Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2010

MATHEMATICS – ORDINARY LEVEL

PAPER 1 (300 marks)

FRIDAY, 11 JUNE – AFTERNOON, 2:00 to 4:30

Attempt **SIX QUESTIONS** (50 marks each)

WARNING: Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

1. **(a)** Express 40 metres as a fraction of 1 kilometre. Give your answer in its simplest form.

(b) **(i)** Calculate the value of

$$\frac{57{\cdot}6 + 80{\cdot}44}{1{\cdot}3 \times 10^4}$$

and write your answer correct to three decimal places.

(ii) An importer buys an item for £221 sterling when the rate of exchange is €1 = £0·85 sterling.

He sells it at a profit of 14% of the cost price.

Calculate, in euro, the price for which he sells the item.

(c) **(i)** What sum of money invested at 5% per annum compound interest will amount to €8682 in 3 years?
Give your answer correct to the nearest euro.

(ii) A sum of €P was invested at r % per annum compound interest.

The interest for the first year was €220.

The interest for the second year was €228·80.

Calculate r and P.

2. **(a)** Find the values of x which satisfy

$$2(3 + 4x) \leq 22, \quad \text{where } x \in \mathbb{N}.$$

(b) Solve for x and y

$$2x - y = 1$$
$$x^2 - xy = -6.$$

(c) **(i)** Show, by division, that $3x + 1$ is a factor of $3x^3 + 4x^2 - 89x - 30$.

(ii) Hence, or otherwise, solve the equation

$$3x^3 + 4x^2 - 89x - 30 = 0.$$

3. **(a)** Given that $3(b + a) = t(6 - a)$,

calculate the value of a when $t = 3$ and $b = -4$.

(b) Solve for x

$$5(x + 1)^2 = 2(x + 1) + 5 .$$

Give your answer correct to two decimal places.

(c) **(i)** $2 + \sqrt{3}$ is a root of the equation $x^2 - 4x + c = 0$, where c is a real number.
Find the value of c and write down the other root.

(ii) The equation $x^2 + 10x + k = 0$ has equal roots.
Find the value of the real number k and write down the value of each root.

4. **(a)** Given that $i^2 = -1$, simplify

$$(4 + 2i)(3 - i)$$

and write your answer in the form $x + yi$, where $x, y \in \mathbb{R}$.

(b) Let $u = 4 + 3i$ and $w = 6 - 8i$.

(i) Find the value of the real number k such that $|u| = k|w|$.

(ii) Express $\frac{w}{u}$ in the form $x + yi$.

(c) Let $z = a + bi$, where $a, b \in \mathbb{R}$.

Find the value of a and the value of b for which

$$3z - 10i = (2 - 3i)z .$$

5. **(a)** The first term of a geometric sequence is 4 and the common ratio is 0·5.

Write down the first five terms of the sequence.

(b) In an arithmetic series, the first term is 6 and the fifth term is 22.

(i) Find d, the common difference.

(ii) Find T_{14}, the fourteenth term.

(iii) Find S_{20}, the sum of the first twenty terms.

(c) In a geometric series, the fourth term is 9 and the seventh term is 243.

(i) Find r, the common ratio.

(ii) Find a, the first term.

(iii) Find S_8, the sum of the first eight terms.

6. **(a)** Let $h(x) = x^2 + 1$, where $x \in \mathbb{R}$.

Write down a value of x for which $h(x) = 50$.

(b) Let $g(x) = \dfrac{1}{x-2}$, where $x \in \mathbb{R}$ and $x \neq 2$.

(i) Copy and complete the following table:

x	0	1	1·5	1·75	2·25	2·5	3	4
$g(x)$		−1		−4		2		

(ii) Draw the graph of the function g in the domain $0 \leq x \leq 4$.

(c) Let $f(x) = x - \dfrac{5}{x}$, where $x \in \mathbb{R}$ and $x \neq 0$.

(i) Find $f'(x)$, the derivative of $f(x)$.

(ii) Find the co-ordinates of the two points at which the tangent to the curve $y = f(x)$ is parallel to the line $y = 6x$.

7. **(a)** Differentiate $x^2 - 6x + 1$ with respect to x.

(b) **(i)** Differentiate $5 - 3x$ with respect to x from first principles.

(ii) Given that $y = (x^2 - 4)(3x - 1)$, find the value of $\frac{dy}{dx}$ when $x = 2$.

(c) The speed, v, of an object at time t is given by

$$v = 96 + 40t - 4t^2$$

where t is in seconds and v is in metres per second.

(i) At what times will the speed of the object be 96 metres per second?

(ii) What will the acceleration of the object be at $t = 2{\cdot}5$ seconds?

(iii) At what value of t will the acceleration become negative?

8. Let $f(x) = x^3 - 3x + 1$, where $x \in \mathbb{R}$.

(i) Find $f(-3)$, $f(-2)$, $f(0)$, $f(2)$ and $f(3)$.

(ii) Find $f'(x)$, the derivative of $f(x)$.

(iii) Find the co-ordinates of the local maximum point and of the local minimum point of the curve $y = f(x)$.

(iv) Draw the graph of the function f in the domain $-3 \leq x \leq 3$.

(v) Find the range of values of k for which the equation

$$x^3 - 3x + 1 = k$$

has three real solutions (roots).

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