = \sqrt{2}$ [2m]

$\left|\frac{4+2i}{3-i}\right|(k+5i) = \frac{1}{\sqrt{2}}(7+(t-1)i).$

$\Rightarrow \sqrt{2}(k+5i) = \frac{1}{\sqrt{2}}(7+(t-1)i)$ [5m]

$\Rightarrow 2k + 10i = 7 + (t-1)i$

Real parts: $2k = 7 \Rightarrow k = 3.5$ [2m]
Imaginary parts: $t - 1 = 10 \Rightarrow t = 11$ [5m]

* No penalty for $\sqrt{2} = 1.414$

* Accept from c(i) candidate's $\left|\frac{4+2i}{3-i}\right|$

Blunders (-3)
B1 Mathematical error e.g. modulus or transposing error
B2 Only finds one value in Step 2

Attempts (2 marks)
A1 States modulus formula and stops – Step 1
A2 Substitutes answer from (i) into (ii) – Step 1
A3 Forms equations and stops – Step 2

QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

* Do not penalise notation

Part (a) **10 marks** **Att 3**

The first term of a geometric sequence is 5 and the common ratio is 2.
Find the first four terms of the sequence.

(a) **10 marks** **Att 3**

I $a = 5 \quad r = 2$

$a = 5, \quad ar = 10, \quad ar^2 = 20, \quad ar^3 = 40$

II List

5, 10, 20 , 40

[3m] [4m] [7m] [10m]

* Accept correct answers without work

Blunders(-3)
B1 Mathematical error e.g. indices error once only if consistent
B2 Each missing term

Slips (-1)
S1 Numerical slips

Attempts (3 marks)
A1 Identifies a as 5 and/or r as 2
A2 States $T_1 = 5$ or similar

Worthless (0)
W1 Treats as an arithmetic sequence but see A1 and A2
W2 Incorrect answer(s) without work

Part (b) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

> The first three terms of an arithmetic series are 7 + 4 + 1 + ...
> **(i)** Find d, the common difference.
> **(ii)** Find T_{15}, the fifteenth term of the series.
> **(iii)** Find S_{15}, the sum of the first fifteen terms of the series.

* Answers to parts of question must be clearly identified, otherwise order as in question

(i) **10 marks** **Att 3**

$T_1 = a = 7$

$T_2 = a + d = 4 \Rightarrow 7 + d = 4 \Rightarrow d = -3$ or $d = T_2 - T_1 = 4 - 7 = -3$

* Accept correct answer without work
* Acceptable formulae – see guidelines

Blunders(-3)
B1 Mathematical error e.g. sign of d

Slips (-1)
S1 Numerical slips

Attempts (3 marks)
A1 States $a = 7$ or $T_1 = 7$

Worthless (0)
W1 Treats as a geometric series but see A1

> Note
> $d = 3$ without work 0 marks
>
> Allow its use in (ii) and (iii)

(ii) **5 marks** **Att 2**

I

$T_{15} = a + 14d = 7 + 14(-3) = 7 - 42 = -35$

or

II List

7,	4,	1,	–2,	–5,	– 8,	–11,	–14,	–17,	–20,	–23,	–26,	–29,	–32,	**–35**
T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	$\mathbf{T_{15}}$

(Assume final term is T_{15}, otherwise must indicate term)

* Accept candidate's d from (i)
* Accept correct answer without work

Blunders(-3)
B1 Mathematical error e.g. sign or altered list
B2 Incorrect term from complete list

Attempts (2 marks)
A1 Identifies a and/or d for this part of question

Worthless (0)
W1 Treats as a geometric series but see A1

(iii) **5 marks** **Att 2**

I

$$S_{15} = \frac{15}{2}(2a + 14d) = \frac{15}{2}(14 - 42) = -210$$

II List

7 + 4 + 1 – 2 – 5 – 8 – 11 – 14 – 17 – 20 – 23 – 26 – 29 – 32 – 35 = – 210

III

$$S_n = \frac{n}{2}(a + l) \qquad a = T_1 = 7, \quad l = T_{15} - 35$$

$$S_{15} = \frac{15}{2}(7 - 35) = -210$$

* Accept correct answer without work if summing list from (ii) but no retrospective marking

* Accept candidate's *d* and /or T_{15} from (ii) but no retrospective marking

Blunders (-3)

B1 Mathematical error

B2 Re-writes list but fails to sum

B3 Incorrect number of terms in re-written list

Attempts (2 marks)

A1 Any relevant step at this point

Part (c) **20 (10, 10) marks** **Att (3, 3)**

> The first three terms of a geometric sequence are
> $$h-1,\quad 2h \text{ and } 5h+3,$$
> where h is a real number greater than 1.
> **(i)** Find the value of h.
> **(ii)** The kth term of the sequence is 486. Find k.

(c) (i) **10 marks** **Att 3**

$$\frac{2h}{h-1}=\frac{5h+3}{2h} \quad [4m]$$

$$\Rightarrow 4h^2=(h-1)(5h+3)=5h^2+3h-5h-3$$

$$\Rightarrow h^2-2h-3=0 \Rightarrow (h-3)(h+1)=0 \Rightarrow h=3, h=-1.$$

[7m] [10m]

* No penalty if single value of h not selected at this point
* Treats as an arithmetic sequence, c(i) and c(ii) Att 3 + Att 3 marks at most
* $h = 3$ by trial and error must be fully justified by $T_2/T_1 = T_3/T_2$ or similar to merit 10 marks otherwise att 3. May be used for full marks in part (ii)

Blunders(-3)

B1 Mathematical error e.g. inverts one of the fractions

Attempts (3 marks)

A1 Some work of merit $T_1 \div T_2$ and stops

(c) (ii) **10 marks** **Att 3**

I $a = h - 1 = 3 - 1 = 2$

$$r=\frac{2h}{h-1}=\frac{6}{2}=3 \quad [4m]$$

$$T_k=ar^{k-1}=486 \Rightarrow 2(3^{k-1})=486 \Rightarrow 3^{k-1}=243=3^5 \Rightarrow k=6$$

[7m] [9m] [10m]

or

II $T_1=3-1=2,\quad T_2=2(3)=6,\quad T_3=5(3)+3=18$ [4m]

$T_4=18\times 3=54,\quad T_5=54\times 3=162$ [7m]

$T_6=162\times 3=486$ [9m]

$k=6$ [10m]

* Accept candidates' value of h from (i) (solution may not be possible); but use of $h = -1$ merits attempt marks at most

Blunders (-3)

B1 Mathematical error e.g. indices error using formula

B2 Selects incorrect term method II

Slips(-1)

S1 Answer $k = 5$ Method 1

Attempts (3 marks)

A1 Finds T_1 and /or T_2

A2 Some relevant work e.g. $T_k = ar^{k-1}$

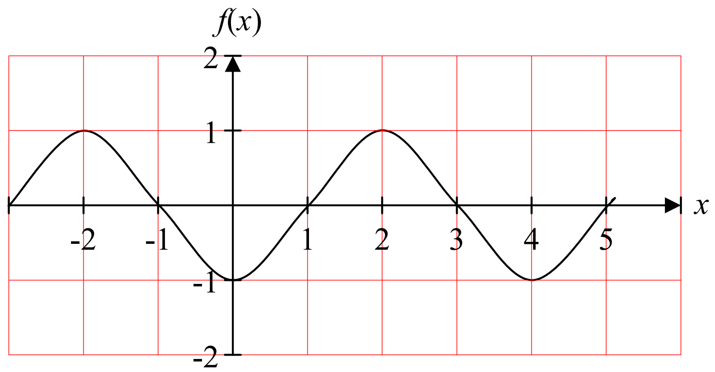
A3 Correct answer without work

QUESTION 6

Part (a)	**15 (10, 5) marks**	**Att (3, 2)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Part (a) **15 marks** **Att (3, 2)**

$f : x \to f(x)$ is a periodic function defined for $x \in \mathbb{R}$.
The period is as indicated in the diagram.



(i) Write down the period and the range of the function.
(ii) Find $f(71)$.

(a) (i) **10 marks** **Att 3**

(i) Period 4, Range [-1, 1]

* Accept correct answers without work
* If answers are unidentified assume first is Period, second Range
* Acceptable notation: Range, $-1 \rightarrow 1$, $[1, -1]$, $-1,1$ or $(-1,1)$

Blunders (-3)
B1 Confuses period and range – once only
B2 Period = –2, 2 or similar
B3 Range : single number 2
B4 Only one correct answer

Attempts (3 marks)
A1 Period and/or range marked on graph but not stated
A2 No period or worthless period and states range –1 or 1

(a) (ii) **5 marks** **Att 2**

(ii)	$f(71) = f(68+3) = f(3) = 0.$
	[2m] [5m]

* Accept correct answer without work
* Accept candidate's value for period from (i)

Blunders(-3)
B1 Mathematical error e.g. incorrect period
B2 Incorrect reading (if work shown)

Attempts (2 marks)
A1 Shows some understanding of period e.g. $f(1) = f(5)$ or similar
A2 Mentions period, word or figure 4, at this stage and stops e.g. $\frac{71}{4}$ and stops

Part (b) **20 (10, 10) marks** **Att (3, 3)**

(i)	Differentiate $(4x-1)(3-2x^2)$ with respect to x and simplify your answer.
(ii)	Given that $y=\dfrac{1}{x^2-3x}$, $x \neq 3$, find the range of values of x for which $\dfrac{dy}{dx}<0$.

(b) (i) **10 marks** **Att 3**

I			II	
$y=(4x-1)(3-2x^2)$		or	$y=12x-8x^3-3+2x^2$	[3m]
$u=4x-1$ $\quad v=3-2x^2$				
$\dfrac{du}{dx}=4$ $\quad \dfrac{dv}{dx}=-4x$	[4m]		$\dfrac{dy}{dx}=12-24x^2+4x$	[10m]
$\dfrac{dy}{dx}=(4x-1)(-4x)+(3-2x^2)(4)$	[7m]			
$=-16x^2+4x+12-8x^2$				
$=-24x^2+4x+12$	[10m]			

* Differentiation 7 marks, simplifying 3 marks
* Method II if over simplified when multiplying out only award attempt mark – (must have at least three terms)
* No use of *uv,* Method I , merits attempt i.e. has (4)(–4*x*)
* Errors in use of *uv* see guidelines

Blunders (-3)
B1 Differentiation once per term (includes sign)
B2 Differentiation, omitted term
B3 Error when tidying up (final step I) first step II – once only. See comment above for II

Attempts(3 marks)
A1 Some relevant work

(b) (ii) **10 marks** **Att 3**

I

$$y = \frac{1}{x^2 - 3x}$$

$u = 1 \qquad v = x^2 - 3x$

$\frac{du}{dx} = 0 \qquad \frac{dv}{dx} = 2x - 3$ [4m] both required

$\frac{dy}{dx} = \frac{(x^2 - 3x)(0) - (1)(2x - 3)}{(x^2 - 3x)^2}$ [7m]

$\frac{dy}{dx} = \frac{-2x + 3}{(x^2 - 3x)^2} < 0 \Rightarrow -2x + 3 < 0 \Rightarrow x > 1.5$ [10m]

II

$y = \frac{1}{x^2 - 3x} = (x^2 - 3x)^{-1}$ [4m]

$\frac{dy}{dx} = -1(2x - 3)(x^2 - 3x)^{-2}$ or $\frac{-1(2x - 3)}{(x^2 - 3x)^2}$ [7m]

$\frac{dy}{dx} = \frac{-2x + 3}{(x^2 - 3x)^2} < 0 \Rightarrow -2x + 3 < 0 \Rightarrow x > 1.5$ [10m]

* Differentiation 7 marks – 3 marks solving $\frac{dy}{dx} < 0$

Blunders (-3)

B1 Differentiation once per term (includes sign)
B2 Differentiation, omitted term
B3 Mathematical error solving inequality – once only

Slips(-1)

S1 Numerical slips

Attempts (3 marks)

A1 Some relevant work e.g. identifies *u* and /or *v* and stops
A2 No quotient or chain rule in differentiation e.g. has $\frac{dy}{dx}$ as $\frac{0}{2x-3}$
A3 Over simplifies to $y = x^2 - 3x$ and continues

Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Let $f(x) = 2x + \dfrac{1}{x}$, where $x \in \mathbb{R}$ and $x \neq 0$.

(i) Find the equation of the tangent to the curve $y = f(x)$ at the point $P(1, 3)$.

(ii) Q is another point on the curve $y = f(x)$ such that the tangent at Q is parallel to the tangent at P. Find the co-ordinates of Q.

(c) (i)	**10 (5, 5) marks**	**Att (2, 2)**

Step 1 Differentiation: 5 marks
Step 2 Equation of tangent: 5 marks

$f(x) = 2x + \dfrac{1}{x} \Rightarrow f'(x) = 2 - \dfrac{1}{x^2}$ or $f'(x) = 2 - x^{-2}$ [5m]

or $f(x) = 2x + \dfrac{1}{x} = \dfrac{2x^2+1}{x} \Rightarrow f'(x) = \dfrac{x(4x)-(2x^2+1)1}{x^2} = \dfrac{4x^2-2x^2-1}{x^2} = 2 - \dfrac{1}{x^2}$ [5m]

$f'(x) = 2 - \dfrac{1}{x^2} \Rightarrow f'(1) = 2 - 1 = 1$ [2m]

$y - 3 = 1(x - 1)$ [5m]

* $f'(x)$ as 2 + 0/1 and continues merits at most Att 2 (Step 1) + Att 2 (Step 2)

Blunders (-3)
B1 Differentiation once per term (includes sign) – Step 1
B2 Differentiation, omitted term – Step 1
B3 Mathematical error simplifying $f(x)$
B4 Error finding slope of tangent e.g. use of *P* – Step 2

Attempts(2 marks)
A1 Simplifies $f(x)$ partially and stops e.g. has $2x + x^{-1}$ Step 1
A2 Some relevant work e.g. states slope of tangent is $f'(x)$
A3 Some effort at finding equation of tangent Step 2 – [Formula of line does not merit attempt mark]

Worthless (0)
W1 Finds $f(1)$ and stops

(c) (ii)	**5 marks**	**Att 2**

$f'(x) = 2 - \dfrac{1}{x^2} = 1 \Rightarrow -\dfrac{1}{x^2} = -1 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$ [2m]

$f(-1) = 2(-1) + \dfrac{1}{-1} = -2 - 1 = -3$. [5m] Point $(-1, -3)$

Blunders (-3)
B1 Mathematical error

Attempts(2 marks)
A1 Some relevant work e.g. sets up $f'(x) = 1$
A2 Finds (1, 3) again
A3 Correct answer without work

QUESTION 7

Part (a)	**15 marks**	**Att 5**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Part (a) **15 marks** **Att5**

Differentiate $x^3 - 7x^2 + 6x$ with respect to x.

(a) **15 marks** **Att 5**

$\frac{dy}{dx} = 3x^2 - 14x + 6$ or $f'(x) = 3x^2 - 14x + 6$

* Correct answer without work or notation: full marks
* If done from first principles, ignore errors in procedure – just mark the answer
* Only one non zero term correct, award 9 marks

Blunders (-3)

B1 Differentiation error once per term, (to a maximum of 2) – includes sign

B2 Term omitted each time

Attempts (5 marks)

A1 A correct step in differentiation from first principles

A2 A correct coefficient or a correct index of x in one of the term(s)

A3 Mentions $\frac{dy}{dx}$ or $f'(x)$

Part (b) **20 (10, 10) marks** **Att (3, 3)**

(i) Differentiate $\dfrac{3x+1}{x-2}$ with respect to x.

Write your answer in the form $\dfrac{k}{(x-2)^n}$, where $k, n \in \mathbb{Z}$.

(ii) Given that $y = (x^2 - 2x - 9)^4$, find the value of $\dfrac{dy}{dx}$ when $x = -2$.

(b) (i) **10 marks** **Att 3**

$$y = \frac{3x+1}{x-2}$$

$u = 3x+1 \qquad v = x-2$

$\dfrac{du}{dx} = 3 \qquad \dfrac{dv}{dx} = 1 \qquad$ [4m]

$\dfrac{dy}{dx} = \dfrac{(x-2)(3) - 1(3x+1)}{(x-2)^2} \qquad$ [9m]

$\Rightarrow \dfrac{dy}{dx} = \dfrac{3x-6-3x-1}{(x-2)^2} = \dfrac{-7}{(x-2)^2} \qquad$ [10m]

Note $\dfrac{dy}{dx} = \dfrac{-7}{x^2-4x+4} \qquad$ [9m]

- * Apply penalties as in guidelines
- * No penalty for missing brackets if multiplication implied (decide by later work)
- * No marks for writing *u/v* formula from tables and stopping
- * No use of *u/v* formula, has $\dfrac{dy}{dx}$ as $\dfrac{3}{1}$ merits attempt mark only

Blunders (-3)

B1 Differentiation errors, once per term

B2 Error in formula - see guidelines

Slips (-1)

S1 Numerical slips

Attempts (3 marks)

A1 *u* and/or *v* correctly identified and stops

A2 Any correct differentiation

(b) (ii) **10 marks** **Att 3**

I

$$y = (x^2 - 2x - 9)^4$$

$\dfrac{dy}{dx} = 4(x^2 - 2x - 9)^3(2x - 2)$ [9m]

$x = -2:$

$\dfrac{dy}{dx} = 4((-2)^2 - 2(-2) - 9)^3(2(-2) - 2) = 24$ [10m]

II

$u = (x^2 - 2x - 9)^4$ $\quad\quad y = u^4$

$\dfrac{du}{dx} = 2x - 2$ $\quad\quad \dfrac{dy}{du} = 4u^3$ [4m]

$\dfrac{dy}{dx} = \dfrac{dy}{du}.\dfrac{du}{dx} = 4u^3(2x - 2) = 4(x^2 - 2x - 9)^3(2x - 2)$ [9m]

$x = -2:$

$\dfrac{dy}{dx} = 4((-2)^2 - 2(-2) - 9)^3(2(-2) - 2) = 24$ [10m]

* Apply penalties as in guidelines for differentiation
* No penalty for missing brackets if multiplication implied (decide by later work)
* Treats $4(x^2 - 2x - 9)^3$ and $(2x - 2)$ as separate parts - see above
* If differentiation correct accept answer **24** with or without work for final marks, answer **24** with <u>no work at all</u> award attempt 3 only

Blunder (-3)
B1 Differentiation error once per part - see parts above e.g. $(2x - 2)$ omitted

Attempts (3 marks)
A1 Some correct element of the chain rule e.g. index 3 or coefficient 4
A2 $u = x^2 - 2x - 9$ and stops
A3 $\dfrac{dy}{dx} = 2x - 2$ and continues or not, only attempt

Worthless (0)
W1 Substitutes $x = -2$ into y and evaluates y

Part (c) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

A ball is rolled in a straight line along a surface. The distance, s metres, the ball travels is given by $$s = 18t - 2t^2$$ where t is the time in seconds from the instant the ball begins to move. **(i)** Find the speed of the ball after 3 seconds. **(ii)** How far is the ball from the starting point when it stops moving? **(iii)** Show that the speed of the ball decreases at a constant rate while it is moving.

* Units: Penalise as per guidelines
* No retrospective marking
* No penalty for incorrect notation
* If parts of (c) are unlabelled, and the context doesn't identify which part is which, assume the questions were answered in sequence from (c)(i) to (c)(iii)

(c) (i) **5 marks** **Att 2**

$\frac{ds}{dt} = 18 - 4t = 18 - 4(3) = 6 \text{ m s}^{-1}$ at $t = 3$

* Correct answer without work: Att 2

Blunders (-3)
B1 Mathematical error e.g. differentiation error
B2 Incorrect or no value of t substituted into $\frac{ds}{dt}$

Slips(-1)
S1 Incorrect or no units (only apply if answer correct)

Attempts (2 marks)
A1 Partial differentiation and stops
A2 $\frac{ds}{dt}$ mentioned

Worthless (0)
W1 $t = 3$ substituted into original equation

(c) (ii) **5 marks** **Att 2**

$$\frac{ds}{dt} = 18 - 4t = 0 \Rightarrow 4t = 18 \Rightarrow t = 4.5 \text{ s}$$
$$s = 18t - 2t^2 = 18(4.5) - 2(4.5)^2 = 40.5 \text{ m}$$

* No use of derivative merits 0 at this part
* Accept candidates derivative from (i)

Blunders(-3)
B1 Mathematical error e.g. solving equation
B2 $\frac{ds}{dt} \neq 0$

Slips (-1)
S1 No units or incorrect unit (only apply if answer correct)

Attempts (2 marks)
A1 Some use or mention of derivative at this part

(c) (iii) **5 marks** **Att 2**

$$\left[\frac{d^2s}{dt^2}\right] = -4 \qquad [5\text{m}]$$

* If candidates 2nd derivative does not give a negative constant apply, slip -1, if explanation not given
* Candidates may use notation $\frac{dv}{dt}$

Blunders (-3)
B1 Error in differentiation

Attempts (2 marks)
A1 Graphical approach or substitution into $\frac{ds}{dt}$
A2 Mentions acceleration or $\frac{d^2s}{dt^2}$ or $\frac{dv}{dt}$

Worthless (0)
W1 $\frac{ds}{dt}$ and stops

QUESTION 8

Part (i)	**15 marks**	**Att 5**
Part (ii)	**10 marks**	**Att 3**
Part (iii)	**10 marks**	**Att 3**
Part (iv)	**5 marks**	**Att 2**
Part (v)	**10 marks**	**Att 3**

Part (i) **15 marks** **Att 5**

Let $f(x) = \dfrac{1}{x+2}$, where $x \in \mathbb{R}$ and $x \neq -2$.

(i) Copy and complete the following table:

x	-5	-4	-3	$-2{\cdot}5$	$-1{\cdot}5$	-1	0	1
$f(x)$		$-0{\cdot}5$	-1	-2				

(i) **15 marks** **Att 5**

x	-5	-4	-3	$-2{\cdot}5$	$-1{\cdot}5$	-1	0	1
$f(x)$	$-\frac{1}{3}$	$-0{\cdot}5$	-1	-2	2	1	0.5	$\frac{1}{3}$

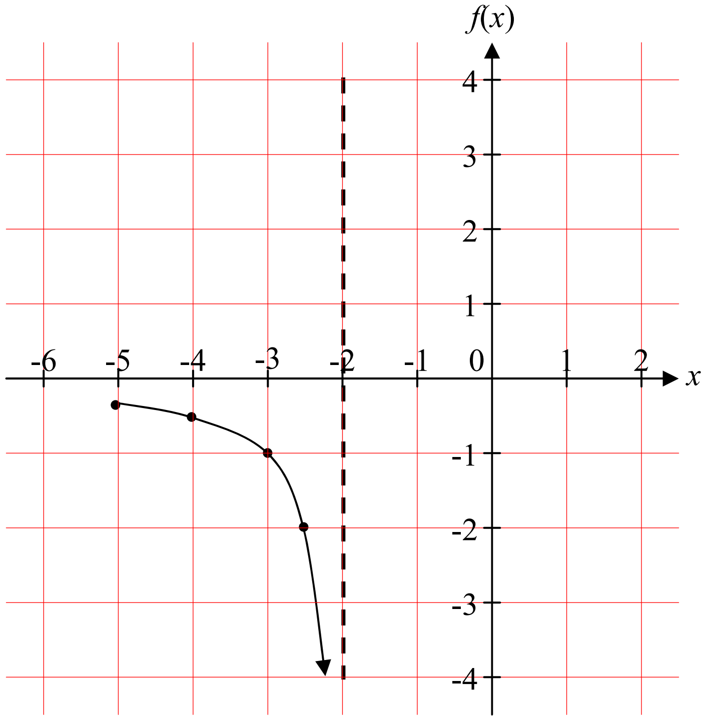
* Values of $f(x) = x + 2$ calculated (all/some correct) misreading which oversimplifies, Att 5
* Accept correct values without work for full marks
* Do not penalise if candidate writes 1/3 as 0.3

Blunder (-3)
B1 Mathematical error – once if consistent
B2 Treats the function as $f(x) = \dfrac{1}{x} + 2$ or $f(x) = \dfrac{1}{x} + \dfrac{1}{2}$, even if $f(x) = \dfrac{1}{x+2}$ written
B3 Where no work shown, each missing or incorrect value

Attempts (5 marks)
A1 Some relevant work e.g. one non-given value correct

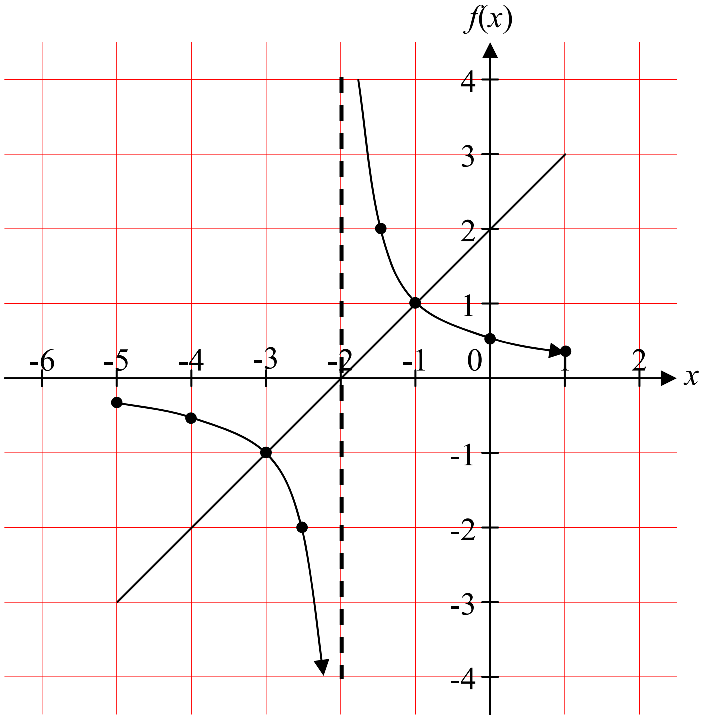
Part (ii)	**10 marks**	**Att 3**
Part (iii)	**10 marks**	**Att 3**

(ii) The diagram shows part of the graph of the function f.
Copy and complete the graph from $x = -5$ to $x = 1$.



(iii) On the same diagram, draw the graph of the function $g(x) = x + 2$ in the domain $-5 \le x \le 1$, where $x \in \mathbb{R}$.

(ii) $f(x)$	**10 marks**	**Att 3**
(iii) $g(x)$	**10 marks**	**Att 3**



(ii) $f(x)$ — 10 marks — Att 3

* Accept candidates values from (i) if not oversimplified
* If candidates work in section (i) merits 0 marks, award attempt mark at most in section (ii)
* If candidates re–do without reference to part(i) mark as above – no retrospective marking

Blunders (-3)
B1 Joins both sides of graph i.e. ignoring asymptote
B2 Plots points but does not join or joined incorrectly
B3 Error in plotting once if consistent
B4 Error in scales

Attempts(3)
A1 One point correctly plotted and stops

Worthless (0)
W1 Free hand sketch with no correct points

(iii) $g(x)$ — 10 marks — Att 3

x	–5	–4	–3	–2	–1	0	1
$g(x) = x + 2$	–3	–2	–1	0	1	2	3

[For reference only]

* Only two points required
* Two (2) correct points and correct graph award 10 marks
* Two (2) correct points not resulting in a line graph blunder – 7 marks
* No penalty for not drawing on same diagram

Blunders(-3)
B1 Points plotted but not joined
B2 Incomplete domain

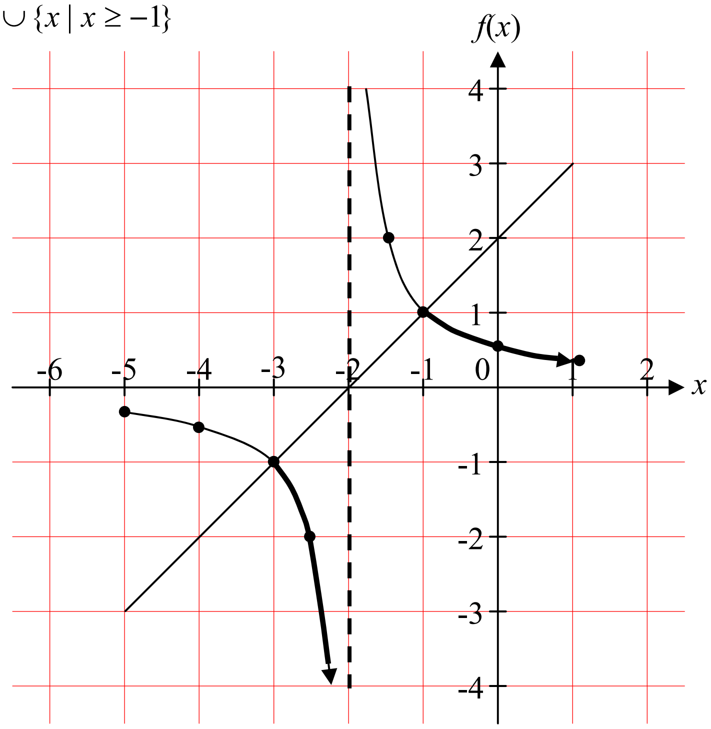
Attempts (3 marks)
A1 Finds point or plots one correct point

Part (iv) **5 marks** **Att 2**

Use your graphs to estimate the range of values of x for which $f(x) \le g(x)$.

(iv) **5 marks** **Att2**

$\{x \mid -3 \le x < -2\} \cup \{x \mid x \ge -1\}$



Blunders (-3)

B1 Mathematical error

B2 $f(x) \ge g(x)$

Attempts (2 marks)

A1 Shows on graph only

A2 States/mentions $f(x)$ below $g(x)$

A3 Finds $f(x) \cap g(x)$

Part (v) **10 marks** **Att 3**

Prove that the curve $y = f(x)$ has no turning points.

(v) **10 marks** **Att 3**

$$f(x) = \frac{1}{(x+2)} = (x+2)^{-1} \qquad [4m]$$

$$\Rightarrow \; f'(x) = -1(x+2)^{-2} = \frac{-1}{(x+2)^2} \qquad [7m]$$

$$f'(x) = \frac{-1}{(x+2)^2} = 0 \text{ Impossible or } f'(x) \neq 0 \text{ or } f'(x) < 0 \qquad [10m]$$

or

$$f(x) = \frac{1}{(x+2)}$$

$$u = 1, \qquad v = x + 2$$

$$\Rightarrow \frac{du}{dx} = 0, \qquad \frac{dv}{dx} = 1 \qquad [4m]$$

$$f'(x) = \frac{(x+2)(0) - (1)(1)}{(x+2)^2} \qquad [7m]$$

$$f'(x) = \frac{-1}{(x+2)^2} = 0 \text{ Impossible or } f'(x) \neq 0 \text{ or } f'(x) < 0 \qquad [10m]$$

* Finds $f'(x)$ correctly 7 marks, conclusion 3 marks

* No quotient or chain rule in differentiation, merits attempt mark at most

Blunders (-3)

B1 Differentiation once per term (includes sign)

B2 Differentiation, omitted term

B3 None or incorrect conclusion

Attempts (3 marks)

A1 Mentions $f'(x)$

A2 States function is always "decreasing"

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2011

Marking Scheme

MATHEMATICS – Paper 2

Ordinary Level

GENERAL GUIDELINES FOR EXAMINERS – PAPER 2

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

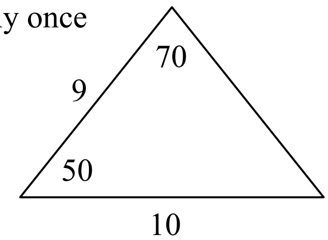
12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

Application of penalties throughout scheme

Penalties are applied subject to marks already secured.

Blunders - examples of blunders are as follows:

- Algebraic errors: $8x + 9x = 17x^2$ *or* $5p \times 4p = 20p$.
- Sign error: $-3(-4) = -12$ *or* $(-3)^2 = -9$.
- Fraction error: Incorrect fraction inversion etc. apply once.
- Cross-multiplication error.
- Error in misplacing the decimal point.
- Transposing error: $-2x - k + 3 = 0 \Rightarrow -2x = 3 + k$ *or* $-3x = 6 \Rightarrow x = 2$.
  or $4x = 12 \Rightarrow x = 8$ each type once per section.
- Distributive law errors (once per pair of brackets)
  $\frac{1}{2}(3 - x) = 6 \Rightarrow 6 - 2x = 6$ *or* $-(4x + 3) = -4x + 3$ *or* $3(2x + 4) = 6x + 4$
- Expanding brackets incorrectly: $(2x - 3)(x + 4) = 8x^2 - 12x$.
- Omission, if work not oversimplified, unless directed otherwise.
- Index error, each time unless directed otherwise.
- Factorisation: error in one or both factors of a quadratic, apply once
  $2x^2 - 2x - 3 = (2x - 1)(x + 3)$.
- Root errors from candidate's factors, error in one or both roots, apply once
- Incorrect substitution into formulae (where not an obvious slip):
  e.g. $2x^2 + 3x + 4 = 0 \Rightarrow x = \dfrac{-3 \pm \sqrt{9 - 4(2)(4)}}{2(3)}$
  or $\dfrac{10}{\sin 70} = \dfrac{9}{\sin 50}$.



- Incorrectly treating co-ordinates as (x_1, x_2) and (y_1, y_2) when using co-ordinate geometry formula.
- Errors in formula for example:
  $\dfrac{y_2 + y_1}{x_2 + x_1}$ *or* $A = P\left(1 + \dfrac{n}{100}\right)^r$ *or* $a^2 = b^2 + c^2 + bc \cos A$ *or* $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$, except as indicated in scheme.

Note: A correct relevant formula isolated and stops is awarded the attempt mark if the formula is not in the *Formulae and Tables* booklet.

Slips – examples are as follows:

- Numerical slips such as: 4 + 7 = 10 *or* $3 \times 6 = 24$ *but* $5 + 3 = 15$ is a blunder.
- An omitted round-off to a required level of accuracy or an incorrect round-off to the incorrect accuracy or an early round-off that affects accuracy are penalised as a slip once in each section.
- However, an early round-off which has the effect of simplifying the work is at least a blunder.
- The omission of the units of measurement in an answer or giving the incorrect units of measurement is treated as a slip once in each section where the candidate would otherwise have obtained full marks in that section. This applies to Q1 (a) (i), (ii), (b) (i) and (c) (i), (ii) and to Q5 and (b) (i), (ii).

Misreadings

- Examples such as 436 for 346 will not alter the nature of the question and are penalised -1.
- However, writing 5026 as 5000 would alter the work and is penalised as at least a blunder.

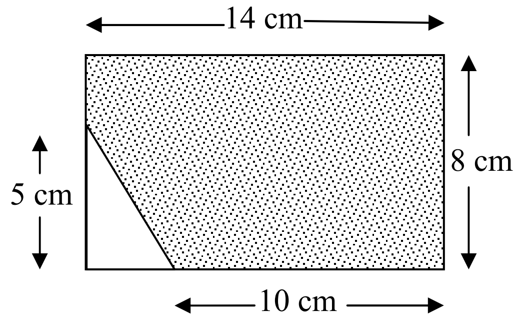
QUESTION 1

Part (a)	**10 (5, 5) marks**	**Att (2, -)**
Part (b)	**20 (15, 5) marks**	**Att (5, 2)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a)	**10 (5, 5) marks**	**Att (2, -)**

(i) Calculate the area of the rectangle shown in the diagram.

(ii) Hence, calculate the area of the shaded region.



(a) (i)	**5 marks**	**Att 2**

(i) Area of rectangle = $14 \times 8 = 112$ cm^2.

* Accept correct answer without work, including an answer written on a diagram.
* Only penalise missing/incorrect unit where answer given would otherwise merit full marks.

Award the following marks only:

5 marks Correct answer .
4 marks One slip or misreading or incorrect/missing units. [see 2nd *].
2 marks One blunder, or two slips, or some work of merit, e.g. 14×8 or ½×5.
0 marks Incorrect answer without work, use of perimeter/volume formulae, subject to work of merit.

(a) (ii)	**5 marks**	**Att -**

(ii) Area of unshaded region = $0{\cdot}5 \times 4 \times 5 = 10$ cm^2
Area of shaded region = 102 cm^2

* Accept correct answer without work, including an answer written on a diagram.
* Accept in section (ii) an answer consistent with the candidate's answer to section (i).
* Only penalise missing/incorrect unit where answer given would otherwise merit full marks.

Award the following marks only:

5 marks Correct or consistent answer in (a) (ii) (with work).
4 marks One obvious slip or misreading or incorrect/missing units. [see 3rd *]
0 marks Otherwise.

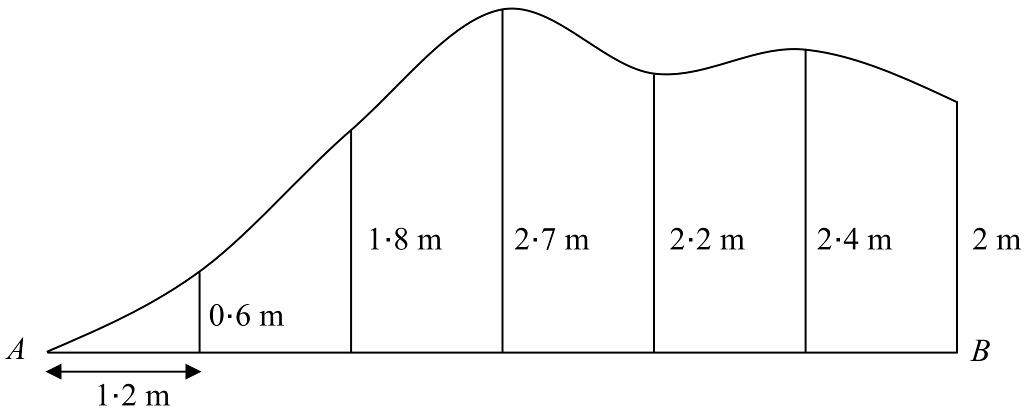
Treat as separate blunders:

- Wrong dimension
- Incorrect operation.

Part (b) **20 (15, 5) marks** **Att (5, 2)**

The sketch shows a section of a wall that is to be painted.

At equal intervals of 1·2 m along the bottom of the wall, [*AB*], perpendicular measurements are made to the uneven edge, as shown on the sketch.



(i) Use Simpson's rule to estimate the area of the section of the wall.

(ii) How many litres of paint are required to paint the section of the wall, if 1 litre of paint covers an area of 2·2 m^2? Give your answer correct to the nearest litre.

(b) (i) Use of formula **10 marks** **Att 3**
Calculations **5 marks** **Att 2**

$$\text{Area} \approx \tfrac{h}{3}\left(F + L + 2\Sigma O + 4\Sigma E\right)$$
$$= \tfrac{1\cdot 2}{3}\left[0 + 2 + 2(1\cdot 8 + 2\cdot 2) + 4(0\cdot 6 + 2\cdot 7 + 2\cdot 4)\right] \qquad [\textit{10 marks}]$$
$$= 0{\cdot}4[2 + 8 + 22{\cdot}8]$$
$$= 0{\cdot}4[32{\cdot}8]$$
$$= 13{\cdot}12 \text{ m}^2 \qquad [5 \textit{ marks}]$$

* Allow $^{h}/_{3}$ = (F + L + TOFE) and penalise in calculations if formula not used correctly.

* Only penalise missing/incorrect unit where answer given would otherwise merit full marks.

Award the following marks only:
Substitution:

10 marks Fully correct substitution.
3 marks Any work of merit e.g. one or two of: ($\tfrac{h}{3}$) or (F and L) or (TOFE) correct.
0 marks Otherwise.

Treat as separate blunders:

- Incorrect $^{h}/_{3}$
- Incorrect F and/or L or extra terms with F and/or L.
- Incorrect or omitted TOFE
- Use of Trapezoidal rule: (Answer = 12·84m^2).

Calculations:

5 marks Correct or consistent answer (accept without work if substitution done).
4 marks Slip in calculations, misreading, incorrect/no units.
2 marks Omitted TOFE or blunder in calculations, some work of merit.
0 marks Incorrect or inconsistent answer without work.

Note: Where calculation work not shown.			
I	**II**	**III**	**IV**
No Substitution Ans: $13{\cdot}12\,m^2$ (3 marks + 2marks)	Substitution (mark =*) Ans:$13{\cdot}12m^2$/ consistent (* marks + 5 marks)	Substitution (mark =*) Ans: 13·12/consistent (* marks + 4 marks)	Substitution (mark =*) Ans: #/not consistent (* marks + 0 marks)

(b) (ii) **5 marks** **Att 2**

Number of litres = $\frac{13\cdot12}{2\cdot2}$ = 5·9 ≈ 6 litres.

* Accept correct or consistent answer without work.

Award the following marks only:

5 marks Correct or consistent answer.
4 marks One slip or misreading or no rounding off.
2 marks Some work of merit.
0 marks Incorrect or inconsistent answer withoutwork.

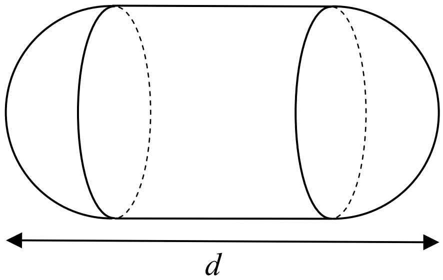
Part (c) **20 (10, 10) marks** **Att (3, 3)**

A solid object consists of a cylinder with hemispherical ends, as shown.
The cylinder and hemispheres have the same radius.

The volume of each hemisphere is 144π cm^3.

(i) Find the radius of each hemisphere.

(ii) The total volume of the object is 720π cm^3.
Find the length, *d*, of the object.



(c) (i) **10 marks** **Att 3**

(c) (ii) **10 marks** **Att 3**

(c) (i)

Volume of hemisphere = $\frac{2}{3}\pi r^3 = 144\pi$

$r^3 = 216$

$r = 6$ cm

(c) (ii)	(c) (ii)
Volume of cylinder = $720\pi - 288\pi = 432\pi$ $\pi r^2 h = 432\pi$ $36h = 432$ $h = 12$cm Length $d = 12 + 6 + 6 = 24$cm	Volume = $\frac{2}{3}\pi r^3 + \pi r^2 h + \frac{2}{3}\pi r^3$ $\Rightarrow 720\pi = \frac{2}{3}\pi 6^3 + \pi(6)^2 h + \frac{2}{3}\pi 6^3$ $\Rightarrow 720\pi = \frac{2}{3}(216)\pi + 36\pi h + \frac{2}{3}(216)\pi$ $\Rightarrow 720\pi = 144\pi + 36h\pi + 144\pi$ $\Rightarrow 720\pi - 288\pi = 36h\pi$ $\Rightarrow 36h = 432$ $\Rightarrow h = 12$ cm Length $d = 12 + 6 + 6 = 24$cm

* Accept an answer in section (ii) consistent with the candidate's answer to section (i).

* Only penalise missing/incorrect unit where answer given would otherwise merit full marks.

Award the following marks only:

10 marks Fully correct or consistent answer.
9 marks One slip, misreading or no units. [see *above]
7 marks One blunder, or one blunder and one slip, or two slips, e.g. $r = 72(^{216}/_{3})$ with work.
4 marks Two blunders, or one blunder and two slips.
3 marks Some work of merit.
0 marks Incorrect or inconsistent answer without work.

Treat as separate blunders:

- incorrect sphere equation in (i), e.g. volume of sphere = 144π
- incorrect volume of sphere formula and continues in (i), e.g. $k(\pi r^3)$, $k \neq \frac{4}{3}$ *or* $\frac{2}{3}$
- transposition error
- incorrect use of $\sqrt[3]{\ }$
- $\pi r^2 h = 720\pi$ in section (ii) and continues.

Attempts (3 marks)

- Some work of merit e.g. equation set up or r substituted into relevant volume formula (c)(ii).
- Correct formula with any correct substitution.
- Correct answer without work in each section.
- $\pi r^2 h = K\pi$ and continues.

Worthless (0 marks)

- Use of any area formula, e.g. $4\pi r^2$, $2\pi rh, \frac{4}{3}\pi r^2$.
- Non sphere formula used in (i), subject to work of merit.
- Non cylinder formula used in (ii), subject to work of merit.

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, -, 2, 2)**
Part (c)	**20 (10, 10) marks**	**Att (3, -)**

Apply the following to each section of question 2 and question 3.
If the correct formula is not written, any sign or substitution error is at least a blunder.

Blunders (-3)
B_a Two or more incorrect substitutions.
B_b Switches x and y in substituting or treats as a pair of couples (x_1, x_2) and (y_1, y_2).
B_c Error in the central sign in a formula.

Slips (-1)
S_a One incorrect non-central sign.
S_b One incorrect substitution in the formula.
S_c Obvious misreading of one co-ordinate.

Attempts
A_a The correct relevant formula written and stops. [If not transcribed from tables]
A_b The co-ordinates of a relevant point written with x_1 and y_1 identified.
A_c A correct substitution into relevant formula and stops.

Worthless (0 Marks)
W_a Correct formula transcribed from tables and stops.

Part (a) **10 marks** **Att 3**

Verify that the point (2, –4) is on the line $3x - y = 10$.

(a) **10 marks** **Att 3**

$3x - y = 10 \Rightarrow 3(2) - (-4) = 1 \downarrow_{7 marks} 0 \Rightarrow 6 + 4 = 10 \downarrow_{9 marks} \Rightarrow 10 = 10$ or "pt on line".
[Hence (2, –4) is on the line].
or
May substitute $x = 2$ to find $y = -4$ or *visa versa*.

Blunders (-3)
B1 Substitution but work not completed to arrive at LHS = RHS.

Slips(-1)
S1 No conclusion if L.H.S.$\neq$ R.H.S.

Attempts (3 marks)
A1 Some substitution attempted or some correct work with the equation e.g. $3x - y - 10 = 0$.
A2 Plots (2, –4) for this section.

Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, -, 2, 2)**

> $P(2, 8)$, $Q(4,-1)$ and $R(6, 0)$ are three points.
> **(i)** Find the slope of PR.
> **(ii)** Show that PR is perpendicular to RQ.
> **(iii)** Find the equation of RQ.
> **(iv)** Find the co-ordinates of the point at which RQ intersects the y-axis.

(b) (i)	**5 marks**	**Att 2**

Slope of $PR = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0-8}{6-2} \downarrow_{2\,marks} = \dfrac{-8}{4}$ or -2

* Accept consistent answers in this and subsequent sections

(b) (ii)	**5 marks**	**Att (-)**

Slope of $RQ = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0+1}{6-4} = \dfrac{1}{2}$ $m_{PR} \times m_{RQ} = -2 \times \dfrac{1}{2} = -1$ $[\Rightarrow PR \text{ is perpendicular to } RQ]$	Using Pythagoras: $\lvert PQ\rvert^2 = \lvert PR\rvert^2 + \lvert RQ\rvert^2$ i.e $\sqrt{85}^2 = \sqrt{80}^2 + \sqrt{5}^2$ $[\Rightarrow PR \text{ is perpendicular to } RQ]$

Award the following marks only: **[Section (b) (ii)]**
5 marks Correct or consistent answer (with work).
4 marks One obvious slip or misreading that does not oversimplify.
0 marks Otherwise.

Note: Required for full marks, either:
- $-2 \times \dfrac{1}{2} = -1$
- $m_{PR} \times m_{RQ} = -1$
- Statement or text to include sign and inversion.

(b) (iii)	**5 marks**	**Att 2**

Slope of $RQ = \dfrac{1}{2}$ & point $(6, 0)$
$y - y_1 = m(x - x_1)$
$y - 0 = \frac{1}{2}(x - 6)$ or $x - 2y - 6 = 0$.

* Accept candidate's slope from (b) (ii).

(b) (iv)	**5 marks**	**Att 2**

Solving equation of RQ:$x - 2y - 6 = 0$ and equation of y-axis:$x = 0$,
gives $0 - 2y - 6 = 0 \quad \Rightarrow \quad y = -3$ $[\Rightarrow$point $= (0, -3)]$

Award the following marks only: **[Section (b) (i), (b) (iii), (b) (iv)]**
5 marks Fully correct or consistent answer.
4 marks One slip.
2 marks One blunder, or one blunder and one slip, or two slips, or some work of merit.
0 marks Incorrect or inconsistent answer without work.

Treat as separate blunders:

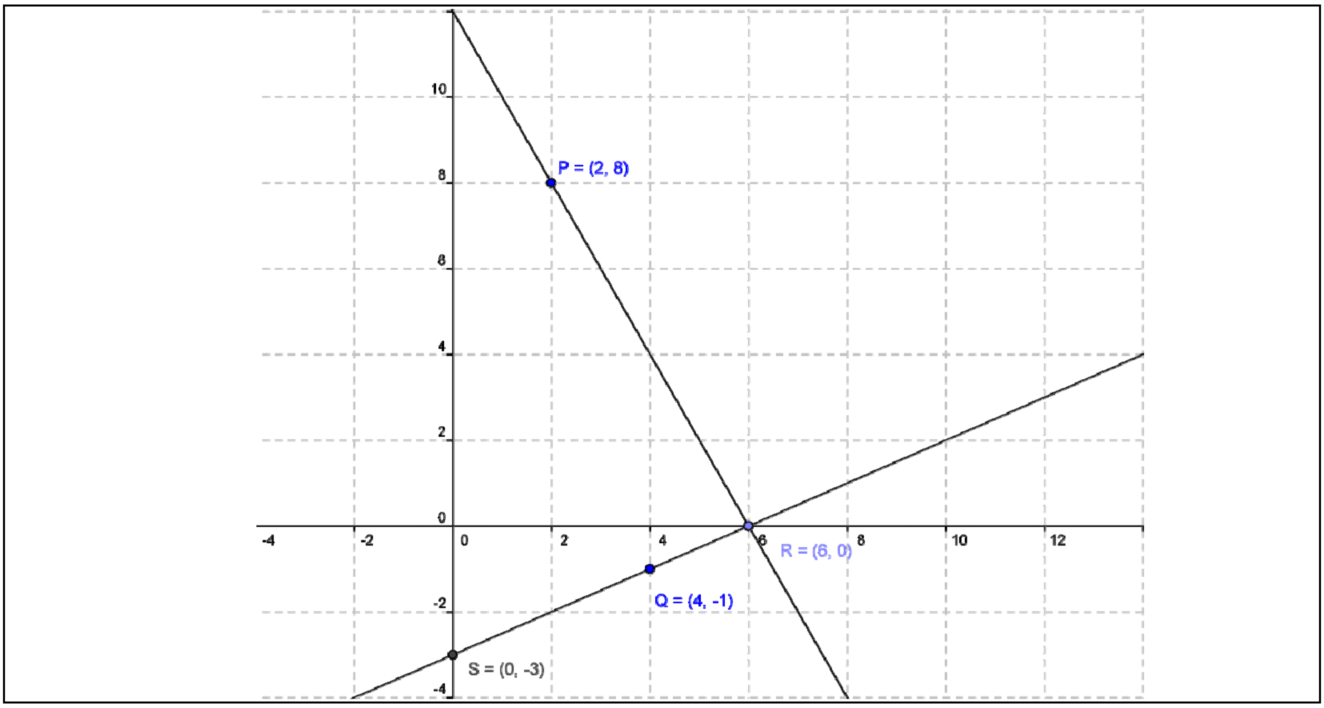
- Mathematical error. Incorrect relevant formula with substitution and continues e.g. $\dfrac{x_2 - x_1}{y_2 - y_1}$
- Fails to finish e.g. slope = $\dfrac{0-8}{6-2}$.
- Finds intercept on the x-axis i.e. $(6, 0)$ (with work).

Misreadings(-1)
- Finds slope of PQ.

Slips(-1)
- No conclusion or incorrect conclusion.

Attempts (2 marks)
- Some correct substitution attempted.
- Some work of merit e.g. writes $x = 0$ and stops.
- Correct answer without work [applies to section (b) (i), (b) (ii), (b) (iv)].
- Plots a correct point/s per section. [except (b)(ii)].

Worthless (0 marks)
- Irrelevant formula, even if substituted but subject to A_b.
- Writes $y = 0$ and stops.



Part (c)	**20 (10, 10) marks**	**Att (3, -)**

$A(-1, -6)$, $B(6, 8)$ and $C(2, 5)$ are three points.
(i) Find the area of the triangle ABC.
(ii) Find the co-ordinates of two possible points D on the x-axis such that area of triangle ABD = area of triangle ABC.

(c) (i)	**10 marks**	**Att 3**

$A(-1, -6)$, $B(6, 8)$, $C(2, 5)$
$(0, 0)$, $(7, 14)$, $(3, 11)$
Area

$$= \tfrac{1}{2}|x_1y_2 - x_2y_1| = \tfrac{1}{2}|7(11) - (3)(14)|\downarrow_{4\,marks} = \tfrac{1}{2}|77 - 42|\downarrow_{7\,marks} = \tfrac{1}{2}|35|\downarrow_{7\,marks} = 17\cdot 5.$$

or

$$\text{Area} = \tfrac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$
$$= \tfrac{1}{2}|-1(8-5) + 6(5+6) + (2)(-6-8)|\downarrow_{4\,marks} = \tfrac{1}{2}|-3 + 66 - 28|\downarrow_{7\,marks} = \tfrac{1}{2}|35|\downarrow_{7\,marks}$$
$$= 17\cdot 5$$

or

$$\text{Area} = \tfrac{1}{2}[x_1y_2 + x_2y_3 + x_3y_1 - x_1y_3 - x_3y_2 - x_2y_1]$$
$$= \tfrac{1}{2}|-1(8) + 6(5) + 2(-6) - (-1)5 - (2)8 - 6(-6)|\downarrow_{4\,marks}$$
$$= \tfrac{1}{2}|-8 + 30 - 12 + 5 - 16 + 36|\downarrow_{7\,marks} = 17\cdot 5$$

* $\tfrac{1}{2}|-35| = -17\cdot 5$ incurs no penalty.

Blunders (-3)
B1 Incorrect relevant formula and continues e.g. $\tfrac{1}{2}|x_1y_2 + x_2y_1|$ or omits the $\tfrac{1}{2}$.
B2 An incorrect or no translation.

Attempts (3 marks)
A1 Uses the distance formula or the perpendicular distance formula.
A2 Plots one or more points on a scaled diagram.
A3 A correct answer without work.

Worthless (0 marks)
W1 Irrelevant formula and stops e.g. $\tfrac{1}{2}$ on its own.

(c) (ii)	**10 marks**	**Att -**

$A(-1, -6)$, $B(6, 8)$, $D(k, 0)$
$(0, 0)$, $(7, 14)$, $(k+1, 6)$

$$Area = \tfrac{1}{2}|x_1y_2 - x_2y_1| = \tfrac{1}{2}|7(6) - (k+1)(14)|\downarrow_{7\,marks} = \tfrac{1}{2}|42 - 14k - 14|$$
$$\Rightarrow \tfrac{1}{2}|28 - 14k| = 17\cdot 5\downarrow_{7\,marks} \Rightarrow |28 - 14k| = 35$$
$$\Rightarrow 28 - 14k = 35 \text{ or } 28 - 14k = -35$$
$$\Rightarrow k = -\frac{1}{2} \text{ or } k = \frac{63}{14} \text{ or } \frac{9}{2}\downarrow_{10\text{ marks}}$$
$$[Pt = \left(-\frac{1}{2}, 0\right) \text{ and } Pt\left(\frac{9}{2}, 0\right)]$$

Award the following marks only:

10 marks Fully correct or consistent answer.
9 marks One obvious slip, misreading.
7 marks Work of merit.
0 marks No work of merit.

Blunders (-3)

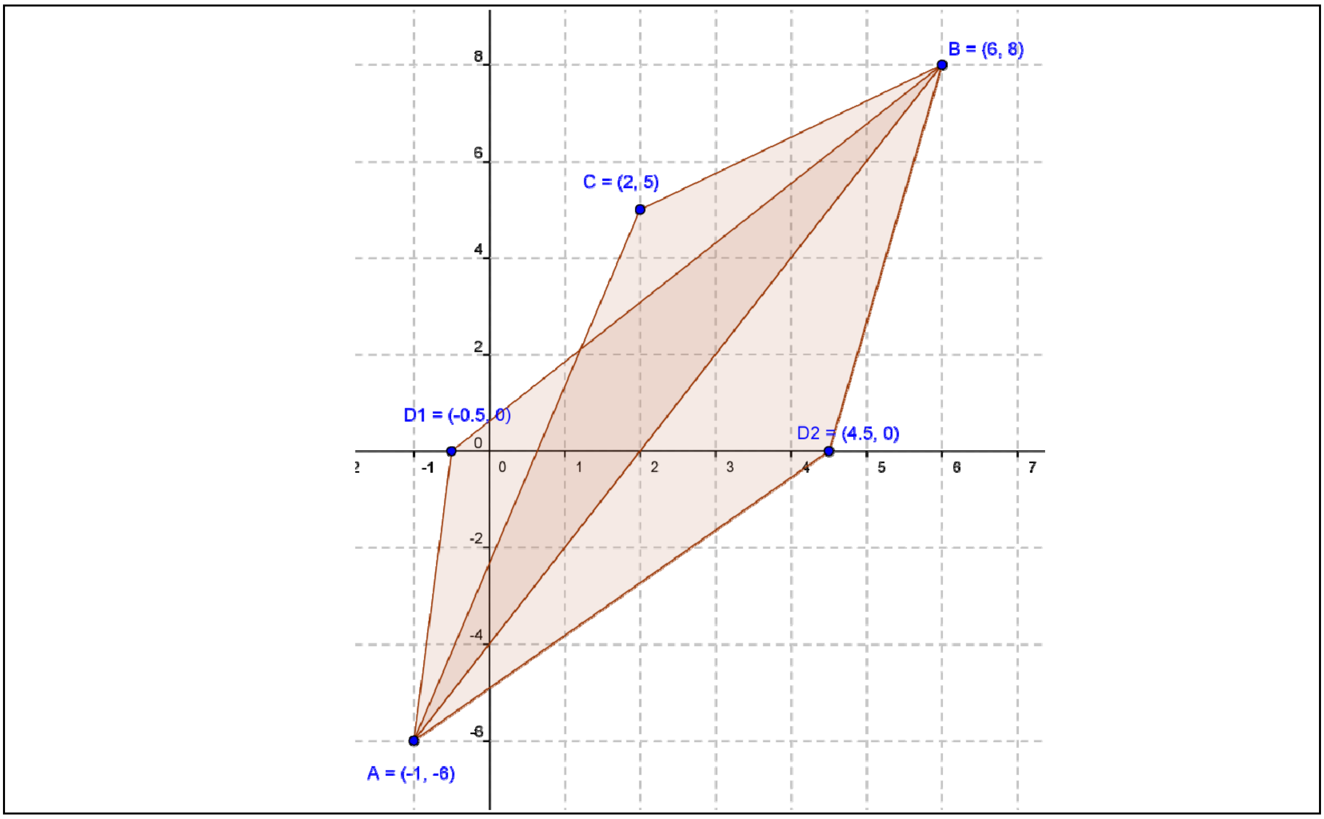
B1 Incorrect relevant formula and continues e.g. $\frac{1}{2}|x_1y_2 + x_2y_1|$ or omits the $\frac{1}{2}$.
B2 An incorrect translation.
B3 Points considered on y-axisi.e. $(0, k)$ with work.
B4 Finds one point only.

Award 7 marks.

- Some correct substitution into relevant formula and stops.
- Plots one or more points on a scaled diagram.
- Some work of merit in forming an equation e.g. writes $y = 0$ or tests an arbitrary point on the x-axis.
- Correct answer without work.

Worthless (0 marks)

W1 Irrelevant formula and stops e.g. $\frac{1}{2}$ on its own.



QUESTION 3

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, -)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

A circle has equation $x^2 + y^2 = 81$.

(i) Write down the co-ordinates of the centre of the circle.

(ii) Find the radius of the circle.

(a) (i) **5 marks** **Att 2**

(a) (ii) **5 marks** **Att 2**

(i) Centre = (0, 0)

(ii) Radius $\sqrt{81} = 9$

* Any error other than an obvious slip merits the attempt mark at most.

* Accept correct answer without work.

Attempts (2marks)

A1 Any work of merit e.g. draws a circle at the origin.

A2 Radius $= \sqrt{81}$.

A3 Tries to identify a point that is on the circle.

A4 Any mention of $x = 0$ or $y = 0$ or $x^2 + y^2 = r^2$.

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

The circle c has equation $(x-3)^2 + (y+1)^2 = 17$.

(i) Verify that the point (7,–2) is on c.

(ii) On a co-ordinate diagram, mark the centre of c and draw c.

(iii) Find, using algebra, the co-ordinates of the two points at which c intersects the x-axis.

(b) (i) **5 marks** **Att 2**

$(x-3)^2 + (y+1)^2 = 17$ $\Rightarrow (7-3)^2 + (-2+1)^2 = 17$ $\Rightarrow (4)^2 + (-1)^2 = 17$ $\Rightarrow 16+1=17$ $\Rightarrow 17=17$ $[Hence\ (7,-2)\ is \in c]$	Centre of c is $(3,-1)$ Radius = $\sqrt{(7-3)^2 + (-2+1)^2} = \sqrt{17}$ $[Hence\ (7,-2)\ is \in c]$

* Any error other than an obvious slip merits the attempt mark at most.

* Accept statement "Distance from (7,–2) to (3,–1) is $\sqrt{17}$ which is radius".

Award the following marks only:

5 marks Correct or consistent answer.

4 marks One slip.

2 marks Some work of merit.

0 marks Incorrect or inconsistent answer without work.

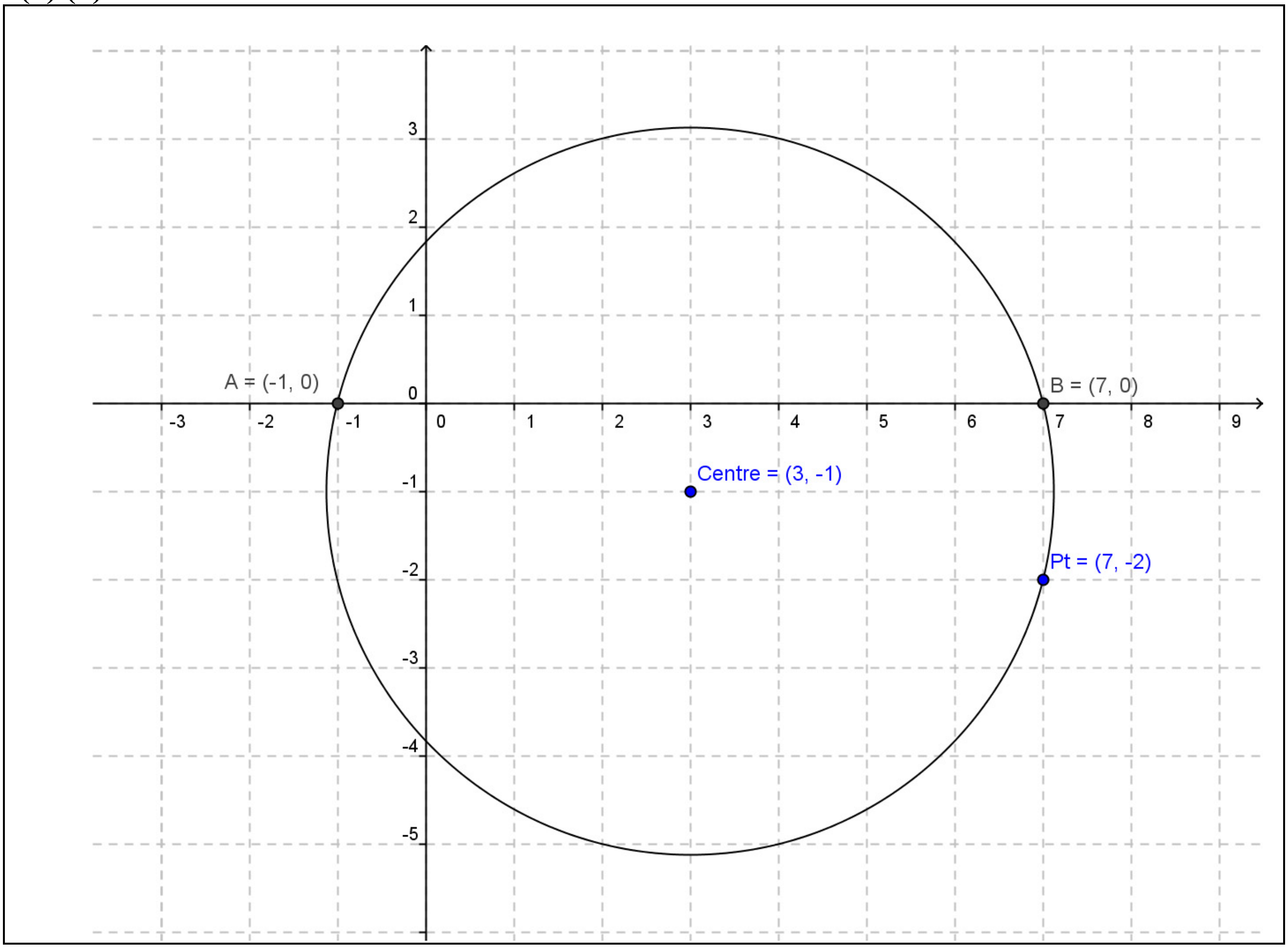
Slips(-1)

- Incorrect or omitted conclusion.

Attempts (2 marks)

- Any work of merit e.g. $x_1 = 7$.
- Circle with centre (3, –1) drawn.
- States radius $= \sqrt{17}$.
- States centre = (3,–1).

(b) (ii) **5 marks** **Att2**



* Accept a free hand diagram of a circle with correct centre of *c* marked clearly and reasonably drawn on a co-ordinate diagram. [Must extend into each quadrant and include (7, –2)].

* Scales must be indicated or implied for full marks.

Blunders (-3)

B1 Centre other than (3, –1).
B2 Correct centre indicated but no circle drawn.
B3 Scales unreasonably inconsistent. (to the eye).
B4 Different scales on x and y axes. (to the eye).
B5 Draws x-axis vertical and a y-axis horizontal.

Attempts (2 marks)

A1 Draws scaled axes and stops.
A2 Plots one correct point and stops e.g. (7, –2).

b (iii) Formation of quadratic **5 marks** **Att 2**
Solving quadratic **5 marks** **Att 2**

$x-\text{axis} \Rightarrow y=0$	$x-\text{axis} \Rightarrow y=0$
$y=0 \cap (x-3)^2+(y+1)^2=17$	$y=0 \cap (x-3)^2+(y+1)^2=17$
$(x-3)^2+(0+1)^2=17$	$(x-3)^2+(0+1)^2=17$
$x^2-6x-7=0$ [5 *marks*]	$(x-3)^2=16$ [5 *marks*]
$(x-7)(x+1)=0$	$x-3=\pm 4$
$x=7$ *and* $x=-1$ [5 *marks*]	$\Rightarrow x=7$ *and* $x=-1$ [5 *marks*]
$[\Rightarrow Pts\ (7,0)\ and\ (-1,0)]$	$[\Rightarrow Pts\ (7,0)\ and\ (-1,0)]$

* If there is no equation in the first part → zero marks in second part. (Subject to A3)

Blunders(-3)
B1 Finds co-ordinates of the points where *c* cuts the *y*-axis.
B2 Blunder in applying the quadratic formula.
B3 Mathematical error.

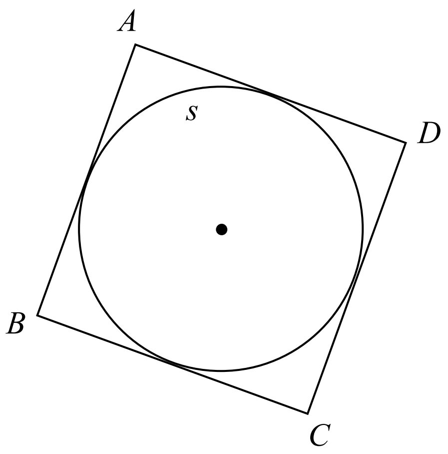
Slips (-1)
S1 Numerical slips to a maximum of 3.

Attempts (2 marks)
A1 Any work of merit.
A2 Correct substitution of $y=0$ into the equation of *c*.
A3 Graphical solution or the correct answer without work. [Merits Att (2 + 2)].
A4 $y=0$ and stops.

Part (c) **20 (5, 5, 10) marks** **Att (2, 2, -)**

The points $A(-1, 2)$, $B(-3,-4)$, $C(3, -6)$ and $D(5, 0)$ are the vertices of a square.

The sides of the square are tangents to the circle s, as shown.



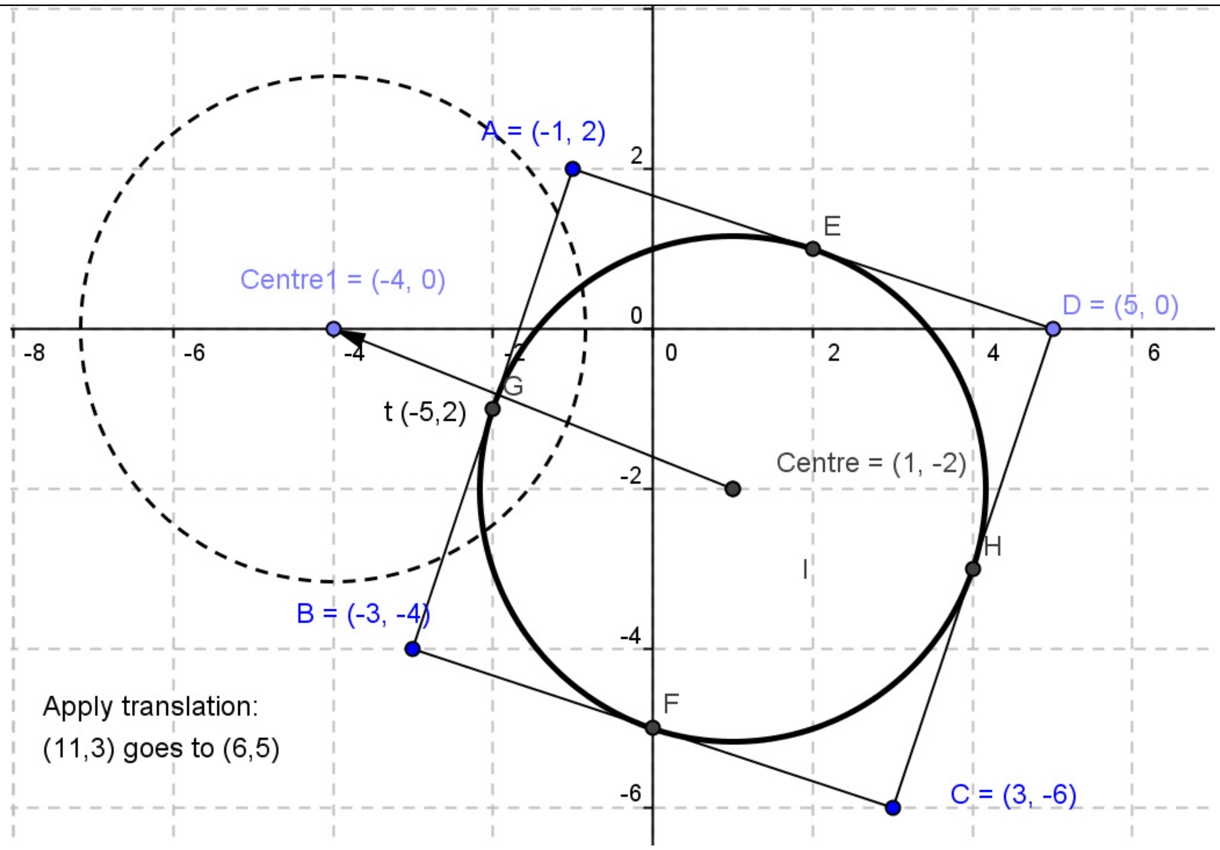
(i) Find the co-ordinates of the centre of s.

(ii) Find the equation of s.

(iii) The circle $(x+4)^2 + y^2 = 10$ is the image of s under the translation $(p, q) \to (6, 5)$. Find the value of p and the value of q.

(c) (i) **5 marks** **Att 2**

*Centre = mid-point of diagonal $[AC]$	**Centre = mid-point of $[EF]$
$\left(\frac{-1+3}{2}, \frac{2-6}{2}\right) = \left(\frac{2}{2}, \frac{-4}{2}\right) = (1, -2)$	$\left(\frac{2+0}{2}, \frac{1-5}{2}\right) = \left(\frac{2}{2}, \frac{-4}{2}\right) = (1, -2)$
Centre = mid-point of diagonal $[BD]$	Centre = mid-point of $[GH]$
$\left(\frac{-3+5}{2}, \frac{-4-0}{2}\right) = \left(\frac{2}{2}, \frac{-4}{2}\right) = (1, -2)$	$\left(\frac{-2+4}{2}, \frac{-1-3}{2}\right) = \left(\frac{2}{2}, \frac{-4}{2}\right) = (1, -2)$
*** Translation Method.	
$(-1,2) \to (3,-6) \Rightarrow t = (4,-8)\ \tfrac{1}{2}t \Rightarrow t_{new} = (2,-4)$ *Apply to* $(-1,2) \to (1,-2)$	



* Any error other than an obvious slip merits attempt marks at most.

Award the following marks only:
5 marks Fully correct or consistent answer.
4 marks One slip.
2 marks One blunder or one blunder + one slip or two slips or some work of merit.
0 marks Incorrect or inconsistent answer without work.

Treat as separate blunders:
- Mathematical error.
- Incorrect end point used to get centre.

Attempts (2 marks)
- Any work of merit e.g. plots a point correctly.
- Graphical solution to give (1, –2).
- Correct answer without work.

(c) (ii) **5 marks** **Att 2**

Radius = $\frac{1}{2}[AB]$	Radius = distance from (1, –2) to E(2,1)
$\frac{1}{2}\sqrt{(-1+3)^2+(2+4)^2} = \frac{1}{2}\sqrt{(2)^2+(6)^2}$ $=\frac{1}{2}\sqrt{40} = \sqrt{10}$	$\sqrt{(2-1)^2+[1-(-2)]^2} = \sqrt{(1)^2+(3)^2} = \sqrt{10}$
Radius = $\frac{1}{2}[AD]$	Radius = distance from (1, –2) to F(0, –5)
$\frac{1}{2}\sqrt{[5-(-1)]^2+(0-2)^2} = \frac{1}{2}\sqrt{(6)^2+(-2)^2}$ $=\frac{1}{2}\sqrt{40} = \sqrt{10}$	$\sqrt{(0-1)^2+[-5-(-2)]^2} = \sqrt{(-1)^2+(-3)^2} = \sqrt{10}$
Radius = $\frac{1}{2}[DC]$	Radius = distance from (1, –2) to G(–2, –1)
$\frac{1}{2}\sqrt{(3-5)^2+(-6-0)^2} = \frac{1}{2}\sqrt{(-2)^2+(-6)^2}$ $=\frac{1}{2}\sqrt{40} = \sqrt{10}$	$\sqrt{(-2-1)^2+[-1-(-2)]^2} = \sqrt{(-3)^2+(1)^2} = \sqrt{10}$
Radius = $\frac{1}{2}[BC]$	Radius = distance from (1, –2) to H(4, –3)
$\frac{1}{2}\sqrt{[3-(-3)]^2+[-6-(-4)]^2}$ $=\frac{1}{2}\sqrt{(6)^2+(-2)^2} = \frac{1}{2}\sqrt{40} = \sqrt{10}$	$\sqrt{(4-1)^2+[-3-(-2)]^2} = \sqrt{(3)^2+(-1)^2} =$ $\sqrt{10}$
Equation of $s : (x-1)^2+(y+2)^2 = 10$	

* Accept answers consistent with candidate's work in (i).
* Perpendicular distance from line to point could be used here.

Award the following marks only:
5 marks Fully correct or consistent answer.
4 marks One slip.
2 marks One blunder or one blunder and one slip, or two slips, or some work of merit.
0 marks Incorrect or inconsistent answer without work.

Treat as separate blunders:
- Mathematical error e.g. mishandles square root
- Radius = $\frac{1}{2}|BD|$ or $\frac{1}{2}|AC|$.
- Failed to get r^2
- $(x-1)^2 + (y+2)^2 = \sqrt{10}$
- $(x-1)^2 + (y+2)^2 = r^2$

Attempts (2 marks)
- Any work of merit e.g. indicates a correct radius on a diagram.
- Uses an arbitrary radius and continues.
- Work to find the equation of any of the four sides of the given square.
- Use of perpendicular distance formula with any correct substitution.
- Plots one or more of points A, B, C and D.
- Correct answer without work. (If worked from translation, candidate must indicate.)

(c) (iii) **10 marks** **Att -**

Centre of image = $(-4, 0)$
$(1, -2) \rightarrow (-4, 0)$ => x co-ordinate moves down 5, y co-ordinate moves up 2
$(p, q) \rightarrow (6, 5)$
$p = 11$ and $q = 3$

* Accept answers consistent with candidate's previous work.

Award the following marks only:
10 marks Fully correct or consistent answer.
9 marks One obvious slip, misreading.
7 marks Work of merit.
0 marks No work of merit.

Blunders (-3)
B1 Incorrect translation used with both circles.
B2 Error in the use of a correct translation. Note S1.
B3 Incorrect centre of image and continues.

Slips (-1)
S1 One correct and one incorrect co-ordinate having used correct translation.

Award 7 marks:
- Any work of merit e.g. indicates centre of s (in this section).
- Finds the centre of image of s.
- Correct answer without work.
- Fails to use both circles.

Worthless (0 marks)
W1 Substitutes (6, 5) into the equation of either circle and makes no progress.

QUESTION 4

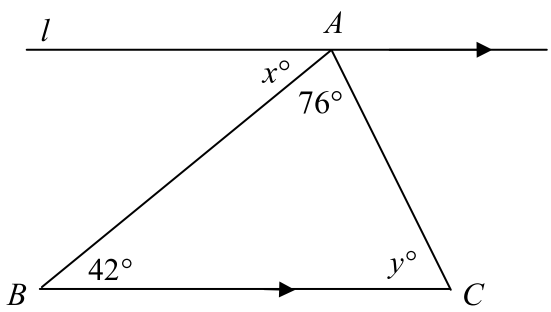
Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

In the diagram, the line l passes through the point A and is parallel to BC.

(i) Find x.

(ii) Find y.



(a) (i) **5 marks** **Att 2**
(a) (ii) **5 marks** **Att 2**

(i) $x = 42^\circ$

(ii) $y + 42^\circ + 76^\circ = 180^\circ \Rightarrow y = 180^\circ - 118^\circ = 62^\circ$

* Accept correct answers without work or an answer clearly indicated on a diagram.
* Accept answers in any order based on work presented.

Blunders (-3)
B1 Mathematical error in a(ii).
B2 Incorrect geometrical result, e.g. ΣAngles $\neq 180°$ or Straight line angle $\neq 180°$

Slips (-1)
S1 Each numerical slip to a maximum of 3.

Attempts (2 marks)
A1 Works to $180^\circ - 42^\circ = 138^\circ$ or similar and stops.
A2 Alternate angle y° noted and stops.
A3 Some work of merit e.g. ΣAngles $\neq 180°$ and stops or any mention of 180°
A4 Any mention of "corresponding angles" or "alternate angles".

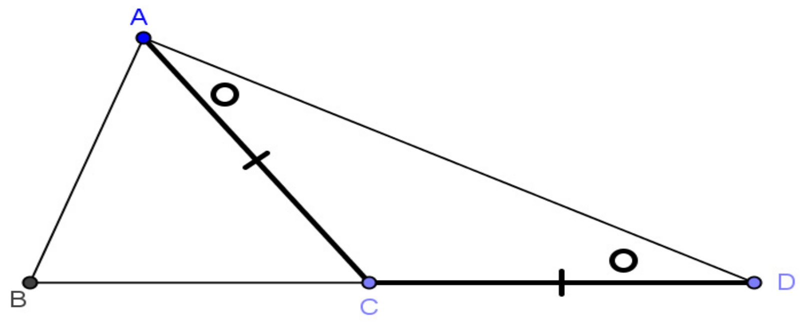
Worthless (0 marks)
W1 Incorrect answer without work shown.

Part (b) **20 marks** **Att 7**

Prove that the sum of the lengths of any two sides of a triangle is greater than that of the third side.

(b) **20 marks** **Att 7**

To Prove $|BC| + |AC| > |AB|$



Construction: Produce BC to D so that $|CD| = |AC|$. Join A to D $\downarrow_{7\ marks}$

Proof: In ΔACD, $|AC| = |CD| \Rightarrow |\angle DAC| = |\angle CDA|$ ……..ACD is isosceles $\downarrow_{10\ marks}$

$|\angle DAC| + |\angle CAB| > |\angle CDA|$ $\downarrow_{13\ marks}$

$\Rightarrow |BD| > |AB|$…….side opposite greatest angle $\downarrow_{16\ marks}$

But $|BD| = |BC| + |CD|$

Thus $|BC| + |CD| > |AB|$ $\downarrow_{19\ marks}$

Thus $|BC| + |AC| > |AB|$ $\downarrow_{20\ marks}$

* If work presented is not worthless, then Att 7 at least must be awarded.
* Proof without a diagram merits Att 7 if a complete proof can be reconciled with the aid of a diagram.
* A correct diagram with the relevant angles and sides indicated correctly merits 10 marks.

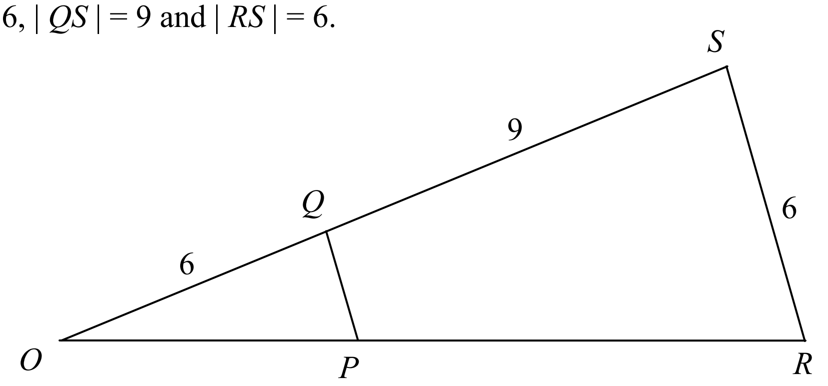
Blunders (-3)
B1 Each step omitted, incorrect or incomplete (except the last).
B2 Steps written in an illogical order. [Penalise once only.]

Attempts (7 marks)
A1 Outline diagram of a triangle with any additional relevant information.
A2 Any work of merit e.g. longest side opposite greatest angle in any triangle.
A3 Attempt at proof using special case. [Diagram with sides measured.]

Worthless (0 marks)
W1 Any irrelevant theorem, subject to the attempt mark.
W2 Triangle only.

Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

The triangle ORS is the image of the triangle OPQ under an enlargement of centre O.
$|OQ| = 6$, $|QS| = 9$ and $|RS| = 6$.



(i) Find the scale factor of the enlargement.
(ii) Find $|PQ|$.
(iii) Given that the area of the triangle OPQ is $7{\cdot}2$ square units, find the area of the triangle ORS.
(iv) Find the area of the quadrilateral $PRSQ$.

(c) (i)	**5 marks**	**Att 2**

Scale factor $= \dfrac{6+9}{6} = \dfrac{15}{6}$ or $2{\cdot}5$ or similar

* Accept a correct answer without work shown.

Blunders (-3)
B1 Incorrect ratio when finding the scale factor.
B2 Scale factor as $1\cdot 5$ i.e. $^9/_6$
B3 Mathematical error e.g. incorrect cancelling of 6's to get $k = 9$ or 10.

Attempts (2 marks)
A1 Any work with ratio $|OQ| : |OS|$.
A2 Some work of merit e.g. $6 + 9$.

(c) (ii)	**5 marks**	**Att 2**

$|PQ| = \dfrac{6}{2\cdot 5} = 2{\cdot}4$ or $\dfrac{|SR|}{|PQ|} = \dfrac{|OS|}{|OQ|} \Rightarrow \dfrac{|6|}{|PQ|} = \dfrac{15}{6} \Rightarrow |PQ| = \dfrac{36}{15} = 2\cdot 4$

* Accept answer consistent with candidate's previous work.

Blunders (-3)
B1 Incorrect or inconsistent scale factor.
B2 Incorrect ratio.
B3 Mathematical error.

Attempts (2 marks)
A1 Any work of merit, e.g. $|OP| : |OR|$ written.
A2 Correct answer without work.

(c) (iii) **5 marks** **Att 2**

Area of triangle $ORS = (2{\cdot}5)^2 \times 7{\cdot}2 = 45$

* Accept answer consistent with candidate's previous work.

Blunders (-3)

B1 Incorrect geometrical result e.g. area = $k(7\cdot 2)$ or $k * (7\cdot 2)$ where $* \neq$ multiplication.

B2 Mathematical blunder e.g. $(2\cdot 5)^2 = 5$.

Attempts (2 marks)

A1 Some substitution into correct formula.

A2 Some work of merit e.g. works with k^2 and stops.

A3 Correct answer without work.

A4 Treats triangle as right angled or works area formula. (Area $= \frac{1}{2}(6)(15) = 45$).

(c) (iv) **5 marks** **Att 2**

Area of quadrilateral $PRSQ = 45 - 7{\cdot}2 = 37{\cdot}8$

* Accept without work an answer consistent with candidate's previous work.

Blunders (-3)

B1 Leaves as $45 - 7{\cdot}2$.

Attempts (2 marks)

A1 Work of merit, e.g. states area = difference of the two triangles.

Worthless (0 marks)

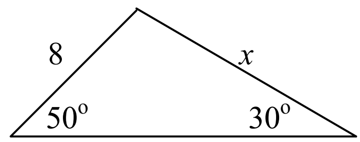
W1 Area = 9×6.

QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a) **10 marks** **Att 3**

Use the sine rule to calculate the value of x in the diagram.
Give your answer correct to the nearest integer.



(a) **10 marks** **Att 3**

$$\frac{x}{\sin 50^\circ} = \frac{8}{\sin 30^\circ}\downarrow_{4\,marks}$$

$$x = \frac{8 \times \sin 50^\circ}{\sin 30^\circ}\downarrow_{7\,marks}$$

$$x = \frac{8 \times 0\cdot 7660...}{0\cdot 5}\downarrow_{7\,marks}$$

$$x = 12\cdot 25\downarrow_{9\,marks} \approx 12$$

* Calculates x correctly without use of the sine rule award <u>7</u> marks.

Blunders (-3)
B1 Mathematical error.
B2 Uses radians (or gradient) mode incorrectly - apply once in each part.
B3 Error in the use of the sine rule.
B4 Incorrect substitution into sine rule and continues.
B5 Incorrect function read e.g. reads $Cos\,\theta$ instead of $Sin\,\theta$ and continues.
B6 Misplaced decimal point.

Slips (-1)
S1 Each numerical slip to a maximum of 3.
S2 Early round-off that affects the accuracy of the answer.
S3 Failure to round-off to the required level of accuracy (with work shown) e.g. $(12\cdot 3)$.

Attempts (3 marks)
A1 Some correct substitution into incorrect relevant formula e.g. $\frac{1}{2}ab\sin C$.
A2 Some correct use of sine rule e.g. $\frac{x}{Sin\,50^\circ} = \frac{*}{Sin\,\#}$.
A3 Answer of $12\cdot 2$ or 12 without work shown.
A4 Some work of merit e.g. third angle = 100°

Worthless (0 marks)
W1 Incorrect answer without work e.g. "15 without work". Otherwise A4 may apply.
W2 Measurement from the diagram.
W3 Incorrect use of the cosine rule.
W4 Formula transcribed from table and stops.

Part (b) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

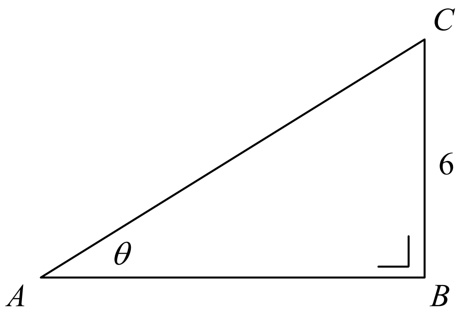
In the triangle ABC, $|BC| = 6$ cm, $|\angle ABC| = 90°$

$|\angle CAB| = \theta$ and $\sin\theta = \dfrac{3}{5}$.

(i) Find $|AC|$.

(ii) Find $|AB|$.

(iii) Verify that $\cos^2\theta + \sin^2\theta = 1$.



(b) (i) **10 marks** **Att 3**

$Sin\,\theta = \dfrac{6}{\lvert AC\rvert} = \dfrac{3}{5}$ $\Rightarrow 3\lvert AC\rvert = 30$ $\lvert AC\rvert = 10\,\text{cm}$	$Sin\,\theta = \dfrac{3}{5} \Rightarrow \theta = 36^\circ\ 52$ $\Rightarrow Tan\,36^\circ\ 52' = \dfrac{6}{\lvert AB\rvert}$ $\dfrac{3}{4} = \dfrac{6}{\lvert AB\rvert} \Rightarrow \lvert AB\rvert = 8$ $\Rightarrow \lvert AC\rvert = \sqrt{36+64} = 10\,\text{cm}$	$\lvert AC\rvert = \dfrac{6}{Sin\theta} = \dfrac{6}{3/5} = \dfrac{6\times 5}{3} = 10$ cm

* Accept correct answer without work.
* Units required for b (i) and b (ii) if answers are otherwise correct.

Blunders (-3)
B1 Mathematical error e.g. error in manipulation of fractions.
B2 Incorrect ratio and continues e.g. ratio inverted.

Slips (-1)
S1 Each numerical slip to a maximum of 3.
S2 Units omitted. [Apply only if answer would have secured full marks].
S3 Rounding-off that affects the final answer.

Attempts (3 marks)
A1 An exact scaled diagram giving the correct answer of 10.
A2 Trigonometric function correctly defined or found e.g. $Tan\,\theta = 3/4$.
A3 Attempt at constructing trigonometric fractions.
A4 Incorrect relevant formula with some correct substitution.
A5 Indicates $\theta = 36^\circ\ 52'$ $(36\cdot 86..)$ (i).
A6 Some work of merit e.g. states the theorem of Pythagoras, e.g. $h^2 = o^2 + a^2$.
A7 $|AC| = 5$.

Worthless (0 marks)
W1 Measurement from the diagram. [$6\,\text{cm}$]
W2 Incorrect answer without work. Note: A7 above.

(b) (ii) **5 marks** **Att 2**

$\|AB\|^2 = \|AC\|^2 - \|BC\|^2$ $\Rightarrow \|AB\|^2 = 10^2 - 6^2$ $\Rightarrow \|AB\| = \sqrt{100 - 36}$ $\Rightarrow \|AB\| = \sqrt{64} = 8\text{ cm}$	$Sin\theta = \frac{3}{5} \Rightarrow \theta = 36^\circ 52'$ $\Rightarrow Tan 36^\circ 52' = \frac{6}{\|AB\|}$ $\frac{3}{4} = \frac{6}{\|AB\|} \Rightarrow \|AB\| = 8\text{ cm}$	$Sin\ \theta = \frac{3}{5} \Rightarrow \theta = 36^\circ 52'$ $\Rightarrow Cos 36^\circ 52' = \frac{\|AB\|}{10}$ $\frac{4}{5} = \frac{\|AB\|}{10} \Rightarrow \|AB\| = 8\text{ cm}$
$\|AB\|^2 = 6^2 + 10^2 - 2(6)(10)Cos(53^\circ 08')$ $\|AB\|^2 = 36 + 100 - 71\cdot 994...$ $\|AB\| = \sqrt{64\cdot 005}$ $\|AB\| = 8\text{ cm}$	$\frac{\|AB\|}{Sin\ 53^\circ 08''} = \frac{6}{Sin\ 36^\circ 52''}$ $\Rightarrow \frac{\|AB\|}{0\cdot 800...} = \frac{6}{0\cdot 599...}$ $\Rightarrow \|AB\| = 8\text{ cm}$	

* Accept answers consistent with candidate's work in previous section (i).
* Accept correct answer without work.

Blunders (-3)
B1 Mathematical error e.g. $6^2 = 12$.
B2 Error in the use of Pythagoras e.g. $|AB|^2 = 10^2 + 6^2$.
B3 Incorrect use of Pythagoras.
B4 Incorrect ratio.
B5 Incorrect trigonometric function and continues.
B6 Incorrect function read e.g. reads $Cos\,\theta$ instead of $Sin\,\theta$.
B7 Error in the manipulation of fractions.
B8 Error in the use of inverse function.

Slips (-1)
S1 Each numerical slip to a maximum of 3.
S2 Rounding-off that affects the final answer.

Attempts (2 marks)
A1 States the theorem of Pythagoras e.g. $h^2 = o^2 + a^2$.
A2 Any work of merit e.g. $6^2 = 36$, $Sin\,\theta = \frac{opposite}{hypotenuse}$.
A3 An exact scaled diagram giving correct answer.
A4 States $\theta = 36\cdot 869^\circ$ and stops.
A5 $|AB| = 3$ or 4.

Worthless (0 marks)
W1 Incorrect answer without work. Note: A5 above.
W2 $10 + 6 = 16$ or $10 = x + 6$.
W3 Measurement from the diagram. [$5\cdot 1\text{cm}$].

(b) (iii) **5 marks** **Att 2**

*	**
$\cos\theta = \frac{8}{10} \Rightarrow \cos^2\theta = \frac{64}{100}$ $\sin\theta = \frac{6}{10} \Rightarrow \sin^2\theta = \frac{36}{100}$ $\Rightarrow \cos^2\theta + \sin^2\theta = \frac{64+36}{100} = 1$	$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ $\underline{\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)}$ $\Rightarrow \cos^2\theta + \sin^2\theta = 1$

* Accept in method ** θ as θ or an angle from the triangle given.
* Accept answers consistent with candidate's work in previous section. (ii).

Blunders (-3)
B1 Mathematical error e.g. error in squaring, $\left(\frac{8}{10}\right)^2 = \frac{16}{20}$ or similar.
B2 Error in the manipulation of fractions.
B3 $Cos^2\theta$ treated as $2Cos\theta$ and continues.

Slips (-1)
S1 Conclusion not stated.

Attempts (2 marks)
A1 Some correct substitution into given identity.
A2 Some work of merit e.g. $10^2 = 100$.
A3 $Cos^2\theta = Cos\theta \times Cos\theta$ and stops.
A4 Correct answer without work.

Worthless (0 marks)
W1 $Cos^2\theta$ treated as $2Cos\theta$ and stops.

Part (c) **20 (10, 10) marks** **Att (3, 3)**

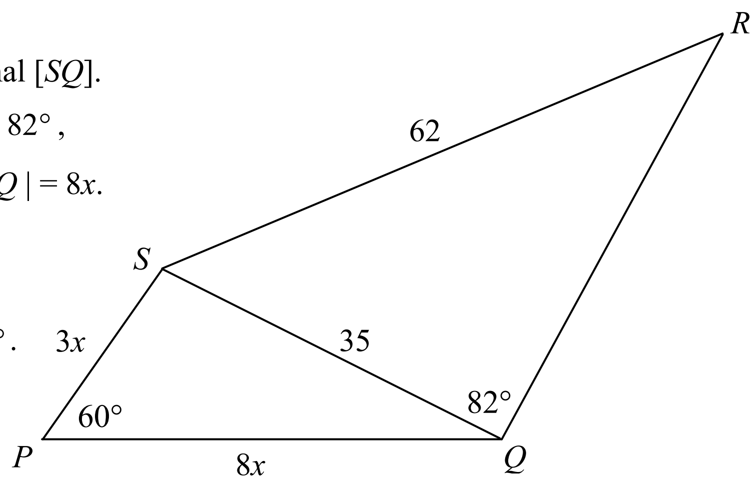
PQRS is a quadrilateral with diagonal [*SQ*].

$|RS| = 62$, $|SQ| = 35$, $|\angle SQR| = 82^\circ$,

$|\angle SPQ| = 60^\circ$, $|SP| = 3x$ and $|PQ| = 8x$.

(i) Find $|\angle QRS|$, correct to the nearest degree, given that $0^\circ \le |\angle QRS| \le 90^\circ$.

(ii) Find the value of x.



* Incorrectly working with two triangles merits at most Att 3 marks.

(c) (i) **10 marks** **Att 3**

$$\frac{35}{\sin|\angle QRS|} = \frac{62}{\sin 82^\circ} \Rightarrow \sin|\angle QRS| = \frac{35 \times \sin 82^\circ}{62} \downarrow_{4\,marks} = \frac{35 \times 0\cdot 9902...}{62} \downarrow_{7\,marks} = 0\cdot 5590\ldots$$

$$|\angle QRS| = 33\cdot 98 \downarrow_{9\,marks} \approx 34^\circ.$$

Blunders (-3)

B1 Mathematical error e.g. mishandles inverse trigonometric function.
B2 Uses radians (or gradient) mode incorrectly – apply once in each part in which it occurs.
B3 Error in the use of the sine rule.
B4 Incorrect substitution into sine rule and continues.
B5 Incorrect function read e.g. reads $Cos\,\theta$ instead of $Sin\,\theta$ and continues.
B6 Error in the use of the inverse trigonometric function.
B7 Misplaced decimal point.

Slips (-1)

S1 Each numerical slip to a maximum of 3.
S2 Early round-off that affects the accuracy of the answer.
S3 Failure to round-off to the required level of accuracy (with work shown).
S4 Obtuse angle as answer (146°).

Attempts (3 marks)

A1 Some correct substitution into incorrect relevant formula e.g. $\frac{1}{2}ab\sin C$.
A2 Some correct use of sine rule e.g. $\dfrac{Sin|\angle QRS|}{35} = \dfrac{*}{Sin\#}$.
A3 Answer of $33\cdot 98^\circ$ or 34° without work shown.
A4 Some work of merit e.g. total angle measure in triangle $= 180^\circ$.

Worthless (0 marks)

W1 Incorrect answer without work e.g. "28 without work". Otherwise A4 may apply.
W2 Measurement from the diagram (39°).
W3 Incorrect use of the cosine rule.
W4 Treats triangle *QRS* as a right-angled triangle. [Subject to attempt marks.]

(c) (ii) **10 marks** **Att 3**

$$(35)^2 = (3x)^2 + (8x)^2 - 2(3x)(8x)\cos 60^{\circ}$$
$$1225 = 9x^2 + 64x^2 - 48x^2(\tfrac{1}{2}) \downarrow_{4\,marks}$$
$$1225 = 73x^2 - 24x^2 = 49x^2 \downarrow_{7\,marks}$$
$$25 = x^2$$
$$x = 5 \downarrow_{10\,marks}$$

Blunders (-3)

B1 Mathematical error e.g. error in use of cosine rule.
B2 Incorrect trigonometric function and continues.
B3 Incorrect function read e.g. *Sin* instead of *Cos* and continues.
B4 Misplaced decimal point.
B5 Transposition error.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

Attempts (3 marks)

A1 Correct answer without work shown.
A2 Incorrect relevant formula with some correct substitution.
A3 Some work of merit e.g. any correct use of the cosine rule, $(3x)^2$ or $(8x)^2$.

Worthless (0 marks)

W1 Measurement from the diagram i.e. $[8x = 6 \cdot 2]$.
W2 Treats triangle *SPQ* as right-angled triangle or uses sine rule inappropriately in (ii), (subject to attempt marks) & (subject to marks already secured).

QUESTION 6

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (-, -, -, -)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (-, -, -, -)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

(i) Find 4!

(ii) Simplify $\frac{6(5!)}{5(4!)}$.

(a) (i)	**5 marks**	**Att 2**
(a) (ii)	**5 marks**	**Att 2**

(i) $4! = 4.3.2.1 = 24.$

(ii) $\frac{6(5!)}{5(4!)} = \frac{6.5.4.3.2.1}{5.4.3.2.1} = 6$

* Accept correct answer without work in each case.

Blunders (-3)

B1 Treats factorial as a combination. [Apply once in (a) e.g. $\binom{6}{5}$ or $\binom{5}{4}$ in (a) (ii)].

B2 Blunder in evaluating or expanding term.

Attempts (2 marks)

A1 Attempt at expanding term.
A2 Gives a complete list of correct numbers.
A3 Any three correct or consecutive numbers with multiplication indicated.

Worthless (0 marks)

W1 Incorrect answer without work shown e.g. writes $\frac{6}{4}$ and stops.

Part (b)	**20 (5, 5, 5, 5) marks**	**Hit/miss**

The letters in the word FERMAT are arranged taking all of the letters each time.
How many different arrangements are possible if

(i) there are no restrictions
(ii) the arrangements begin with the letter F
(iii) the arrangements begin with the letter F and end with a vowel
(iv) the two vowels are together?

(b) (i)	**5 marks**	**Hit/miss**

$6! = 6.5.4.3.2.1 = 720$

(b) (ii)	**5 marks**	**Hit/miss**

$5! = 5.4.3.2.1 = 120$

(b) (iii)	**5 marks**	**Hit/miss**

$2 \times 4! = 2 \times 4.3.2.1 = 48$

(b) (iv)	**5 marks**	**Hit/miss**

$5! \times 2! = 5.4.3.2.1 \times 2 = 240$

* If the sections of (b) and (c) are not identified, and it is not obvious which section is being attempted treat each section in order.

Award marks as follows, in each section:

5 marks Fully correct answer with or without work.
0 marks Incorrect answer even if consistent.

Part (c) **20 (5, 5, 5, 5) marks** **Hit/miss**

The table below shows how the students in a school usually travel to school.

	Walk	Cycle	Other
Boys	157	123	166
Girls	184	91	172

(i) A student is picked at random. What is the probability that the student is a boy?
(ii) A student is picked at random. What is the probability that the student walks to school?
(iii) A boy is picked at random. What is the probability that he cycles to school?
(iv) A girl is picked at random. What is the probability that she does not walk to school?

(c)(i) **5 marks** **Hit/miss**

$$\text{P(boy)} = \frac{157+123+166}{157+123+166+184+91+172} = \frac{446}{893} \textit{ or } 0\cdot4994 \textit{ or } 49\cdot94\%$$

(c)(ii) **5 marks** **Hit/miss**

$$\text{P(Student walks)} = \frac{157+184}{157+123+166+184+91+172} = \frac{341}{893} \textit{ or } 0\cdot38185 \textit{ or } 38\cdot18585\%$$

(c)(iii) **5 marks** **Hit/miss**

$$\text{P(boy who cycles)} = \frac{123}{157+123+166} = \frac{123}{446} \textit{ or } 0\cdot27578 \textit{ or } 27\cdot578\%$$

(c)(iv) **5 marks** **Hit/miss**

$$\text{P(girl not walking)} = \frac{91+172}{184+91+172} = \frac{263}{447} \textit{ or } 0\cdot58836 \textit{ or } 58\cdot8366\%$$

* Award **5** marks for each correct answer without work shown.

Award marks as follows, in each section:

5 marks Fully correct answer with or without work.
0 marks Incorrect answer even if consistent.

QUESTION 7

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 10, 5) marks**	**Att (2, 3, 2)**
Part (b)	**20 (5, 10, 5) marks**	**Att (2, 3, 2)**

Part (a)	**10 marks**	**Att 3**

Calculate the mean of the numbers 8, 6, 1, 3, 7, 8, 2.

(a)	**10 marks**	**Att 3**

$$\frac{8+6+1+3+7+8+2}{7}\downarrow_{7\ marks} = \frac{35}{7}\downarrow_{9\ marks} = 5$$

* Accept correct answer without work.

Blunders (-3)
B1 Mathematical error.
B2 Denominator $\neq$ 7.

Slips (-1)
S1 Each numerical error to a maximum of 3.

Attempts (3 marks)
A1 Any work of merit e.g. indicates addition of numbers given and stops.
A2 Answer = 35 without work.
A3 Writes 7 as answer. (Σn).
A4 Writes 6 as answer (median).
A5 Writes 8 as answer (mode).

Part (b) **20 (5, 10, 5) marks** **Att (2, 3, 2)**

An information evening was held at a school. The number of people who entered the school during 20 minute intervals, beginning at 18:00, is given in the following table:

Time	18:00 - 18:20	18:20 - 18:40	18:40 - 19:00	19:00 - 19:20	19:20 - 19:40	19:40 - 20:00
Number of people	35	55	190	140	110	70

[Note: 18:20 - 18:40 means 18:20 or later, but before 18:40, etc.]

(i) Copy and complete the following cumulative frequency table:

Time	Before 18:20	Before 18:40	Before 19:00	Before 19:20	Before 19:40	Before 20:00
Number of people						

(ii) Draw the cumulative frequency curve (ogive).

(iii) Use your curve to estimate the interquartile range.

(b) (i) **5 marks** **Att 2**

Time	Before 18:20	Before 18:40	Before 19:00	Before 19:20	Before 19:40	Before 20:00
Number of people	35	90	280	420	530	600

* Deduct 1 mark for each incorrect, inconsistent or omitted entry subject to blunders and attempt marks.

Blunders (-3)

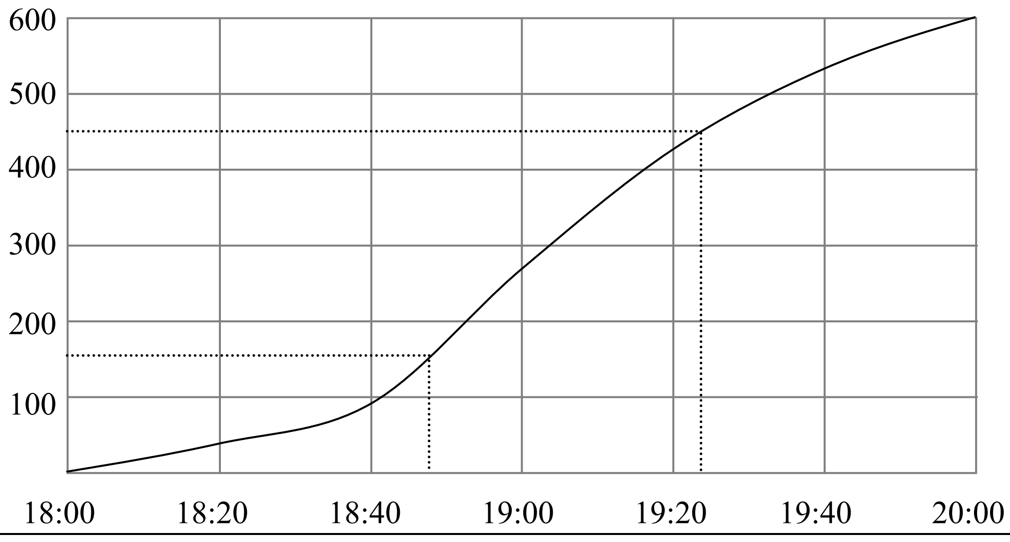
B1 Subtracts instead of adds.

Attempts (2 marks)

A1 Any work of merit e.g. one correct frequency and stops.

A2 Copies the given table and stops.

(b) (ii) **10 marks** **Att 3**



* Accept frequency on horizontal axis.

* Accept curve consistent with candidates work in (b) (i). Subject to A3.

Blunders (-3)
B1 Scale irregular (apply once).
B2 Draws a cumulative frequency polygon – apply slips also. [B1 may also apply]
B3 Draws a cumulative cumulative curve – apply slips also. [B1+ B2 may also apply]

Slips (-1)
S1 Each point omitted or incorrectly plotted (to the eye). [B1 may also apply]
S2 Each pair of points not joined – including (18:00,0) to (18:20, 35).
[Note: a point omitted may incur two penalties, S1 and S2.]

Attempts (3 marks)
A1 One correct step e.g. draws scaled axes and stops.
A2 Draws histogram correctly instead of ogive.
A3 The original table plotted.

(b) (iii) **5 marks** **Att 2**

Interquartile range = 19:25 − 18:46 = 39 minutes

* Accept without work answer consistent with candidates curve.
* Accept a tolerance of $\pm$ 4 minutes. (Based on candidate's graph).
* Accept a tolerance of $\pm$ 10 from vertical point.

Blunders (-3)
B1 Works from the time axis to the number of people axis. (Range 470 – 60 = 410).
B2 Each incorrect or omitted quartile.
B3 Fails to subtract. (Answer as 19:25 – 18:46).
B4 Treats time values as decimal. (19:25 – 18:46 = $0 \cdot 79$).

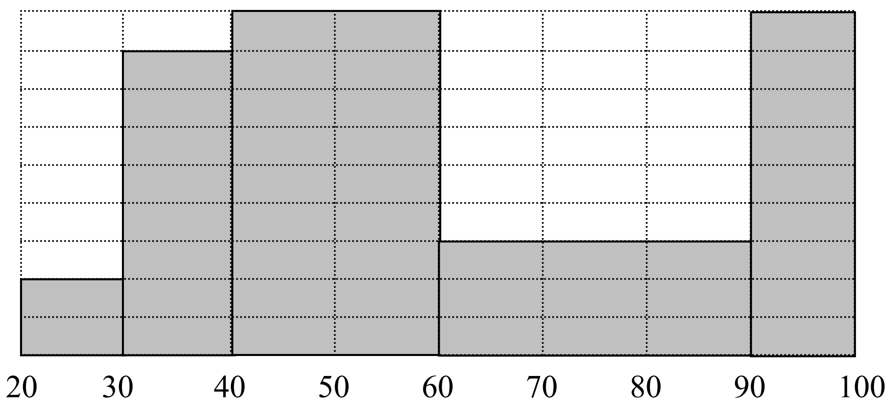
Slips (-1)
S1 Writes answer as " Before 39 minutes" or " ≤ 39 ".

Attempts (2marks)
A1 A relevant line drawn on the graph or an indication of subtraction, (subject to tolerance above).
A2 19:25 or 18:46 or similar without work, (subject to tolerance above).
A3 Some relevant statement about interquartile range e.g. 150 or 450 stated or shown.
A4 Reads from time axis only.

Worthless (0 marks)
W1 Horizontal or vertical line drawn outside tolerance e.g. draws median, (subject to work of merit).

Part (c)	**20 (5, 10, 5) marks**	**Att (2, 3, 2)**

The histogram represents the marks obtained by candidates in an examination.



(i) Completethe following frequency table:

Marks	20 - 30	30-40	40-60	60-90	90-100
Number of candidates	4				

(ii) The mean mark was 60.Taking the mid-interval values of the completed frequency table, find the standard deviation, correct to the nearest integer.

(iii) Find the maximum possible number of candidates whose marks were within one standard deviation of the mean.

(c) (i)	**5 marks**	**Att 2**

Marks	20 - 30	30-40	40-60	60-90	90-100
Number of candidates	4	16	36	18	18

Blunders (-3)

B1 General blunders. Writes table entry as 4, 8, 18, 9, 9.

Slips (-1)

S1 Each incorrect frequency, (subject to A1).

Attempts (2 marks)

A1 One correct frequency.

A2 Any work of merit on the histogram, (e.g. one rectangle marked as 2 or similar).

A3 Copies the given table and stops.

(c) (ii) **10 marks** **Att 3**

Deviations: $x - \mu$:25 – 60 = –35,35 – 60 = –25,50 – 60 = – 10,75 – 60 =15,95 – 60=35$\downarrow_{3\,marks}$.

$$\sigma = \sqrt{\frac{4(-35)^2 + 16(-25)^2 + 36(-10)^2 + 18(15)^2 + 18(35)^2}{4 + 16 + 36 + 18 + 18}} \downarrow_{4\,marks}$$

$$= \sqrt{\frac{4900 + 10000 + 3600 + 4050 + 22050}{92}} \downarrow_{7\,marks} = \sqrt{\frac{44600}{92}} or \sqrt{484 \cdot 7826087..} \downarrow_{7\,marks}$$

$$= 22 \cdot 017..... \downarrow_{9\,marks} = 22$$

or

x	f	d	d^2	fd^2
25	4	35	1225	4 900
35	16	25	625	10 000
50	36	10	100	3 600
75	18	15	225	4 050
95	18	35	1225	22 050
	92			44 600$\downarrow_{4\,marks}$

Standard Deviation = $\sqrt{\frac{44600}{92}}$ *or* $\sqrt{484 \cdot 78...} \downarrow_{7\,marks} = 22{\cdot}01 \downarrow_{9\,marks} \ldots \approx 22$

* Accept correct or consistent answer without work i.e. uses calculator.
* Accept either positive or negative deviations (provided not oversimplified).
* Accept candidate's values from (i).

Award the following marks only:

10 marks Correct or consistent answer.
9 marks Obvious slip or failure to round-off.
7 marks One blunder or one blunder and one slip or 2/3 slips.
4 marks Further blunder.
3 marks Some relevant step e.g. 92 written without further work in this part.
0 marks Worthless work e.g. formula from Tables without further work.

Treat as separate blunders:

- Blunders in the determination of mid-interval values (once).
- Reads d values as f values or x values as d or f (once).
- Working from a cumulative table or similar.
- Blunder in the numerator.
- Blunder in the denominator.
- Inconsistent or no fraction.
- Omits square root, or blunder in square root, or incomplete square root.

Slips (-1) (to a maximum of 3)
- Numerical error.
- Candidate uses sample standard deviation on calculator (Answer $22 \cdot 7566..$).
  [Note: may also incur round-off error].
- Failure to round-off.
- Misreading.

Attempts (3 marks)
- Any relevant step, e.g. finds a mid-interval value.
- A correct multiplication and stops.
- Any correct deviation.
- 4 + 16 + 36 + 18 + 18 and stops.
- $\Sigma f = 92$.
- Accept a reasonable estimate, $21 \le \sigma \le 23$.
- Calculates μ.

Special Case: (Oversimplification in use of formula) :	**Award Att 3 marks**
$\sqrt{\dfrac{\sum f(x-\mu)^2}{\sum f}} = \sqrt{\dfrac{92(280-60)^2}{92}} = \sqrt{(280-60)^2} = \sqrt{(220)^2} = 220$	

(c) (iii) **5 marks** **Att 2**

60 – 22 to 60 + 22 marks, i.e. 38 to 82 marks Maximum number of candidates = 16 + 36 + 18 = 70

* Accept answer consistent with answer in (c) (i) and (c) (ii).
* If no standard deviation is found in (c) (ii), then award zero marks for (c) (iii).

Blunders (-3)
B1 Deals with either $\mu + \sigma$ or $\mu - \sigma$.
B2 Omits a group.

Slips (-1).
S1 Leaves as 16+36+18 and stops.

Attempts (2 marks)
A1 Correct or consistent answer without work.
A2 Writes 16 or 36 or 18 or consistent values.

QUESTION 8

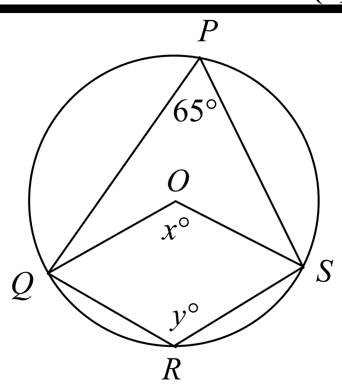
Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a)	**10 marks**	**Att (2, 2)**

The points P, Q, R and S lie on a circle, centre O.
$|\angle SPQ| = 65°$.

(i) Find the value of x.

(ii) Find the value of y.



(a) (i)	**5 marks**	**Att 2**
(ii)	**5 marks**	**Att 2**

(i) $x = 2 \times 65^\circ = 130^\circ$

(ii) $y = 180^\circ - 65^\circ = 115^\circ$

* Accept correct answers without work or an answer clearly indicated on a diagram.
* In section (ii) accept candidate's answer from section (i).

Blunders (-3)
B1 Incorrect operation in (a) (i) or (a) (ii).

Attempts (2 marks)
A1 Some statement that the angle at the centre of the circle equals twice the angle at the circle standing on the same arc.
A2 Some statement that the opposite angles in a cyclical quadrilateral sum to 180°.
A3 Writes 230°.

Worthless (0 marks)
W1 Incorrect answer without work shown.
W2 $x = 65^\circ$ or $y = 65^\circ$.

Part (b) **20 marks** **Att 7**

Prove that if $[AB]$ and $[CD]$ are chords of a circle and the lines AB and CD meet at the point K, where K is inside the circle, then $|AK| . |KB| = |CK| . |KD|$.

(b) **20 marks** **Att 7**

To Prove $|AK| . |KB| = |CK| . |KD|$

Construction: Join A to C and D to B. $\downarrow_{7\ marks}$

Proof: In triangles ACK and DBK

$|\angle KAC| = |\angle BDK| \downarrow_{8\ marks}$

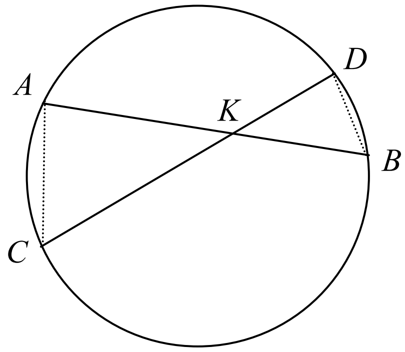
$|\angle ACK| = |\angle KBD| \downarrow_{11\ marks}$

$|\angle CKA| = |\angle DKB| \downarrow_{14\ marks}$

Triangles are similar $\downarrow_{17\ marks}$

$$\frac{|AK|}{|KD|} = \frac{|CK|}{|KB|} \downarrow_{19\ marks}$$

$|AK|. |KB| = |CK| . |KD| \downarrow_{20\ marks}$



* If work presented is not worthless, then Att 7 at least must be awarded.
* Proof without a diagram merits Att 7 if a complete proof can be reconciled with the aid of a diagram.
* A correct diagram with the relevant angles indicated correctly merits **<u>14 marks</u>**.

Blunders (-3)
B1 Each step omitted, incorrect or incomplete (except the last).
B2 Steps written in an illogical order. [Penalise once only.]
[Note: Some of the steps above may be interchanged.]

Misreadings(-1)
M1 Chords intersecting at the point K, where K is outside the circle.

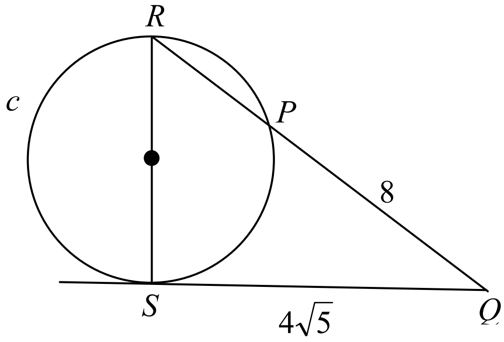
Attempts (7 marks)
A1 Outline diagram with a circle and chord drawn. (Minimum required circle and chord).
A2 Any work of merit e.g. mention that vertically opposite angles are equal.
A3 Attempt at proof using special case. [Both chords are diameters].

Worthless (0 marks)
W1 Any irrelevant theorem, subject to the attempt mark.
W2 Circle only.

Part (c)	**20 (10, 10) marks**	**At (3, 3)**

The line QS is a tangent to the circle c.
$[RS]$ is a diameter of the circle.
$[QR]$ cuts the circle at P.
$|QP| = 8$ and $|QS| = 4\sqrt{5}$.

(i) Calculate $|RP|$.
(ii) Hence, calculate $|RS|$ and give your answer in the form $a\sqrt{b}$, where $a, b \in \mathbb{N}$ and $a > 1$.



(c) (i)	**10 marks**	**Att 3**

$|QP|.|QR| = |QS|^2 \downarrow_{3\ marks} \Longrightarrow 8\,|QR| = \left(4\sqrt{5}\right)^2 \downarrow_{7\ marks} = 80 \Rightarrow |QR| = 10 \downarrow_{9\ marks} |RP| = 2$

Blunders (-3)
B1 Error in theorem e.g. $|RP|\,|PQ| = |SQ|^2$.
B2 Error in expansion.
B3 Mathematical error e.g transposition, error in squaring…..

Misreading (-1)
M1 Leaves as $|QR| = 10$.

Attempts (3 marks)
A1 Any work of merit e.g. $4\sqrt{5} = \sqrt{80}$ or $\left(4\sqrt{5}\right)^2 = 80$.
A2 $|RQ| = |RP| + |PQ|$ or similar.

(c) (ii)	**10 marks**	**Att 3**

$|QR|^2 = |RS|^2 + |QS|^2 \downarrow_{3\ marks}$
$|RS|^2 = |QR|^2 - |QS|^2 \downarrow_{4\ marks} = 100 - 80 = 20 \downarrow_{7\ marks}$
$|RS| = \sqrt{20} = 2\sqrt{5} \downarrow_{10\ marks}$

* Accept in part (c) (ii) an answer consistent with the candidate's answer to part (c) (i).

Blunders (-3)
B1 Error in the use of Pythagoras.
B2 Mathematical error.
B3 Answer not in required format.

Attempts (3 marks)
A1 Statement or reference to or some use of Pythagoras.

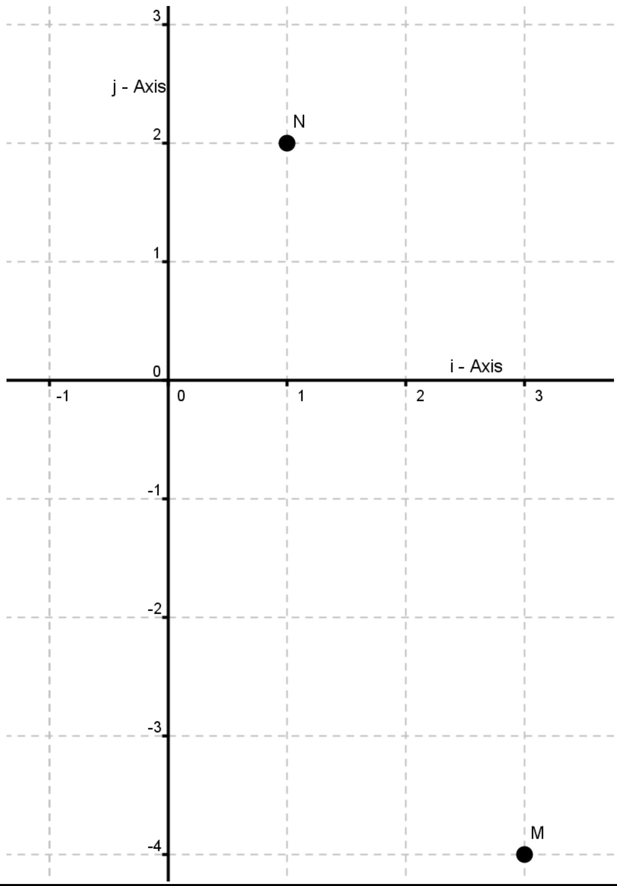
QUESTION 9

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

$\overrightarrow{OM} = 3\vec{i} - 4\vec{j}$ and $\overrightarrow{ON} = \vec{i} + 2\vec{j}$, where O is the origin.
Plot the points M and N on a co-ordinate diagram.

(a) **10 (5, 5) marks** **Att (2, 2)**



* No penalty if points are not labelled.

Blunders (-3)

B1 Draws i-axis as vertical and j-axis as horizontal [apply once].
B2 Point/s labelled incorrectly [apply once].
B3 Point/s plotted incorrectly [each time, if not consistent].
B4 Incorrect scaling of one or both axes. (Same scaling is required on both axes).

Attempts (2 marks)

A1 Draws scaled axes and stops.

Part (b) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

$\overrightarrow{OP} = 5\vec{i} + 3\vec{j}$ and $\overrightarrow{OQ} = -4\vec{i} + \vec{j}$, where O is the origin.

(i) Express $2\overrightarrow{OP} - \overrightarrow{OQ}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Express $\overrightarrow{PQ}$ in terms of $\vec{i}$ and $\vec{j}$.

(iii) Find the real numbers k and t such that $k\overrightarrow{OP} + t\overrightarrow{OQ} = 6\vec{i} + 7\vec{j}$.

(b) (i) **5 marks** **Att 2**

$$2\overrightarrow{OP} - \overrightarrow{OQ} = 2(5\vec{i} + 3\vec{j}) - (-4\vec{i} + \vec{j}) = 10\vec{i} + 6\vec{j} + 4\vec{i} - \vec{j} = 14\vec{i} + 5\vec{j}$$

Blunders(-3)

B1 Confuses $\vec{i}$'s and $\vec{j}$'s.

B2 Mathematical error.

B3 Fails to combine.

Slips(-1)

S1 Numerical slips to a maximum of 3.

Misreadings(-1)

M1 Expresses $2\overrightarrow{OQ} - \overrightarrow{OP}$ in terms of $\vec{i}$ and $\vec{j}$ [answer $-13\vec{i}$ $-1\vec{j}$].

M2 Obvious misread that does not oversimplify the task.

Attempts (2 marks)

A1 Correct answer without work.

Worthless(0 marks)

W1 Incorrect answer without work.

(b) (ii) **5 marks** **Att 2**

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (-4\vec{i} + \vec{j}) - (5\vec{i} + 3\vec{j}) = -4\vec{i} + \vec{j} - 5\vec{i} - 3\vec{j} = -9\vec{i} - 2\vec{j}$$

Blunders(-3)

B1 Confuses $\vec{i}$'s and $\vec{j}$'s.

B2 Mathematical error.

B3 Fails to combine.

B4 $\overrightarrow{PQ} = \overrightarrow{OP} - \overrightarrow{OQ}$ *or* $\overrightarrow{PQ} = \overrightarrow{OQ} + \overrightarrow{OP}$.

B5 $\overrightarrow{PQ} = \overrightarrow{OP}.\overrightarrow{OQ}$ i.e. the scalar product, [answer $-20+3= -17$].

Slips(-1)

S1 Numerical slips to a maximum of 3.

Misreadings(-1)

M1 Reads *PQ* as *QP* and continues correctly.

M2 Obvious misread that does not oversimplify the task. (Provided not already penalised).

Attempts (2 marks)

A1 Correct answer without work.

Worthless(0 marks)

W1 Incorrect answer without work.

(b) (iii) **10 marks** **Att 3**

$k(5\vec{i} + 3\vec{j}) + t(-4\vec{i} + \vec{j}) = 6\vec{i} + 7\vec{j}\downarrow_{3\,marks} \Rightarrow 5k\vec{i} + 3k\vec{j} - 4t\vec{i} + t\vec{j} = 6\vec{i} + 7\vec{j}\downarrow_{4\,marks}$

$$\left.\begin{array}{l} 5k - 4t = 6 \\ 3k + t = 7 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 5k - 4t = 6 \\ 12k + 4t = 28 \end{array}\right\} \Rightarrow 17k = 34 \Rightarrow k = 2\downarrow_{7\,marks}, \quad t = 1\downarrow_{10\,marks}$$

Blunders(-3)

B1 Confuses $\vec{i}$'s and $\vec{j}$'s.

B2 Mathematical error.

B3 One solution only.

Slips(-1)

S1 Numerical slips to a maximum of 3.

Misreadings(-1)

M1 Obvious misread that does not oversimplify the task. (Provided not already penalised.)

Attempts (3 marks)

A1 Correct answer without work.

Worthless(0 marks)

W1 Incorrect answer without work.

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

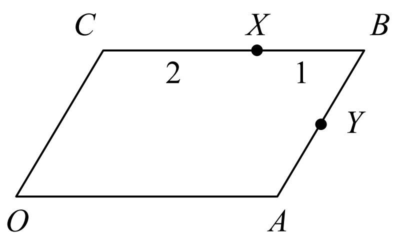
OABC is a parallelogram.
OABC is a parallelogram.
X is a point on $[CB]$ such that $|CX| : |XB| = 2 : 1$.
Y is the mid-point of $[AB]$.
Express, in terms of $\overrightarrow{OA}$ and $\overrightarrow{OC}$,

(i) $\overrightarrow{OB}$,

(ii) $\overrightarrow{OX}$,

(iii) $\overrightarrow{OY}$,

(iv) $\overrightarrow{XY}$.



(c) (i) **5 marks** **Att 2**

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{OC}$ *or* $\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB} = \overrightarrow{OC} + \overrightarrow{OA}$

(c) (ii) **5 marks** **Att 2**

$\overrightarrow{OX} = \overrightarrow{OC} + \frac{2}{3}\overrightarrow{CB} = \overrightarrow{OC} + \frac{2}{3}\overrightarrow{OA}$ *or* $\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BX} = \overrightarrow{OA} + \overrightarrow{OC} - \frac{1}{3}\overrightarrow{OA} = \frac{2}{3}\overrightarrow{OA} + \overrightarrow{OC}$

(c) (iii) **5 marks** **Att 2**

$\overrightarrow{OY} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{OC}$ *or* $\overrightarrow{OY} = \overrightarrow{OC} + \overrightarrow{CB} + \overrightarrow{BY} = \overrightarrow{OC} + \overrightarrow{OA} - \frac{1}{2}\overrightarrow{OC} = \frac{1}{2}\overrightarrow{OC} + \overrightarrow{OA}$

(c) (iv) **5 marks** **Att 2**

$\overrightarrow{XY} = \overrightarrow{OY} - \overrightarrow{OX} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{OC} - \overrightarrow{OC} - \frac{2}{3}\overrightarrow{OA} \;\; = \;\; \frac{1}{3}\overrightarrow{OA} - \frac{1}{2}\overrightarrow{OC}$ *or*
$\overrightarrow{XY} = \overrightarrow{XB} + \overrightarrow{BY} = \frac{1}{3}\overrightarrow{OA} - \frac{1}{2}\overrightarrow{OC}$

* Accept $\vec{A}$ for $\overrightarrow{OA}$ etc.
* Accept correct answer without work.
* Accept OA without arrows.

[Apply the following to **part (c) sections (i), (ii), (iii) and (iv)**.]

Award the following marks only:

5 marks Fully correct answer.
4 marks Obvious misread that does not oversimplify the task.
2 marks A correct combination of non-trivial vectors with correct starting point and correct finishing point written or clearly indicated on a diagram and stops,
or an incorrect combination of vectors correctly expressed in terms of $\overrightarrow{OA} + \overrightarrow{OC}$
e.g. $\overrightarrow{OX} = \overrightarrow{OC} + \frac{1}{2}\overrightarrow{CB} = \overrightarrow{OC} + \frac{1}{2}\overrightarrow{OA}$ [(c) (ii)],
or a correct combination finished incorrectly.
0 marks Incorrect or inconsistent answer without work.

Treat as separate blunders:
- Incorrect direction (once per section).
- Error in using the triangle law or the parallelogram law.
- Does not express answer in terms of $\overrightarrow{OA}$ and $\overrightarrow{OC}$.
- Fails to gather $\overrightarrow{OA}$ and $\overrightarrow{OC}$ correctly (once per section).

Misreadings(-1)
- Any obvious misreading which does not oversimplify the task.

Attempts (2 marks)
- A correct linear combination of at least 2 vectors. (Starting point and end point must be correct.)

Worthless (0 marks)
W1 Copies the diagram and stops.

Note: Incorrect answers without work:

Answer NOT in terms of $\overrightarrow{OA}$ and $\overrightarrow{OC}$ **<u>Zero</u> marks**	Answer is in terms of $\overrightarrow{OA}$ and $\overrightarrow{OC}$ with **<u>one</u>** correct **<u>2</u> marks**	Answer is in terms of $\overrightarrow{OA}$ and $\overrightarrow{OC}$ with **<u>none</u>** correct **<u>Zero marks</u>** (Subject to incorrect direction)

QUESTION 10

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

(i) Write out the first three terms in the expansion of $(1+x)^4$ in ascending powers of x.

(ii) Calculate the value of the third term when $x = 0{\cdot}2$.

(a) (i) **5 marks** **Att 2**

$$(1+x)^4 = 1 + \binom{4}{1}x + \binom{4}{2}x^2 + \ldots = 1 + 4x + 6x^2 + \ldots$$

* Accept long multiplication or Pascal's triangle.
* Accept correct answer without work.
* Ignore extra terms.

Blunders (-3)
B1 Stops at $1 + 4x$.
B2 Incorrect index.
B3 Incorrect coefficient.
B4 Incorrect sign or sign between coefficient and variable.

Misreadings(-1)
M1 Expands $(1-x)^4$.

Slips (-1)
S1 Numerical slips to a maximum of 3.

Attempts (2 marks)
A1 Any term, including first term, written down correctly.
A2 Gives part of Pascal's triangle or effort at Pascal's triangle.
A3 Gives coefficients only.
A4 Any step towards getting a binomial coefficient e.g. $\binom{4}{2}$.
A5 Any correct step towards long multiplication.

Worthless (0 marks)
W1 Writes $4(1+x)^3$.

(a) (ii)	**5 marks**	**Att 2**

$x = 0{\cdot}2, \qquad 6x^2 = 6(0{\cdot}2)^2 = 6(0{\cdot}04) = 0{\cdot}24$ or $\frac{6}{25}$

* Accept correct or consistent answer without work (if not oversimplified).

Blunders (-3)

B1 Decimal error.
B2 Incorrect use of index.
B3 Failing to complete calculations.

Attempts (2 marks)

A1 Work of merit such as $(0\cdot 2)^2 = (0\cdot 04)$ or $6(0\cdot 2)^2$.
A2 Answer of $(1\cdot 2)$ and stops.
A3 Substitution into 2nd term.

Worthless (0 marks)

W1 Incorrect answer without work.
W2 $12x$.
W3 $(1+0\cdot 2)^4 = 2\cdot 07$.

Part (b)	**20 (10, 10) marks**	**Att (3, 3)**

(i) Find S, the sum to infinity of the geometric series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$.

(ii) The sum to infinity of another geometric series is also S.
The common ratio of the series is 0·4.
Find the first term.

(b) (i)	**10 marks**	**Att 3**

$$S = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}}\downarrow_{4\,marks} = \frac{1}{\frac{1}{2}}\downarrow_{7\,marks} = 2$$

Blunders (-3)

B1 Incorrect a.
B2 Incorrect r.
B3 Error in formula e.g. $1+r$ as denominator.
B4 Error in substitution.
B5 Mathematical error e.g. $1-\frac{1}{2}=\frac{3}{4}$.
B6 Error in dividing by ½.

Slips (-1)

S1 Numerical slips to a maximum of 3.

Attempts (3 marks)

A1 Work of merit such as $1-\frac{1}{2}$.
A2 Clearly identifying a or r.
A3 Finds S_4 by formula or otherwise ....getting answer of $1\cdot 875$ or equivalent.
A4 Correct answer without work.

Worthless (0 marks)

W1 Incorrect answer without work.
W2 Incorrect formula and stops.

(b) (ii) **10 marks** **Att 3**

$$S_\infty = \frac{a}{1-r} = 2 \downarrow_{3\text{ marks}}$$

$$\frac{a}{1-0\cdot 4} = 2 \downarrow_{4\,marks} \Rightarrow a = 2(1-0\cdot 4) \downarrow_{7\,marks} \quad or \quad a = 2-0.8 \quad \Rightarrow a = 1.2 \downarrow_{10\,marks}$$

* Accept answer consistent with previous work.

Blunders (-3)
B1 Incorrect r.
B2 Transposition error.
B3 Incorrect relevant formula e.g. $\frac{a}{1+r} \Rightarrow a = 2\cdot 8$.

Slips (-1)
S1 Numerical slips to a maximum of 3.

Attempts (3 marks)
A1 Work of merit such as $1-0\cdot 4$ and stops.
A2 Correct answer without work.

Worthless (0 marks)
W1 Incorrect answer without work.
W2 Correct formula and stops.
W3 Formula for a GP and stops.

Part (c) **20 (10, 10) marks** **Att (3, 3)**

(i)	Equipment costing €15 000 depreciates at the compound rate of 12% per annum. Find the value of the equipment at the end of seven years, correct to the nearest euro.
(ii)	A company invests €15 000 in equipment at the beginning of each year for seven consecutive years. The equipment depreciates at the rate of 12% per annum compound depreciation. Using the formula for the sum of the first *n* terms of a geometric series, find the total value of the machinery at the end of the seven years, correct to the nearest euro.

(c) (i) **10 marks** **Att 3**

$$F = P(1-i)^t$$
$$F = 15\,000(1-0\cdot 12)^7$$
$$F = 15\,000(0\cdot 40867..)$$
$$F = 6130\cdot 133... = €\,6130$$

Year	Initial(€)	Depreciation(€)	Final(€)
1	15000	1800	13200
2	13200	1584	11616
3	11616	1393·92	10222·08
4	10222·08	1226·65	8995·43
5	8995·43	1079·45	7915·98
6	7915·98	949·92	6966·06
7	6966·06	835·93	6130·13 €6130

Blunders (-3)

B1 Uses $A = P(1 + r/100)^n$ or $F = P(1+i)^t$.
B2 Uses the calculated depreciation as the principal for the next year and continues.
B3 Each year omitted. (Subject to attempt mark 3).

Slips (-1)

S1 Numerical slips to a maximum of 3.
S2 Failure to round-off.
S3 Early round-off that affects the answer.

Attempts (3 marks)

A1 Some work of merit e.g. states the value for *P*, or the value for *i*, or $A = P(1 - r/100)^n$.
A2 12% of €15 000 and stops.
A3 $1 - 0\cdot 12 = 0\cdot 88$ and stops.
A3 One correct step in multiplication or division e.g. $15\,000(12)$ or $15\,000/100$
A4 Uses straight line depreciation.
A5 Correct answer without work.

Worthless (0 marks)

W1 Incorrect answer without work.

(c) (ii)	10 marks	Att 3

Total value = $15\,000(0{\cdot}88)^7 \downarrow_{3\,marks} + 15\,000(0{\cdot}88)^6 + 15\,000(0{\cdot}88)^5 + \ldots + 15\,000(0{\cdot}88) \downarrow_{4\,marks}$

$= 15\,000\{(0\cdot 88) + (0\cdot 88)^2 + (0\cdot 88)^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots(0\cdot 88)^7\}$

$= 15\,000\ \{S_n\}$ where $S_n = \dfrac{a(1-r^n)}{1-r}$

$a = 0{\cdot}88,\quad r = (1\cdot 0 - 0\cdot 12) = 0{\cdot}88,\quad n = 7$

$= 15\,000\{\dfrac{0\cdot 88[1-(0\cdot 88)^7]}{1-0\cdot 88}\} \downarrow_{7\,marks} = 15\,000\{\dfrac{0\cdot 88[1-0\cdot 40867....]}{0\cdot 12}\}$

$= 15\,000[\dfrac{0\cdot 88(0\cdot 591\ ......)}{0\cdot 12}]$

$= 15\,000[\dfrac{0\cdot 520......}{0\cdot 12}]$

$= 15\,000[4\cdot 336...]$

$= €\ 65\,045{\cdot}68 = €65\,046$

Blunders (-3)
B1 Error in substitution each time (subject to attempt mark 3).
B2 Error in multiplication (apply once to each step).
B3 Error in using indices e.g. $(0\cdot 88)^3 = 2\cdot 64$, or similar.
B4 Decimal errors.

Slips (-1)
S1 Numerical slips to a maximum of 3.

Attempts (3 marks)
A1 Some work of merit e.g. states the value for *a* or the value for *r*, or works correctly with $F = P(1-i)^t$.
A2 Adds 2 or more of the given terms e.g. $S_2 = €13\,200 + 11\,616 = €24\,816$ or similar.
A3 One correct step in setting up geometric sequence.
A4 Correct answer without work/calculations on a yearly basis.

Worthless (0 marks)
W1 Formula for arithmetic series and stops.
W2 Incorrect answer without work.

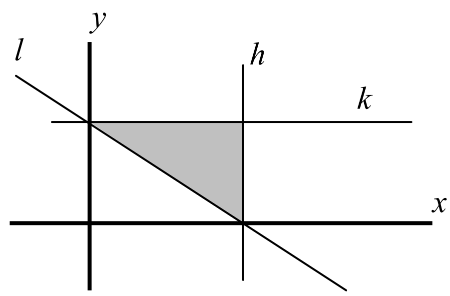
QUESTION 11

Part (a)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**
Part (b)	**35 (15, 10, 10) marks**	**Att (6, 4, -)**

Part (a) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

The diagram shows the lines l: $2x + 3y - 6 = 0$, h: $x - 3 = 0$ and k: $y - 2 = 0$.

Write down the three inequalities that together define the shaded region in the diagram.



(a) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

l:	$2x + 3y \geq 6$ (or equivalent)	[*5 marks*]
h:	$x \leq 3$	[*5 marks*]
k:	$y \leq 2$	[*5 marks*]

* Accept correct inequalities without work shown.
* Accept < for ≤ and > for ≥ without penalty.
* Accept $2x + 3y - 6 = 0 \rightarrow 0 + 0 - 6 \leq 0 \rightarrow 2x + 3y - 6 \geq 0$ for 5 marks.
* It is possible to award only 2 marks for part (a).

Award the following marks only:
5 marks Fully correct answer (each inequality).
4 marks One obvious slip or misreading (each inequality).
2 marks Incorrect direction of inequality symbol (each inequality).
0 marks Any other error subject to B2.

Blunders (–3)
B1 Incorrect inequality symbol, each inequality.
B2 Both directions of inequality given (each inequality).

Part (b)	**35 (15, 10, 10) marks**	**Att (2, 2, 2, 2, 2, -)**

A garage is starting a van rental business. The garage will rent out two types of vans, small vans and large vans.
To set up the business, each small van costs €20 000 and each large van costs €40 000.
The garage has at most €800 000 to purchase the vans.
Each small van requires 18 m^2 of parking space and each large van requires 24 m^2 of parking space. The garage has at most 576 m^2 of parking space available for the vans.

(i) Taking x as the number of small vans and y as the number of large vans, write down two inequalities in x and y and illustrate these on graph paper.
(ii) The garage charges €40 a day to rent a small van and €50 a day to rent a large van. How many of each should the garage rent to maximise rental income, assuming that all vans are rented?
(iii) The garage incurs daily expenses of €12 for each van. Calculate the maximum daily profit from renting the vans.

(b) (i) Inequalities	**10 (5, 5) marks**	**Att (2, 2)**

Cost: $20\,000x + 40\,000y \le 800\,000$ or $x + 2y \le 40$ or similar

Time: $18x + 24y \le 576$ or $3x + 4y \le 96$ or similar

* Accept correct multiples or fractions of inequalities or the use of different letters.
* Apply (-3), once, if no inequality sign or the incorrect inequality sign is written the first time it occurs.
* Accept $<$ for $\le$.
* Table as is, worth 7 marks.

	Small x	Large y	Maximum
Cost	20000	40000	800000
Space	18	24	576

7 Marks

Blunders (-3)
B1 Mixes up x's and y's (once if consistent error), e.g. $24x + 18y \le 576$.
B2 Confuses rows and columns in table, e.g. $20000x + 18y \le 576$ (once if consistent).
B3 Decimal blunder applies for error with zeros in equation, unless an obvious misreading.

Attempts (2 marks)
A1 Incomplete relevant data in table and stops e.g. $20\,000x$ or $18x$ or $40\,000y$ or $24y$ or $\le 800\,000$ or ≤ 576 (each inequality).
A2 Any other correct inequality e.g. $x \ge 0$, $y \ge 0$ (each time).

(b) (i) Graph **5 marks** **Att 2**

Points or scales required. Scales need not be the same.

- * Correct shading over-rules arrows or correct arrows overrule shading.
- * Inequalities not written but correct graph drawn – award 0 + 5 marks.
- * Two lines drawn and no shading indicated, only one of the following cases applies:

Case 1:	Two sets of arrows in expected direction	5 marks
Case 2:	Two sets of arrows in unexpected direction	5 marks
Case 3:	One set of arrows "correct", the other "incorrect"	2 marks
Case 4:	One line with and the other without arrows	2 marks
Case 5:	No arrows	2 marks
Case 6:	Half-planes consistent with incorrect, penalised inequalities.	5 marks

Blunders (-3)

B1 Blunder in plotting a line or calculations.

B2 Incorrect shading e.g. one or both of the small triangles shaded (subject to case 6 above).

B3 Vertical *x*-axis, horizontal *y*-*a*xis.

Attempts (2 marks)

A1 Some relevant work towards a point on a line.

A2 Draws scaled axes or axes and one line.

(b) (ii) Intersection **5 marks** **Att 2**
Rental Income **5 marks** **Att 2**

$$\begin{array}{lcl} 2x + 4y = 80 & \Rightarrow & 12x + 24y = 480 \\ \underline{18x + 24y = 576} & & \underline{18x + 24y = 576} \\ & & 6x \qquad = 96 \Rightarrow x = 16 \Rightarrow y = 12 \end{array}$$

[*5 marks*]

Step1	Vertices	$40x + 50y$	Income
	(0,0)	0 + 0	0
Step2	(0,20)	0 + 1000	1000
Step3	(16, 12)	640 + 600	1240
Step4	(32, 0)	1280 + 0	1280
Step 5	32 small vans and 0 large vans to maximise income [*5 marks*]		

* Accept candidate's own equations from previous sections.
* If solving incorrect equations, the point found may be outside the feasible set; accept for correct work and accept in later sections.
* If no marks have been awarded for intersection of lines and this point is written here award Att 2 for the previous work and also reward it here if the step is correct.
* Information does not have to be in table form.
* Award 1 mark for each consistent step, subject to the attempt mark.
* Accept any correct multiple or fraction of $40x + 50y$ here.
* Accept work on a feasible set of points (at least three relevant points) formed by axes and one line without further penalty.
* Accept only vertices consistent with previously accepted work, not arbitrary ones.
* If (40, 0) or (0, 24) is tested and result is used to give maximum income, award zero for step 5, otherwise ignore.
* Step 5 must be explicitly written to gain full marks.

Blunders (-3)
B1 Fails to multiply / divide both sides of equation(s) correctly when eliminating variable.
B2 x or y value only found.

Slips (-1)
S1 Numerical slips to a maximum of 3.
S2 Each arithmetic slip to a maximum of 3.
S3 Each step of the solution omitted, subject to the attempt mark [Step 1 may be implied].

Attempts (2 marks)
A1 Correct or consistent answer without work or from a graph.
[Should get the *exact same* values from graph as if they had been found algebraically.]
A2 Any relevant step towards solving equations.
A3 Any relevant work involving x or y and/or 40, 50 or similar.
A4 Any attempt at substituting co-ordinates into some relevant expression.

Worthless (0 marks)
W1 Incorrect answer without work and inconsistent with graph.
W2 Writing €40 or €50 without further work.

(b) (iii) Profit **10 marks** -

Step 1	Vertices	Income	Expenses	Profit
	(0,0)	0	0	0
Step 2	(0, 20)	1000	240	760
Step 3	(16,12)	1240	336	904
Step 4	(32,0)	1280	384	896
Step 5	Maximum daily profit is €904			

OR

Step 1	Vertices	$28x + 38y$	Profit
	(0,0)	0 + 0	0
Step 2	(0, 20)	0 + 760	760
Step 3	(16,12)	448 + 456	904
Step 4	(32,0)	896 +0	896
Step 5	Maximum daily profit is €904		

* Information does not have to be in table form.
* Accept only vertices consistent with previously accepted work, not arbitrary ones.
* Maximum profit must be written or highlighted to gain full marks.
* Testing only (16,12) to get 904 merits 7 marks even if the candidate writes €904 maximum profit.

Award the following marks only:

10 marks Fully correct or consistent answer.
9 marks One slip, misreading. [see *Misreading* below]
7 marks Work of merit.
0 marks No work of merit.

Treat as separate blunders:
- Mathematical error e.g. 38(0) = 38.
- Each step of the solution omitted, subject to the attempt mark [step 1 may be implied].
- Mishandles the objective function.

Slips (-1)
- Numerical slips to a maximum of 3.

Misreadings (-1)
- Indicates or writes (16 small, 12 large) instead of profit.

Award 7 Marks
- Any correct step, e.g. 40 – 12 = 28 and stops.
- Any work involving $28x$ or 38 y.
- Uses $12x + 12y$ for profit expression.
- Any relevant work involving x or y and / or 12, 28 or 38 or similar.
- Any attempt at substituting co-ordinates into some relevant expression.

Worthless (0 marks)
- Simply writing down €40 or €50 and no other work.

MARCANNA BREISE AS UCHT FREAGAIRT TRÍ GHAEILGE

(Bonus marks for answering through Irish)

Ba choir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba choir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. 198 marc $\times$ 5% = 9·9 $\Rightarrow$ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle [300 – bunmharc] $\times$15%, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0