Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2011

MATHEMATICS – ORDINARY LEVEL

PAPER 1 (300 marks)

FRIDAY, 10 JUNE – AFTERNOON, 2:00 to 4:30

Attempt **SIX QUESTIONS** (50 marks each).

WARNING: Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

1. **(a)** Aoife and Brian share a prize fund in the ratio 4 : 3. Aoife gets €56.

(i) Find the total prize fund.

(ii) How much does Brian get?

(b) The cost of staying for three nights in a hotel in England is £231 sterling.

(i) Find that cost in euro, given that €1 = £0·88 sterling?

(ii) This cost is 5% more than the cost a year ago.
Find, in euro, the cost a year ago.

(c) The speedometer in a car is faulty. When the car is actually travelling at 57 km/h, the speedometer reads 60 km/h.

(i) Calculate the percentage error, correct to one decimal place.

(ii) If the percentage error is the same at all speeds, at what speed is the car actually travelling when the speedometer reads 110 km/h?
Give your answer correct to one decimal place.

(iii) The driver is not aware of the fault. He calculates that if he travels at an average speed of 80 km/h as shown on the speedometer, he will reach his destination in four hours.
How long, correct to the nearest minute, will it actually take him to reach his destination?

2. **(a)** Given that $3a(x + 5) = 114$, find the value of x when $a = 4$.

(b) **(i)** Find A, the solution set of $3x - 5 < 7, \quad x \in \mathbb{Z}$.

(ii) Find B, the solution set of $\frac{-2 - 3x}{4} \leq 1, \; x \in \mathbb{Z}$.

(iii) List the elements of $A \cap B$.

(c) Let $f(x) = x^3 - 2x^2 + cx + d$.

(i) Given that $f(0) = 6$, find the value of d.

(ii) Given that $f(3) = 0$, find the value of c.

(iii) Hence, solve the equation $f(x) = 0$.

3. **(a)** Multiply $(3x - 1)(2x^2 + 5x - 4)$ and simplify your answer.

(b) **(i)** Solve for x and y

$$2x = 13 + 3y$$
$$\frac{x}{2} = \frac{2 - y}{5}.$$

(ii) Hence, find the value of $4(x - y^2)$.

(c) **(i)** Solve for x

$$\frac{x-1}{x} + \frac{x}{x+1} = \frac{1}{2}, \quad x \neq 0, \ x \neq -1.$$

(ii) Verify **one** of your solutions.

4. **(a)** Let $u = 1 + 2i$, where $i^2 = -1$.
Plot on an Argand diagram

(i) u

(ii) $u - 3$.

(b) Let $z = 2 + 3i$.

(i) Find z^2 in the form $x + yi$, where $x, y \in \mathbb{R}$.

(ii) Show that $z^2 = 4z - 13$.

(iii) Show that $\bar{z}^2 + 13 = 4\bar{z}$, where $\bar{z}$ is the complex conjugate of z.

(c) **(i)** Express $\frac{4 + 2i}{3 - i}$ in the form $x + yi$, where $x, y \in \mathbb{R}$.

(ii) Hence, or otherwise, find the real numbers k and t such that

$$\left|\frac{4 + 2i}{3 - i}\right|(k + 5i) = \frac{1}{\sqrt{2}}(7 + (t - 1)i).$$

5. **(a)** The first term of a geometric sequence is 5 and the common ratio is 2.
Find the first four terms of the sequence.

(b) The first three terms of an arithmetic series are $7 + 4 + 1 + \ldots$.

(i) Find d, the common difference.

(ii) Find T_{15}, the fifteenth term of the series.

(iii) Find S_{15}, the sum of the first fifteen terms of the series.

(c) The first three terms of a geometric sequence are

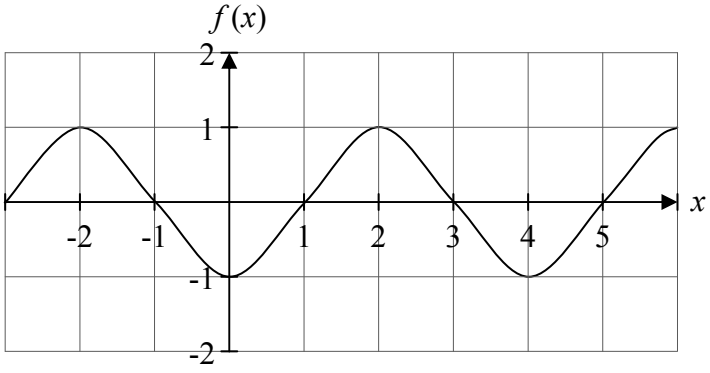
$$h - 1, \quad 2h \text{ and } 5h + 3,$$

where h is a real number greater than 1.

(i) Find the value of h.

(ii) The kth term of the sequence is 486. Find k.

6. **(a)** $f : x \to f(x)$ is a periodic function defined for $x \in \mathbb{R}$.
The period is as indicated in the diagram.



(i) Write down the period and the range of the function.

(ii) Find $f(71)$.

(b) **(i)** Differentiate $(4x-1)(3-2x^2)$ with respect to x and simplify your answer.

(ii) Given that $y = \dfrac{1}{x^2 - 3x}$, $x \neq 3$, find the range of values of x for which $\dfrac{dy}{dx} < 0$.

(c) Let $f(x) = 2x + \dfrac{1}{x}$, where $x \in \mathbb{R}$ and $x \neq 0$.

(i) Find the equation of the tangent to the curve $y = f(x)$ at the point $P(1, 3)$.

(ii) Q is another point on the curve $y = f(x)$ such that the tangent at Q is parallel to the tangent at P. Find the co-ordinates of Q.

7. **(a)** Differentiate $x^3 - 7x^2 + 6x$ with respect to x.

(b) **(i)** Differentiate $\frac{3x+1}{x-2}$ with respect to x.

Write your answer in the form $\frac{k}{(x-2)^n}$, where $k, n \in \mathbb{Z}$.

(ii) Given that $y = (x^2 - 2x - 9)^4$, find the value of $\frac{dy}{dx}$ when $x = -2$.

(c) A ball is rolled in a straight line along a surface.
The distance, s metres, the ball travels is given by

$$s = 18t - 2t^2$$

where t is the time in seconds from the instant the ball begins to move.

(i) Find the speed of the ball after 3 seconds.

(ii) How far is the ball from the starting point when it stops moving?

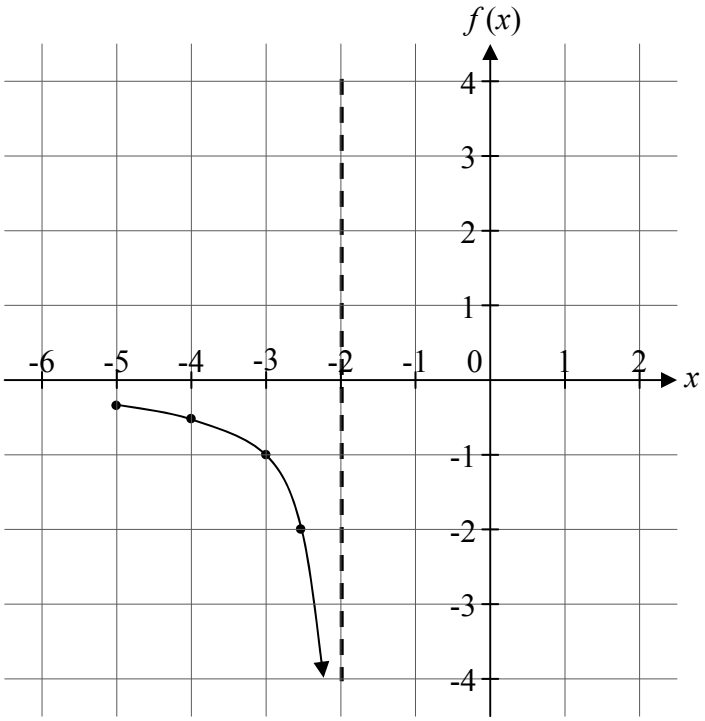
(iii) Show that the speed of the ball decreases at a constant rate while it is moving.

8. Let $f(x) = \dfrac{1}{x+2}$, where $x \in \mathbb{R}$ and $x \neq -2$.

(i) Copy and complete the following table:

x	-5	-4	-3	$-2{\cdot}5$	$-1{\cdot}5$	-1	0	1
$f(x)$		$-0{\cdot}5$	-1	-2				

(ii) The diagram shows part of the graph of the function f.
Copy and complete the graph from $x = -5$ to $x = 1$.



(iii) On the same diagram, draw the graph of the function $g(x) = x + 2$ in the domain $-5 \leq x \leq 1$, where $x \in \mathbb{R}$.

(iv) Use your graphs to estimate the range of values of x for which $f(x) \leq g(x)$.

(v) Prove that the curve $y = f(x)$ has no turning points.

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