



Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE 2010

MARKING SCHEME

**MATHEMATICS
(PROJECT MATHS)**

ORDINARY LEVEL

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Introduction

The Ordinary Level Mathematics examination for candidates in the 24 initial schools for *Project Maths* shared a common Paper 1 and one common question on Paper 2 with the examination for all other candidates. The marking scheme used for these common elements was identical for the two groups.

This document contains the complete marking scheme for both papers for the candidates in the 24 schools.

Readers should note that, as with all marking schemes used in the state examinations, the detail required in any answer is determined by the context and the manner in which the question is asked, and by the number of marks assigned to the question or part. Requirements and mark allocations may vary from year to year.

Marking scheme for Paper 1

GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates' work as follows:
 - Blunders - mathematical errors/omissions (-3)
 - Slips - numerical errors (-1)
 - Misreadings (provided task is not oversimplified) (-1).

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,..., S1, S2,..., M1, M2,...etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
 - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
 - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
 - a mark between zero and the attempt mark is never awarded.
3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,...etc.
4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.
5. The phrase “and stops” means that no more work is shown by the candidate.
6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.
7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.
8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.
9. The *same* error in the *same* section of a question is penalised *once* only.
10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.
11. A serious blunder, omission or misreading results in the attempt mark at most.
12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

APPLYING THE GUIDELINES

Examples (**not exhaustive**) of the different types of error:

Blunders (i.e. mathematical errors) (-3)

- Algebraic errors : $8x + 9x = 17x^2$ or $5p \times 4p = 20p$ or $(-3)^2 = 6$
- Sign error $-3(-4) = -12$
- Decimal errors
- Fraction error (incorrect fraction, inversion etc); apply once.
- Cross-multiplication error
- Operation chosen is incorrect, (e.g. multiplication instead of division)
- Transposition error, e.g. $-2x - k + 3 \Rightarrow -2x = 3 + k$ or $-3x = 6 \Rightarrow x = 2$ or
- $4x = 12 \Rightarrow x = 8$; each time.
- Distribution error (once per term, unless directed otherwise) e.g. $3(2x + 4) = 6x + 4$ or $\frac{1}{2}(3 - x) = 5 \Rightarrow 6 - x = 5$
- Expanding brackets incorrectly, e.g. $(2x - 3)(x + 4) = 8x^2 - 12$
- Omission, if not oversimplified.
- Index error, each time unless directed otherwise
- Factorisation: error in one or both factors of a quadratic: apply once, e.g. $2x^2 - 2x - 3 = (2x - 1)(x + 3)$
- Root errors from candidate's factors: error in one or both roots: apply once.
- Error in formula
- Error(s) in transcribing formulae from tables (assuming it generates mathematical acceptable answer(s)) Serious errors or over simplifications will merit Attempt marks at most (check relevant section of scheme)
- Central sign error in uv or u/v formulae
- Omission of $\div v^2$ or division not done in u/v formula (apply once)
- Vice-versa substitution in uv or u/v formulae (apply once)
- Quadratic formula (acceptable) and its application apply a maximum of two blunders

Slips (-1)

- Numerical slips: $4 + 7 = 10$ or $3 \times 6 = 24$, but $5 + 3 = 15$ is a blunder.
- An omitted round-off or incorrect round off to a required degree of accuracy, or an early round off, is penalised as a slip each time.
- However an early round-off which has the effect of simplifying the work is at least a blunder
- Omission of units of measurement or giving the incorrect units of measurement in an answer is treated as a slip, once per part (a), (b) and (c) of each question. Only applies where a candidate would otherwise have achieved full marks

Misreadings (-1)

- Writing 2436 for 2346 will not alter the nature of the question so M(-1)
However, writing 5000 for 5026 will simplify the work and is penalised as at least a blunder.

QUESTION 1

Part (a)	10 marks	Att 3
Part (b)	25 (15, 5, 5) marks	Att (5, 2, 2)
Part (c)	15 (10, 5) marks	Att (3, 2)

Part (a) 10 marks Att 3

Express 40 metres as a fraction of 1 kilometre. Give your answer in its simplest form.

(a) 10 marks Att 3

1 km = 1000 m [3]	or	
$\frac{40}{1000}$ [7] = $\frac{2}{50}$ [9] = $\frac{1}{25}$ [10]		0.04 [7] = $\frac{4}{100}$ [9] = $\frac{1}{25}$ [10]

* Accept correct answer without work for full marks. Accept 1:25

* Accept without work 0.04, 4%, 25:1 or $\frac{25}{1}$ for [7] marks

* Accept without work $\frac{40}{100}$, $\frac{20}{50}$, $\frac{4}{10}$, $\frac{2}{5}$ or 0.4 for [4] marks – **these only**

Blunders (-3)

B1 Mathematical error e.g. conversion/decimal error

B2 Fraction error

B3 No simplification

Slips (-1)

S1 Simplification not completed to simplest form, between $\frac{40}{1000}$ and $\frac{1}{25}$

Attempts (3 marks)

A1 $\frac{1}{40}$ or $\frac{40}{1}$

A2 Some effort at conversion

A3 Mentions 25 without supporting work

Worthless (0)

W1 Incorrect answer with no work

Part (b)**25 (15, 5, 5) marks****Att (5, 2, 2)**

(i) Calculate the value of

$$\frac{57.6 + 80.44}{1.3 \times 10^4}$$

and write your answer correct to three decimal places.

- (ii) An importer buys an item for £221 sterling when the rate of exchange is €1 = £0.85 sterling.
He sells it at a profit of 14% of the cost price.
Calculate, in euro, the price for which he sells the item.

(i)**15 marks****Att 5**

$$\frac{57.6 + 80.44}{1.3 \times 10^4} = \frac{138.04}{13\,000} = 0.0106 = 0.011$$

$$57.6 + 80.44 = 138.04 \quad [5]$$

$$\text{or } 1.3 \times 10^4 = 13\,000 \quad [5] \quad \text{Both } [9]$$

$$\frac{138.04}{13\,000} \quad [12] = 0.0106184154 \quad [14]$$

$$= 0.011 \quad [15]$$

* Accept correct answer without work for 15 marks 0.01.....[14 marks] without work

* Accept without work for 12 marks :

57.606, 94.1756 - 94.176, 618826.8307- 618826.831 , **These only**
 [12] [11] [12] [11] [12]

Blunders (-3)

B1 Mathematical error

Slips (-1)

S1 Incorrect or no rounding off

S2 Numerical slips which are not mathematical errors

Misreading (-1)

M1 Must not make work easier – see guidelines

(ii) Exchange
Percentage

5 marks
5 marks

Att 2
Att 2

$$\frac{\pounds 221 \times 1.14}{0.85} = \frac{251.94}{0.85} = \text{€}296.40$$

[2] + [2] [5] + [2]

Sterling to Euro exchange

$$\pounds 221 = \text{€} \frac{221}{0.85} = \text{€}260 \quad [5]$$

Percentage

$$14\% \text{ of } \text{€}260 = \text{€}36.40 \quad [4]$$

$$\text{€}260 + \text{€}36.4 = \text{€}296.40 \quad [5]$$

or

$$\text{€}260 \times 1.14 = \text{€}296.40 \quad [5]$$

Percentage

$$\pounds 221 \times 0.14 = \pounds 30.94 \text{ (14\% of } \pounds 221) \quad [4]$$

$$\pounds 221 + \pounds 30.94 = \pounds 251.94 \quad [5]$$

or

$$\pounds 221 \times 1.14 = \pounds 251.94 \quad [5]$$

Sterling to Euro exchange

$$\frac{\pounds 251.94}{0.85} = \text{€}296.40 \quad [5]$$

* Accept correct answer without work for full marks [5] + [5]

* No penalty if € not included

Blunders (-3)

B1 Error in finding percentage e.g. decimal or inversion

B2 Error in currency conversion e.g. incorrect operation

Slips (-1)

S1 Fails to add percentage profit

Attempts (2 marks)

A1 Any relevant step, may get both

Worthless (0)

W1 Incorrect answer without work

- (i) What sum of money invested at 5% per annum compound interest will amount to €8682 in 3 years?
Give your answer correct to the nearest euro.
- (ii) A sum of € P was invested at r % per annum compound interest.
The interest for the first year was €220
The interest for the second year was €228.80
Calculate r and P .

(c) (i)

10 marks

Att 3

I

$$F = P(1+i)^t \Rightarrow 8682 = P(1.05)^3 \Rightarrow \frac{8682}{1.157625} = 7499.83 \Rightarrow P = \text{€}7500$$

[4] [7] [9] [10]

II

$$P = \frac{F}{(1+i)^n} = \frac{8682}{(1+0.05)^3} \quad [4] = \frac{8682}{1.157625} \quad [7] = 7499.83 = \text{€}7500$$

III

€8682 at end year 3

$$P \text{ year 3} = \frac{8682}{1.05} = 8268.57$$

$$P \text{ year 2} = \frac{8268.57}{1.05} = 7874.83$$

$$P \text{ year 1} = \frac{7874.83}{1.05} = 7499.83 = \text{€}7500$$

IV

$$P \text{ year 1} = 100\%; \quad P \text{ year 2} = 105\%; \quad P \text{ year 3} = 110.25\%; \quad P \text{ year 4} = 115.7625\%$$

$$115.7625\% = \text{€}8682 \quad [7]$$

$$100\% = \frac{8682}{1.157625} \times 100 = 7499.83 = \text{€}7500$$

* Candidates may offer other correct versions

* *Formulae and Tables*, page 30, use F for A and i for $\frac{r}{100}$ *Blunders (-3)*B1 Mathematical error e.g. percentages or index Note $8682(1.05)^3 = 10\,050.50 = 10\,051$ [7]

B2 Incorrect number of years

B3 Fails to finish method IV

Slips (-1)

S1 Incorrect or no rounding off

Attempts (3 marks)

A1 No compounding of interest - offers €8682 – 15% (€7380) Work must be shown

A2 Answer found by trial and error

A3 5% or 15% of 8682 or mentions 1.05 or 1.15

A4 7499.83 or 7500 without work

Worthless (0)

W1 Incorrect answer without work

(c) (ii)

5 marks

Att 2

Finding r

I

$$F = P(1+i)^t \Rightarrow 220(1+i) = 228.80 \Rightarrow (1+i) = 1.04 \Rightarrow r = 4$$

II

$$\text{Interest on } \text{€}220 = 228.80 - \text{€}220 = \text{€}8.80$$

$$\frac{8.80}{220} \times 100 = 4$$

Finding P

$$P(0.04) = 220 \Rightarrow P = 5500$$

$$4\% = 220$$

$$1\% = 55$$

$$100\% = 5500$$

* Candidates may offer other correct versions

* *Formulae and Tables*, page 30, use F for A and i for $\frac{r}{100}$

Blunders (-3)

B1 Mathematical error

B2 Error in finding % from 1.04, method I

Attempts (2 marks)

A1 Finds €8.80

A2 Finds by “trial and error” or $r = 4\%$ verified

A3 Correct answer without work

Worthless (0)

W1 Incorrect answer without work

Note

Award **5 marks** for fully correct with work

Award **2 marks** for some relevant work

Otherwise **0 marks**

Part (b)

25 (10, 5, 5, 5) marks

Att (3, 2, 2, 2)

Solve for x and y

$$2x - y = 1$$

$$x^2 - xy = -6.$$

(b)

25 (10, 5, 5, 5) marks

Att (3, 2, 2, 2)

$$2x - y = 1 \Rightarrow y = 2x - 1$$

Step 1 Isolates x or y

[10]

$$x^2 - xy = -6$$

Step 2 Forms quadratic equation

[5]

$$\Rightarrow x^2 - x(2x - 1) = -6$$

(Penalise error in simplification at Step 3)

$$x^2 - 2x^2 + x + 6 = 0$$

Step 3 Roots of quadratic

[5]

$$\Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow (x - 3)(x + 2) = 0$$

$$\Rightarrow x = 3 \quad \text{or} \quad x = -2$$

$$y = 5 \quad \text{or} \quad y = -5$$

Step 4 Values of other coordinate

[5]

* Error(s) in simplification of quadratic equation apply at the Step 3

* If equation at Step 2 becomes linear award at most Att 2 + Att 2 for Steps 3 and 4

* Apply similar scheme if candidate isolates x at Step 1

* Random value(s) of x award attempt marks at most (Step 4) if no work of merit in previous steps

Blunders (-3)

B1 Mathematical error – apply at relevant step – see note

B2 Incorrect factors – Step 3

B3 Incorrect roots from factor – Step 3

B4 Only finds one value of x – Step 3 Note B5 will also apply at Step 4

B5 Only finds one value of y

Attempts (3 or 2 marks)

A1 Some relevant work

Note: Don't award multiple Attempts to the same piece of work

Part (c)

10 (5, 5) marks

Att (2, 2)

- (i) Show, by division, that $3x + 1$ is a factor of $3x^3 + 4x^2 - 89x - 30$.
(ii) Hence, or otherwise, solve the equation $3x^3 + 4x^2 - 89x - 30 = 0$.

(c) (i)

5 marks

Att2

$$\begin{array}{r} x^2 + x - 30 \\ 3x + 1 \overline{) 3x^3 + 4x^2 - 89x - 30} \\ \underline{3x^3 + x^2} \\ 3x^2 - 89x \\ \underline{3x^2 + x} \\ -90x - 30 \\ \underline{-90x - 30} \\ 0 \quad 0 \end{array}$$

Blunders (-3)

B1 Each error in division

B2 Shows clearly $f(-\frac{1}{3}) = 0$

Attempts (2 marks)

A1 Some correct division and stops

A2 Substitutes $-\frac{1}{3}$ into expression or mentions $f(-\frac{1}{3})$

A3 Sets up division correctly

(c) (ii)

5 marks

Att 2

$$\begin{aligned}3x^3 + 4x^2 - 89x - 30 &= 0 \\ \Rightarrow (3x+1)(x^2 + x - 30) &= 0 \\ \Rightarrow (3x+1)(x-5)(x+6) &= 0 \\ \Rightarrow x = -\frac{1}{3}, x = 5, x = -6\end{aligned}$$

* Accept candidates answer from part (i) provided it does not over simplify question

* Accept $f(5)$ and $f(-6)$ fully verified for 4 marks

Blunders (-3)

B1 Incorrect factors of quadratic

B2 Incorrect or missing roots from factors, but see S1

Slips (-1)

S1 Omits $x = -\frac{1}{3}$ as a root, if left out [4] at most

Attempts (2 marks)

A1 States $x = -\frac{1}{3}$ is a root and stops in part (ii)

A2 Attempt at factorising quadratic from (i)

A3 Some correct use of “ $-b$ ” formula [Note: Stating formula does not merit attempt mark]

A4 Correct answers without relevant work

A5 Sets up using answer from (i)

A6 Finds $f(k)$, $k \neq 5$, $k \neq -6$

Worthless (0 marks)

W1 Attempts at factorising $3x^3 + 4x^2 - 89x - 30 = 0$ such as $x^2(3x + 4) = 89x + 30$

W2 Differentiation

QUESTION 3

Part (a)	15 (10, 5) marks	Att (3, 2)
Part (b)	20 (10, 10) marks	Att (3, 3)
Part (c)	15 (10, 5) marks	Att (3, 2)

Part (a) **15 (10, 5) marks** **Att (3, 2)**

Given that $3(b + a) = t(6 - a)$, calculate the value of a when $t = 3$ and $b = -4$.

(a) **15 (10, 5) marks** **Att 3, 2**

I	Substitution for t and b :	10 marks	
	Evaluation of a :	5 marks	
	$3(b + a) = t(6 - a)$		
	$3(-4 + a) = 3(6 - a) \Rightarrow -12 + 3a = 18 - 3a \Rightarrow 6a = 30 \Rightarrow a = 5$		
II	$3b + 3a = 6t - at$		substitution merits [10]
	$3a + at = 6t - 3b$		
	$a(3 + t) = 6t - 3b$		
	$a = \frac{6t - 3b}{3 + t} = \frac{6 \times 3 - 3 \times -4}{3 + 3} = \frac{18 + 12}{6} = 5$		rest of work [5]

* Accept correct answer without work.

* Once a candidate has substituted correctly for t and b he/she is entitled to [10] marks

Blunders (-3)

- B1 Mathematical error e.g transposition, distribution, from 5 marks
- B2 Substitution error
- B3 Substitutes one value only
- B4 Interchanges t and b

Attempts (2 marks)

- A1 Some correct effort at isolating /evaluating a , from 5 marks

Worthless (0)

- W1 Incorrect answer without work

Solve for x

$$5(x + 1)^2 = 2(x + 1) + 5.$$

Give your answer correct to two decimal places.

(b)

20 (10, 10) marks

Att (3, 3)

Step 1, forming quadratic equation: 10 marks

Step 2, solving quadratic equation: 10 marks

I

$$5(x + 1)^2 = 2(x + 1) + 5$$

$$\text{Let } y = x + 1$$

$$5y^2 = 2y + 5 \Rightarrow 5y^2 - 2y = 5 \quad [10]$$

$$5y^2 - 2y - 5 = 0$$

$$\Rightarrow y = \frac{2 \pm \sqrt{4 - 4(5)(-5)}}{2(5)} [4] = \frac{2 \pm \sqrt{104}}{10} = \frac{2 \pm 10.198}{10} [7] = \frac{12.198}{10} \text{ or } \frac{-8.198}{10}$$

$$\Rightarrow y = 1.2198 \text{ or } y = -0.8198 \quad [9]$$

$$\Rightarrow x = 0.22 \text{ or } x = -1.82 \quad [10]$$

II

$$5(x + 1)^2 = 2(x + 1) + 5$$

$$\Rightarrow 5x^2 + 10x + 5 = 2x + 7$$

$$\Rightarrow 5x^2 + 8x = 2 \quad [10]$$

$$5x^2 + 8x - 2 = 0$$

$$\Rightarrow x = \frac{-8 \pm \sqrt{64 - 4(5)(-2)}}{2(5)} [4] = \frac{-8 \pm \sqrt{104}}{10} = \frac{-8 \pm 10.198}{10} [7] = \frac{2.198}{10} \text{ or } \frac{-18.198}{10}$$

$$\Rightarrow x = 0.22 \text{ or } x = -1.82 \quad [10]$$

* Accept candidate's quadratic equation for second 10 marks if not factorisable

* If quadratic equation reduced to a linear attempt marks at most in Step 2

Blunders (-3)

B1 Mathematical error each time

B2 Error in use of quadratic formula to a maximum of 2 (Step 2)

Slips (-1)

S1 Fails to round off or rounds off incorrectly – once only

S2 Early rounding off that affects answer

S3 Fails to find x from y in method **I***Attempts (3 marks)*A1 Some effort at multiplying out equation - Step 1 Method **11**

A2 If equation becomes linear, maximum possible mark from Step 2 is Attempt

A3 Solves a factorisable quadratic equation even if they use formula

A4 Attempts to factorise the quadratic

Part (c)**15 (10, 5) marks****Att (3, 2)**

- (i) $2 + \sqrt{3}$ is a root of the equation $x^2 - 4x + c = 0$, where c is a real number.
Find the value of c and write down the other root.
- (ii) The equation $x^2 + 10x + k = 0$ has equal roots.
Find the value of the real number k and write down the value of each root.

(i)**10 marks****Att 3**

$$x^2 - 4x + c = 0$$

$$\Rightarrow (2 + \sqrt{3})^2 - 4(2 + \sqrt{3}) + c = 0 \quad [4]$$

$$\Rightarrow 4 + 4\sqrt{3} + 3 - 8 - 4\sqrt{3} + c = 0$$

$$\Rightarrow c = 1 \quad [7]$$

$$\text{Other root: } 2 - \sqrt{3} \quad [10]$$

* Accept any valid method

Blunders (-3)

- B1 Mathematical error
B2 Using decimals $c \neq 1$

Attempts (3 marks)

- A1 Some correct substitution
A2 Some correct substitution into “ $-b$ ” formula
A3 States 2nd root is $2 - \sqrt{3}$ and stops must be in surd form
A4 $c = 1$ without work even if second root found

(ii)**5 marks****Att 2**

I
Let root = p
 $(x - p)(x - p) = 0 \Rightarrow x^2 - 2px + p^2 \Rightarrow -2p = 10 \Rightarrow p = -5 \Rightarrow k = (-5)^2 = 25 \quad [4]$

II
 $b^2 - 4ac = 0 \Rightarrow 100 - 4(1)(k) = 0 \Rightarrow k = 25 \quad [4]$
 $x = -5, \quad [5]$

* Accept any valid method

Blunders (-3)

- B1 Mathematical error

Note:

$x^2 + 10x + 25 \quad [\text{Att } 2]$

$(x + 5)(x + 5) \quad [\text{Att } 2]$

Slips (-1)

- S1 Value of root omitted

$k = 25 \quad [4]$

$x = -5 \quad [5]$

Attempts (3 marks)

- A1 Correct answer for k without work
A2 Roots found without work
A3 Correct answer without work

QUESTION 4

Part (a)	15 marks	Att 5
Part (b)	20 (10, 10) marks	Att (3, 3)
Part (c)	15 (5, 5, 5) marks	Att (2, 2, 2)

Part (a) **15 marks** **Att 5**

Given that $i^2 = -1$, simplify $(4 + 2i)(3 - i)$
and write your answer in the form $x + yi$, where $x, y \in \mathbb{R}$.

(a) **15 marks** **Att 5**

$$(4 + 2i)(3 - i) = 4(3 - i) + 2i(3 - i) = 12 - 4i + 6i - 2i^2 = 12 + 2i + 2 = 14 + 2i$$

[9] [12] [14] [15]

Blunders (-3)

- B1 Mathematical error
- B2 Error in multiplication – maximum of 2 blunders
- B3 $i^2 \neq -1$, mis-use of i^2 or avoids use of i^2 B1 and B2 can apply
- B4 Mixes up real and imaginary terms

Slips (-1)

- S1 Numerical slips

Attempts (5 marks)

- A1 Any correct relevant multiplication

Worthless (0)

- W1 Incorrect answer without work

Part (b)

20 (10, 10) marks

Att (3, 3)

Let $u = 4 + 3i$ and $w = 6 - 8i$

(i) Find the value of the real number k such that $|u| = k|w|$.

(ii) Express $\frac{w}{u}$ in the form $x + yi$.

(b) (i)

10 marks

Att 3

$$|u| = k|w|$$

$$\Rightarrow |4 + 3i| = k|6 - 8i|$$

$$\Rightarrow \sqrt{16 + 9} = k\sqrt{36 + 64}$$

$$\Rightarrow \sqrt{25} = k\sqrt{100}$$

$$\Rightarrow k = \frac{1}{2} \quad \text{accept } k = \frac{5}{10} = \frac{\sqrt{25}}{\sqrt{100}}$$

Note modulus: One correct $\sqrt{25}$ or $\sqrt{100}$ [4]

Two correct $\sqrt{25}$ and $\sqrt{100}$ [7]

* No penalty for using 8 for -8 in formula

* Accept distance from $(4, 3)$ to $(0, 0)$ or $(6, -8)$ to $(0, 0)$

Blunders (-3)

B1 Incorrect formula e.g. $\sqrt{\quad}$ omitted

B2 Incorrect substitution e.g. has $(3i)^2$ and /or $(8i)^2$ in $\sqrt{a^2 + b^2}$ - once only

B3 Mathematical error

Attempts (3 marks)

A1 Incorrect formula with some correct substitution

A2 Plots u and/or w

A3 Correct answer without work

A4 Correct modulus formula and stops

A5 Correct substitution for u and/or v

Worthless (0)

W1 Incorrect answer without work

(b) (ii)

10 marks

Att 3

$$\frac{w}{u} = \frac{6-8i}{4+3i} = \frac{6-8i}{4+3i} \times \frac{4-3i}{4-3i} \quad [3]$$

$$= \frac{24-18i-32i+24i^2}{16+9} \quad [7]$$

$$= \frac{0-50i}{25} \quad [9]$$

$$= 0-2i \quad \text{or} \quad = 0-\frac{50i}{25} \quad [10]$$

Note: 0 required in answer

* Can use multiple of conjugate i.e. $n(4-3i)$, n a real number, $n \neq 0$

* Calculates numerator or denominator, merits 4 marks

* Calculates numerator and denominator, merits 7 marks

Blunders (-3)

B1 $i^2 \neq -1$ or misuse of i^2

B2 Mathematical error in multiplying out numerator – maximum 1 blunder

B3 Mathematical error in multiplying out denominator – maximum 1 blunder

B4 Error in formation of $\frac{w}{u}$ at final stage e.g. may multiply numerator and denominator

Attempts (3 marks)

A1 Substitutes for u and/or w and stops

A2 Finds conjugate of u and stops

A3 Any correct relevant multiplication

Part (c)

15 (5, 5, 5) marks

Att (2, 2, 2)

Let $z = a + bi$, where $a, b \in \mathbb{R}$.
Find the value of a and the value of b for which
 $3z - 10i = (2 - 3i)z$.

(c)

15 (5, 5, 5) marks

Att (2, 2, 2)

I

$$3z - 10i = (2 - 3i)z$$
$$\Rightarrow 3(a + bi) - 10i = (2 - 3i)(a + bi) \quad [5]$$

$$\Rightarrow 3a + 3bi - 10i = 2a + 2bi - 3ai - 3bi^2$$
$$\Rightarrow 3a + 3bi - 10i = 2a + 2bi - 3ai + 3b \quad [5]$$

Real parts: $3a = 2a + 3b \Rightarrow a = 3b$
Imaginary parts: $3b - 10 = 2b - 3a \Rightarrow 3a + b = 10$
 $3a + b = 10 \Rightarrow 10b = 10 \Rightarrow b = 1$
 $\Rightarrow a = 3 \quad [5]$

II

$$3z - 10i = (2 - 3i)z$$
$$\Rightarrow z = 10i - 3zi$$
$$\Rightarrow a + bi - 10i = -3i(a + bi) \quad [5]$$

$$\Rightarrow a + bi = 10i - 3ai - 3bi^2$$
$$\Rightarrow a + bi = 10i - 3ai + 3b \quad [5]$$

Real parts: $3a = 2a + 3b \Rightarrow a = 3b$
Imaginary parts: $3b - 10 = 2b - 3a \Rightarrow 3a + b = 10$
 $3a + b = 10 \Rightarrow 10b = 10 \Rightarrow b = 1$
 $\Rightarrow a = 3 \quad [5]$

III

$$3z - 10i = (2 - 3i)z$$
$$\Rightarrow z = 10i - 3zi \quad \Rightarrow z + 3zi = 10i \quad \Rightarrow z(1 + 3i) = 10i \quad [5]$$

$$\Rightarrow z = \frac{10i}{1 + 3i} \quad [5]$$

$$\Rightarrow z = 3 + i = a + bi$$
$$\Rightarrow a = 3 \text{ and } b = 1 \quad [5]$$

Blunders (-3)

B1 Mathematical error - once per step

Attempts (2 marks)

A1 Any relevant work for a given step

QUESTION 5

Part (a)	10 marks	Att 3
Part (b)	20 (10, 5, 5) marks	Att (3, 2, 2)
Part (c)	20 (10, 5, 5) marks	Att (3, 2, 2)

* Do not penalise notation

Part (a)	10 marks	Att 3
-----------------	-----------------	--------------

The first term of a geometric sequence is 4 and the common ratio is 0.5.
Write down the first five terms of the sequence.

(a)	10 marks	Att 3
------------	-----------------	--------------

I

$$T_1 = a = 4,$$

$$T_2 = ar = 4 \times 0.5 = 2$$

$$T_3 = ar^2 = 4 \times 0.5^2 = 1 \text{ or } [2 \times 0.5]$$

$$T_4 = ar^3 = 4 \times 0.5^3 = 0.5 \text{ or } [1 \times 0.5]$$

$$T_5 = ar^4 = 4 \times 0.5^4 = 0.25 \text{ or } [0.5 \times 0.5]$$

II

List 4, 2, 1, 0.5, 0.25

* Accept correct answers with no work

* Accept in fractional form

Blunders (-3)

B1 Decimal error – once if consistent e.g. 0.5 taken as 5 or $r = 2$

B2 Indices error – each time

B3 Error in formula – see guidelines

Misreading (-1)

M1 r taken as 0.05

Attempts (3 marks)

A1 Identifies a as 4 and/or r as 0.5 and stops

A2 States $T_1 = 4$

Worthless (0)

W1 Treats as an arithmetic sequence but see A1 and A2

W2 Incorrect answer(s) without work

Note: Answers without work

1 term correct 3 marks

2 terms correct 4 marks

3 terms correct 4 marks

4 terms correct 7 marks

5 terms correct 10 marks

Part (b)**20 (10, 5, 5) marks****Att (3, 2, 2)**

In an arithmetic series, the first term is 6 and the fifth term is 22.

- (i) Find d , the common difference.
- (ii) Find T_{14} , the fourteenth term.
- (iii) Find S_{20} , the sum of the first twenty terms .

* Answers to parts of questions must be clearly identified

(i)**10 marks****Att 3****I**

$$T_1 = a = 6 \quad [3]$$

$$T_5 = a + 4d = 22 \quad [4]$$

$$\Rightarrow 4d = 22 - 6 \quad [7]$$

$$\Rightarrow d = 4 \quad [10]$$

II

$$6, 10, 14, 18, 22 \quad [7]$$

* Accept correct answer without work

* Acceptable formula - see guidelines

Blunders (-3)

B1 Mathematical error

Slips (-1)

S1 Numerical slips

Attempts (3 marks)

A1 Correct relevant work

A2 $22 - 4 = 16$ and stops or $d = 16$

(ii)**5 marks****Att 2****I**

$$T_{14} = a + 13d = 6 + 13(4) = 6 + 52 = 58$$

II

$$\text{List: } 6 + 10 + 14 + 18 + 22 + 26 + 30 + 34 + 38 + 42 + 46 + 50 + 54 + 58$$

(Assume final term is answer, otherwise must indicate term 14)

* Accept candidates d from (i)

* Accept correct answer without work

Blunders (-3)

B1 Mathematical error

B2 Incorrect term from list

B3 Finds S_{14} by formula

Slips (-1)

S1 Numerical slips

Attempts (2 marks)

A1 Identifies a as 6 for this part of question

Worthless (0)

W1 Treats as a geometric series but may have identified a as 6 as part of this question

(iii)

5 marks

Att 2

I

$$S_{20} = \frac{20}{2}(2a + 19d) = 10(12 + 76) = 10(88) = 880$$

II

$$\text{List: } 6+10+14+18+22+26+30+34+38+42+46+50+54+58+62+66+70+74+78+82 = 880$$

* Accept candidate's answers from (i) and (ii)

Blunders (-3)

- B1 Finds T_{20} and stops
- B2 Writes complete list but fails to sum
- B3 Finds S_{14} from (ii)
- B4 Incorrect number of terms in list

Slips (-1)

- S1 Numerical slips

Attempts (2 marks)

- A1 Identifies a and/or d
- A2 Correct answer without work.

Worthless (0)

- W1 Treats as a geometric series but identification of a will merit A1

Part (c)**20 (10, 5, 5) marks****Att (3, 2, 2)**

In a geometric series, the fourth term is 9 and the seventh term is 243.

- (i) Find r , the common ratio.
 (ii) Find a , the first term.
 (iii) Find S_8 , the sum of the first eight terms.

(c) (i)**10 marks****Att 3**

- | | |
|---|--|
| <p>I</p> <p>$T_4 = ar^3 = 9$ [3]</p> <p>$T_7 = ar^6 = 243$ [4]</p> <p>$\frac{ar^6}{ar^3} = \frac{243}{9} \Rightarrow r^3 = 27$ [7]</p> <p>$\Rightarrow r = 3$ [10]</p> <p>II</p> <p>List [$\frac{1}{3}$, 1, 3,] 9, 27, 81, 243 [7]</p> <p>$\Rightarrow r = 3$ [10]</p> | <p>Note: $ar^4 = 9$ for T_4
 $ar^7 = 243$ for T_7
 $r^3 = 27$ etc. Accept</p> |
|---|--|

Blunders (-3)

- B1 Mathematical error
 B2 Error in use of formula

Attempts (3 marks)

- A1 T_4 or T_7 expressed in algebraic form and stops
 A2 Finds $243/9 = 27$ and stops
 A3 Correct answer without work
 A4 Partial list

(c) (ii)**5 marks****Att 2**

- | | |
|--|--|
| <p>I</p> <p>$ar^3 = 9 \Rightarrow a(3^3) = 9 \Rightarrow 27a = 9 \Rightarrow a = \frac{1}{3}$</p> <p>[2] [5]</p> | <p>II</p> <p>$\frac{1}{3}, 1, 3, 9 \Rightarrow a = \frac{1}{3}$</p> <p>[2] [5]</p> |
|--|--|

* Accept candidate's r from (i) as long as it does not oversimplify work

Blunders (-3)

- B1 Mathematical error

Attempts (2 marks)

- A1 Any relevant step
 A2 Correct answer without work but allow if full list given in (i)

(c) (iii)

5 marks

Att 2

I

$$S_8 = \frac{a(r^n - 1)}{r - 1} = \frac{\frac{1}{3}(6561 - 1)}{3 - 1} = \frac{\frac{1}{3}(6560)}{2} = \frac{3280}{3} = 1093\frac{1}{3}$$

[2] [5]

II

List: $\frac{1}{3} + 1 + 3 + 9 + 27 + 81 + 243 + 729 = 1093.3333$

[2] [5]

* Accept candidate's a and r from (i) and (ii) provided they do not over simplify work

Blunders (-3)

- B1 Mathematical error
- B2 Fails to sum list in method II
- B3 Missing or extra terms in list method

Slips (-1)

- S1 Numerical slips

Attempts (2 marks)

- A1 Finds T_8
- A2 Identifies a as $1/3$ in this part
- A3 Correct answer without work

Worthless (0)

- W1 Treats as an arithmetic series but identification of a will merit A2

QUESTION 6

Part (a)	10 marks	Att 3
Part (b)	20 (10, 10) marks	Att (3, 3)
Part (c)	20 (10, 10) marks	Att (3, 3)

Part (a) **10 marks** **Att 3**

Let $h(x) = x^2 + 1$, where $x \in \mathbb{R}$.

Write down a value of x for which $h(x) = 50$.

Part (a) **10 marks** **Att 3**

$$h(x) = 50 \Rightarrow x^2 + 1 = 50 \Rightarrow x^2 = 49 \Rightarrow x = \pm 7$$

[3] [7] [10]

* Accept correct answer without work. Accept $\sqrt{49}$

* Only one value for x is required.

Blunders (-3)

B1 Mathematical errors

B2 Evaluates $h(50) = 2501$

Attempts (3 marks)

A1 Unsuccessful trial and error, e.g. $h(5) = 25 + 1$

A2 Any correct relevant step

Worthless (0)

W1 $50(x^2 + 1)$ whether continues or not

W2 Incorrect answer with no work

W3 Differentiates

Part (b)**20 (10, 10) marks****Att (3, 3)**

Let $g(x) = \frac{1}{x-2}$, where $x \in \mathbb{R}$ and $x \neq 2$.

(i) Copy and complete the following table:

x	0	1	1.5	1.75	2.25	2.5	3	4
$g(x)$		-1		-4		2		

(ii) Draw the graph of the function g in the domain $0 \leq x \leq 4$.

(b) (i)**10 marks****Att 3**

x	0	1	1.5	1.75	2.25	2.5	3	4
$g(x)$	-0.5	-1	-2	-4	4	2	1	0.5

* Values of $g(x) = \frac{1}{x-2}$ calculated (all/some correct) misreading which oversimplifies, Att 3

* Accept values as fractions; must be $\frac{1}{\text{Singleton}}$ or $\frac{1}{\text{Single number}}$

e.g. for $x = 1.5$ accept $\frac{1}{-0.5}$ but not $\frac{1}{1.5-2}$

Blunders (-3)

B1 Treats the function as $f(x) = \frac{1}{x} - 2$, even if $g(x) = \frac{1}{x-2}$ written.

The relevant values for $f(x) = \frac{1}{x} - 2$ are:

$(0, \text{undefined}), (1.5, -\frac{4}{3}), (2.25, -\frac{14}{9}), (3, -\frac{5}{3}), (4, -\frac{7}{4})$

B2 Treats as $g(x) = \frac{1}{x+2}$ avoids error with $-$ sign

Slips (-1)

S1 Each un-simplified value to a maximum of 3

Attempts (3 marks)

A1 Copies table and stops

A2 Treats $g(x)$ as $x-2$

Note: Answers without work

1 value correct 3 marks

2 values correct 4 marks

3 values correct 4 marks

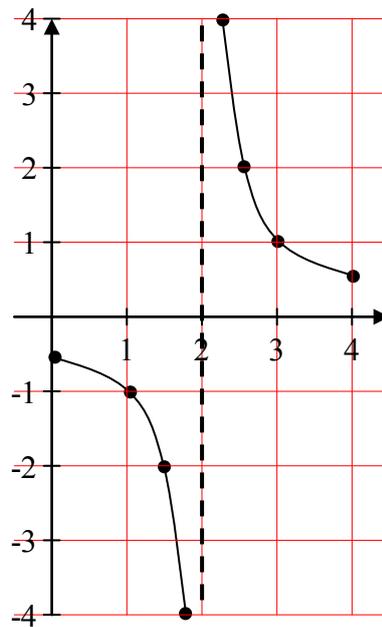
4 values correct 7 marks

5 values correct 10 marks

(b) (ii)

10 marks

Att 3



- * Consider graph as having 3 features LHS/branch, asymptote (actual or implied) and RHS/branch.
- * Asymptote $x = 2$ need not be drawn; an implied vertical asymptote (or visible gap) will suffice
- * Has graph of $x - 2$: oversimplified Att 3
- * Accept candidate's values from (i) if not over simplified
- * Ignore any graph errors outside the given range e.g. graph cutting the horizontal asymptote
- * Points plotted and not joined and not showing asymptote – [4] marks
- * Only one branch without a vertical asymptote – [4] marks at most

Blunders (-3)

- B1 Left and right branches joined
- B2 Points joined incorrectly
- B3 LHS or RHS branch missing or asymptote missing or not implied
- B4 Serious incorrect scaling of axes e.g. equal distance on x -axis for given values

Slips (-1)

- S1 Each point clearly incorrectly plotted or each point clearly omitted to a maximum of 3 per side

Attempts (3 marks)

- A1 Draws axes and stops
- A2 One point correctly plotted
- A3 Any mention of asymptotes
- A4 Table from (i) does not give rise to two branches

Note: If B1 or B2 applied at (i) graph at (ii) will merit attempt mark at most

(c) (ii)

10 marks

Att 3

$$f'(x) = 6 \Rightarrow 1 + \frac{5}{x^2} = 6 \Rightarrow 5 = 5x^2 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

[3] [7]

$$f(1) = 1 - \frac{5}{1} = 1 - 5 = -4. \quad \text{Point } (1, -4)$$

$$f(-1) = -1 - \frac{5}{-1} = -1 + 5 = 4. \quad \text{Point } (-1, 4) \quad [10]$$

* Accept candidates answer from (i) unless it is oversimplified

* Penalise simplification of $f'(x)$ errors in this part if necessary

Blunders (-3)

B1 Mathematical errors

B2 $f'(x) \neq 6$

B3 Only one solution found for x , B4 will also apply

B4 Only one value of $f(x)/y$ found

Slips (-1)

S1 Numerical slips

Attempts (3 marks)

A1 Mentions slope of $y = 6x$ is 6

A2 Answer from (i) = 6 and stops

A3 Mentions connection of slope and derivative and stops

QUESTION 7

Part (a)	15 marks	Att 5
Part (b)	20 (5, 15) marks	Att (2, 5)
Part (c)	15 (5, 5, 5) marks	Att (2, 2, 2)

Part (a) **15 marks** **Att 5**

Differentiate $x^2 - 6x + 1$ with respect to x .

(a) **15 marks** **Att 5**

$$\frac{dy}{dx} = 2x - 6$$

- * Correct answer without work or notation: full marks.
- * If done from first principles, ignore errors in procedure – just mark the answer.
- * Only one non zero term correct, award 12 marks

Blunders (-3)

B1 Differentiation error once per term

Attempts (5 marks)

A1 A correct step in differentiation from 1st principles

A2 A correct coefficient or a correct index of x in one of the term(s)

A3 Mentions $\frac{dy}{dx}$ or $f'(x)$

Worthless (0)

W1 No differentiation

Part (b)

20 (5, 15) marks

Att (2, 5)

- (i) Differentiate $5 - 3x$ with respect to x from first principles.
(ii) Given that $y = (x^2 - 4)(3x - 1)$, find the value of $\frac{dy}{dx}$ when $x = 2$.

(b)

5 marks

Att 2

<p>I $f(x) = 5 - 3x$ $f(x + h) = 5 - 3(x + h)$ $= 5 - 3x - 3h$</p> <hr/> <p>II $f(x + h) - f(x) = 5 - 3x - 3h - (5 - 3x) = -3h$</p> <hr/> <p>III $\frac{f(x + h) - f(x)}{h} = \frac{-3h}{h} = -3$ $\lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h} = -3$</p>	<p>$y = 5 - 3x$</p> <p>I $y + \Delta y = 5 - 3(x + \Delta x)$ $= 5 - 3x - 3\Delta x$ $y = 5 - 3x$</p> <hr/> <p>II $\Delta y = -3\Delta x$ $\frac{\Delta y}{\Delta x} = -3$</p> <hr/> <p>III $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = -3$</p>
--	--

* Accept use of $(x - h)$

Blunders (-3)

B1 Any error once per step I, II or III

Note: Must have correct LHS and RHS

Attempts (2 marks)

A1 $f(x \pm h)$ on LHS or some substitution of $x \pm h$ for x on RHS, or equivalent; these only

A2 Mentions Δx or Δy or similar

Worthless (0)

W1 Answer -3 without work; no attempt at first principles

(b) (ii)

15 marks

Att 5

I	or	II	
$y = (x^2 - 4)(3x - 1)$		$y = (x^2 - 4)(3x - 1)$	
$u = x^2 - 4$	$v = 3x - 1$	$y = 3x^3 - x^2 - 12x + 4$	[9]
$\frac{du}{dx} = 2x$	$\frac{dv}{dx} = 3$		[9]
$\frac{dy}{dx} = (3x - 1)(2x) + (x^2 - 4)(3)$		$\frac{dy}{dx} = 9x^2 - 2x - 12$	[12]
At $x = 2$		At $x = 2$	
$\frac{dy}{dx} = (6 - 1)(4) + (4 - 4)(3) = 20$		$\frac{dy}{dx} = 9(4) - 2(2) - 12 = 36 - 4 - 12 = 20$	[15]

* Uses $\frac{u}{v}$ merits 9 marks at most – allow for $u = \Rightarrow \frac{du}{dx} = \dots$ and $v = \Rightarrow \frac{dv}{dx} = \dots$ better than A5

Blunders (-3)

B1 Differentiation error

B2 Errors in expanding brackets once only unless over simplifies.

B3 Error in substitution, once only

Slips (-1)

S1 Numerical slips

Attempts (5 marks)

A1 u and/or v correctly identified and stops (I)

A2 Any correct differentiation

A3 At least one term multiplied correctly

A4 Uses $3x^3 + 4$ even if completed correctly

A5 $\frac{dy}{dx} = (2x)(3)$

Worthless (0)

W1 Substitutes $x = 2$ into y and stops

W2 uv formula written and stops

Part (c)

15 (5, 5, 5) marks

Att (2, 2, 2)

The speed, v , of an object at time t is given by

$$v = 96 + 40t - 4t^2$$

where t is in seconds and v is in metres per second.

- (i) At what times will the speed of the object be 96 metres per second?
- (ii) What will the acceleration of the object be at $t = 2.5$ seconds?
- (iii) At what value of t will the acceleration become negative?

* Units: Penalise as per guidelines.

* No retrospective marking.

* No penalty for incorrect notation.

* If parts of (c) are unlabelled, and the context doesn't identify which part is which, assume the questions were answered in sequence from (c) (i) to (c) (iii).

(c) (i)

5 marks

Att 2

$$96 = 96 + 40t - 4t^2$$

$$\Rightarrow 4t^2 - 40t = 0$$

$$\Rightarrow t(t - 10) = 0$$

$$\Rightarrow t = 0, t = 10 \text{ s}$$

* One or both answers correct without work, Att 2

Blunders (-3)

B1 Equation $\neq 96$

B2 Incorrect factors

B3 Incorrect roots from factors but see S2

Slips (-1)

S1 No units or incorrect units

S2 $t = 0$ not included

Attempts (2 marks)

A1 Attempt at factorising

A2 Trial and error on $96 + 40t - 4t^2$ even if correct

Worthless (0)

W1 Differentiation

(c) (ii)

5 marks

Att 2

$$a = \frac{dv}{dt} = 40 - 8t \quad [4]$$

$$\text{At } t = 2.5 \quad a = 40 - 8(2 \cdot 5) = 20 \text{ m s}^{-2} \quad [5]$$

* Acceleration as second derivative of v i.e correct d^2v/dt^2 merits 4

Blunders (-3)

B1 Differentiation error

Slips (-1)

S1 No units or incorrect units

S2 Substitution error

Attempts (2 marks)

A1 Mentions dv/dt or similar

Worthless (0)

W1 Substitutes $t = 2.5$ into v

(c) (iii)

5 marks

Att 2

I

$$\frac{dv}{dt} < 0 \Rightarrow 40 - 8t < 0 \Rightarrow -8t < -40 \Rightarrow t > 5$$

or

II

“Acceleration negative (deceleration) after velocity reaches its maximum” or similar

$$\frac{dv}{dt} = 0 \Rightarrow 40 - 8t = 0 \Rightarrow t = 5$$

Acceleration negative after $t = 5$

* Correct answer without work, Att 2.

Blunders (-3)

B1 Error solving inequality (I) or equation (II)

Slips (-1)

S1 $t \leq 5$

Attempts (2 marks)

A1 Any correct value offered

A2 Has acceleration $d^2v/dt^2 = -8$, therefore acceleration is always negative

Worthless (0)

W1 $t = 8$ from $d^2v/dt^2 = -8$

W2 Attempts to solve $96 + 40t - 4t^2 < 0$

QUESTION 8

Part (i)	15 marks	Att 5
Part (ii)	10 marks	Att 3
Part (iii)	10 marks	Att 3
Part (iv)	10 marks	Att 3
Part (v)	5 marks	Att 2

* Assume answering in order (i)(v) No retrospective marking

Part (i) **15 marks** **Att 5**

Let $f(x) = x^3 - 3x + 1$, where $x \in \mathbb{R}$.

(i) Find $f(-3)$, $f(-2)$, $f(0)$, $f(2)$ and $f(3)$.

(i) **15 marks** **Att 5**

$$f(x) = x^3 - 3x + 1$$

$$f(-3) = (-3)^3 - 3(-3) + 1 = -27 + 9 + 1 = -17 \quad [5]$$

$$f(-2) = (-2)^3 - 3(-2) + 1 = -8 + 6 + 1 = -1 \quad [6]$$

$$f(0) = (0)^3 - 3(0) + 1 = 0 + 0 + 1 = 1 \quad [9]$$

$$f(2) = (2)^3 - 3(2) + 1 = 8 - 6 + 1 = 3 \quad [12]$$

$$f(3) = (3)^3 - 3(3) + 1 = 27 - 9 + 1 = 19 \quad [15]$$

* Correct answers without work, full marks.

* Don't penalise extra values e.g $f(1)$ and/or $f(-1)$.

Blunders (-3)

B1 Mathematical errors, each time if different

B2 Use x^2 for x^3

Slips (-1)

S1 Arithmetic slips to maximum of 3

Attempts (5 marks)

A1 Only finds one value and stops

A2 Some correct substitution into $f(x)$

A3 $f'(x)$ with some correct substitution

Worthless (0)

W1 All incorrect answers without work

Note: Answers without work

1 point/value correct 5 marks

2 points/values correct 6 marks

3 points/values correct 9 marks

4 points/values correct 12 marks

5 points/values correct 15 marks

Part (ii)

10 marks

Att 3

Find $f'(x)$, the derivative of $f(x)$.

(ii)

10 marks

Att 3

$$f'(x) = 3x^2 - 3$$

- * Correct answer without work or notation, full marks.
- * If done from first principles, ignore errors in procedure – just mark the answer.
- * Only one non zero term correct, award 7 marks

Blunders (-3)

B1 Differentiation error once per term.

Attempts (3 marks)

- A1 A correct step in differentiation from 1st principles
- A2 A correct coefficient or a correct index of x .

Part (iii)

10 marks

Att 3

Find the co-ordinates of the local maximum point and of the local minimum point of the curve $y = f(x)$.

(iii)

10 marks

Att 3

$$f'(x) = 3x^2 - 3 = 0 \quad [3]$$

$$\Rightarrow x^2 - 1 = 0 \Rightarrow (x+1)(x-1) = 0 \Rightarrow x = -1 \text{ or } x = 1. \quad [7]$$

$$f(x) = x^3 - 3x + 1$$

$$f(-1) = (-1)^3 - 3(-1) + 1 = -1 + 3 + 1 = 3$$

$$f(1) = (1)^3 - 3(1) + 1 = 1 - 3 + 1 = -1$$

Local maximum $(-1, 3)$, local minimum $(1, -1)$. [10]

- * Accept candidate's $f'(x)$ from (ii) but see A1
- * Accept implied ' $= 0$ ' if subsequent work supports it.
- * Accept distinguishing max from min by comparing y -ordinates. Second derivative not required.
- * Correct answers without calculus, Att 3 at most. May be from graph.

Blunders (-3)

B1 $f'(x) \neq 0$ (but see 2nd asterisk)

B2 Error finding roots

B3 Only finds one root (B4 will also apply)

B4 Error finding $f(x)$ value e.g. fails to find $f(x)$ value or only finds one value or does not use $f(x)$

Slips (-1)

S1 Numerical slips

S2 Does not distinguish between maximum and minimum, or identifies incorrectly

Attempts (3 marks)

A1 $f'(x)$ linear and continues

A2 $f''(x)$

Worthless (0)

W1 $f(x) = 0$, whether continues or not

Part (iv)

10 marks

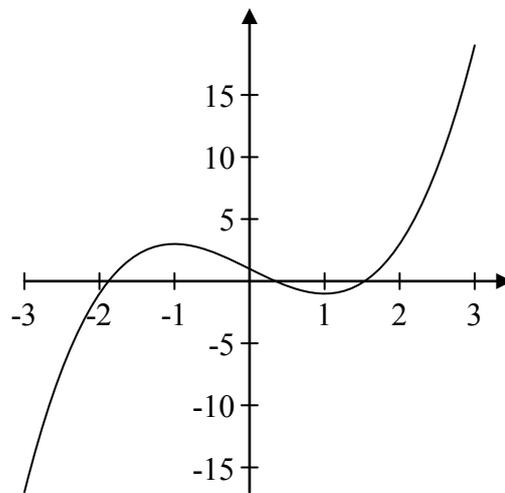
Att 3

Draw the graph of the function f in the domain $-3 \leq x \leq 3$.

(iv)

10 marks

Att 3



- * Accept candidate's values of $(x, f(x))$ from previous parts unless oversimplified.
- * If candidate recalculates points, apply slips and blunders as per guidelines.
- * Seven (7) points required Only uses 5 points from (i) [8]

Blunders (-3)

B1 Scale error, serious

B2 Points not joined or joined incorrectly or joined with a series of straight lines

B3 Axes not in standard form

Slips (-1)

S1 Each point incorrectly plotted or omitted

Attempts (3 marks)

A1 Plots $f'(x)$ or graph of a non-cubic function

A2 Answers from part (iii) transferred to this part, carries forward max and min values

A3 Effort at calculation of a point with some substitution e.g. $f(0)$

A4 Scaled and labelled axes and stops

Part (v)

5 marks

Att 2

Find the range of values of k for which the equation $x^3 - 3x + 1 = k$ has three real solutions (roots).

(v)

5 marks

Att 2

$$-1 \leq k \leq 3$$

- * Accept answer consistent with candidate's graph if cubic
- * Accept any valid solution
- * Accept answer clearly indicated on graph
- * Accept answer using words rather than symbols and $[3, -1]$ or $[-1, 3]$
- * Accept $-1 < k < 3$

Blunders (-3)

B1 Inequalities not as stated

Attempts (2 marks)

A1 One correct end-point identified

A2 Solves $f(x) = 0$ or finds one correct value of k

A3 Mentions local maximum or local minimum or max. and min.



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State Examinations Commission

LEAVING CERTIFICATE 2010

MARKING SCHEME

**MATHEMATICS
(PROJECT MATHS)
PAPER 2**

ORDINARY LEVEL



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2010

Mathematics (Project Maths)

Paper 2

Ordinary Level

Monday 14 June Morning 9:30 – 12:00

300 marks

Model Solutions – Paper 2

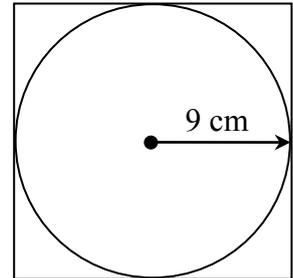
Note that the model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

Answer **Question 1** from this section.

Question 1**(50 marks)**

- (a) A circle is inscribed in a square as shown.
The radius of the circle is 9 cm.

- (i) Find the perimeter of the square.

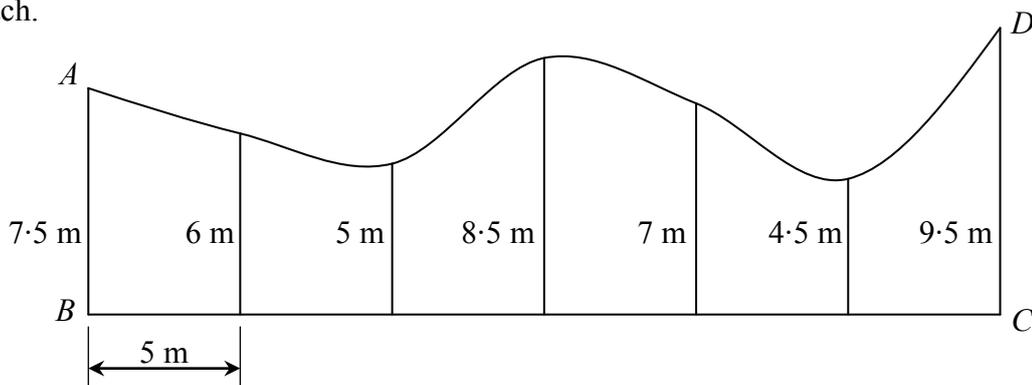


$$l = 9 \times 8 = 72 \text{ cm} \quad \text{or} \quad l = 18 \times 4 = 72 \text{ cm} \quad [5 \text{ marks}]$$

- (ii) Calculate the area of the square.

$$A = (18)^2 = 324 \text{ cm}^2 \quad [5 \text{ marks}]$$

- (b) The diagram shows a sketch of a field $ABCD$ that has one uneven edge. At equal intervals of 5 m along $[BC]$, perpendicular measurements are made to the uneven edge, as shown on the sketch.



- (i) Use Simpson's rule to estimate the area of the field.

$$\begin{aligned} \text{Area} &\approx \frac{h}{3}(F + L + 2\Sigma O + 4\Sigma E) \\ &= \frac{5}{3}(7.5 + 9.5 + 2(5 + 7) + 4(6 + 8.5 + 4.5)) && [10 \text{ marks}] \\ &= \frac{5}{3}(17 + 2(12) + 4(19)) = \frac{5}{3}(17 + 24 + 76) = \frac{5}{3}(117) \\ &= 195 \text{ m}^2. && [5 \text{ marks}] \end{aligned}$$

- (ii) The actual area of the field is 200 m^2 .
Find the percentage error in the estimate.

$$\text{Percentage error: } \frac{5}{200} \times 100 = 2.5\%.$$

[5 marks]

- (c) (i) The diameter of a solid metal sphere is 9 cm .
Find the volume of the sphere in terms of π .

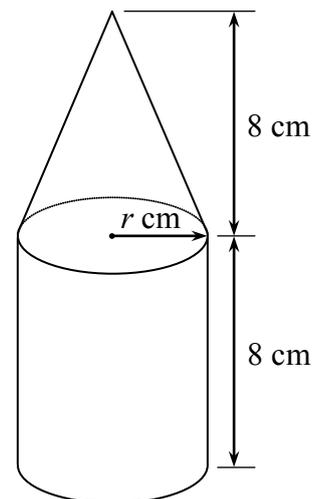
$$\begin{aligned} V &= \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (4.5)^3 \\ &= 121.5\pi \text{ cm}^3 \end{aligned}$$

[10 marks]

- (ii) The sphere is melted down. All of the metal is used to make a solid shape which consists of a cone on top of a cylinder, as shown in the diagram.

The cone and the cylinder both have height 8 cm .
The cylinder and the base of the cone both have radius $r \text{ cm}$.

Calculate r , correct to one decimal place.



$$V = \frac{1}{3} \pi r^2 h + \pi r^2 h = 121.5\pi$$

$$\Rightarrow \frac{4}{3} r^2 (8) = 121.5$$

$$\Rightarrow r^2 = \frac{121.5 \times 3}{4 \times 8} = 11.39$$

$$\Rightarrow r = 3.37$$

$$\therefore r \approx 3.4 \text{ cm}$$

[10 marks]

Answer **all five** questions from this section.

Question 2**(25 marks)**

- (a) A line crosses the x -axis at $x = 3$ and the y -axis at $y = 2$.

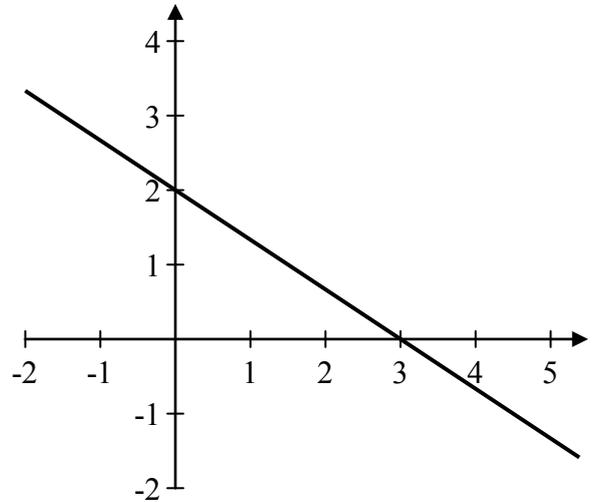
Find the equation of the line.

$$\text{Slope} = -\frac{2}{3}$$

OR

Points $(3,0)$ and $(0,2)$

$$\text{Slope} = \frac{2-0}{0-3} = -\frac{2}{3} \quad [15 \text{ marks}]$$



$$\text{Equation: } y = -\frac{2}{3}x + 2$$

OR

$$y - 0 = -\frac{2}{3}(x - 3)$$

$$3y = -2x + 6$$

$$2x + 3y = 6$$

[5 marks]

- (b) The equations of two lines l_1 and l_2 are:

$$l_1: x + 3y = 8$$

$$l_2: 6x - 2y = 15.$$

Determine whether these lines are perpendicular. Justify your answer clearly.

$$\text{Slope } l_1 = -\frac{1}{3} \quad \text{Slope } l_2 = 3$$

$$m_1 \times m_2 = \left(-\frac{1}{3}\right) \times (3) = -1 \quad \Rightarrow l_1 \perp l_2 \quad [5 \text{ marks}]$$

Question 3**(25 marks)**

(a) A circle has centre (0, 0) and passes through the point (3, 4).

(i) Find the equation of the circle.

$$x^2 + y^2 = r^2$$

$$(3)^2 + (4)^2 = r^2$$

$$25 = r^2$$

$$\text{Equation : } x^2 + y^2 = 25$$

[10 marks]

(ii) Find the co-ordinates of the two points at which the circle crosses the y-axis.

$$\text{Let } x = 0 \Rightarrow y^2 = 25$$

$$y = \pm 5$$

Points are (0, 5) and (0, -5)

[5 marks]

(b) A circle has centre (2, 4) and touches the y-axis.
Find the equation of the circle.

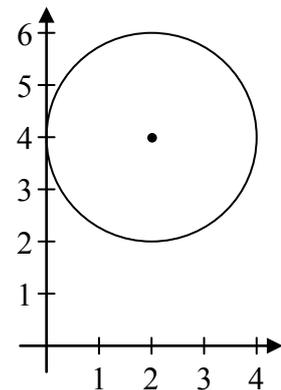
$$\text{Radius} = 2$$

Equation:

$$(x - 2)^2 + (y - 4)^2 = (2)^2$$

$$\Rightarrow (x - 2)^2 + (y - 4)^2 = 4$$

[10 marks]



Question 4**(25 marks)**

- (a) Using a calculator, or otherwise, find the mean and standard deviation of the data in the following frequency table.

x	20	30	40	50
f	16	38	26	20

Mean = 35	Standard deviation = 9.848857802 (using n)	
	= 9.898474528 (using $n - 1$)	
		[5 marks]

- (b) Below is a stem-and-leaf plot of the heights of a group of students, in centimetres.

13	3						
13	5	6					
14	0	0	1				
14	6	6	7	8			
15	0	1	2	2	3	3	
15	5	5	6	7			

Key: 13 | 3 means 133 cm.

- (i) How many students are in the group?

20 students

[10 marks]

- (ii) What is the *range* of heights in the group?

157 cm – 133 cm = 24 cm

[5 marks]

- (iii) What percentage of the students are between 145 cm and 154 cm in height?

$$\frac{10}{20} = 50\%$$

[5 marks]

Question 5**(25 marks)**

- (a) Helen has enough credit to download three songs from the internet. There are seven songs that she wants.

- (i) How many different possible selections of three songs can she make?

$${}^7C_3 = 35$$

- (ii) If there is one particular song that she definitely wants, how many different selections can she now make?

$${}^6C_2 = 15$$

[5 marks (parts (i) and (ii))]

- (b) (i) Two fair coins are tossed. What is the probability of getting two heads?

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

[5 marks]

- (ii) Two fair coins are tossed 1000 times. How often would you expect to get two heads?

$$1000 \times \frac{1}{4} = 250 \text{ times}$$

[5 marks]

- (c) Síle hands Pádraig a fair coin and tells him to toss it ten times. She says that if he gets ten heads then she will give him a prize. The first nine tosses are all heads. How likely is it that the last toss will also be a head? Tick the correct answer, and give a reason.

Extremely unlikely

Fairly unlikely

50-50 chance

Fairly likely

Almost certain

Reason:

e.g.:

Independent Trials

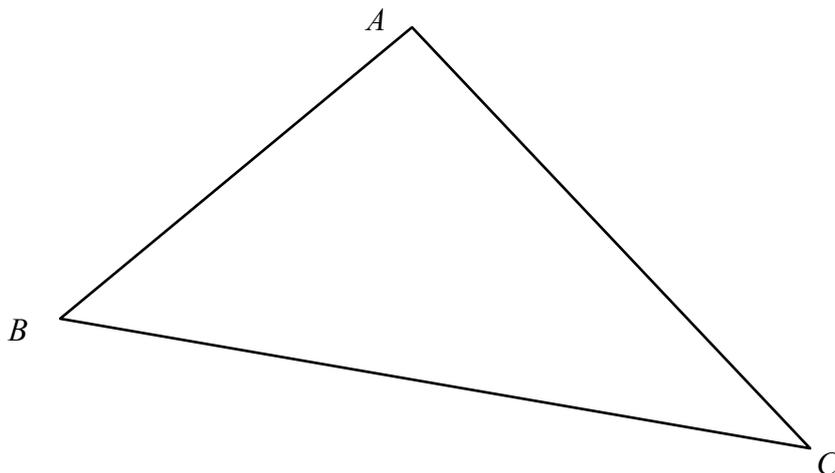
Fair coin

Not influenced by previous tosses

[10 marks]

Question 6**(25 marks)**

The diagram shows a triangle ABC in which $|AB| = 6$ cm, $|CB| = 10$ cm, and $|\angle ABC| = 50^\circ$.



- (a) Calculate the area of triangle ABC , correct to the nearest cm^2 .

$$\begin{aligned} \text{Area} &= \frac{1}{2}(6)(10)\sin 50^\circ \\ &= 30 \sin 50^\circ \\ &= 22.98133 \\ &\approx 23 \text{ cm}^2 \end{aligned}$$

[15 marks]

- (b) Calculate the length of $[AC]$, correct to one decimal place.

$$\begin{aligned} |AC|^2 &= (10)^2 + (6)^2 - 2(10)(6)\cos 50^\circ \\ &= 136 - 120\cos 50^\circ \\ &= 58.86548684 \end{aligned}$$

$$\begin{aligned} \Rightarrow |AC| &= 7.67238 \\ &\approx 7.7 \text{ cm} \end{aligned}$$

[5 marks]

- (c) The triangle $A'BC'$ is the image of triangle ABC under the enlargement with centre B and scale factor 3. Find the area of $A'BC'$, correct to the nearest cm^2 .

$$\text{Image Area} = (3)^2[23] = 9[23] = 207 \text{ cm}^2$$

OR

$$|BA'| = 18 \text{ cm}, |BC'| = 30 \text{ cm},$$

$$\Rightarrow \text{Area} = \frac{1}{2}(18)(30)\sin 50^\circ \approx 207 \text{ cm}^2$$

[5 marks]

Answer Question 7, Question 8, and **either** Question 9A **or** Question 9B.

Question 7**Probability and Statistics****(40 marks)**

The table below gives motor insurance information for fully licensed, 17 to 20-year-old drivers in Ireland in 2007. All drivers who had their own insurance policy are included.

	Number of drivers	Number of claims	Average cost per claim
Male	9634	977	€6108
Female	6743	581	€6051

(Source: adapted from: Financial Regulator. *Private Motor Insurance Statistics 2007*.)

Questions (a) to (e) below refer to drivers in the table above only.

- (a) What is the probability that a randomly selected **male** driver made a claim during the year? Give your answer correct to three decimal places.

$$\frac{977}{9634} \approx 0.101 \quad [10 \text{ marks}]$$

- (b) What is the probability that a randomly selected **female** driver made a claim during the year? Give your answer correct to three decimal places.

$$\frac{581}{6743} \approx 0.086 \quad [10 \text{ marks}]$$

- (c) What is the *expected value* of the cost of claims on a male driver's policy?

$$0.101 \times €6108 = €616.91 \quad [5 \text{ marks}]$$

- (d) What is the *expected value* of the cost of claims on a female driver's policy?

$$0.086 \times €6051 = €520.39 \quad [5 \text{ marks}]$$

- (e) The male drivers were paying an average of €1688 for insurance in 2007 and the female drivers were paying an average of €1024. Calculate the average surplus for each group, and comment on your answer.

(Note: the *surplus* is the amount paid for the policy minus the expected cost of claims.)

Male	Female
$€1688 - €616.91 = €1071.09$	$€1024 - €520.39 = €503.61$
Comment: e.g. Male drivers are generating a much higher surplus. Insurance companies are making far more money from male drivers. [5 marks]	

- (f) A 40-year-old female driver with a full license has a probability of 0.07 of making a claim during the year. The average cost of such claims is €3900. How much should a company charge such drivers for insurance in order to show a surplus of €175 per policy?

Expected claims value = $€3900 \times 0.07 = €273$	
Charge: $€273 + €175 = €448$	[5 marks]

Question 8

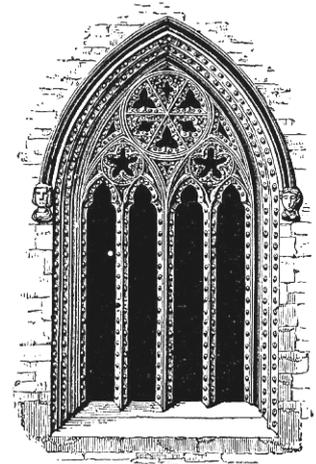
Geometry and Trigonometry

(40 marks)

Windows are sometimes in the shape of a pointed arch, like the one shown in the picture.

A person is designing such an arched window. The outline is shown in the diagram below the picture.

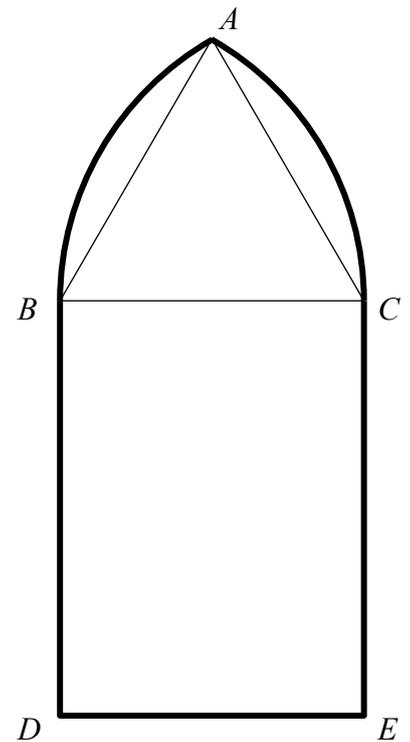
The centre for the arc AB is C and the centre for the arc AC is B . $|BD| = 2.4$ metres and $|DE| = 1.8$ metres.



- (a) Show that $|\angle ABC| = 60^\circ$.

$$\begin{aligned}
 |AB| &= |BC| \quad (\text{Radius of arc } AC) \\
 |AC| &= |BC| \quad (\text{Radius of arc } AB) \\
 &\Rightarrow ABC \text{ is an equilateral triangle} \\
 \therefore |\angle ABC| &= 60^\circ
 \end{aligned}$$

[10 marks]



- (b) Find the length of the arc AB .
Give your answer in metres, correct to three decimal places.

$$\begin{aligned}
 \text{Length} &= \left(\frac{60}{360}\right) \times 2\pi(1.8) \\
 &= \frac{1}{6} \times 11.30973355 \\
 &= 1.884955 \approx 1.885 \text{ m}
 \end{aligned}$$

[5 marks]

- (c) Find the length of the perimeter of the window.
Give your answer in metres, correct to two decimal places.

$$\begin{aligned}
 \text{Perimeter} &= 2(2.4) + 1.8 + 2(1.885) \\
 &= 4.8 + 1.8 + 3.77 \\
 &= 10.37 \text{ m}
 \end{aligned}$$

[10 marks]

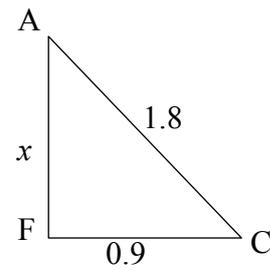
- (d) Find the height of the window.
Give your answer in metres, correct to two decimal places.

$$x^2 + (0.9)^2 = (1.8)^2$$

$$x^2 + 0.81 = 3.24$$

$$x^2 = 2.43$$

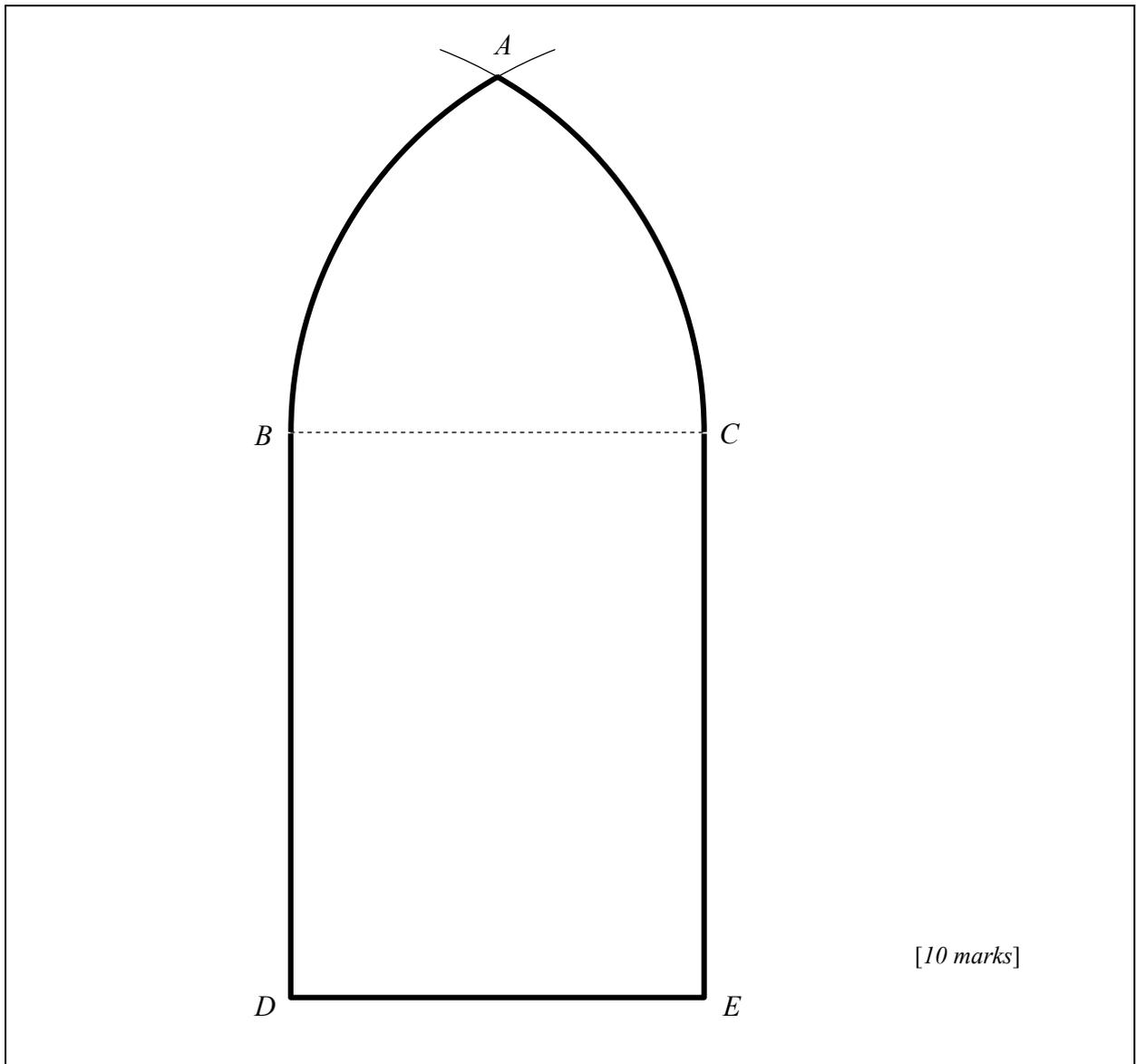
$$x = 1.5558 \approx 1.56 \text{ m}$$



$$\text{Height of Window} = 1.56 + 2.4 = 3.96 \text{ m}$$

[5 marks]

- (e) Make an accurate scaled drawing below of the outline of the window, using the scale 1:30. That is, 1 cm on your diagram should represent 30 cm in reality.



[10 marks]

Question 9A

Probability and Statistics

(45 marks)

Students in two schools – one in County Kerry and the other in County Offaly – were arguing about which county had the nicest weather in the summer. They agreed to record the highest temperature at each school on ten randomly selected days during the summer of 2009. The results were as follows:

Temperature at Kerry school ($^{\circ}\text{C}$)			Temperature at Offaly school ($^{\circ}\text{C}$)		
18.5	17.2	17.8	22.1	18.0	19.1
17.6	17.5	17.2	17.2	18.4	18.6
17.1	16.9	16.9	19.8	19.0	17.6
17.1			17.0		

(a) Construct a back-to-back stem-and-leaf plot of the above data.

					9	9		16				
8	6	5	2	2	1	1		17	0	2	6	
						5		18	0	4	6	
								19	0	1	8	
								20				
								21				
								22	1			

key: | 17 | 0 means 17.0

[15 marks]

(b) State **two differences** between the two distributions.

Examples

- Kerry temperatures are generally lower
- Greater range of temperatures in Offaly
- Offaly has an outlier
- Temperatures in Kerry are largely in the 17 $^{\circ}\text{C}$ bracket

[5 marks]

(c) Perform a *Tukey Quick Test* on the data, stating clearly what can be concluded.

Lower Tail = 2	Upper Tail = 5
Tail Count = 7	

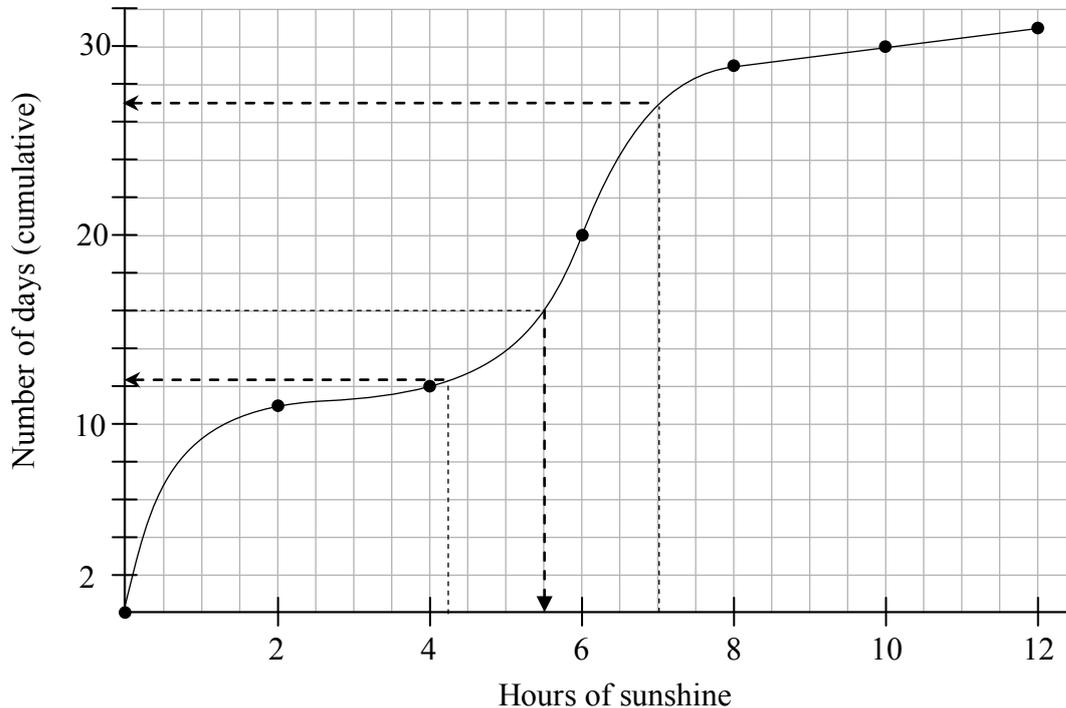
Conclusion: In general, in the summer of 2009, temperatures in Offaly were higher than in Kerry.

[5 marks]

- (d) The students in Offaly looked also at the amount of sunshine. They recorded the number of hours of sunshine each day in July 2009. The data are summarised in the table below.

Hours of sunshine	≤ 2	≤ 4	≤ 6	≤ 8	≤ 10	≤ 12
Number of days	11	12	20	29	30	31

Draw a cumulative frequency curve to represent this data, using the scale indicated.



[10 marks]

- (e) Use your cumulative frequency curve to estimate:

(i) the median number of hours of sunshine

5.5

(ii) the number of days with **more than** 7 hours of sunshine.

4

[5 marks]

- (f) The mean amount of sunshine per day in Offaly in July generally is 4.24 hours. A day is chosen at random from the days in July 2009, as described in part (d) above. What is the probability that the amount of sunshine on that day was less than the mean?

$$\text{Probability} \approx \frac{12}{31}$$

[5 marks]

(Data in this question adapted from *Monthly Weather Bulletin, July 2009*, at www.met.ie.)

Question 9B

Geometry and Trigonometry

(45 marks)

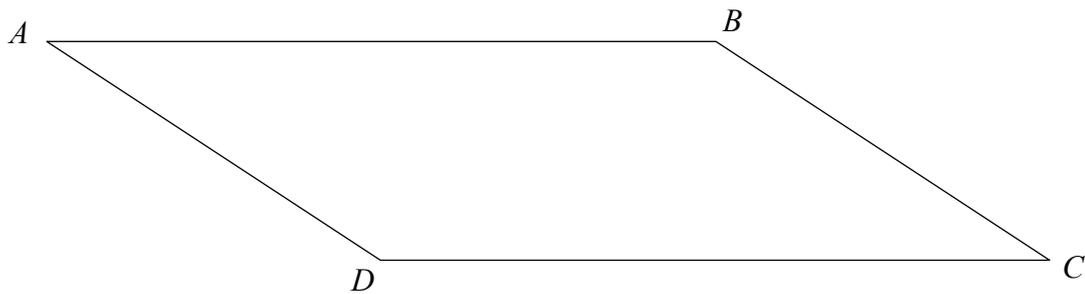
- (a) The photograph shows the *Dockland* building in Hamburg, Germany.

The diagram below is a side view of the building. It is a parallelogram.

The parallelogram is 29 metres high.
The top and bottom edges are 88 metres long.



Photo by NatiSythen. Wikipedia Commons. License: CC-SA



- (i) Find the area of this side of the building.

Area = $29 \times 88 = 2552 \text{ m}^2$ [15 marks]

- (ii) If $|BD| = |AD|$, find $|BC|$.

Let $|BC| = x$

$x^2 = (29)^2 + (44)^2$

$x^2 = 2777$

$x = \sqrt{2777}$

$|BC| = 52.6972485$

$|BC| \approx 52.697 \text{ m}$

[5 marks]

(iii) The lines BC and AD are parallel. Find the distance between these parallel lines.

Let $[BC]$ = base and let y = perpendicular height (distance between BC and AD)

$$|BC| \times y = 2552$$

$$52.697 \times y = 2552$$

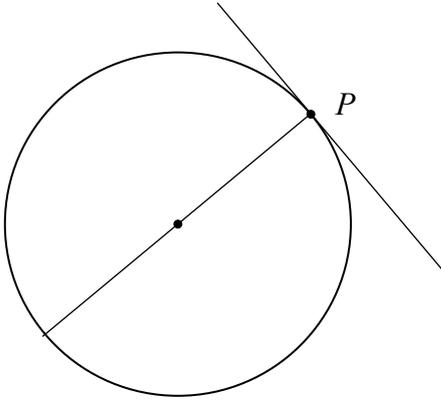
$$y = 48.427 \text{ m}$$

[5 marks]

(b) There is a theorem on your geometry course that can be used to construct the tangent to a circle at a given point on the circle. State this theorem and use it to construct the tangent to the circle shown at the point P .

Theorem:

Each tangent is perpendicular to the radius that goes to the point of contact. [5 marks]



[5 marks]

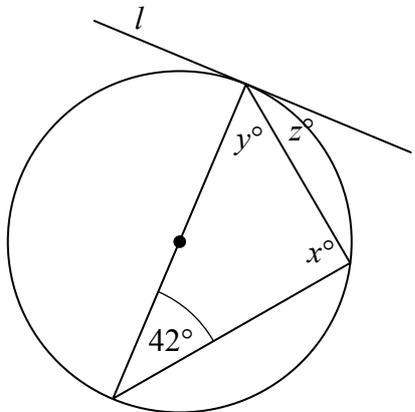
(c) In the diagram, the line l is a tangent to the circle. Find the values of x , y and z .

$x = 90$

$y = 48$

$z = 42$

[10 marks]



Marking scheme – Paper 2, Section 0 (Question 1)

N.B. This page applies only to Question 1.

The scheme for this question is identical to that used for candidates who are not involved in Project Maths.

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:

- Blunders - mathematical errors/omissions (-3)
- Slips - numerical errors (-1)
- Misreadings (provided task is not oversimplified) (-1).

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,..., S1, S2,..., M1, M2,...etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that

- any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
- if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
- a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,...etc.

4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase “and stops” means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

Application of penalties

N.B. This page applies only to Question 1.

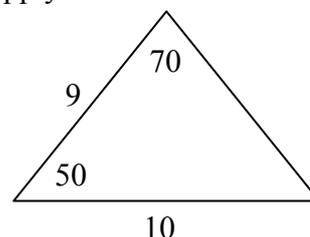
Penalties are applied subject to marks already secured.

Blunders - examples of blunders are as follows:

- Algebraic errors: $8x + 9x = 17x^2$ or $5p \times 4p = 20p$.
- Sign error: $-3(-4) = -12$ or $(-3)^2 = -9$.
- Fraction error: Incorrect fraction inversion etc. apply once.
- Cross-multiplication error.
- Error in misplacing the decimal point.
- Transposing error: $-2x - k + 3 = 0 \Rightarrow -2x = 3 + k$ or $-3x = 6 \Rightarrow x = 2$.
or $4x = 12 \Rightarrow x = 8$ each type once per section.
- Distributive law errors (once per pair of brackets)
 $\frac{1}{2}(3 - x) = 6 \Rightarrow 6 - 2x = 6$ or $-(4x + 3) = -4x + 3$ or $3(2x + 4) = 6x + 4$
- Expanding brackets incorrectly: $(2x - 3)(x + 4) = 8x^2 - 12x$.
- Omission, if work not oversimplified, unless directed otherwise.
- Index error, each time unless directed otherwise.
- Factorisation: error in one or both factors of a quadratic, apply once per section.
 $2x^2 - 2x - 3 = (2x - 1)(x + 3)$.
- Root errors from candidate's factors, error in one or both roots, apply once
- Incorrect substitution into formulae (where not an obvious slip):

$$\text{e.g. } 2x^2 + 3x + 4 = 0 \Rightarrow x = \frac{-3 \pm \sqrt{9 - 4(2)(4)}}{2(2)}$$

$$\text{or } \frac{10}{\sin 70} = \frac{9}{\sin 50}$$



- Incorrectly treating co-ordinates as (x_1, x_2) and (y_1, y_2) when using co-ordinate geometry formula.
- Errors in formula for example: $\frac{y_2 + y_1}{x_2 + x_1}$ or $A = P\left(1 + \frac{n}{100}\right)^r$ or $a^2 = b^2 + c^2 + bc \cos A$
or $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$, except as indicated in scheme.

Note: A correct relevant formula isolated and stops is awarded the attempt mark if the formula is not in the *Formulae and Tables* booklet.

Slips – examples are as follows:

- Numerical slips such as: $4 + 7 = 10$ or $3 \times 6 = 24$ but $5 + 3 = 15$ is a blunder.
- An omitted round-off to a required level of accuracy or an incorrect round-off to the incorrect accuracy or an early round-off that affects accuracy are penalised as a slip once in each section.
- However, an early round-off which has the effect of simplifying the work is at least a blunder.
- The omission of the units of measurement in an answer or giving the incorrect units of measurement is treated as a slip once in each section where the candidate would otherwise have obtained full marks in that section. This applies to **Q1** (a) (i), (ii), (b) (i) and (c) (i), (ii) and to **Q5** (a), and (c) (i), (ii).

Misreadings

- Examples such as 436 for 346 will not alter the nature of the question and are penalised -1.
- However, writing 5026 as 5000 would alter the work and is penalised as at least a blunder.

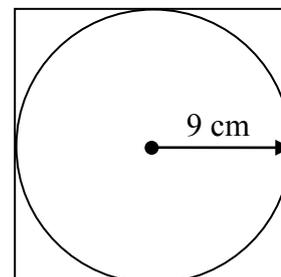
QUESTION 1

Part (a)	10 (5, 5) marks	Att (2, 2)
Part (b)	20 (15, 5) marks	Att (5, 2)
Part (c)	20 (10, 10) marks	Att (3, 3)

Part (a) **10 (5, 5) marks** **Att (2, 2)**

A circle is inscribed in a square as shown.
The radius of the circle is 9 cm.

- (i) Find the perimeter of the square.
- (ii) Calculate the area of the square.



(a) (i) **5 marks** **Att 2**

$$l = 9 \times 8 = 72 \text{ cm} \quad \text{or} \quad l = 18 \times 4 = 72 \text{ cm.}$$

(a) (ii) **5 marks** **Att 2**

$$A = 18^2 = 324 \text{ cm}^2$$

- * Accept correct answer without work, including an answer written on a diagram.
- * Accept in section (ii) an answer consistent with candidate's answer to section (i).

- 5 marks Correct answer.
- 4 marks One slip or misreading.
- 2 marks Work of some merit, otherwise 0 marks.e.g. 2×9 or 18 or 81.

Worthless (0 marks)

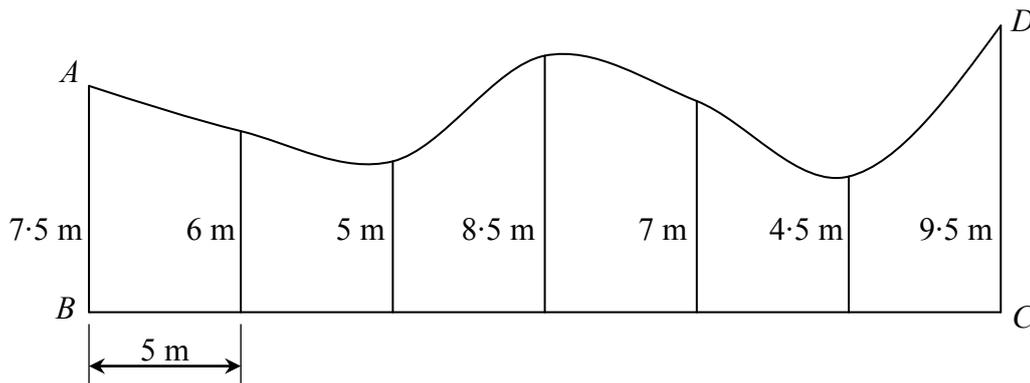
W1 Any incorrect answer without work- subject to work of some merit.

Note: Exception (i) = 324 (4 marks), (ii) = 72 cm² (5 marks).

Case 1	Case 2
$l = (18)^2 = 324 \text{ cm}$ (4 marks). $A = (18 \times 4) = 72 \text{ cm}^2$ (5 marks).	$l = 4 \times 9 = 36$ (2 marks) $A = 9 \times 9 = 81 \text{ cm}^2$ (5 marks)

Case 3	Not Obvious (wrong formula)
$Circle = 2\pi r = 18\pi \text{ cm}$ (4 marks) $Area = \pi r^2 = 81\pi \text{ cm}^2$ (5 marks)	(i) = $2\pi r = 18\pi \text{ cm}$ (2 marks) (ii) = $Area = \pi r^2 = 81\pi$ (2 marks)

The diagram shows a sketch of a field $ABCD$ that has one uneven edge. At equal intervals of 5 m along $[BC]$, perpendicular measurements are made to the uneven edge, as shown on the sketch.



- (i) Use Simpson's rule to estimate the area of the field.
 (ii) The actual area of the field is 200 m^2 .
 Find the percentage error in the estimate.

(b) (i) Use of formula

10 marks

Calculations

5 marks

(b) (ii) %

5 marks

- (i) $\text{Area} \approx \frac{h}{3}(F + L + 2\sum O + 4\sum E)$
 $= \frac{5}{3}(7.5 + 9.5 + 2(5 + 7) + 4(6 + 8.5 + 4.5))$ [10 marks]
 $= \frac{5}{3}(17 + 2(12) + 4(19)) = \frac{5}{3}(17 + 24 + 76) = \frac{5}{3}(117) = 195 \text{ m}^2$. [5 marks]
- (ii) Percentage error: $\frac{5}{200} \times 100 = 2.5\%$

- * Allow $\frac{h}{3} = (F + L + \text{TOFE})$ and penalise in calculations if formula not used correctly.
- * Accept correct TOFE *or* TOFE consistent with candidates F and L.
- * Accept 2.5 or consistent answer without work. (ii)

Substitution:

- 10 marks Fully correct substitution
- 7 marks One blunder in substitution
- 4 marks Two blunders in substitution
- 3 marks Some merit in candidates work, otherwise 0 marks.

Treat as separate blunders

- Incorrect $\frac{h}{3}$ (once).
- Incorrect F and/or L or extra terms with F and/or L (once).
- $\sum E$ **or** $\sum O$ omitted (once).
- Incorrect TOFE (once), if not consistent with candidates F and L.
 e.g. $\frac{5}{3}(0 + 9.5 + 2(6 + 8.5 + 4.5) + 4(7.5 + 5 + 7))$

Calculation

- 5 marks Correct or consistent answer.
- 4 marks One slip or misreading.
- 2 marks Work of some merit, otherwise 0 marks

Section (ii)

5 marks Correct answer

4 marks One slip or misreading.

2 marks Work of some merit, otherwise 0 marks.

NOTE:			
I	II	II	IV
No Substitution Ans: 195 (3marks + 2marks)	Substitution (mark =*) Ans: 195m ² /consistent (* marks + 5 marks)	Substitution (mark =*) Ans: 194/consistent (* marks + 4 marks)	Substitution (mark =*) Ans: #/not consistent (* marks + 0 marks)

Part (c)

20 (10, 10) marks

Att (3, 3)

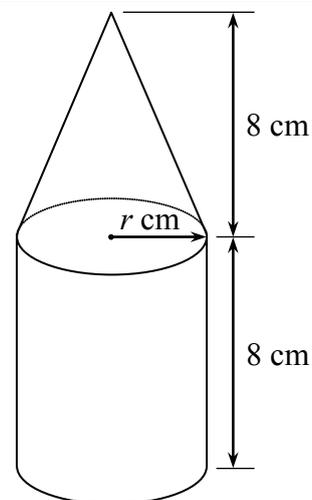
A solid metal sphere has diameter 9 cm.

(i) Find the volume of the sphere in terms of π .

The sphere is melted down. All of the metal is used to make a solid shape which consists of a cone on top of a cylinder, as shown in the diagram.

The cone and the cylinder both have height 8 cm.

The cylinder and the base of the cone both have radius r cm.



(ii) Calculate r , correct to one decimal place.

(c) (i)

10 marks

(c) (ii)

10 marks

$$(i) \quad V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (4.5)^3 \downarrow_{4 \text{ marks}} = \frac{243}{2} \pi \text{ cm}^3 \text{ or } 121.5 \pi \text{ cm}^3$$

$$V = \frac{1}{3} \pi r^2 h + \pi r^2 h \downarrow_{3 \text{ marks}} = 121.5 \pi \downarrow_{4 \text{ marks}} \Rightarrow \frac{4}{3} r^2 (8) = 121.5 \downarrow_{4 \text{ marks}}$$

$$(ii) \quad \Rightarrow r = \sqrt{\frac{121.5 \times 3}{4 \times 8}} = \sqrt{\frac{364.5}{32}} = \sqrt{11.3906} \downarrow_{7 \text{ marks}} = 3.375 = 3.4 \text{ cm}$$

$$\text{or } r^2 = \frac{121.5 \times 3}{4 \times 8} = \frac{364.5}{32} = 11.3906 \downarrow_{7 \text{ marks}} \Rightarrow r = 3.375 = 3.4 \text{ cm}$$

* Accept an answer in section (ii) consistent with the candidate's answer to section (i).

- 10 marks Fully correct answer
9 marks One slip or misreading.
7 marks One blunder. e.g. $381 \cdot 7$ or $381 \cdot 7\pi$ with work.
4 marks Two blunders
3 marks Some merit in candidates work, otherwise 0 marks.

Treat as separate blunder.

- Incorrect relevant volume of sphere formula i.e. $k(\pi r^3)$ where $k \neq \frac{4}{3}$ and continues.

Attempts (3 marks)

- A1 Some merit in work. e.g. equation set up or h substituted into relevant volume formula in (ii).
A2 Correct formula with any correct substitution.
A3 Correct answer without work in each section.
A4 $\pi r^2 h = 121 \cdot 5\pi$ even if completed.

Worthless (0 marks)

- W1 Use of any area formula. e.g. $4\pi r^2$.
W2 Non sphere formula.e.g. $\pi r^2 h$

Marking scheme – Paper 2, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5 mark scale	0, 5	0, 3, 5	0, 3, 4, 5	
10 mark scale		0, 8, 10	0, 5, 8, 10	
15 mark scale			0, 7, 12, 15	0, 7, 9, 12, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C** indicates that 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Section A

Question 2

- (a) slope: 15D
eqt: 5C
- (b) 5C

Question 3

- (a) (i) 10C
(ii) 5C
- (b) 10C

Question 4

- (a) 5C
- (b) (i) 10B*
(ii) 5C*
(iii) 5C

Question 5

- (a) 5C
- (b) (i) 5B
(ii) 5B
- (c) 10C

Question 6

- (a) 15C*
- (b) 5C*
- (c) 5C*

Section B

Question 7

- (a) 10C*
- (b) 10C*
- (c) 5C
- (d) 5C
- (e) 5C
- (f) 5C

Question 8

- (a) 10C
- (b) 5C*
- (c) 10C*
- (d) 5C*
- (e) 10C

Question 9A

- (a) 15C*
- (b) 5C
- (c) 5C
- (d) 10C
- (e) 5C
- (f) 5C

Question 9B

- (a) (i) 15C*
(ii) 5C*
(iii) 5C*
- (b) Theorem: 5A
Construction 5B
- (c) 10C

Detailed marking notes

Section A

Question 2

(a) Slope Scale 15D

High partial credit: Slope = $\frac{2}{3}$ or $-\frac{3}{2}$ or $\frac{2-0}{0-3}$ and fails to finish correctly

Middle partial credit: Slope = $\frac{3}{2}$ or identifies 2 correct points on line

Low partial credit: Identifies one correct point only
Correct relevant formula

Note: Accept correct answer without work

Equation Scale 5C

High partial credit: Correct substitution into equation of line
Substitutes for x and y instead of x_1 and y_1 and finishes correctly

Low partial credit: Some correct substitution into equation of line

Note: Accept correct answer without work

(b) Scale 5C

High partial credit: Both slopes correct but no/incorrect conclusion

Low partial credit: One slope correct
Any correct attempt at finding a slope
Condition given for two lines being perpendicular
Effort to find 2 points on either l_1 or l_2

No Credit Simultaneous equations only

Question 3

(a) (i) Scale 10C

High partial credit: Finds correct radius and stops (no equation of circle given)
 $(0 - 3)^2 + (0 - 4)^2 = r^2$ and correctly finds r^2
 $(3 - 0)^2 + (4 - 0)^2 = r^2$ and squares incorrectly

Low partial credit: Any correct substitution into circle equation.
 $x^2 + y^2 = r^2$ and stops.
Attempts to find distance between (0, 0) and (3, 4)

Note: Accept correct answer without work

(ii) Scale 5C

High partial credit: $y^2 = 25$ and stops
Correct answer from graph

Low partial credit: $x = 0$ and stops
 $y = 0$ and continues to $x^2 = 25$

Note: Accept correct answer without work

(b) Scale 10C

High partial credit: Radius = 2 but fails to finish correctly
 $(x - 2)^2 + (y - 4)^2 = r^2$

Low partial credit: Correctly identifies point on circle eg. (0,4)
Equation of circle with any correct substitution for h , k , x or y

Question 4

(a) Scale 5C

High Partial credit: Correctly calculates one answer only

Low Partial credit: Gives answer as 25 for mean

(b) (i) Scale 10B*

Partial credit: Gives answer as 26 [Stem values included]

* Gives answer as either 19 or 21 merits 9 marks

(ii) Scale 5C*

High partial credit: Max and min given but fails to subtract

Low partial credit: 157 or 133 only given

Lists all entries but doesn't identify max/min

(iii) Scale 5C

High partial credit: Correct fraction but fails to convert into percentage

Low partial credit: 10 students only written down (no fraction/percentage)

Writes answer as $\frac{k}{20}$ where $k \neq 10$

Question 5

(a) Scale 5C

High partial credit: Writes both answers as combinations but not evaluated

Low partial credit: One correct answer only

Uses permutations (3 choices only) for (i) and/or (ii)
Attempt at listing

(b) (i) Scale 5B

Partial credit: Answer given as $\frac{1}{2}$ or $\frac{1}{3}$

Attempt at Sample Space

(ii) Scale 5B

Partial credit: Describes probability e.g. 'Not very often'; 'Some of the time'
Any effort at combining a fraction with 1000

No credit: Answer > 1000

Note: Allow candidate to use an incorrect answer from (b) (i) without further penalty

(c) Scale 10C

High partial credit: Correct answer with no reason given

Low partial credit: Correct reason with no box ticked

No credit: Incorrect box ticked

Question 6

(a) Scale 15C*

High partial credit: Correct answer by accurate measurement
 $\frac{1}{2}(6)(10)\sin 50$

Low partial credit: $\frac{1}{2}(6)(10)$

Some correct substitution into $\frac{1}{2}ab\sin C$

Transfers all 3 pieces of given information correctly to diagram

(b) Scale 5C*

High partial credit: Correct work as far as $|AC|^2 = 58.86548684$

Low partial credit: Correct substitution into cosine rule
Answer by accurate measurement in the range [7.6 to 7.8]
Treats as a right angled triangle

(c) Scale 5C*

High partial credit: Area = $\frac{1}{2}(18)(30)\sin 50$ but fails to finish correctly
Area = $(3)^2[23]$ and fails to finish correctly

Low partial credit: Any use of 3 and the answer to (a)
Finds $|BA'|$ (18) or $|BC'|$ (30)
 $k^2 = 3^2$

Section B

Question 7

(a) Scale 10C*

High partial credit: $\frac{977}{16377}$ or $\frac{9634}{977}$

Low partial credit: Any use of 977 or 9634

* *Note:* $\frac{977}{9634}$ merits at least 9 marks

Full Credit for: 0.101; 10.1%; 10.141%

(b) Scale 10C*

High partial credit: $\frac{581}{16377}$ or $\frac{6743}{581}$

Low partial credit: Any use of 581 or 6743

**Note:* $\frac{581}{6743}$ merits at least 9 marks

Full Credit for: 0.086; 8.6%; 8.616%

(c) Scale: 5C

High partial credit: 6108×0.101 but fails to finish

Low partial credit: Writes down 6108

Note: Accept correct answer or answer consistent with candidate's answer from (a)

(d) Scale 5C

High partial credit: 6051×0.086 but fails to finish

Low partial credit: Writes down 6051

Note: Accept correct answer or answer consistent with candidate's answer from (b)

(e) Scale 5C

High partial credit: €10,294,676 and €3,389,201 and appropriate comment
Correct answers with no/incorrect comment

Low partial credit: Any use of relevant information e.g. 1688 in Male column

(f) Scale 5C

High partial credit: €273 but fails to finish correctly

Low partial credit: Any use of relevant data e.g. €3900 + €175

Question 8

(a) Scale 10C

High partial credit: Writes the reason as 'Isosceles Triangle'
Shows 1.8 on all 3 sides of triangle in diagram

Low partial credit: Puts 60° into all 3 angles in diagram but no reference to side length

(b) Scale 5C*

High partial credit: Uses a radius of 0.9 and finishes correctly
 $\frac{1}{6} \times 2\pi \times 1.8$ and fails to finish

Low partial credit: Uses $l = r\theta \Rightarrow l = (1.8)(60)$
Writes $\frac{60}{360}$ or $\frac{1}{6}$

(c) Scale 10C*

High partial credit: Uses at least 3 correct sides in addition

Low partial credit: Any relevant addition

(d) Scale 5C*

High partial credit: Correct height of triangle (1.56) but fails to add 2.4

Low partial credit: 2.4 added to some other number
Any relevant attempt at getting height of triangle
Measures height from part (e)

(e) Scale 10C

High partial credit: 3 sides of rectangle or 2 arcs constructed
Correct construction outside agreed tolerance [$\pm 3\text{mm}$]

Low partial credit: Any partial construction other than above

Calculates 8 cm and 6 cm

Question 9A

(a) Scale 15C*

High partial credit: Leaves 20 or 21 out of stem
Omits 2 or 3 leaves

Low partial credit: Omits more than 3 leaves
Any relevant attempt

Note: Accept leaves unordered.

* Fully correct but no key merits 14 marks

* Omits only one leaf merits 14 marks

(b) Scale 5C

High partial credit: One correct difference identified

Low partial credit: Reference to individualised data but not referring to distribution

(c) Scale 5C

High partial credit: TC = 7 but incorrect conclusion

Low partial credit: 2 and/or 5 written but not added
Identifies tails correctly
Any work of merit

(d) Scale 10C

High partial credit: 2 or less errors

Low partial credit: >2 errors

Errors: Inaccurate scale, joins points with lines, plots point incorrectly

Note: Curve does not need to be joined to (0,0)

[Number of days with 0 hours sunshine unknown]

(e) Scale 5C

High partial credit: One correct answer

Low partial credit: Any work of merit e.g. Fails to subtract from 31, draws line up from 6

Note: Accept answers consistent with candidate's graph with a tolerance of ± 0.3 on horizontal axis and ± 1 on vertical axis.

Accept use of 15.5 or 16 to find median.

(f) Scale 5C

High partial credit: Writes 12, 13 or 14 but no fraction
Writes answer as a percentage [38% to 45%] without work

Low partial credit: Writes $\frac{k}{31}$ where $k \neq 12, 13, 14$

Descriptive answer within required range e.g. Moderately likely
Any work of merit

Question 9B

(a) (i) Scale 15C*

High partial credit: $\frac{1}{2}(29)(88)$

Low partial credit: Any use of 88 and 29

(ii) Scale 5C*

High partial credit: $x^2 = 277$

Low partial credit: Use of 44
Joins B to D in diagram

(iii) Scale 5C*

High partial credit: Sets up equation correctly: $|BC| \times y = 2552$

Low partial credit: Uses answer from (i)
Mentions Base \times Height

(b) **Theorem:** Scale 5A

Construction: Scale 5B

Partial credit: Tangent drawn with no radius/diameter
Joins centre to P

(c) Scale 10C

High partial credit: 2 correct answers only

Low partial credit: 1 correct answer only
Gives incorrect answers for y and z that sum to 90°
Gives incorrect answers for x and y that sum to 138°

Marcanna breise as ucht freagairt trí Ghaeilge

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. $198 \text{ marc} \times 5\% = 9.9 \Rightarrow$ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle $[300 - \text{bunmharc}] \times 15\%$, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0

