Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE 2008

MARKING SCHEME

MATHEMATICS

ORDINARY LEVEL

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LEAVING CERTIFICATE EXAMINATION 2008

MATHEMATICS – ORDINARY LEVEL – PAPER 1

GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates' work as follows:

• Blunders	- mathematical errors/omissions	(-3)
• Slips	- numerical errors	(-1)
• Misreadings	(provided task is not oversimplified)	(-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

APPLYING THE GUIDELINES

Examples of the different types of error:

Blunders (i.e. mathematical errors) (-3)

- Algebraic errors : $8x + 9x = 17x^2$ or $5p \times 4p = 20p$ or $(-3)^2 = 6$
- Sign error -3(-4) = -12
- Decimal errors
- Fraction error (incorrect fraction, inversion etc); apply once.
- Cross-multiplication error
- Operation chosen is incorrect. (e.g., multiplication instead of division)
- Transposition error: e.g. $-2x - k + 3 \Rightarrow -2x = 3 + k$ *or* $-3x = 6 \Rightarrow x = 2$ *or* $4x = 12 \Rightarrow x = 8$; each time.
- Distribution error (once per term, unless directed otherwise) e.g. $3(2x + 4) = 6x + 4$ *or* $\frac{1}{2}(3 - x) = 5 \Rightarrow 6 - x = 5$
- Expanding brackets incorrectly: e.g. $(2x - 3)(x + 4) = 8x^2 - 12$
- Omission, if not oversimplified.
- Index error, each time unless directed otherwise
- Factorisation: error in one or both factors of a quadratic: apply once $2x^2 - 2x - 3 = (2x - 1)(x + 3)$
- Root errors from candidate's factors: error in one or both roots: apply once.
- Error in formulae: e.g. $T_n = 2a + (n - 1)d$
- Central sign error in *uv* or *u/v* formulae
- Omission of $\div v^2$ or division not done in *u/v* formula (apply once)
- Vice-versa substitution in *uv* or *u/v* formulae (apply once)

Slips (-1)

- Numerical slips: 4 + 7 = 10 or 3×6 = 24, *but* 5 + 3 = 15 is a blunder.
- An omitted round-off or incorrect round off to a required degree of accuracy, or an early round off, is penalised as a slip each time.
- However an early round-off which has the effect of simplifying the work is at least a blunder
- Omission of units of measurement or giving the incorrect units of measurement in an answer is treated as a slip, once per part (a), (b) and (c) of each question. (Deduct at first non-zero or non-attempt-mark section, where applicable.

Misreadings (-1)

- Writing 2436 for 2346 will not alter the nature of the question so M(-1)
  However, writing 5000 for 5026 will simplify the work and is penalised as at least a blunder.

Note: Correct relevant formula *isolated* and stops: if formula is *not* in Tables, award attempt mark.

QUESTION 1

Part (a)	**15 marks**	**Att 5**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**15 (10, 5) marks**	**Att (3, 2)**

* Incorrect or omitted units: penalise as per guidelines.

Part (a)	**15 marks**	**Att 5**

1. (a) John works from 09:00 hours to 13:00 hours and again from 14:00 hours to 17:30 hours. He is paid €18·50 per hour. Find his total pay for the day.

(a)	**15 marks**	**Att 5**

1. (a)	**or**
09:00 to 13:00 is 4 hours. 14:00 to 17:30 is 3.5 hours } [9m]	
Hours worked 7.5 hours. [12m]	4×€18.50 = €74.00 [9m] 3.5×€18.50 = €64.75 [12m] } interchangeable
Total pay = 7.5 × €18.50 = €138.75.[15m]	74.00 + 64.75 = €138.75 [15m]

* Accept correct answer without work for full marks.

Blunders (-3)

B1 1h = 100 min. e.g. 3.3 hours

B2 Uses 8½ hours

Slips (-1)

S1 Numerical slips (each time)

Attempts (5 marks)

A1 Finds 4 or 3.5 only and stops

A2 [1h = €18.50] ⇒ ½ h = €9.25 and stops

A3 Any time ×18.50, but see B2

Worthless (0)

W1 Incorrect answer with no work

Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

1. (b) Alice frequently travels from her home to Cork, a distance of 85 km. The journey usually takes 1 hour 15 minutes.

(i) Find her average speed in km per hour for the journey.

(ii) On a day of very heavy rain her average speed on a 28 km section of the journey was reduced to 35 km per hour.
How long did this section of the journey take on that day?

(iii) How much longer did the total journey take on that day, if she completed the rest of the journey at her usual average speed?
Give your answer correct to the nearest minute.

(i) **10 marks** **Att 3**
(ii) **5 marks** **Att 2**

1. (b) (i)

$$\frac{85}{1.25}[7m] = 68 \text{ km/h.}[10m]$$

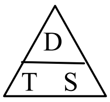
or $^{5}/_{4}$ h = 85km $\Rightarrow$ ¼ = $^{85}/_{5}$ $\Rightarrow$ $^{4}/_{4}$ = $^{85}/_{5}$ × 4 [7m] = 68km/h [10m]

(ii) Time taken = $\frac{28}{35}$ [2m] = 0.8 hours *or* 48 minutes. [5m]

* Accept correct answer without work for full marks in both (i) and (ii).

Blunders (-3)
B1 Error in D-S-T formula
B2 1h = 100 min e.g. 1.15h
B3 Answer in km/min (i) (= $1.1\dot{3}$ km/min)

Attempts (3 marks in (i), 2 marks in (ii))

A1 

or equivalent and stops

A2 15 mins. = 0.25h or similar e.g. 1h 15min = 75min

Worthless (0)
W1 Incorrect answer without work

(b) (iii) **5 marks** **Att 2**

1. (b) (iii)	**or**
Remainder of journey = 85 – 28 = 57 km.[2m] Time taken = $\frac{57}{68} \times 60 = 50.29$ min. Time for total journey = 48 + 50 = 98 min. Extra time taken = 98 – 75 = 23 min.[5m]	Time for 28km at usual speed = $\frac{28}{68} \times 60$ = 24.7 = 25 min Extra time taken 48 – 25= 23min [5m]

* Accept candidate's answers from (i) and (ii).
* Correct answer without work: Att2.

Blunders (-3)
B1 Fails to calculate time difference
B2 Error in D-S-T formula

Slips (-1)
S1 Incorrect or no rounding off

Attempts (2 marks)
A1 85 −28 and stops
A2 1h 15min = 75 min
A3 Answer from b(ii) <u>used</u>

Worthless (0)
W1 Incorrect answer without work

Part (c) **15 (10, 5) marks** **Att (3, 2)**

1. (c)

A retailer buys an item for €73. She wants to apply a mark-up of 40% of the cost price of the item. She must then add VAT at 21% to this amount to find the price that she would need to charge the customer.

(i) Find this price, correct to the nearest cent.

The retailer adjusts the price charged to the customer so that it is 1 cent less than a multiple of 10, while keeping the mark-up as close as possible to 40%.

(ii) Using this adjusted price, calculate the actual percentage mark-up achieved, correct to the nearest percent.

(c) (i) **10 marks** **Att 3**

1. (c) (i)

Price = (€73 × 1.40) = €102.20 [4m] × 1.21 [7m] = €123.662 [9m] = €123.66 [10m]

* Correct answer without work: Att 3.

Blunders (-3)

B1 VAT calculated but not added
B2 Mathematical errors
B3 VAT calculated on increase only

Slips (-1)

S1 Incorrect or no rounding

Attempts (3 marks)

A1 73×1.4 and stops or similar
A2 Finds 61% or 161% of €73

(c) (ii) **5 marks** **Att 2**

1. (c) (ii) Adjusted price = €119.99 ………………………………..[2m]

Price before VAT = $\frac{119.99}{1.21}$ = 99.165 = €99.17.

Mark-up = $\frac{99.17}{73}$ = 1.3584.

Percentage mark-up = 35.84% = 36%...................................[5m]

* Accept candidate's answer from (i).

Blunders (-3)

B1 Price incorrectly adjusted, but see S2
B2 VAT not removed
B3 Fails to use C.P.

Slips (-1)

S1 Incorrect or no rounding
S2 Price = €129.99

Attempts (2 marks)

A1 73 used in some calculation
A2 Calculates any '1c less' than a multiple of €10, e.g. €9.99, and stops.

QUESTION 2

Part (a)	**15 marks**	**Att 5**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Part (a) **15 marks** **Att 5**

2. (a) Simplify $3(4x+5)-2(6x+4)$.

(a) **15 marks** **Att 5**

2. (a) $12x + 15 - 12x - 8$ [12m} $= 7$ [15m]

* Correct answer without work: Full marks.

Blunders (-3)

B1 Distribution error, once per term

B2 $3(4x+5)=2(6x+4)$ and continues

Attempts (5 marks)

A1 Any correct relevant multiplication or addition

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

2. (b) **(i)** Solve $x^2-4x+1=0$.
Write your solutions in the form $a \pm \sqrt{b}$, where $a, b \in \mathbf{N}$.

(ii) Find the value of x for which

$$\frac{5^x}{3}=\frac{5^6}{75}.$$

(i) Substitution	**5 marks**	**Att 2**
Simplify	**5 marks**	**Att 2**
Required form	**5 marks**	**Att 2**

2. (b) (i)

$$x=\frac{4\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)} \text{ [1st 5m] } =\frac{4\pm\sqrt{16-4}}{2}=\frac{4\pm\sqrt{12}}{2} \text{ [2nd 5m]}.$$

$$=\frac{4\pm 2\sqrt{3}}{2}=2\pm\sqrt{3} \text{ [3rd 5m]}$$

Blunders (-3)

B1 Error in quadratic formula

B2 Error in substitution

B3 Error in surd

Attempts (2 marks)

A1 Correct quadratic formula and stops

A2 *a, b* or *c* correctly identified

A3 1 error in formula with some correct substitution and stops

A4 Attempt at finding factors

A5 Answers correctly worked out as decimal (last part)

A6 Oversimplification due to no surd (last part)

Worthless (0 marks)

W1 Correct/incorrect answer without work

(b) (ii) **5 marks** **Att 2**

2.(b) (ii)	$\frac{5^x}{3}=\frac{5^6}{75} \Rightarrow 5^x=\frac{5^6\times 3}{75} \Rightarrow 5^x=5^4$ or $x=4$[5m]
or	$\frac{5^x}{3}=208.\dot{3} \Rightarrow 5^x=625 \Rightarrow 5^x=5^4$ or $x=4$ [5m]

* Correct answer by T+E, verified: full marks; unverified: Att 2.

Blunders (-3)
B1 Error in dealing with denominators
B2 Errors with indices, each time

Attempts (2 marks)
A1 Any correct relevant use of indices and stops e.g. 5^6 =15625 or 75 =3×5×5
A2 75÷3 = 25 and stops
A3 Incorrect T + E, with work

Worthless (0 marks)
W1 If 5 is "cancelled" in first line
W2 $375^x = 15^6$

Part (c) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

(i) Factorise x^2+4x+4.

(ii) Simplify $\sqrt{x^2+4x+4} + \sqrt{x^2+2x+1}$, given that $x \geq 0$.

(iii) Given that $x \geq 0$, solve for x

$$\sqrt{x^2+4x+4} + \sqrt{x^2+2x+1} = x^2.$$

(c) (i) **5 marks** **Att 2**

2.(c) (i)	$x^2+4x+4=(x+2)(x+2)$ *or* $(x+2)^2$.

Blunders (-3)
B1 Error in finding factors.

Attempts (2 marks)
A1 One correct element: x or 2
A2 Effort at factors, e.g. 1×4
A3 Correct quadratic formula and stops

(c) (ii) **5 marks** **Att 2**

2.(c) (ii) $\sqrt{(x+2)^2} + \sqrt{(x+1)^2} = x+2+x+1 = 2x+3.$

*Accept candidate's *factors* from (i) written under $\sqrt{\ }$.

Blunders (-3)
B1 Error in factors
B2 Error in finding √

Attempts (2 marks)
A1 Effort at factorising $x^2 + 2x + 1$
A2 $(x + 2)^2$ written in this part

Worthless (0 marks)
W1 Substitutes <u>numerical</u> value(s)

(c) (iii) **5 marks** **Att 2**

2.(c) (iii) $\sqrt{x^2+4x+4} + \sqrt{x^2+2x+1} = x^2$

$\Rightarrow\ 2x+3 = x^2$

$\Rightarrow\ x^2-2x-3=0$

$\Rightarrow\ (x-3)(x+1)=0$

$\Rightarrow\ x=3$ or $x=-1$

Answer $x = 3.$

* Accept candidate's answer from (i) or (ii) if it leads to a trinomial, otherwise Att at most.
* If quadratic formula used, apply blunders as per guidelines.

Blunders (-3)
B1 Error in transposition
B2 Error in factorising
B3 Root errors from candidates factors

Slips (-1)
S1 Does not exclude -1

Attempts (2 marks)
A1 Attempt at factorising x^2+4x+4 or x^2+2x+1 *in this part*
A2 Candidate's answers from (i) and/or (ii) $= x^2$ and stops
A3 $x = 3$ by T + E
A4 Correct quadratic formula and stops

QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (15, 5) marks**	**Att (5, 2)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a) **10 marks** **Att 3**

3. (a) Given that $a(x+5)=8$, express x in terms of a.

(a) **10 marks** **Att 3**

3. (a) **I** $a(x+5)=8$	**II** $a(x+5)=8$
	$\Rightarrow ax+5a=8$ [3m]
$\Rightarrow x+5=\frac{8}{a}$ [7m]	$\Rightarrow ax=8-5a$ [7m]
$\Rightarrow x=\frac{8}{a}-5$ [10m]	$\Rightarrow x=\frac{8-5a}{a}$ [10m]

*Accept correct answer without work.

Blunders (-3)

B1 Transposition error

B2 Distribution error

Misreadings (-1)

M1 $a(x-5)=8$ or similar

Attempts (3 marks)

A1 Expresses a in terms of x (oversimplified)

A2 Any correct multiplication by a in method **II**

Worthless (0)

W1 Incorrect answer without work

Part (b) **20 (15, 5) marks** **Att (5, 2)**

3. (b) **(i)** Solve for x and y

$$x - y = 1$$

$$x^2 + y^2 = 25.$$

(ii) Hence, find the two possible values of $x - y^2$.

(b) (i) **15 marks** **Att 5**

3 (b) (i)

$x - y = 1 \quad \Rightarrow \quad x = y + 1$ [5m]

$x^2 + y^2 = 25$

$\Rightarrow \quad (y+1)^2 + y^2 = 25$ [6m]

$\Rightarrow \quad y^2 + 2y + 1 + y^2 - 25 = 0$

$\Rightarrow \quad 2y^2 + 2y - 24 = 0$ [9m]

$\Rightarrow \quad y^2 + y - 12 = 0$ [9m]

$\Rightarrow \quad (y+4)(y-3) = 0$

$\Rightarrow \quad y = -4 \text{ or } y = 3$ [12m]

$\Rightarrow \quad x = -3 \text{ or } x = 4.$ [15m]

* Apply similar structure if y is isolated first.

* No additional marks from any point where the equation becomes linear; but see A3 below.

* Substitution into quadratic, rather than linear equation: no penalty for excess answers.

Blunders (-3)

B1 Mathematical errors, once per step

B2 Incorrect factors, apply once

B3 If quadratic formula used, apply blunders as per guidelines (1 step in scheme)

Attempts (5 marks)

A1 Effort at isolating x or y

A2 Correct quadratic formula written and stops

A3 Having found the first variable from work of no value, substitutes correctly to find the second variable

A4 Correct answer(s) by T+E or without work, even if verified

Worthless (0)

W1 ‘Invented’ values substituted, even if continues, e.g. $y = 0 \Rightarrow x = 1$ or some such.

(b) (ii) **5 marks** **Att 2**

3. (b) (ii)

$x = -3, \quad y = -4$

$x - y^2 = -3 - (-4)^2 = -3 - 16 = -19.$ [2m]

$x = 4, \quad y = 3$

$x - y^2 = 4 - 3^2 = 4 - 9 = -5.$ [5m]

interchangeable

* Accept candidate's coordinates from (i).

Blunders (-3)

B1 One value only found

B2 Values of *x* and *y* interchanged

Slips(-1)

S1 Use of incorrect coordinates, if excess coordinates found in (i)

Attempts (2 marks)

A1 Some correct substitution

Worthless (0)

W1 Tries to solve $x - y^2 = 0$ or similar

W2 Invented values used

Part (c) **20 (10, 10) marks** **Att (3, 3)**

3. (c) **(i)** Let $f(x) = x^2 + bx + c$, $x \in \mathbf{R}$.
The graph of the function *f* intersects the *y*-axis at 3 and the *x*-axis at -1.
Find the value of *b* and the value of *c*.

(ii) The lengths of the sides of an isosceles triangle are $\sqrt{x^2+1}$, $\sqrt{x^2+1}$ and $2x$.
Taking $2x$ as the base, find the perpendicular height of the triangle.

(c) (i) **10 marks** **Att 3**

3. (c) (i)

$f(x) = x^2 + bx + c$

$f(0) = 0 + 0 + c = 3 \quad [\Rightarrow \quad c = 3.]$ [3m]

$f(-1) = 1 - b + c = 0$ [4m]

interchangable

$f(-1) = 1 - b + 3 = 0$ [7m]

$\Rightarrow \quad -b = -4 \Rightarrow b = 4.$ [10m]

* Substitution into linear expression, Att at most.

Blunders (-3)

B1 (3, 0) for (0, 3) and/or (0, -1) for (-1, 0)

B2 $f(0) \neq 3$ or $f(-1) \neq 0$ each time, subject to B1

Attempts (3 marks)

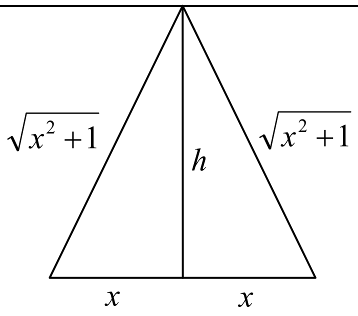
A1 *f(x)* substituted for $x \notin \{0, -1, 3\}$ and stops

A2 States *x* = 0 for *y*-intercept, or *y* = 0 for *x*-intercept or similar and stops

A3 $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \underline{\underline{-1}}$

(c) (ii) **10 marks** **Att 3**

3. (c) (ii)



. $x^2 + h^2 = \left(\sqrt{x^2+1}\right)^2$ [3m]

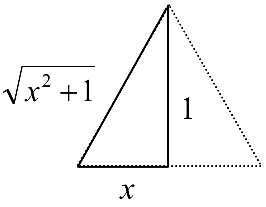
$x^2 + h^2 = x^2 + 1$ [4m]

. $h^2 = 1$ [7m]

$h = \pm 1$ [10m]

Height $h = 1$

* Correct answer without work: Att 3, but correct answer from correctly labelled diagram (as shown): full marks.



* Accept correct answer by substitution of numerical value of x into Pythagoras.

Blunders (-3)

B1 Uses $2x$ in calculation

B2 Mathematical errors

B3 $h^2 = x^2 + \left(\sqrt{x^2+1}\right)^2$

Attempts (3 marks)

A1 Diagram with some correct, relevant information shown and stops

A2 States or implies Pythagoras e.g. $a^2 + b^2 = c^2$

QUESTION 4

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

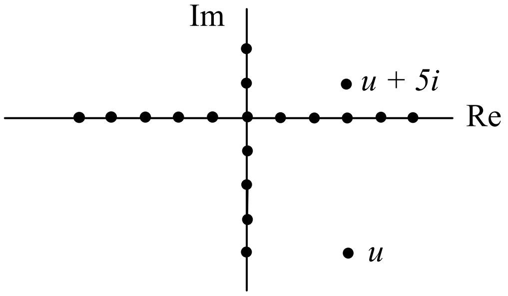
Part (a) **10 (5, 5) marks** **Att (2, 2)**

4. (a) Let $u = 3 - 4i$, where $i^2 = -1$.
Plot on an Argand diagram
(i) u
(ii) $u + 5i$.

(i) **5 marks** **Att 2**
(ii) **5 marks** **Att 2**

4. (a)

$u + 5i$
$3 - 4i + 5i =$
$3 + i$



* Unlabelled axes: assume horizontal axis is real.
* Accept reversed axes if clearly identified, otherwise B(-3).
* If one unnamed point only is plotted, assume it is *u*.

Blunders (-3)
B1 *u* incorrectly plotted
B2 Incorrect calculation of $u + 5i$
B3 $u + 5i$ correctly calculated but not plotted or incorrectly plotted
B4 Incorrect or no scales on axes

Attempts (2 marks)
A1 Scaled axes. Apply once only
A2 Any correct step in finding $u + 5i$ (applies to 2nd 5 marks)

Part (b) **20 (10, 10) marks** **Att (3, 3)**

4. (b) Let $w = 2 + 5i$.

(i) Express w^2 in the form $x + yi$, where $x, y \in \mathbf{R}$.

(ii) Verify that $|\,w^2\,| = |\,w\,|^2$.

(b) (i) **10 marks** **Att 3**

4. (b) (i)

$$(2+5i)^2 = (2+5i)(2+5i) \quad [3\text{m}] \quad = 2(2+5i)+5i(2+5i) = 4+10i+10i+25i^2 \ [7\text{m}]$$

$$= 4+20i+25(-1) \quad = -21+20i. \quad [10\text{m}]$$

Blunders (-3)

B1 Each incorrect or omitted term when expanding brackets to max of 2×B

B2 $i^2 \neq -1$

B3 Real and imaginary terms confused

B4 $w\overline{w}$ used

Attempts (3 marks)

A1 $w^2 = w \times w$

A2 Any correct, relevant multiplication

A3 Finds $2w$

(b) (ii) **10 marks** **Att 3**

4. (b) (ii)

$$\begin{array}{ccccl} |w^2| & & = & |w|^2 & \\ |-21+20i| & \text{and/or} & & |2+5i|^2 & \ldots.[3\text{m}] \\ \sqrt{(-21)^2+(20)^2} & \text{or} & & \left(\sqrt{2^2+5^2}\right)^2 & \ldots[4\text{m}] \quad \text{Both} \quad [7\text{m}] \\ \sqrt{441+400} & & = & \left(\sqrt{4+25}\right)^2 & \\ \sqrt{841} & & = & \left(\sqrt{29}\right)^2 & \\ 29 & & = & 29 & \ldots\ldots[10\text{m}] \end{array}$$

* Accept candidate's answer from (i) for w^2.

* Accept use of distance formula or $z\bar{z} = |z|^2$.

* Stated conclusion not necessary unless $|\,w^2\,| \neq |\,w\,|^2$ then S(-1).

* No penalty for omission of $(\sqrt{\ })^2$ on RHS.

Case:
One side only found, or
one side repeated: 4 marks

Blunders (-3)

B1 Error in modulus formula

B2 Mathematical errors

B3 Errors in substitution into formula e.g. $(20i)^2$ but accept $(21)^2$

Attempts (3 marks)

A1 $\sqrt{a^2+b^2}$ or distance formula correct and stops

A2 Mod formula or distance formula with at most 1 error and some correct substitution, and stops

Worthless (0)

W1 Incorrect formula (other than A2) with or without substitution

Part (c) **20 (10, 10) marks** **Att (3, 3)**

4. (c) Let $z = 6 - 4i$.

(i) Find the real number k such that $k(z + \bar{z}) = 24$,
where $\bar{z}$ is the complex conjugate of z.

(ii) Find the real numbers s and t such that $\frac{s+ti}{4+3i} = z$.

(c) (i) **10 marks** **Att 3**

4. (c)(i) $\bar{z} = 6 + 4i$ [3m]

$k(6 - 4i + 6 + 4i) = 24$ [4m] $\Rightarrow$ $k(12) = 24$ [7m] $\Rightarrow$ $k = 2$ [10m].

Blunders (-3)

B1 Incorrect conjugate
B2 Algebraic errors
B3 Confuses real and imaginary parts

Attempts (3 marks)

A1 $k(z + \bar{z}) = kz + k\bar{z}$ and stops
A2 Some correct transposition e.g. $\frac{24}{k}$
A3 Substitutes 6 – 4i for z

Worthless (0)

W1 $k(z - z) = 24$ and stops

(c) (ii) **10 marks** **Att 3**

4. (c) (ii) $\frac{s+ti}{4+3i} = 6 - 4i$ [3m]	$\frac{s+ti}{4+3i} = 6 - 4i$ [3m]
$s + ti = (4 + 3i)(6 - 4i)$ [4m]	$\frac{s+ti}{4+3i} \times \frac{4-3i}{4-3i} = 6 - 4i$ [4m]
$= 24 - 16i + 18i - 12i^2$.	$\frac{4s - 3si + 4ti - 3ti^2}{16 - 9i^2} = 6 - 4i$
$= 36 + 2i$ [7m]	$= \frac{4s - 3si + 4ti + 3t}{16 + 9} = \frac{4s + 3t - (3s - 4t)i}{25} = 6 - 4i$
	$4s + 3t = 150$ and $3s - 4t = 100$ [7m]
$s = 36$, $t = 2$. [10m]	$s = 36$, $t = 2$. [10m]

Blunders (-3)

B1 Incorrect conjugate
B2 $i^2 \neq -1$
B3 Real and imaginary terms confused
B4 Fails to identify s and t explicitly

Attempts (3 marks)

A1 Correct conjugate and stops
A2 Any correct, relevant multiplication
A3 Substitutes 6 – 4i for z

QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**
Part (c)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

* Error in formula: if one error only, then 1×B. Otherwise it is not a valid formula.
* Do not penalise notation

Part (a) **10 marks** **Att 3**

5. (a) Find the eleventh term of the arithmetic sequence 5, 14, 23 …

(a) **10 marks** **Att 3**

5. (a)
I $a = 5$ or $d = 14 - 5 = 9$ or $T_{11} = a + 10d$ [3m] $= 5 + (11 - 1)9$ [7m] $= 95$ [10m]

II List: 5, 14, 23, 32, 41, 50, 59, 68, 77, 86, **95** … [10m]
III Uses $S_{11} - S_{10}$: $550 - 455 = 95$

* In method **II**, answer must be clearly indicated
* Accept correct answer with no work

Blunders (-3)
B1 Incorrect *a*
B2 Incorrect *d,* unless an obvious slip from shown work $\Big\}$ but *a* and *d* interchanged, penalise once
B3 Mathematical errors, each time
B4 In **II**: answer 86 or 104 with/without work

Attempts (3 marks)
A1 Correct *a*, *d*, or *n* stated or clearly indicated (includes correct substitution of these *a*, *d*, or *n* into a formula.)
A2 14 – 5 and stops, or similar
A3 $a + (n - 1)d$ and stops
A4 $^{n}/_{2}\{2a + (n - 1)d\}$ and stops
A5 Continues sequence correctly for at least 1 further term

Worthless (0)
W1 Incorrect answer without work, except for B4
W2 Any GP formula but note A1
W3 11 or T_{11} and stops

Part (b) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

5. (b) The *n*th term of a geometric sequence is

$$T_n = \frac{3^n}{27}.$$

(i) Find *a*, the first term.
(ii) Find *r*, the common ratio.
(iii) The *k*th term of the sequence is 243. Find *k*.

(i) **5 marks** **Att 2**
(ii) **5 marks** **Att 2**

5. (b) (i) $a = T_1 \text{ [2m]} = \frac{3^1}{27} \text{ [5m]} = \frac{3}{27} \text{ [5m]} = \frac{1}{9} \text{ [5m]}.$

(ii) $T_2 = \frac{3^2}{27} = \frac{9}{27} = \frac{1}{3}.$

. $r = \frac{T_2}{T_1} = \frac{{}^1\!/_3}{{}^1\!/_9} = 3.$

* Accept correct answer without work (both parts)
* If an incorrect value of *a* from (i) is used in (ii), penalise in (ii) <u>if not already penalised.</u>
* Accept any correct $T_{n+1} \div T_n$ worked out.(ii)

Blunders (-3)
B1 Index errors
B2 Any $T_n \div T_{n+1} \Rightarrow r = ⅓$, in (ii)

Slips (-1)
S1 $^1/_3 = 0.3..$ and/or $^1/_9 = 0.1..$ used

Attempts (2 marks)
A1 $n = 1$ and stops
A2 $27 \div 3$ (i)
A3 $T_n = ar^{n-1}$
A4 Calculates any T_n correctly using $n \in \mathbf{N}, n \neq 1$, in (i) or (ii).
A5 $n = 2$ and stops, in (ii)

Worthless (0)
W1 3 ± 27
W2 Incorrect answer without work

(iii) **10 marks** **Att 3**

5. (b) (iii)

$T_k = \frac{3^k}{27} \text{ [3m]} = 243 \text{ [4m]} \Rightarrow 3^k = 27 \times 243 = 3^3 \times 3^5$ or $6561 \text{ [7m]} = 3^8$ or $k = 8$ [10m]

or $T_k = \frac{3^k}{3^3}$ [3m] $= 243$ [4m] => $3^{k-3} = 243 = 3^5$ [7m] => $k - 3 = 5$ => $k = 8$ [10m]

or $^1/_9$, $^1/_3$, 1, 3, 9, 27, 81, **243**. [7m] $\therefore$ 243 = T_8 or $k = 8$ [10m]

* Correct answer without work: Att 3.
* Ignore notation but $\frac{3^n}{27}$, and stops is worthless.

Blunders (-3)
B1 Mathematical errors each time

Attempts (3 marks)
A1 Minimum of 2 correct consecutive terms written
A2 ar^{n-1} in this part

Worthless (0)
W1 Incorrect answer without work

Part (c) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

The sum of the first n terms of an arithmetic series is given by $S_n = n^2 - 16n$.

(i) Use S_1 and S_2 to find the first term and the common difference.

(ii) Find T_n, the nth term of the series.

(iii) Find the values of $n \in \mathbf{N}$ for which $S_n = -63$.

(c) (i) **10 marks** **Att 3**

$S_n = n^2 - 16n$.

$$\left.\begin{array}{ll} a = S_1 = 1^2 - 16(1) = 1 - 16 = -15 & [3m] \\ S_2 = 2^2 - 16(2) \text{ or } 4 - 32 \text{ or } -28 & [4m] \end{array}\right\} \text{interchangeable}$$

$T_2 = S_2 - S_1 = -28 - (-15) = -13$ [7m]

$d = T_2 - T_1 = -13 - (-15) = 2$. [10m]

Blunders (-3)

B1 Sign errors

B2 $T_2 = S_1 - S_2$ ($= 13 \Rightarrow d = 28$)

B3 Index errors

Misreading(-1)

M1 Finds a and d not using S_1 and S_2

Attempts (3 marks)

A1 States or substitutes $n = 1$ or $n = 2$ and stops.

A2 Substitutes any other number for $n \in N$ in $n^2 - 16n$

Worthless (0)

W1 $n(n - 16)$

(c) (ii) **5 marks** **Att 2**

5. (c) (ii) **I** $T_n = a + (n-1)d$ [2m] $= -15 + (n-1)2$ or $-15 + 2n - 2$ or $2n - 17$ [5m].

or **II** $T_n = S_n - S_{n-1}$ [2m]

$= n^2 - 16n - \left((n-1)^2 - 16(n-1)\right)$

or $n^2 - 16n - n^2 + 2n - 1 + 16n - 16$ or $2n - 17$ [5m]

* Accept candidate's values for a and d from (i).

Blunders (-3)

$$\left.\begin{array}{ll} \text{B1} & \text{Incorrect } a \\ \text{B2} & \text{Incorrect } d \end{array}\right\} \text{but } a \text{ and } d \text{ interchanged, penalise once}$$

B3 Mathematical errors – each time

Attempts (2 marks)

A1 $a = -15$ and/or $d = 2$ and stops

A2 Some correct substitution into an AP formula

Worthless (0)

W1 Irrelevant formula or incorrect relevant formula with no correct substitution

(c) (iii) **5 marks** **Att 2**

5. (c) (iii) $S_n = n^2 - 16n = -63$ [2m]

$\Rightarrow n^2 - 16n + 63 = 0 \quad \Rightarrow (n-7)(n-9) = 0 \quad \Rightarrow n = 7$ and $n = 9$ [5m]

or

$-15, -28, -39, -48, -55, -60,$ **−63**, $-64,$ **−63**, [4m]

S_7 and $S_9 = -63 \quad \Rightarrow n = 7$ or $n = 9$ [5m]

* If quadratic formula used, apply guidelines.

Blunders (-3)

B1 Mathematical errors (each time)

B2 One value of *n* only found

Attempts (2 marks)

A1 $T_n = -63$ and continues correctly

A2 Quadratic formula written and stops

A3 S_n formula written and stops

A4 S_3 or T_3 or any subsequent term worked out

Worthless (0)

W1 $n(n - 16)$

QUESTION 6

Part (a)	**15 marks**	**Att 5**
Part (b)	**20 marks**	**Att 7**
Part (c)	**15 (10, 5) marks**	**Att (3, 2)**

Part (a) **15 marks** **Att 5**

6. (a) Let $g(x) = 2x - 5$, where $x \in$ R.
Find the value of x for which $g(x) = 19$.

Part (a) **15 marks** **Att 5**

6. (a) $2x - 5 = 19$ [9m] $\Rightarrow$ $2x = 5 + 19$ *or* 24 [12m] $\Rightarrow$ $x = 12$ [15m]

* Accept correct answer without work.

Blunders (-3)
B1 Mathematical errors
B2 Evaluates $g(19)$

Attempts (5 marks)
A1 Unsuccessful T+E e.g. $g(1) = 2 - 5$

Worthless (0)
W1 Incorrect answer without work
W2 $19(2x - 5)$ whether continues or not
W3 Differentiates

Part (b) **20 marks** **Att 7**

6. (b) Differentiate $3x^2+5$ with respect to x from first principles.

(b) **20 marks** **Att 7**

6. (b)

$$\begin{aligned} f(x) &= 3x^2+5 \\ f(x+h) &= 3(x+h)^2+5 && [\ 8\text{m}] \\ &= 3x^2+6xh+3h^2+5 && [11\text{m}] \\ f(x+h)-f(x) &= 3(x+h)^2+5-(3x^2+5) && [11\text{m}] \\ &= 3x^2+6xh+3h^2+5-3x^2-5 \\ &= 6xh+3h^2 && [14\text{m}] \\ \frac{f(x+h)-f(x)}{h} &= \frac{6xh+3h^2}{h} = 6x+3h && [17\text{m}] \\ \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= 6x && [20\text{m}] \end{aligned}$$

or

$$\begin{aligned} y &= 3x^2+5 \\ y+\Delta y &= 3(x+\Delta x)^2+5 && [8\text{m}] \\ &= 3x^2+6x\Delta x+3(\Delta x)^2+5 && [11\text{m}] \\ \underline{y} &\underline{= 3x^2 \qquad\qquad +5} \\ \Delta y &= 6x\Delta x+3(\Delta x)^2 && [14\text{m}] \\ \frac{\Delta y}{\Delta x} &= 6x+3\Delta x && [17\text{m}] \\ \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} &= 6x && [20\text{m}] \end{aligned}$$

or

$$\begin{aligned} \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\frac{3(x+h)^2+5-(3x^2+5)}{h} \\ &= \lim_{h\to 0}\frac{3x^2+6hx+3h^2+5-3x^2-5}{h} \\ &= \lim_{h\to 0}\frac{6xh+3h^2}{h} \\ &= \lim_{h\to 0}(6x+3h) \\ &= 6x \end{aligned}$$

*Accept $h = 0$ or $\Delta x = 0$ in limit.

Blunders(-3)

B1 Mathematical errors: once per step on RHS

B2 Omits $f(x+h)$ and/or $f(x+h)-f(x)$ on LHS, or equivalent

B3 Omits $\{f(x+h)-f(x)\}/h$ on LHS, or equivalent

B4 Omitted or incorrect indication of limit on LHS, and/or error in evaluating candidate's limit

Misreading(-1)

M1 Uses $3x^2-5$ or $3x^2 \pm 5x$

Attempts (7marks)

A1 $f(x \pm h)$ on LHS or some substitution of $x \pm h$ for x on RHS, or equivalent; these only

A2 Linear function used (oversimplification)

Worthless (0)

W1 Answer $6x$ without work

Part (c) **15 (10, 5) marks** **Att (3, 2)**

6. (c) Let $f(x)=\dfrac{x^2-x}{1-x^3}$, $x\in R$, $x\neq 1$.

(i) Find $f'(x)$, the derivative of $f(x)$.

(ii) Show that the tangent to the curve $y=f(x)$ at the point (0, 0) makes an angle of 135° with the positive sense of the x-axis.

(c) (i) **10 marks** **Att 3**

6. (c) (i) .

$$[f'(x)]=\frac{(1-x^3)(2x-1)-(x^2-x)(-3x^2)}{(1-x^3)^2}\text{ [10m]}\quad=\left[\frac{x^4-2x^3+2x-1}{(1-x^3)^2}\right].$$

* Apply penalties as in guidelines.

* If errors made in simplification, apply in (ii) if appropriate.

Blunders (-3)

B1 Differentiation errors, once per term

Attempts (3 marks)

A1 u and/or v correctly identified and stops

A2 Any correct differentiation

A3 Numerator and/or denominator factorised correctly and stops

Worthless (0)

W1 $\frac{u}{v}$ written and stops

W2 $f'(x)$ or $\frac{dy}{dx}$

Note

If simplification done first: apply blunders to simplification to max 1×B, provided it does not oversimplify quotient.

$$f(x)=\frac{x^2-x}{1-x^3}=\frac{-x}{1+x+x^2}\qquad\text{[3m]}$$

$$f'(x)=\frac{(1+x+x^2)(-1)-(-x)(1+2x)}{(1+x+x^2)^2}\text{ [10m]}$$

$$=\frac{x^2-1}{(1+x+x^2)^2}$$

(c) (ii) **5 marks** **Att 2**

6. (c) (ii) $f'(0)=\dfrac{0-2(0)+2(0)-1}{(1-0)^2}=\left[\dfrac{-1}{1}\right]=-1$ [2m]

[tan $\theta=-1$] $\Rightarrow$ $\theta=\tan^{-1}(-1)=135°$ or $\tan 135°=-1$ [5m]

* Accept candidate's $f'(x)$ from (i), but see 2nd asterisk in (c)(i).

Blunders (-3)

B1 Mathematical errors

B2 Finds tan(−1) = −0.0174…

Slips(-1)

S1 If $\theta\neq 135°$ and correct conclusion not stated.

Attempts (2 marks)

A1 States tan = $f'(x)$ or slope, or similar

A2 Any use of $f'(x)$ in this part, and stops

A3 slope = tan 135

Worthless (0)

W1 Finds $f(0)$

QUESTION 7

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

7. (a) Differentiate with respect to x

(i) x^7

(ii) $5x - 3x^4$.

(a) (i) **5 marks** **Att 2**

7. (a) (i) $\frac{dy}{dx} = 7x^6$

(a) (ii) **5 marks** **Att 2**

7. (a) (ii) $\frac{dy}{dx} = 5 - 12x^3$

* Correct answer without work or notation: full marks.
* If done from first principles, ignore errors in procedure – just mark the answer.
* Only one term correctly differentiated (in part (ii)): award 2 marks.

Blunders (-3)
B1 Differentiation error

Attempts (2 marks)
A1 A correct step in differentiation from 1st principles
A2 A correct coefficient or a correct index of x.

Worthless (0)
W1 No correct differentiation, but check attempts first

Part (b)	**20 (10, 10) marks**	**Att (3, 3)**

7. (b) **(i)** Differentiate $(1+3x)(4-x^2)$ with respect to x.

(ii) Given that $y=(3x^2-4x)^8$, find $\frac{dy}{dx}$ when $x = 1$.

(i)	**10 marks**	**Att 3**
(ii)	**10 marks**	**Att 3**

7. (b) (i) $f'(x)=(4-x^2)(3)+(1+3x)(-2x)$ or $12-3x^2-2x-6x^2$ or $-9x^2-2x+12$.

or $f(x)=4+12x-x^2-3x^3 \quad \Rightarrow f'(x)=12-2x-9x^2$

(ii) $y=(3x^2-4x)^8$.

$\frac{dy}{dx}=8(3x^2-4x)^7(6x-4)$ [7m]

$=8(3(1)^2-4(1))^7(6(1)-4)=8(-1)(2)=-16$ [10m] at $x=1$.

* Apply penalties as in the guidelines.
* No penalty for omission of brackets if multiplication implied. (Decide by later work.)
* No marks for writing *uv* formula from tables (part (i)), and stopping.
* Treat $8(3x^2-4x)^7$ and $(6x-4)$ as separate parts in (ii).
* If differentiation correct, accept −16 without work in (ii), but −16 with <u>no work at all</u> ⇒ Att 3.
* $\frac{u}{v}$ used instead of *uv*: 2×B.

Blunders (-3)
B1 Differentiation errors, once per term
B2 Errors in expanding brackets to max of 2 ×B
B3 Error in substitution, once only (ii)

Case: $\frac{dy}{dx}=6x-4$,
whether continues or not: Att3

Slips (-1)
S1 Numerical slips

Attempts (3 marks)
A1 *u* and/or *v* correctly identified and stops (i)
A2 Any correct differentiation
A3 At least one term multiplied correctly
A4 Some correct element of chain rule e.g. index = 7 or coefficient = 8
A5 $u = 3x^2 - 4x$ and stops (ii)

Worthless (0)
W1 Substitutes $x = 1$ into $f(x)$ and stops
W2 *uv* or *u/v* written and stops

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

7. (c) (iii)

A distress flare is tested by firing it vertically upwards from the top of a tower.
The height, *h* metres, of the flare above the ground is given by

$$h = 20 + 90t - 5t^2$$

where *t* is the time in seconds from the instant the flare is fired.
The flare is designed to explode 7 seconds after firing.

(i) Find the height above the ground at which the flare explodes.
(ii) Find the speed of the flare at the instant it explodes.
(iii) If the flare failed to explode, find the greatest height above the ground it would reach before falling back down.

* Units: Penalise as per guidelines.
* No retrospective marking.
* No penalty for incorrect notation.
* If parts of (c) are unlabelled, and the context doesn't identify which part is which, assume the questions were answered in sequence from (c)(i) to (c)(iii).

(c) (i) **5 marks** **Att 2**

7. (c) (i) $h = 20 + 90(7) - 5(7)^2 = 20 + 630 - 245 = 405\,\text{m}.$

* Correct answer without work: Att 2

Blunders (-3)

B1 Errors in substitution
B2 Mathematical errors

Slips (-1)

S1 Numerical slips

Attempts (2 marks)

A1 Any correct substitution

Worthless (0)

W1 Differentiates, with or without subsequent substitution

(c) (ii) **5 marks** **Att 2**

$$\frac{dh}{dt} = 90 - 10t$$
$$= 90 - 10(7) = 90 - 70 = 20 \ \text{ms}^{-1} .$$

* Correct answer without work: Att 2.

Blunders (-3)

B1 Differentiation errors

B2 Incorrect or no value of *t* substituted into *dh/dt* equation

Attempts (2 marks)

A1 *dh/dt* or *dy/dx* or *f'(x)* or similar mentioned.

Worthless (0 marks)

W1 $t = 7$ substituted into original equation (ie repeat of part (i))

W2 Incorrect answer without work

W3 States speed = d^2h/dt^2 and stops

W4 Effort to use Speed = Distance ÷ Time

(c)(iii) Speed at max height **5 marks** **Att 2**

Height **5marks** **Att 2**

7. (c) (iii) $\frac{dh}{dt} = 90 - 10t \ = 0$ [1st 5 marks]

$\Rightarrow \ 10t = 90 \ \Rightarrow \ t = 9$ s.

$h = 20 + 90(9) - 5(9)^2 = 20 + 810 - 405 = 425$ m [2nd 5 marks]

* Correct answer without work: 2×Att 2.

* If $t = 9$ is not fully justified: 2×Att 2.

Blunders (-3)

B1 Mathematical errors

Slips (-1)

S1 Numerical slips

Attempts (2 marks)

A1 Speed = 0, *dh/dt* = *0* or similar

A2 $90 - 10t$ written

A3 $20 + 90t - 5t^2$ written

Worthless (0)

W1 Incorrect answer without work

W2 Finds d^2h/dt^2

QUESTION 8

Part (i)	**10 (5, 5) marks**	**Att (2, 2)**
Part (ii)	**10 marks**	**Att 3**
Part (iii)	**10 marks**	**Att 3**
Part (iv)	**5 marks**	**Att 2**
Part (v)	**5 marks**	**Att 2**
Part (vi)	**10 marks**	**Att 3**

Part (i) **10 (5, 5) marks** **Att (2, 2)**

8. (i) Let $f(x) = x^3 - 9x^2 + 24x - 18$, where $x \in$ **R**.
(i) Find $f(1)$ and $f(5)$.

(i) **10 (5, 5) marks** **Att (2, 2)**

8. (i) $f(x) = x^3 - 9x^2 + 24x - 18$.

$f(1) = 1^3 - 9(1)^2 + 24(1) - 18$ or $1 - 9 + 24 - 18$ [2m] $= -2$ [5m]

$f(5) = 5^3 - 9(5)^2 + 24(5) - 18$ or $125 - 225 + 120 - 18$ [2m] $= 2$ [5m]

* Correct answers without work: full marks.

Blunders (-3)
B1 Mathematical errors, each time if different
B2 $f(-1)$ and/or $f(-5)$ and continues

Slips (-1)
S1 Arithmetic error

Attempts (2 marks)
A1 Some correct substitution into *f(x)*
A2 Any other number substituted for *x,* whether evaluated or not

Worthless (0)
W1 Incorrect answer(s) without work

Part (ii) **10 marks** **Att 3**

8. (ii) Find $f'(x)$, the derivative of $f(x)$.

(ii) **10 marks** **Att 3**

8. (ii). $f'(x) = 3x^2 - 18x + 24$

* Correct answer without work or notation: full marks.
* If done from first principles, ignore errors in procedure – just mark the answer.
* Only one term correctly differentiated award 3 marks.

Blunders (-3)
B1 Differentiation error, each time

Attempts (3 marks)
A1 A correct step in differentiation from 1st principles

Worthless (0)
W1 No correct differentiation

Part (iii) **10 marks** **Att 3**

8. (iii) Find the co-ordinates of the local maximum point and of the local minimum point of the curve $y = f(x)$.

(iii) **10 marks** **Att 3**

8. (iii) $f'(x) = 3x^2 - 18x + 24 = 0$ [3m]

. $\Rightarrow x^2 - 6x + 8 = 0 \quad \Rightarrow (x-2)(x-4) = 0 \quad \Rightarrow x = 2$ and $x = 4$. [4m]

$f(x) = x^3 - 9x^2 + 24x - 18$.

$$\left.\begin{array}{ll} f(2) = 2^3 - 9(2)^2 + 24(2) - 18 = 8 - 36 + 48 - 18 = 2 \text{ .} & [7\text{m}] \\ f(4) = 4^3 - 9(4)^2 + 24(4) - 18 = 64 - 144 + 96 - 18 = -2 & [9\text{m}] \end{array}\right\} \text{interchangeable}$$

$$\left[\begin{array}{l} f''(x) = 6x - 18 \text{ .} \\ f''(2) = 12 - 18 = -6 < 0 \quad \Rightarrow \text{ maximum at } x = 2. \\ f''(4) = 24 - 18 = 6 > 0 \quad \Rightarrow \text{ minimum at } x = 4. \end{array}\right. \quad \left| \quad \textit{or} \quad 2 > -2 \Rightarrow \right]$$

Local maximum (2, 2), local minimum (4, -2). [10m]

* Accept candidate's $f'(x)$ from (ii).

* Accept implied '= 0' if subsequent work supports it.

* Accept distinguishing max from min by comparing y-ordinates .

* Correct answers without calculus: Att 3 at most.

Blunders (-3)

B1 $f'(x) \neq 0$ (but see 2nd asterisk)

B2 Algebraic errors

Slips (-1)

S1 Numerical errors

S2 Does not distinguish between max. and min.

Attempts (3 marks)

A1 Correct quadratic formula and stops

Worthless (0)

W1 *f(x)* = 0, whether continues or not

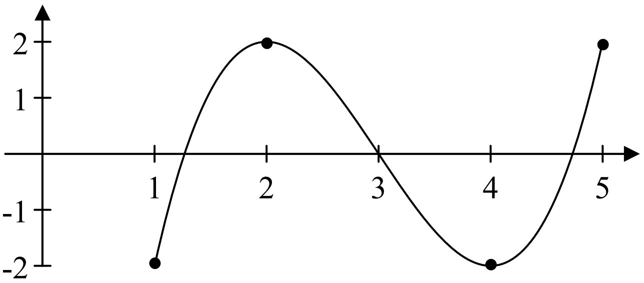
Part (iv) **5 marks** **Att 2**

8. (iv)	Draw the graph of the function f in the domain $1 \le x \le 5$.

(iv) **5 marks** **Att 2**

8. (iv)

Points
(1, -2)
(2, 2)
(4, -2)
(5, 2)



* Accept candidate's values of $(x, f(x))$ from previous parts unless oversimplified.
* 4 points adequate; it is not necessary to find $f(3)$ (= 0).
* If candidate recalculates points, apply slips and blunders as per guidelines.

Blunders (-3)
B1 Scale error

Slips (-1)
S1 Each of candidate's points incorrectly plotted (to max 3×S(-1))
S2 Points not joined

Attempts (2 marks)
A1 $f'(x)$ plotted
A2 One or more of candidate's points transferred correctly to this part and stops
A3 Effort at calculating a point e.g. $f(1)$ with some substitution
A4 Scaled and labelled axes and stops

Part (v) **5 marks** **Att 2**

8.(v)	Use your graph to write down the range of values of x for which $f'(x) < 0$.

(v) **5 marks** **Att 2**

8.(v)	$2 < x < 4$.

* Accept answer consistent with candidate's graph
* Accept answer clearly indicated on graph
* Ignore inclusion of equal sign. ($\le$)
* Accept answer using words rather than symbols, and [2 4], [4 2], (2, 4) or (4, 2)

Blunders (-3)
B1 $f'(x) > 0$
B2 $f(x)$ values indicated on graph but corresponding x-values not indicated

Misreading(-1)
M1 Algebraic solution

Attempts (2 marks)
A1 One correct end-point identified

Worthless (0)
W1 $1 < x < 5$
W2 $f(x) < 0$

Part (vi) **10 marks** **Att 3**

8.(vi) The line $y = -3x + c$ is a tangent to the curve $y = f(x)$. Find the value of c.

(vi) **10 marks** **Att 3**

8. (vi) $[\, y = -3x + c \,] \Rightarrow$ slope $m = -3$. [3m]

. $f'(x)$ *or* $3x^2 - 18x + 24 = -3$ [4m]

. $\Rightarrow 3x^2 - 18x + 27 = 0 \quad \Rightarrow x^2 - 6x + 9 = 0 \quad \Rightarrow (x-3)^2 = 0 \quad \Rightarrow x = 3$ [7m]

$f(3) = 3^3 - 9(3)^2 + 24(3) - 18 = 27 - 81 + 72 - 18 = 0$ [9m]

. $y = -3x + c \quad \Rightarrow 0 = -3(3) + c \quad \Rightarrow c = 9$ [10m]

* If derivative not used: Att at most.

Blunders (-3)

B1 Algebraic errors

Attempts (3 marks)

A1 Tries to solve: $y = f(x) \cap \; y = -3x + c$

A2 $f'(x)$, $\frac{dy}{dx}$, slope or $3x^2 - 18x + 24$ written in this part

Worthless (0)

W1 *f*(*0*) found

Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE 2008

MARKING SCHEME

MATHEMATICS – PAPER 2

ORDINARY LEVEL

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2008

MATHEMATICS – ORDINARY LEVEL – PAPER 2

GENERAL GUIDELINES FOR EXAMINERS – PAPER 2

1. Penalties of three types are applied to candidates' work as follows:

• Blunders	- mathematical errors/omissions	(-3)
• Slips	- numerical errors	(-1)
• Misreadings	(provided task is not oversimplified)	(-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

Application of penalties throughout scheme

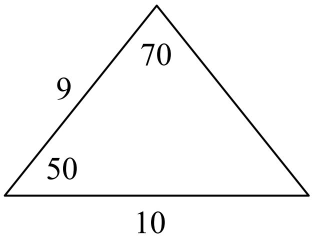
Penalties are applied subject to marks already secured.

Blunders - examples of blunders are as follows:

- Algebraic errors: $8x + 9x = 17x^2$ *or* $5p \times 4p = 20p$
- Sign error: $-3(-4) = -12$ *or* $(-3)^2 = 6$.
- Fraction error: Incorrect fraction inversion etc. apply once
- Cross-multiplication error.
- Error in misplacing the decimal point.
- Transposing error: $-2x - k + 3 = 0 \Rightarrow -2x = 3 + k$ *or* $-3x = 6 \Rightarrow x = 2$ *or* $4x = 12 \Rightarrow x = 8$ each time.
- Distributive law errors (once per term, unless otherwise directed) $\frac{1}{2}(3 - x) = 6 \Rightarrow 6 - 2x = 6$ *or* $-(4x + 3) = -4x + 3$ *or* $3(2x + 4) = 6x + 4$
- Expanding brackets incorrectly: $(2x - 3)(x + 4) = 8x^2 - 12x$
- Omission, if work not oversimplified, unless directed otherwise.
- Index error, each time unless directed otherwise.
- Factorisation: error in one or both factors of a quadratic, apply once $2x^2 - 2x - 3 = (2x - 1)(x + 3)$.
- Root errors from candidate's factors, error in one or both roots, apply once
- Incorrect substitution into formulae (where not an obvious slip):

  e.g. $2x^2 + 3x + 4 = 0 \Rightarrow x = \dfrac{-3 \pm \sqrt{9 - 4(2)(4)}}{2(3)}$

  or $\dfrac{10}{\sin 70} = \dfrac{9}{\sin 50}$



- Incorrectly treating co-ordinates as (x_1, x_2) and (y_1, y_2) when using co-ordinate geometry formula.
- Errors in formula for example: $\dfrac{y_2 + y_1}{x_2 + x_1}$ *or* $A = P\left(1 + \dfrac{n}{100}\right)^r$ *or* $a^2 = b^2 + c^2 + bc\cos A$

  or $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$, except as indicated in scheme.

Note: A correct relevant formula isolated and stops is awarded the attempt mark if the formula is not in the Tables.

Slips – examples are as follows:

- Numerical slips such as: $4 + 7 = 10$ *or* $3 \times 6 = 24$ *but* $5 + 3 = 15$ is a blunder.
- An omitted round-off to a required level of accuracy or an incorrect round-off to either the incorrect accuracy or an early round-off are penalised as a slip once in each section. This applies to **Q1** (a) (i) and (ii), (c) (i) and (ii), **Q5** (a), (b) (i) and (ii), (c) (i) and (ii).
- However, an early round-off which has the effect of simplifying the work is at least a blunder.
- The omission of the units of measurement in an answer or giving the incorrect units of measurement is treated as a slip once per part (a), (b) and (c) of each question where appropriate and at the first place where it matters. This applies to **Q1** (a), (b) and (c) to **Q4** (c) and to **Q5** (a), (b) and (c).

Misreadings

- Examples such as 436 for 346 will not alter the nature of the question and are penalised -1.
- However, writing 5026 as 5000 would alter the work and is penalised as at least a blunder.

QUESTION 1

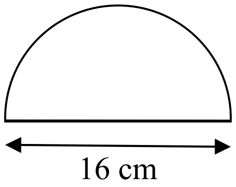
Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**25 (20, 5) marks**	**Att (7, 2)**
Part (c)	**15 (10, 5) marks**	**Att (3, 2)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

The semicircular shape shown in the diagram has diameter 16 cm.

(i) Find the length of the perimeter of the shape, correct to the nearest centimetre.

(ii) Find the area of the shape, correct to the nearest square centimetre.



(a) (i) **5 marks** **Att 2**

$L = 2r + \pi r = 16 + 8\pi \approx 41$ cm.

(a) (ii) **5 marks** **Att 2**

$A = \frac{1}{2}\pi r^2 = \frac{1}{2}(\pi)(8)^2 = 32\pi \approx 101$ cm^2.

* Accept any value of π that gives the correct answer, otherwise apply blunder (-3).
* Accept correct answer without work.
* Any error other than an obvious slip merits the attempt mark at most.

Blunders (-3)
B1 Omits part of perimeter e.g. diameter.
B2 Uses radius of 16 cm or other incorrect value – apply once in part (a).
B3 Gives answer in terms of π.
B4 Works $2\pi r$ in (i) or πr^2 in (ii).

Attempts (2 marks)
A1 Some relevant work, e.g. finds radius or some relevant substitution.

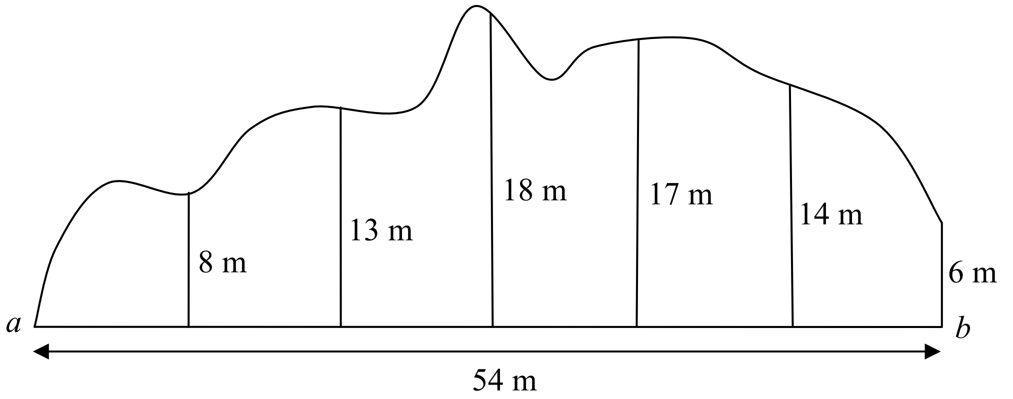
Worthless (0)
W1 Incorrect answer without work, except 8 or 16 or 25.

Part (b) **25 (20, 5) marks** **Att (7, 2)**

The sketch shows a piece of land which borders the side of a straight road $[ab]$.

The length of $[ab]$ is 54 m.

At equal intervals along $[ab]$, perpendicular measurements are made to the boundary, as shown on the sketch.



(i) Use Simpson's Rule to estimate the area of the piece of land.

(ii) The land is valued at €480 000 per hectare. Find the value of the piece of land.
Note: 1 hectare = 10 000 m^2.

(b) (i) Use of formula **15 marks** **Att 5**
Calculations **5 marks** **Att 2**

$h = 54 \div 6 = 9$

Area $= \frac{h}{3}\left(F + L + 2\Sigma O + 4\Sigma E\right)$

$= \frac{9}{3}\left(0 + 6 + 2(13 + 17) + 4(8 + 18 + 14)\right)$ [*15 marks*]

$= 3(6 + 60 + 160) = 3(226) = 678 \text{ m}^2.$ [*20 marks*]

(b) (ii) **5 marks** **Att 2**

Value $= 480000 \times \frac{678}{10000}\downarrow = €32\,544.\downarrow$

[*2 marks*] [*5 marks*]

- * Allow $^h/_3$ = {F + L + TOFE} and penalise in calculations if formula not used correctly.
- * Accept correct TOFE *or* TOFE consistent with candidates F and L.
- * Accept correct or consistent answer without work in section (ii).

Blunders (-3)

B1 Incorrect $^h/_3$ (once).
B2 Incorrect F and / or L or extra terms with F and / or L (once).
B3 Incorrect TOFE (once), if not consistent with candidates F and L.
B4 E **or** O omitted (once).
B5 Mathematical error in applying conversion factor in (ii).

Attempts [5 marks for substituting into formula, 2 marks for calculations in (i), 2 marks in (ii)]

A1 Some relevant step, e.g. identifies F and/or L or odds or evens and stops: 5 marks.
A2 Statement of Simpson's Rule not transcribed from tables: 5 marks.
A3 E **and** O omitted (candidate may be awarded attempt 5 at most and/or attempt 2 marks).
A4 Correct answer without work in (i): 5 marks + 2 marks.
A5 Some correct relevant calculation only: 2 marks.

Worthless (0)
W1 Incorrect answer without work.
W2 Formula transcribed from tables and stops.

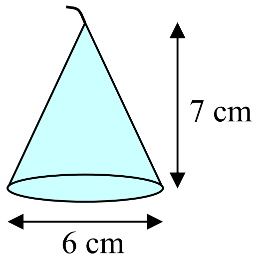
Part (c) **15 (10, 5) marks** **Att (3, 2)**

A wax candle is in the shape of a right circular cone.
The height of the candle is 7 cm and the diameter of the base is 6 cm.

(i) Find the volume of the wax candle, correct to the nearest cm^3.

(ii) A rectangular block of wax measuring 25 cm by 12 cm by 12 cm is melted down and used to make a number of these candles.

Find the maximum number of candles that can be made from the block of wax if 4% of the wax is lost in the process.



(c) (i) **10 marks** **Att 3**

Volume of candle $= \frac{1}{3}\pi r^2 h = \frac{1}{3}(\pi)(3)^2(7)\downarrow = 21\pi \approx 66\ \text{cm}^3.\downarrow$

[*3 marks*] [*10 marks*]

(c) (ii) **5 marks** **Att 2**

Volume of block of wax $= 25 \times 12 \times 12$.
96% of wax used $= 25 \times 12 \times 12 \times 0.96$.
Number of candles $= \dfrac{25 \times 12 \times 12 \times 0.96}{66} = 52.36 = 52$ candles.

* Accept any value of π that gives the correct answer, otherwise apply blunder (-3).
* Accept an answer in section (ii) consistent with the candidate's answer to section (i).

Blunders (-3)
B1 Radius taken as 6 cm.
B2 Incorrect relevant cone formula e.g. $\frac{4}{3}\pi r^2 h$ and continues.
B3 Gives answer in terms of π.
B4 Incorrect relevant volume formula of block of wax and continues.
B5 Calculates the number of candles made from 4% of the block.
B6 Fails to deal with the 0.96 correctly.

Slips (-1)
S1 Each slip to a maximum of 3 in each section.

Attempts (3 marks section (i), 2 marks section (ii))
A1 Some relevant step, e.g. radius found.
A2 Correct answer without work in each section.

QUESTION 2

Part (a)	**5 marks**	**Att 2**
Part (b)	**25 (5, 10, 5, 5) marks**	**Att (2, 3, 2, 2)**
Part (c)	**20 (5, 15) marks**	**Att (2, 5)**

Apply the following to each section of question 2 and question 3.
If the correct formula is not written, any sign or substitution error is at least a blunder.

Blunders (-3)

B_a Two or more incorrect substitutions if the formula is written.

B_b Switches x and y in substituting or treats as a pair of couples (x_1, x_2) and (y_1, y_2).

Slips (-1)

S_a One incorrect non-central sign in the formula, if the formula is written.

S_b One incorrect substitution in the formula, if the formula is written.

S_c Obvious misreading of one co-ordinate.

Attempts

A_a An incorrect relevant formula, partially substituted.

A_b The co-ordinates of a relevant point written with x_1 and y_1 identified.

A_c The correct relevant formula written and stops.

Part (a)	**5 marks**	**Att 2**

Find the area of the triangle with vertices (0, 0), (8, 6) and (−2, 4).

(a)	**5 marks**	**Att 2**

$$\text{Area} = \tfrac{1}{2}\,|\,x_1y_2 - x_2y_1\,| = \tfrac{1}{2}\,|\,8\times4-(-2)\times6\,| = \tfrac{1}{2}\,|\,32+12\,| = \tfrac{1}{2}\,|\,44\,| = 22$$

or

$$\begin{aligned}\text{Area} &= \tfrac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\\ &= \tfrac{1}{2}\,|\,0(6-4)+8(4-0)-2(0-6)\,|\\ &= \tfrac{1}{2}\,|\,0+32+12\,| = \tfrac{1}{2}\,|\,44\,| = 22\end{aligned}$$

or

$$\begin{aligned}\text{Area} &= \tfrac{1}{2}[x_1y_2+x_2y_3+x_3y_1-x_1y_3-x_3y_2-x_2y_1]\\ &= \tfrac{1}{2}\,|\,0\times6+8\times4+(-2)\times0-0\times4-(-2)\times6-8\times0\,|\\ &= \tfrac{1}{2}\,|\,0+32+0+0+12-0\,| = \tfrac{1}{2}\,|\,44\,| = 22\end{aligned}$$

* $\tfrac{1}{2}\,|-44\,| = -22$ incurs no penalty.

Blunders (-3)

B1 Incorrect relevant formula and continues e.g. $\tfrac{1}{2}\,|\,x_1y_2+x_2y_1\,|$ or omits the $\tfrac{1}{2}$.

Attempts (2 marks)

A1 Correct answer without work.

A2 Uses the distance formula or the perpendicular distance formula.

A3 Plots one or more points, to the eye.

Worthless (0 marks)

W1 Irrelevant formula and stops e.g. $\tfrac{1}{2}$ on its own.

Part (b) **25 (5, 10, 5, 5) marks** **Att (2, 3, 2, 2)**

L is the line $y-6=-2(x+1)$.

(i) Write down the slope of L.

(ii) Verify that $p(1, 2)$ is a point on L.

(iii) L intersects the y-axis at t. Find the co-ordinates of t.

(iv) Show the line L on a co-ordinate diagram.

(b) (i) **5 marks** **Att 2**

$y-y_1=m(x-x_1)$: $y-6=-2(x+1) \Rightarrow m=-2$.

or

$y=mx+c$: $y=-2x+4 \Rightarrow m=-2$.

or

Slope of pt = $m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{2-4}{1-0} = -2$.

* Accept correct answer without work.

Blunders (-3)

B1 Incorrect relevant formula e.g. $\dfrac{y_2+y_1}{x_2+x_1}$ or $\dfrac{y_2-y_1}{x_1-x_2}$ or $\dfrac{x_2-x_1}{y_2-y_1}$ and continues.

B2 Answer given is $m = 2$.

Attempts (2 marks)

A1 Work towards writing the equation in the form $y = mx + c$ or $ax + by + c = 0$.

A2 $m = \tan\theta$ or m = vertical/horizontal or $m = -a/b$.

(b) (ii) **10 marks** **Att 3**

$y-6=-2(x+1) \Rightarrow 2-6=-2(1+1) \Rightarrow -4=-4$. Hence, $p \in L$

* Accept consistent answers in this and subsequent sections.

* Award 7 marks for correct substitution of both ordinates and 3 marks for finishing correctly.

Blunders (-3)

B1 Substitution, but work not completed to arrive at LHS = RHS.

B2 Conclusion not stated if error in work results in LHS $\neq$ RHS.

Attempts (3 marks)

A1 Some substitution attempted or some work at simplifying the equation.

(b) (iii) **5 marks** **Att 2**

$y-6=-2(x+1)$.

$x=0 \Rightarrow y-6=-2(0+1) \Rightarrow y=6-2=4 \Rightarrow t(0, 4)$.

* Accept a correct answer without work.

Blunders (-3)

B1 Finds intercept on the x-axis.

Attempts (2 marks)

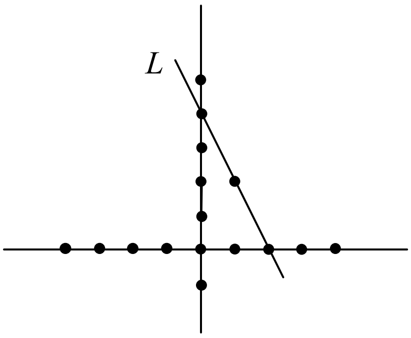
A1 Some relevant step e.g. writes $x = 0$ and stops.

A2 Finds a random point on the line.

Worthless (0 marks)

W1 Writes $y = 0$ and stops.

(b) (iv) **5 marks** **Att 2**



* Accept use of candidate's co-ordinates of t.

* Intervals should be indicated or implied.

* If section (iii) not answered but $t(0, 4)$ labelled on the graph award 5 marks for (iii).

* Work must be shown if diagram is not consistent with (iii).

Blunders (-3)

B1 Scales unreasonably inconsistent (to the eye).

B2 Different scales on x and y axes.

B3 Uses a vertical x-axis and a horizontal y-axis.

B4 Plots t on the x-axis.

B5 Points plotted but not joined.

Attempts (2 marks)

A1 Draws scaled axes and stops.

Worthless (0 marks)

W1 Draws an arbitrary line, subject to A1.

Part (c) **20 (5, 15) marks** **Att (2, 5)**

$o(0, 0)$, $a(5, 2)$, $b(1, 7)$ and $c(-4, 5)$ are the vertices of a parallelogram.

(i) Verify that the diagonals $[ob]$ and $[ac]$ bisect each other.

(ii) Find the equation of ob.

(c) (i) **5 marks** **Att 2**

Midpoint of $[ob] = \left(\frac{0+1}{2}, \frac{0+7}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right) m$

Midpoint of $[ac] = \left(\frac{5-4}{2}, \frac{2+5}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right)$

or

Translation $a(5, 2) \rightarrow m(1/2, 7/2)$ maps $m(1/2, 7/2) \rightarrow c(-4, 5)$

Blunders (-3)

B1 An incorrect translation used.
B2 Blunder in use of translation, e.g. two incorrect co-ordinates, having used correct translation.
B3 Incorrect relevant midpoint formula and continues.
B4 No conclusion given when error in work results in different midpoints.
B5 Candidate does not name the diagonal or it is not clear from the work which diagonal is being worked.

Slips (-1)

S1 One correct and one incorrect ordinate having used correct translation.

Attempts (2 marks)

A1 Plots parallelogram *oabc* on a co-ordinate diagram.
A2 Diagonal with correct midpoint indicated.

(c) (ii) **15 marks** **Att 5**

Slope *ob* $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0-7}{0-1} = 7$

Equation *ob*: $y - y_1 = m(x - x_1) \Rightarrow y - 0 = 7(x - 0) \Rightarrow y = 7x$

or

Equation *ob*: $y - 7 = 7(x-1) \Rightarrow y - 7 = 7x - 7 \Rightarrow y = 7x$

or

Equation *ob*: $y = 7x + c$ But (0, 0) on *ob* $\Rightarrow c = 0 \Rightarrow y = 7x$.

* Do not penalise for errors in simplifying equation of the line.

Award marks as follows:

15 marks: fully correct answer.
14 marks: one slip or one misreading in answer with work shown.
12 marks: a blunder in the slope or in the point.
9 marks: a blunder in the slope and in the point.
5 marks: attempt mark for relevant work.
0 marks: worthless work.

Blunders (-3)

B1 Incorrect relevant formula e.g. $\dfrac{y_2 + y_1}{x_2 + x_1}$ or $\dfrac{y_2 - y_1}{x_1 - x_2}$ or $\dfrac{x_2 - x_1}{y_2 - y_1}$ and continues.
B2 Incorrect relevant formula e.g. $y + y_1 = m(x + x_1)$ [Both signs incorrect].
B3 Uses an arbitrary point for the line.

Misreading (-1)

M1 Find the equation of *oa*, *oc* or *ab* correctly.

Attempts (5 marks)

A1 Gives correct relevant formula and stops e.g. $m = \tan\theta$ or $m =$ vertical/horizontal.

QUESTION 3

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**25 (5, 5, 5, 5, 5)marks**	**Att (2, 2, 2, 2, 2)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

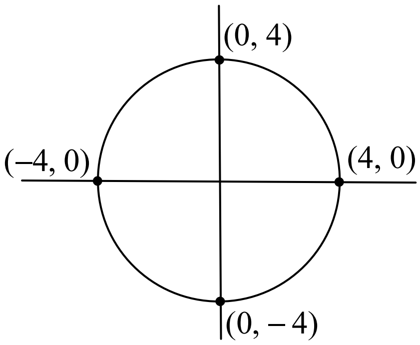
Part (a) **10 (5, 5) marks** **Att (2, 2)**

A circle has equation $x^2 + y^2 = 16$.

(i) Show the circle on a co-ordinate diagram.

(ii) Mark the four points at which the circle intersects the axes and label them with their co-ordinates.

(a) (i) **5 marks** **Att 2**
(a) (ii) **5 marks** **Att 2**



- * Accept a free-hand diagram of a circle, reasonably drawn.
- * Scales must be indicated or implied for full marks.
- * Accept co-ordinates consistent with circle drawn, in (ii).

Blunders (-3)
B1 Centre not at (0, 0) in section (i).
B2 Radius not 4.
B3 Co-ordinates switched in section (ii).

Slips (-1)
S1 The co-ordinates of a point omitted or incorrect to a maximum of 3.

Attempts (2 marks)
A1 Relevant work, e.g. states centre is (0, 0) or scaled axes drawn in section (i).
A2 Points of intersection of circle and axes marked but not labelled.
A3 Indication that 4 is one of the co-ordinates in section (ii).

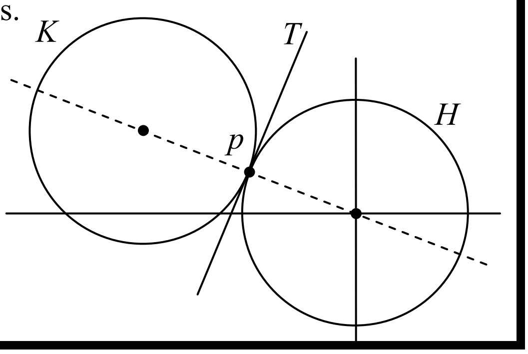
Part (b)	**25 (5, 5, 5, 5, 5) marks**	**Att (2, 2, 2, 2, 2)**

The diagram shows two circles H and K, of equal radius.
The circles touch at the point $p(-2, 1)$.
The circle H has centre (0, 0).

(i) Find the equation of H.

(ii) Find the equation of K.

(iii) T is a tangent to the circles at p.
Find the equation of T.



(b) (i) Radius	**5 marks**	**Att 2**
Equation	**5 marks**	**Att 2**

Radius of H: $r=\sqrt{(0+2)^2+(0-1)^2}=\sqrt{4+1}=\sqrt{5}$.

Equation of H: $x^2+y^2=5$.

or

Equation of H: $x^2+y^2=r^2$

$p(-2,1)$ on H: $(-2)^2+1^2=r^2 \Rightarrow 4+1=r^2 \Rightarrow x^2+y^2=5$

(b) (ii)	**5 marks**	**Att 2**

Centre of K: $(0,0) \rightarrow (-2,1)$ maps $(-2,1) \rightarrow (-4,2)$.

or

$\frac{1}{2}(0+x)=-2 \Rightarrow x=-4$; $\frac{1}{2}(0+y)=1 \Rightarrow y=2$

Equation of K: $(x+4)^2+(y-2)^2=5$.

(b) (iii) Slope *T*	**5 marks**	**Att 2**
Equation *T*	**5 marks**	**Att 2**

Slope of op $=\dfrac{1-0}{-2-0}=-\dfrac{1}{2}$.

Slope of tangent T: $m_1m_2=-1 \Rightarrow -\frac{1}{2}m_2=-1 \Rightarrow m_2=2$.

Equation of T: $y-1=2(x+2) \Rightarrow y-1=2x+4 \Rightarrow y=2x+5$.

or

$x_1x+y_1y=r^2 \Rightarrow -2x+y=5$.

* In section (ii) accept r^2 from (i).

Blunders (-3)

B1 Any blunder in finding radius in (i), once.
B2 Any blunder in centre in (i).
B3 Incorrect centre used in (ii) e.g. $(-2,1)$.
B4 Error in use of translation, unless an obvious slip.
B5 Uses an arbitrary point for the line, in (iii).
B6 Uses an arbitrary or incorrect slope, e.g. of radius in (iii).

Attempts (2 marks)

A1 Correct or consistent answer without work shown in each section.

A2 Reference to $x = -2$ or $y = 1$ and stops in (i).

A3 Attempt to use the given translation.

A4 Gives equation $x^2 + y^2 + 2gx + 2fy + c = 0$.

Worthless (0 marks)

W1 Gives equation $x^2 + y^2 = r^2$ in (ii), subject to attempt mark.

W2 Equation of line for circle or equation of circle for line, subject to an attempt mark.

Part (c) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

The circle *S* has equation $(x-3)^2 + (y+2)^2 = 40$.
S intersects the *x*-axis at the point *a* and at the point *b*.

(i) Find the co-ordinates *a* and the co-ordinates of *b*.

(ii) Show that $|ab|$ is less than the diameter of *S*.

(iii) Find the equation of the circle with $[ab]$ as diameter.

(c) (i) **5 marks** **Att 2**

$(x-3)^2 + (y+2)^2 = 40$.

$y = 0 \quad \Rightarrow (x-3)^2 + (0+2)^2 = 40$

$\Rightarrow (x-3)^2 = 40 - 4 = 36 \quad \Rightarrow x - 3 = \pm 6 \quad \Rightarrow x = 9$ or $x = -3$.

or

$(x-3)^2 + (y+2)^2 = 40 \quad \Rightarrow x^2 - 6x + 9 + y^2 + 4y + 4 - 40 = 0$

$\Rightarrow x^2 - 6x - 27 = 0 \quad \Rightarrow (x-9)(x+3) = 0 \quad \Rightarrow x = 9$ or $x = -3$.

(c) (ii) **5 marks** **Att 2**

$|ab| = 9 + 3 = 12$.

Radius of *S* $= \sqrt{40} \quad \Rightarrow$ Diameter of *S* $= 2\sqrt{40}$.

$12 < 2\sqrt{40} \quad [= 2(6.32) = 12.64]$ *or* $12 < 2\sqrt{40} \quad [\Rightarrow \sqrt{144} < \sqrt{160}]$.

(c) (iii) **5 marks** **Att 2**

Centre of circle $= \left(\frac{9-3}{2}, \frac{0+0}{2}\right) = (3, 0)$.

Radius of circle $= \frac{1}{2}|ab| = \frac{1}{2}(12) = 6$.

Equation of circle $= (x-3)^2 + y^2 = 6^2 = 36$.

Award marks as follows, in each section:

5 marks: fully correct solution or a solution consistent with previous work.

2 marks: some relevant work.

0 marks: worthless work.

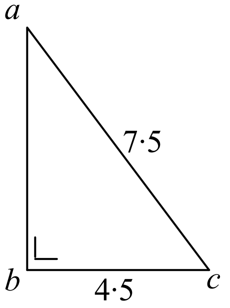
QUESTION 4

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) 10 marks Att 3

In the triangle *abc*,
$|\angle abc| = 90°$, $|bc| = 4{\cdot}5$ and $|ac| = 7{\cdot}5$.

Find $|ab|$.



(a) 10 marks Att 3

$|ab|^2 + 4.5^2 = 7.5^2 \quad \Rightarrow \quad |ab|^2 + 20.25 = 56.25$ [*3 marks*]

$\Rightarrow \quad |ab|^2 = 56.25 - 20.25 = 36 \downarrow \quad \Rightarrow |ab| = 6\,. \downarrow$

[*7 marks*] [*10 marks*]

* Accept a correct trigonometrical method.

Blunders (-3)

B1 Blunder in Theorem of Pythagoras.

Attempts (3 marks)

A1 Statement of or use of any relevant result or any correct step e.g. 4.5^2.
A2 Correct answer without work.
A3 An exact scaled diagram giving the correct answer.

Worthless (0)

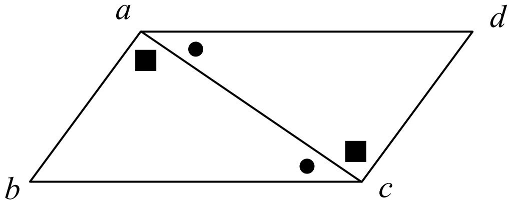
W1 Incorrect answer without work.
W2 Work such as $7.5 - 4.5$.

Part (b) 20 marks Att 7

Prove that the opposite sides of a parallelogram have equal lengths.

(b) 20 marks Att 7

abcd is a parallelogram.
To prove: $|ab| = |dc|$ and $|ad| = |bc|$.
Construction: Join *a* to *c*.



Proof:
In Δabc and Δacd
$|\angle cab| = |\angle acd|$ … alternate angles [*10 marks*]
$|\angle bca| = |\angle dac|$ … alternate angles [*13 marks*]
$|ac| = |ac|$ [*16 marks*]
Thus, Δabc congruent to Δacd
Thus, $|ab| = |dc|$ [*19 marks*]
and $|ad| = |bc|$ [*20 marks*]

* Proof without a diagram merits att 7, if a complete proof can be reconciled with a diagram.

Blunders (-3)

B1 Each step omitted, incorrect or incomplete, except the last.
B2 Steps written in an illogical order. [Penalise once only.]
[Note: Some of the steps above may be interchanged.]

Attempts (7 marks)

A1 Any relevant step, stated or indicated, e.g. parallelogram with additional relevant information.
A2 States or illustrates a special case, e.g. measuring the sides or rectangle used as a rectangle.

Worthless (0 marks)

W1 Any irrelevant theorem, subject to the attempt mark.
W2 Parallelogram only.

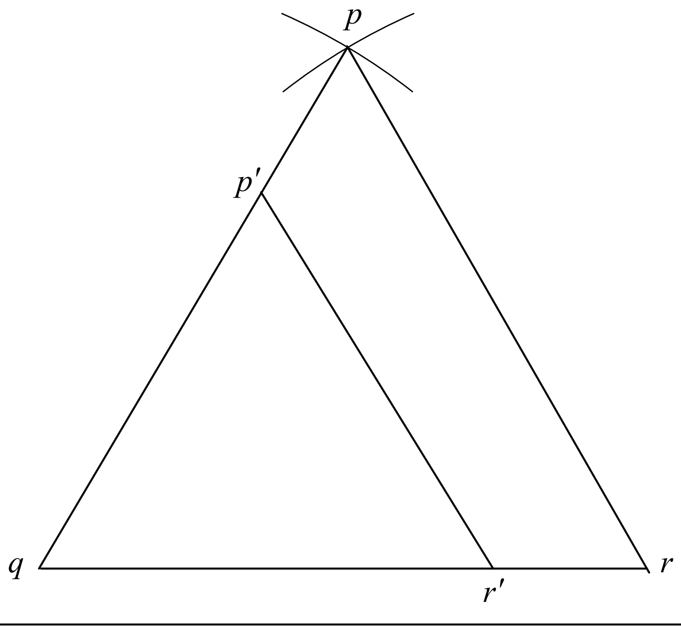
Part (c) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

(i) Construct an equilateral triangle pqr of side 8 cm.

(ii) Construct the image of the triangle pqr under the enlargement of scale factor 0·75 and centre q.

(iii) Given that the area of the triangle pqr is $16\sqrt{3}$ cm^2, find the area of the image triangle in the form $k\sqrt{3}$ cm^2.

(c) (i) **10 marks** **Att 3**
(c) (ii) **5 marks** **Att 2**



Triangle pqr:

one side	[*3 marks*]
two sides and one angle or one side and two arcs	[*7 marks*]
three sides	[*10 marks*]

Image triangle:

one side	[*2 marks*]
two sides	[*3 marks*]
three sides	[*4 marks*]
Centre at q	[*5 marks*]

* Allow tolerance of ± 5 mm in measurements, in each section.
* It is not necessary to label the vertices, or write measurements on the diagram, in each section, subject to S4 in (ii).

Blunders (-3)

B1 Draws the required triangle to scale.
B2 Each side outside tolerance, subject to B1.

Slips (-1)

S1 Side within tolerance but not straight i.e. no straight edge used (once in each section).

S2 Scale factor 1.75.

S3 Centre at p or r.

S4 Image constructed but centre not indicated.

Attempts (3 marks)

A1 Relevant step, e.g. one side drawn or a rough sketch.

A2 A triangle, with no side within tolerance, subject to B1.

Attempts (2 marks)

A1 Some relevant step, e.g. centre clearly indicated.

A2 Scale factor other than 0.75 or 1.75 used.

A3 Centre of enlargement is not a vertex of the triangle.

(c) (iii) **5 marks** **Att 2**

Area of image $\Delta = (0.75)^2$ area of $\Delta pqr = (0.75)^2 \times 16\sqrt{3} = 9\sqrt{3}$ cm^2.

* Accept a correct or consistent answer without work.

Blunders (-3)

B1 Does not square scale factor and continues to $12\sqrt{3}$.

Slips (-1)

S1 Each slip to a maximum of 3.

S2 Error in calculating length of side of image, each side.

Attempts (2 marks)

A1 $(0.75)^2$ or $16\sqrt{3} \div (0.75)^2$ or $(0.75)^2 \div 16\sqrt{3}$ or $16\sqrt{3} \div 0.75$ or $0.75 \div 16\sqrt{3}$.

A2 Some substitution into a correct area formula which is written or clearly obvious.

A3 $(0.75)^2 \times 16 \times 1.732 = 15.58$ or answer of 15.58 without work.

Worthless (0 marks)

W1 Confusing k with scale factor to give $0.75 \times \sqrt{3}$.

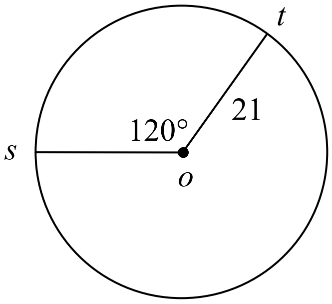
QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

A circle has centre o and radius 21 cm.
s and t are two points on the circle and $|\angle sot| = 120°$.

Find the length of the shorter arc st
correct to the nearest centimetre.



(a) **10 marks** **Att 3**

$$|st| = \frac{120}{360}(2\pi)(r) = \frac{120}{360} \times 2 \times \pi \times 21 = 14\pi \approx 44 \text{ cm.}$$

* Accept any value of π which gives the correct answer, otherwise apply blunder (-3).

Blunders (-3)
B1 Uses radians (or gradient) mode incorrectly – apply once in part (a), in part (b) and in part (c).
B2 Leaves the answer in terms of π.
B3 Uses the formula $|st| = r\theta$ but fails to convert the angle to radians.
B4 Incorrect fraction for the circumference.

Slips (-1)
S1 Each slip to a maximum of 3.

Misreadings (-1)
M1 Finds the length of the other arc.

Attempts (3 marks)
A1 $\frac{1}{3}$ or $\frac{2}{3}$ stated and stops.
A2 Incorrect relevant formula (e.g. sine rule) with some correct substitution.
A3 Some relevant step e.g. fraction 120/360.
A4 Correct answer without work shown.
A5 Chord $|st|$ with correct work.

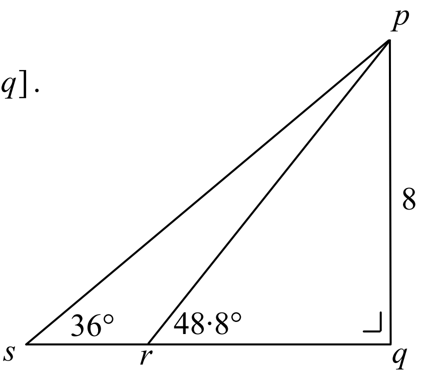
Worthless (0 marks)
W1 Incorrect answer without work shown.

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

In the right-angled triangle psq, p is joined to a point r on $[sq]$.
$|pq| = 8$ cm, $|\angle prq| = 48{\cdot}8°$ and $|\angle psq| = 36°$.

(i) Find $|pr|$, correct to one decimal place.

(ii) Find $|sr|$, correct to the nearest centimetre.



(b) (i) Expression for $|pr|$ **5 marks** **Att 2**
Value for $|pr|$ **5 marks** **Att 2**

$$\sin 48.8° = \frac{8}{|pr|} \quad \Rightarrow \quad |pr| = \frac{8}{\sin 48.8} \quad = \frac{8}{0.7524} = 10.63 = 10.6 \text{ cm.}$$

(b) (ii) **5, 5 marks** **Att 2, 2**

$|\angle rps| = 48.8° - 36° = 12.8°$

or $|\angle rps| = 180° - (|\angle psr| + |\angle srp|) = 180° - (36° + 131.2°) = 12.8°.$

$$\frac{|sr|}{\sin 12.8} = \frac{10.6}{\sin 36}$$ [*5 marks*]

$$\Rightarrow |sr| = \frac{10.6 \times \sin 12.8}{\sin 36} = \frac{10.6 \times 0.2215}{0.5878} = \frac{2.3479}{0.5878} = 3.99 = 4 \text{ cm.}$$ [*5 marks*]

or

$$\tan 36 = \frac{8}{|sq|} \Rightarrow |sq| = \frac{8}{\tan 36} = \frac{8}{0.7265} = 11.01.$$ [5 *marks*]

$$\tan 48.8 = \frac{8}{|rq|} \Rightarrow |rq| = \frac{8}{\tan 48.8} = \frac{8}{1.1423} = 7.003.$$

or $|rq|^2 = |pr|^2 - |pq|^2 = 10.6^2 - 8^2 = 112.36 - 64 = 48.36 \Rightarrow |rq| = 6.95$.

$|sr| = |sq| - |rq| = 11.01 - 7.003 = 4.007 = 4$ cm. [*5 marks*]

* Accept an answer consistent with candidate's work in section (i).

Blunders (-3)
B1 Incorrect ratio and continues.
B2 Incorrect trigonometric function and continues.
B3 Incorrect function read e.g. cosine instead of sine and continues.
B4 Misplaced decimal point.
B5 Error in use of inverse function.
B6 Incorrect substitution into correct formula and continues.
B7 Blunder in finding a necessary angle.

Attempts (2 marks)
A1 Correct answer without work shown.
A2 Trigonometric function correctly defined.
A3 Attempt at constructing trigonometric fractions.
A4 Incorrect relevant formula with some correct substitution.

Worthless (0 marks)

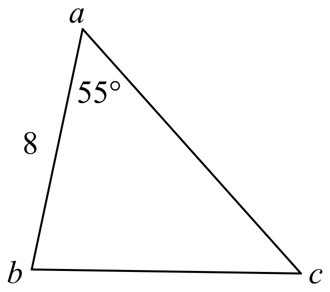
W1 Writes formula from Tables and stops.
W2 Measurement from the diagram.
W3 The triangle *psr* treated as a right-angled triangle.

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

The area of the triangle *abc* is 33 cm^2.
$|ab| = 8$ cm and $|\angle cab| = 55°$.

(i) Find $|bc|$, correct to one decimal place.

(ii) Find $|\angle abc|$, correct to the nearest degree.



(c) (i) Finds |*ac*| **5 marks** **Att 2**
Finds |*bc*| **5 marks** **Att 2**

$$\tfrac{1}{2}|ab|\times|ac|\times\sin|\angle cab| = 33 \;\Rightarrow\; \tfrac{1}{2}(8)\times|ac|\times\sin 55 = 33$$

$$\Rightarrow\; |ac| = \frac{2\times 33}{8\times 0.8192} = \frac{66}{6.5536} = 10.07.$$

$$a^2 = b^2 + c^2 - 2bc\cos A \;\Rightarrow\; |bc|^2 = 8^2 + 10.07^2 - 2(8)(10.07)\cos 55°$$

$$\Rightarrow |bc|^2 = 64 + 101.40 - 161.12(0.5736) = 165.40 - 92.42 = 72.98 \Rightarrow |bc| = 8.54 = 8.5 \text{ cm.}$$

(c) (ii) Expression for $|\angle abc|$ **5 marks** **Att 2**
Value of $|\angle abc|$ **5 marks** **Att 2**

$$\cos|\angle abc| = \frac{a^2 + c^2 - b^2}{2ac} = \frac{8^2 + 8.5^2 - 10.07^2}{2(8)(8.5)}$$

$$= \frac{64 + 72.25 - 101.40}{136} = \frac{34.85}{136} = 0.25625 \;\Rightarrow\; |\angle abc| = 75.15° = 75°.$$

or

$$\frac{\sin b}{10.07} = \frac{\sin 55}{8.5} \;\Rightarrow\; \sin b = \frac{10.07\sin 55}{8.5}$$

$$= 0.9704 \;\Rightarrow\; |\angle abc| = 76.03° = 76°.$$

Blunders (-3)

As in part (b).

Attempts (2 marks)

As in part (b).

Worthless (0 marks)

W1 The triangle *abc* treated as a right-angled triangle.

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

Evaluate $5! + 6!$

(a) **10 marks** **Att 3**

$5! + 6! = 120 + 720 = 840$ ***or*** $5! + 6! = 5!(1 + 6) = 120(7) = 840$

* Multiplication must be clearly indicated, so 5, 4, 3, 2, 1 listed and stops merits Att 3.

Award marks as follows:

10 marks:	Correct answer with or without work shown.
9 marks:	5! and 6! worked correctly but not added.
6 marks:	5! or 6! worked correctly.
5 marks:	5! and/or 6! listed correctly but not worked.
3 marks:	An incorrect answer with some relevant work.
0 marks:	Worthless work.

Attempts (3 marks)

A1 Any relevant partial list beginning with the number 5 or 6 and having at least two numbers e.g. 6×5 but the answer 30, without work shown is worthless.

A2 Writes ${}^5P_5$ or ${}^6P_6$ and stops.

A3 11! worked (answer 39 916 800).

Worthless (0 marks)

W1 Writes ${}^5C_5$ or ${}^6C_5$ or ${}^nC_r$.

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

One shelf of a school library has 70 books. The books are on poetry and on drama and are either hardback or paperback.
The following table shows the number of each type.

	Hardback	Paperback
Poetry	23	17
Drama	14	16

A student selects one book at random from the shelf.
Find the probability that the book selected is

(i) a paperback poetry book
(ii) a hardback book
(iii) a poetry book
(iv) not a paperback drama book.

(b) Each section **5 marks** **Att 2**

(i)	P(paperback poetry book) = $\frac{17}{70}$.
(ii)	P(hardback book) = $\frac{23+14}{70} = \frac{37}{70}$.
(iii)	P(poetry book) = $\frac{23+17}{70} = \frac{40}{70}$.
(iv)	P(not a paperback drama book) = $1 - \frac{16}{70} = \frac{54}{70}$ ***or*** $\frac{23+14+17}{70} = \frac{54}{70}$.

* If the parts of (b) or of (c) are not identified, and it is not obvious which section is being attempted treat each section in order.

* Accept answers consistent with previous work (e.g. incorrect addition of *S*), including decimal and percentage form.

Award 5 marks for each correct answer with or without work shown.

Slips (-1)
S1 Addition or subtraction required for final answer omitted or incorrect in each section.

Attempts (2 marks)
A1 #(E) correctly identified or given as the numerator or
#(S) correctly identified or given as the denominator.
A2 The correct answer inverted each time or partial correct answer e.g. $^{17}/_{70}$ in (iii).
A3 Statement of probability theorem awarded once unless specifically adapted to each section.
A4 Answer inverted, each time.

Worthless (0 marks)
W1 Use of $^{n}P_{r}$ or $^{n}C_{r}$.

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

There are 6 junior-cycle students and 5 senior-cycle students on the student council in a particular school.
A committee of 4 students is to be selected from the students on the council.

In how many different ways can the committee be selected if
(i) there are no restrictions
(ii) a particular student must be on the committee
(iii) the committee must consist of 2 junior-cycle students and 2 senior-cycle students.

The committee of 4 students is chosen at random.
(iv) Find the probability that all 4 students are junior-cycle students.

(c) Each section **5 marks** **Att 2**

(i)	$\binom{11}{4}\underset{[\textit{2 marks}]}{\downarrow} = \frac{11\times10\times9\times8}{1\times2\times3\times4}\underset{[\textit{4 marks}]}{\downarrow} = \frac{7920}{24} = \underset{[\textit{5 marks}]}{\underset{\downarrow}{330}}.$
(ii)	$\binom{10}{3}\underset{[\textit{2 marks}]}{\downarrow} = \frac{10\times9\times8}{1\times2\times3}\underset{[\textit{4 marks}]}{\downarrow} = \frac{720}{6} = \underset{[\textit{5 marks}]}{\underset{\downarrow}{120}}.$
(iii)	$\binom{6}{2}\underset{[\textit{2 marks}]}{\downarrow}\times\binom{5}{2} = \frac{6\times5}{1\times2}\times\frac{5\times4}{1\times2}\underset{[\textit{4 marks}]}{\downarrow} = 15\times10 = \underset{[\textit{5 marks}]}{\underset{\downarrow}{150}}.$
(iv)	Number of ways of all junior cycle = $\binom{6}{4} = \frac{6\times5\times4\times3}{1\times2\times3\times4} = \frac{360}{24}\underset{[\textit{2 marks}]}{\downarrow} = 15.$ Probability (all junior cycle) $= \frac{15}{330}$ or $\frac{1}{22}.\underset{[\textit{5 marks}]}{\downarrow}$

* Accept the correct calculated answer without work in (i), (ii), and (iii).
* Accept an answer in (iv) consistent with candidate's answer in (i).

QUESTION 7

Part (a)	**30 (10, 15, 5) marks**	**Att (3, 5, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) 30 (10, 15, 5) marks Att (3, 5, 2)

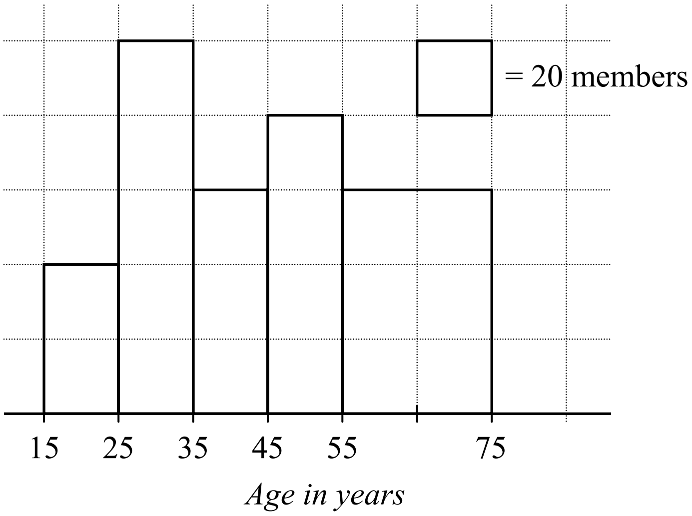
The ages of the members of a sports centre was analysed. The results were:

Age	15 - 25	25 - 35	35 - 45	45 - 55	55 - 75
Number of members	40	100	60	80	120

[Note: 25 - 35 means 25 years old or more but less than 35, etc.]

(i) Draw a histogram to represent the data.
(ii) By taking the data at the mid-interval values, calculate the mean age per member.
(iii) What is the greatest possible number of members who could have been over 60 years of age?

(a) (i) 10 marks Att 3



- * Each rectangle may be blundered only once.
- * Accept areas of rectangles proportional to frequencies, provided bases are correct.
- * Do not penalise a space between 0 -15 on the horizontal axis.

Award marks as follows:

10 marks Correct histogram

7 marks Scale(s) incorrect, rectangles subsequently correct
or scales correct, one rectangle incorrect or omitted
or scales correct, rectangles correct but spaces put between rectangles.

4 marks Scale(s) incorrect, one rectangle incorrect or omitted
or scales correct, two rectangles incorrect or omitted
or scales correct, one rectangle incorrect or omitted and spaces between rectangles.

3 marks Attempt at answer as below.

Attempts (3 marks)

A1 Draws scaled horizontal axis and stops, even without labels.
A2 Treats 0 – 40, 40 – 100 etc. as intervals and 25, 35 etc as frequencies.

(a) (ii) **15 marks** **Att 5**

Mid-interval values 20, 30, 40, 50, 65

$$\text{Mean } \bar{x} = \frac{20\times40+30\times100+40\times60+50\times80+65\times120}{40+100+60+80+120}$$

$$= \frac{800+3000+2400+4000+7800}{400} = \frac{18000}{400} = 45$$

or

Interval	Mid-interval (x)	f	fx
15 – 25	20	40	800
25 – 35	30	100	3000
35 – 45	40	60	2400
45 – 55	50	80	4000
55 – 75	65	120	7800
		400	18000

$$\text{Mean } \bar{x} = \frac{\sum fx}{\sum f} = \frac{18000}{400} = 45$$

* Accept correct answer without work i.e. uses calculator.

* For an incorrect answer consistent with incorrect mid-intervals award marks for relevant work. For an incorrect answer not consistent with incorrect mid-intervals award attempt mark at most.

Award marks as follows:

15 marks: Answer of 45.

14 marks: Answer of 18000/400.

12 marks: Answer of 18000 and 400 without fraction *or* fraction written as 400/18000 *or* 18000/*n* *or* *n*/400, where *n* represents relevant work with frequencies.

9 marks: 18000 *or* 400 *or* incorrect fraction with relevant work.
[Apply maximum of one blunder for numerator and one blunder for denominator].

6 marks: All correct mid-interval values.

5 marks Some relevant attempt as below.

Attempts (5 marks)

A1 Writes formula for mean and stops.
A2 A correct multiplication and stops.
A3 Addition of frequencies indicated and stops.
A4 One or more correct mid-interval values and stops.
A5 Gives a reasonable estimate of $43 \le \bar{x} \le 47$.

(a) (iii) **5 marks** **Att 2**

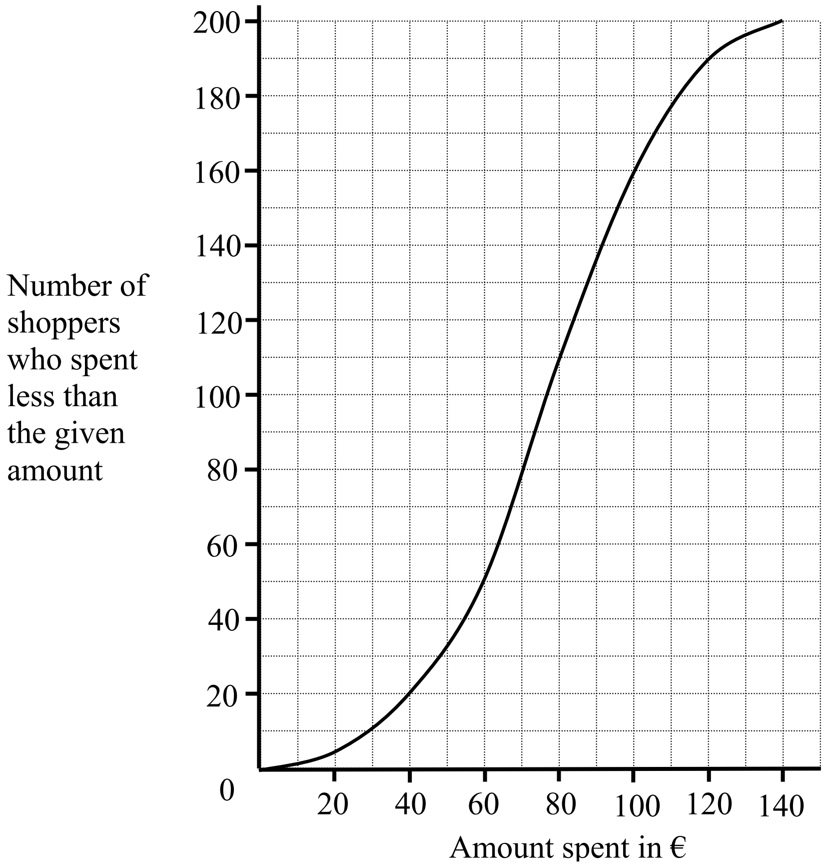
120 members

Attempts (2 marks)

A1 Answer of 0 or 280.

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

The amount of money spent by shoppers in a supermarket over a particular time period was recorded. The results are represented by the ogive below:



(i) Estimate the median amount spent.

(ii) Estimate the interquartile range.

(iii) Estimate the number of shoppers who spent between €40 and €100.

(iv) Given that the mean amount spent was €80 per shopper, estimate the percentage of shoppers who spent more than the mean amount.

(b) (i) **5 marks** **Att 2**

€76

* Accept an answer in the range 70 < median < 80.

Attempts (2 marks)

A1 Answer of 80 or some relevant statement about median.

(b) (ii) **5 marks** **Att 2**

€95 – €60 = €35

* Accept an answer in the range 90 < upper quartile < 100.

Blunders (-3)

B1 Starts on the incorrect axis – range is 170 – 15 = 155.

B2 Each incorrect or omitted quartile or no indication of subtraction.

Slips (-1)
S1 Writes the difference but does not do the subtraction or gives the answer as a range.

Attempts (2 marks)
A1 Answer of 80 – 15
A2 Some relevant statement about interquartile range.
A3 90 < upper quartile <100 without work or 60 without work.

(b) (iii) **5 marks** **Att 2**

€40 ~ 20 shoppers and €100 ~ 160 shoppers
160 – 20 = 140 shoppers

Blunders (-3)
B1 Starts on the incorrect axis – number is 76 – 56 = 20.
B2 Only one value found.
B3 Each incorrect or omitted number or no indication of subtraction.

Slips (-1)
S1 Writes the difference but does not do the subtraction.

Attempts (2 marks)
A1 Some relevant statement on required task.

(b) (iv) **5 marks** **Att 2**

€80 ~ 110 shoppers

$$\frac{90}{200}\times 100 = 45\%$$

Blunders (-3)
B1 Starts on the incorrect axis – 70 which equates to 50%.
B2 Spent less than the mean amount – 55%.

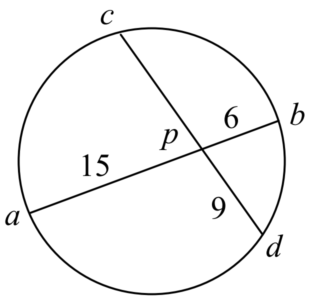
Attempts (2 marks)
A1 Some relevant work or statement on required task.

QUESTION 8

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 2**

Part (a) **10 marks** **Att 3**

The chords $[ab]$ and $[cd]$ of a circle intersect at a point p inside the circle.
$|ap| = 15$, $|pb| = 6$ and $|pd| = 9$.
Find $|cp|$.



(a) **10 marks** **Att 3**

$|cp| \times |pd| = |ap| \times |pb|$ [*3 marks*]

$$|cp| \times 9 = 15 \times 6 \underset{\text{[4 marks]}}{\downarrow} \Rightarrow |cp| = \frac{15 \times 6}{9} \underset{\text{[7 marks]}}{\downarrow} = 10 \underset{\text{[10 marks]}}{\downarrow}$$

* Accept correct answers without work or answer clearly indicated on a diagram.

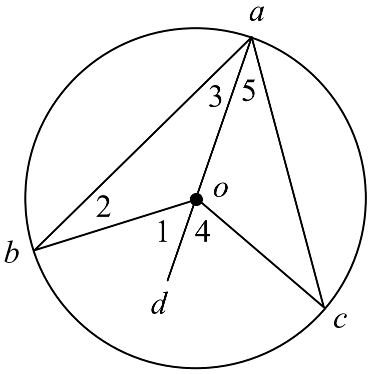
Attempts (3 marks)
A1 Geometrical result indicated on a diagram or stated without numerical data.
A2 Some relevant step, e.g. begins a correct substitution into result, correct or otherwise.
A3 Addition used instead of multiplication (Answer 12).

Worthless (*0 marks*)
W1 Incorrect answer without work shown.

Part (b) **20 marks** **Att 7**

Prove that the degree-measure of an angle subtended at the centre of a circle by a chord is equal to twice the degree-measure of any angle subtended by the chord at a point of the arc of the circle which is on the same side of the chordal line as is the centre.

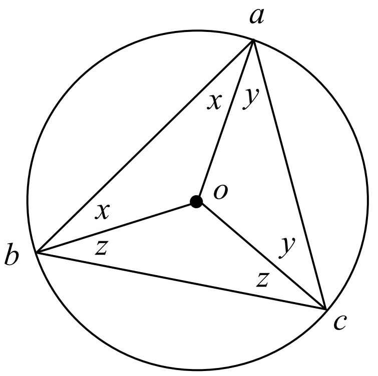
Circle of centre o. Points a, b and c on the circle.
To Prove: $|\angle cob| = 2|\angle cab|$.



Construction: Join a to o and continue to d. [*7 marks*]

Proof:
$|ao| = |ob| \Rightarrow |\angle 2| = |\angle 3|$ [*10 marks*]
$|\angle 1| = |\angle 2| + |\angle 3|$ … exterior angle [*13 marks*]
Hence, $|\angle 1| = 2|\angle 3|$ [*16 marks*]
Similarly, $|\angle 4| = 2|\angle 5|$ [*17 marks*]
Thus, $|\angle 1| + |\angle 4| = 2(|\angle 3| + |\angle 5|)$
i.e. $|\angle cob| = 2|\angle cab|$ [*20 marks*]

or



Construction: Join a, b and c to o. [*7 marks*]

Proof:
$|ao| = |ob| \Rightarrow |\angle oab| = |\angle abo| = x$ [*10 marks*]
Similarly $|\angle cao| = |\angle oca| = y$
and $|\angle obc| = |\angle bco| = z$ [*13 marks*]
$2x + 2y + 2z = 180°$ … in triangle abc
$|\angle boc| + 2z = 180°$ … in triangle obc [*16 marks*]
$|\angle boc| = 180° - 2z = 2x + 2y$ [*19 marks*]
$|\angle boc| = 2|\angle bac|$ [*20 marks*]

* Proof without a diagram merits att 7, if a complete proof can be reconciled with a diagram.

Blunders (-3)
B1 Each step omitted, incorrect or incomplete (except the last in the second method).
B2 Steps written in an illogical order. [Penalise once only.]
[Note: Some of the steps above may be interchanged.]

Attempts (7 marks)
A1 Any relevant step, stated or indicated, e.g. circle with additional relevant information.
A2 States or illustrates a special case, e.g. measuring the angles on a diagram.
A3 Proves an angle on a diameter is a right angle.

Worthless (0 marks)
W1 Any irrelevant theorem, subject to the attempt mark.
W2 Circle only.

Part (c) **20 (5, 5, 5, 5) marks** **Att (-, -, -, 2)**

The points a, b, c and d lie on a circle.

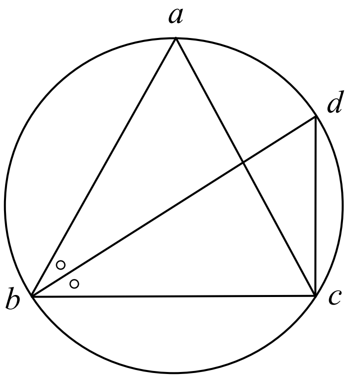
$|ab| = |bc| = |ac|$ and $[bd]$ bisects $\angle abc$.

(i) Find $|\angle cab|$.

(ii) Find $|\angle cdb|$.

(iii) Find $|\angle bcd|$.

(iv) Is $[bd]$ a diameter of the circle?
Give a reason for your answer.



(c) (i) **5 marks** **Hit or miss**

$|\angle cab| = 60°$ …. The triangle abc is equilateral.

(c) (ii) **5 marks** **Hit or miss**

$|\angle cdb| = |\angle cab| = 60°$ …. Angles on the same arc.

(c) (iii) **5 marks** **Hit or miss**

$|\angle dbc| = \frac{1}{2}|\angle abc| = 30°$.

$|\angle bcd| = 180° - (|\angle cdb| + |\angle dbc|) = 180° - (60° + 30°) = 90°$.

(c) (iv) **5 marks** **Att 2**

Yes. The angle in a semicircle is a right angle and $|\angle bcd| = 90°$.

* Accept answer written on a diagram in each section.
* Accept correct or consistent answer without work in each section.

Blunders (-3)
B1 Incorrect or no reason given in section (iv).

QUESTION 9

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**
Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

Let $\vec{v} = 2\vec{i} + 3\vec{j}$ and $\vec{w} = \vec{i} - 4\vec{j}$.

(i) Express $\vec{v} + 2\vec{w}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Express $\overrightarrow{vw}$ in terms of $\vec{i}$ and $\vec{j}$.

(a) (i) **5 marks** **Att 2**

$\vec{v} + 2\vec{w} = 2\vec{i} + 3\vec{j} + 2(\vec{i} - 4\vec{j})$ [*2 marks*]

$= 2\vec{i} + 3\vec{j} + 2\vec{i} - 8\vec{j} = 4\vec{i} - 5\vec{j}.$ [*5 marks*]

(a) (ii) **5 marks** **Att 2**

$\overrightarrow{vw} = \vec{w} - \vec{v} = \vec{i} - 4\vec{j} - (2\vec{i} + 3\vec{j})$ [*2 marks*]

$= \vec{i} - 4\vec{j} - 2\vec{i} - 3\vec{j} = -\vec{i} - 7\vec{j}.$ [*5 marks*]

* Accept correct answer without work shown in sections (i) and (ii).

Blunders (-3)

B1 $\overrightarrow{vw} = \vec{w} + \vec{v}$ or $\vec{v} - \vec{w}$ or $\vec{v}\,.\,\vec{w}$ and continues.

Slips (-1)

S1 Interchanges $\vec{v}$ with $\vec{w}$ and finds $2\vec{v} + \vec{w}$.

Attempts (2 marks)

A1 $4\vec{i}$ or $-5\vec{j}$ without work shown and stops.
A2 Some effort at scalar multiplication or combining components.
A3 Relevant work on a diagram e.g. plots one or more of the vectors.

Worthless (0 marks)
W1 Incorrect answer without work.

Part (b) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

Let $\vec{m} = 4\vec{i} + 3\vec{j}$ and $\vec{n} = 15\vec{i} - 8\vec{j}$.

(i) Find $\vec{m}\,.\,\vec{n}$, the dot product of $\vec{m}$ and $\vec{n}$.

(ii) Calculate $|\vec{m}|$ and $|\vec{n}|$.

(iii) Find the measure of the angle between $\vec{m}$ and $\vec{n}$, correct to the nearest degree.

(b) (i) **5 marks** **Att 2**

$$\vec{m}\,.\,\vec{n} = (4\vec{i} + 3\vec{j}).(15\vec{i} - 8\vec{j})\downarrow = 60 - 24 = 36.\downarrow$$

[*2 marks*] [*5 marks*]

* Accept correct answer without work shown in sections (i) and (ii).

Blunders (-3)

B1 $\vec{i}^{\,2} \neq 1$ or $\vec{j}^{\,2} \neq 1$ or $\vec{i}\,.\,\vec{j} \neq 0$, applied once.

B2 Incorrect relevant formula e.g. $|\vec{m}||\vec{n}|\sin\theta$ or $|\vec{m}| = \sqrt{a^2 - b^2}$.

Attempts (2 marks)
A1 Correct relevant formula and stops.
A2 Finds the length of one vector and stops.
A3 Some correct work in multiplication using *m* and/or *n*.

(b) (ii) **5 marks** **Att 2**

$$|\vec{m}| = |4\vec{i} + 3\vec{j}| = \sqrt{4^2 + 3^2} = \sqrt{25} = 5. \qquad [\textit{2 marks}]$$

$$|\vec{n}| = |15\vec{i} - 8\vec{j}| = \sqrt{15^2 + (-8)^2} = \sqrt{225 + 64} = \sqrt{289} = 17. \qquad [\textit{5 marks}]$$

Blunders (-3)
B1 Blunder in formula e.g. square root omitted or squares omitted or $-$ instead of $+$.

Attempts (2 marks)
A1 Finds the square of the coefficients of any of the given components and stops.
A2 Effort at use of relevant square root.

(b) (iii) **10 marks** **Att 3**

$$\cos\theta = \frac{\vec{m}.\vec{n}}{|\vec{m}|.|\vec{n}|} = \frac{36}{5 \times 17} = 0.4235 \quad \Rightarrow \theta = \cos^{-1} 0.4235 = 64.9 = 65^\circ.$$

* Accept the candidate's answers from sections (i) and (ii).

Blunders (-3)
B1 Incorrect relevant formula e.g. inverted fraction and continues.
B2 Uses a trigonometric or co-ordinate geometry method incorrectly and continues.
B3 Blunder in finding the angle θ.

Attempts (3 marks)
A1 Correct relevant formula and stops.
A2 Finds the slope of *om* and/or of *on*.
A3 Plots one or both vectors.
A4 Correct answer without work shown.
A5 Incorrect formula which oversimplifies the work.

Worthless (0 marks)
W1 Uses an incorrect trigonometric or co-ordinate geometry method to find θ.

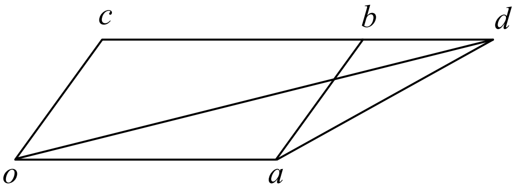
Part (c) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

oabc is a parallelogram. $[cb]$ is produced to d such that $|bd| = \frac{1}{2}|cb|$.

(i) Express $\overrightarrow{cd}$ in terms of $\vec{a}$.

(ii) Express $\vec{d}$ in terms of $\vec{a}$ and $\vec{c}$.

(iii) Express $\overrightarrow{ad}$ in terms of $\vec{a}$ and $\vec{c}$.



(c) (i) **5 marks** **Att 2**

$$\overrightarrow{cd} = \overrightarrow{cb} + \overrightarrow{bd} = \overrightarrow{cb} + \tfrac{1}{2}\overrightarrow{cb} = \vec{a} + \tfrac{1}{2}\vec{a} = \tfrac{3}{2}\vec{a}.$$

(c) (ii) **5 marks** **Att 2**

$$\vec{d} = \vec{c} + \overrightarrow{cd} = \vec{c} + \tfrac{3}{2}\vec{a}.$$

(c) (iii) **10 marks** **Att 3**

$$\overrightarrow{ad} = \overrightarrow{ab} + \overrightarrow{bd} = \vec{c} + \tfrac{1}{2}\vec{a}.$$

or

$$\overrightarrow{ad} = \vec{d} - \vec{a} = \vec{c} + \tfrac{3}{2}\vec{a} - \vec{a} = \vec{c} + \tfrac{1}{2}\vec{a}.$$

- * Allow $\overrightarrow{oa}$ for $\vec{a}$ etc. in each section.
- * Accept correct or consistent answers without work in (i) and (ii).
- * Do not penalise for the omission of arrows.

Award marks as follows in sections (i) and (ii):

5 marks: Answer fully correct.
2 marks: Some relevant work.
0 marks: Worthless work.

Award marks as follows in section (iii):

10 marks: Answer fully correct.
3 marks: Some relevant work.
0 marks: Worthless work.

Attempts (2 or 3 marks)

A1 Relevant work on a diagram.

A2 Correct relevant work e.g $\overrightarrow{ab} = \vec{c}$ or $\overrightarrow{ab} = \vec{b} - \vec{a}$.

Worthless (0 marks)

W1 Diagram reproduced without modifications.

QUESTION 10

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

(i) Write out the first 3 terms in the expansion of $(1-x)^6$, in ascending powers of x.

(ii) Calculate the value of the third term when $x = 0{\cdot}1$.

(a) (i) **5 marks** **Att 2**

$$(1-x)^6 = \binom{6}{0} + \binom{6}{1}(-x) + \binom{6}{2}(-x)^2 + \ldots \quad = \quad 1 - 6x + 15x^2 + \ldots$$

* Accept correct answer without work.

Award marks as follows:

5 marks: Answer fully correct.
4 marks: Correct work except for one obvious slip.
2 marks: Some relevant work.
0 marks: Worthless work.

Slips (-1)

S1 Expands $(1+x)^6$.

Attempts (2 marks)

A1 Any term, including first term, written down correctly.
A2 Answer of $1+x^6$ is attempt mark at most.
A3 Gives part of Pascal's triangle or effort at Pascal's triangle.
A4 Gives coefficients only.
A5 Any step towards getting a binomial coefficient e.g. $\binom{6}{2}$ or writes coefficients i.e.1, 6, 15.
A6 Any correct step towards long multiplication.

Worthless (0 marks)

W1 Writes $6(1-x)^5(-1)$ or similar.

(a) (ii) **5 marks** **Att 2**

$$T_3 = 15x^2 = 15(0.1)^2 = 0.15$$

* Accept an answer consistent with the candidate's answers from the section (i).

Award marks as follows:

5 marks: Answer fully correct.
2 marks: Some relevant work.
0 marks: Worthless work.

Attempts (2 marks)

A1 Identifies the third term and stops.
A2 Some relevant work at substitution.
A3 Correct answer without work.

Worthless (0 marks)

W1 An incorrect answer without work shown.
W2 Substitutes 0.1 into $(1-x)^6$.

Part (b) **20 (10, 10) marks** **Att (3, 3)**

(i) Find the sum to infinity of the geometric series $\frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \dots$.

(ii) Hence, express the recurring decimal 1·777… in the form $\frac{a}{b}$, where $a, b \in \mathbf{N}$.

(b) (i) **10 marks** **Att 3**

$a = \frac{7}{10}, \quad r = \frac{7}{100} \div \frac{7}{10} = \frac{1}{10}$ [*3 marks*]

$$S_\infty = \frac{a}{1-r} = \frac{\frac{7}{10}}{1-\frac{1}{10}} \downarrow = \frac{\frac{7}{10}}{\frac{9}{10}} = \frac{7}{9}. \downarrow$$

[*6 marks*] [*10 marks*]

or

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{\frac{7}{10}\left(1-(\frac{1}{10})^n\right)}{1-\frac{1}{10}} \downarrow = \frac{\frac{7}{10}}{\frac{9}{10}} = \frac{7}{9}. \downarrow$$

[*6 marks*] [*10 marks*]

Blunders (-3)
B1 Incorrect a.
B2 Incorrect r.
B3 Blunder in fractions.
B4 Incorrect relevant formula e.g. $a/(1+r)$ giving answer of $\frac{7}{11}$.
B5 Finds limit as $n \to 0$ in the second method.

Slips (-1)
S1 Numerical slips to a maximum of 3.

Attempts (3 marks)
A1 Correct relevant formula and stops.
A2 Some relevant step e.g. states the value for a or the value for r.
A3 Adds 2 or more of the given terms e.g $S_2 = \frac{77}{100}$ or $S_3 = \frac{777}{1000}$.
A4 One correct step in adding relevant fractions.
A5 Treats as arithmetic series with further work, e.g. identifies a.
A6 Writes $T_n = ar^{n-1}$ or $\frac{7}{10}(\frac{1}{10})^{n-1}$.
A7 Gives $T_4 = \frac{7}{10000}$.
A8 Correct answer without work.

Worthless (0 marks)
W1 Formula for arithmetic series and stops.
W2 $\frac{7}{10} + \frac{7}{100} + \frac{7}{1000} = \frac{21}{1110}$ or similar work.
W3 Incorrect answer without work.

(b) (ii) **10 marks** **Att 3**

$$1.777\ldots = 1 + \frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \downarrow \dots = 1 + \frac{7}{9} \downarrow = \frac{16}{9}. \downarrow$$

[*4 marks*] [*7 marks*] [*10 marks*]

* Accept an answer consistent with candidate's answers from the sections (i).

Blunders (-3)
B1 Blunder in fractions.

Attempts (3 marks)

A1 Effort at writing 1.777… as a series.
A2 Answer given is $^{17}/_{10}$ or $^{7}/_{9}$.
A3 Correct answer without work shown or answer not derived from section (i).

Part (c) **20 (10, 10) marks** **Att (3, 3)**

(i) Tom gave a donation of €200 to a charity in 2004.
Tom agreed to increase his donation by €10 each year for the next 9 years.
Use the relevant series formula to find the total amount Tom will have donated to the charity after the 10 years.

(ii) Kate also gave a donation of €200 to the charity in 2004.
She agreed to increase her donation by a fixed amount each year for the next 9 years.
After the 10 years Kate will have donated €3125.
By how much is Kate increasing her donation each year?

(c) (i) **10 marks** **Att 3**

$200 + 210 + 220 + \ldots \Rightarrow \quad a = 200, \quad d = 10, \quad n = 10.$ [*3 marks*]

$S_n = {}^{n}\!/_{2}\left(2a + (n-1)d\right) \Rightarrow S_{10} = {}^{10}\!/_{2}\left(2(200) + (10-1)10\right)$ [*6 marks*]

$= 5(400 + 90) = 5(490) =$ €2450. [*10 marks*]

(c) (ii) **10 marks** **Att 3**

$a = 200, \quad n = 10, \quad S_n = 3125$ [*3 marks*]

$S_n = {}^{n}\!/_{2}\left(2a + (n-1)d\right) \Rightarrow S_{10} = {}^{10}\!/_{2}\left(2(200) + (10-1)d\right) = 3125$ [*4 marks*]

$\Rightarrow \quad 5(400 + 9d) = 3125 \Rightarrow 400 + 9d = 625 \Rightarrow 9d = 225 \Rightarrow d =$ €25. [*10 marks*]

or

$S = 200 + (200 + x) + (200 + 2x) + \ldots (200 + 9x) = 3125$ [*4 marks*]

$\Rightarrow 2000 + 45x = 3125 \Rightarrow 45x = 1125 \Rightarrow x = 25$ [*10 marks*]

Blunders (-3)

B1 Incorrect *a*.
B2 Incorrect *d*.
B3 Incorrect *n*.
B4 Error in formula (More than one error merits the attempt at most).
B5 Error in substitution (More than one error merits the attempt at most).

Slips (-1)

S1 Numerical slips to a maximum of 3.

Attempts (3 marks)

A1 Correct answer without work or without use of the series formula.
A2 Writes the correct formula and stops.
A3 Treats as geometric series with further work, e.g. identifies *a*.
A4 Relevant work such as correct *a* or *d* or *n* or donation for second year.

Worthless (0 marks)

W1 Incorrect answer without work.
W2 Incorrect formula and stops.
W3 Formula for a GP and stops but subject to A4.
W4 Writes $3125 \div 10$ or gives answer of 312.5.

QUESTION 11

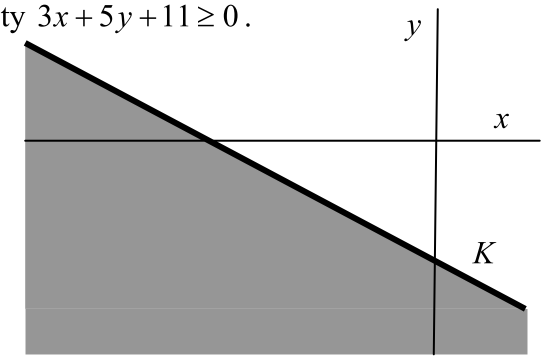
Part (a)	**15 (10, 5) marks**	**Att (3, 2)**
Part (b)	**35 (10, 5, 5, 5, 5, 5) marks**	**Att (3, 2, 2, 2, 2, 2)**

Part (a) **15 (10, 5) marks** **Att 3, 2**

(i) Does the point $(18, -15)$ satisfy the inequality $3x+5y+11 \geq 0$.
Justify your answer.

(ii) The equation of the line K is
$x+2y+4=0.$

Write down the inequality which defines the shaded half-plane in the diagram.



(a) (i) **10 marks** **Att 3**

$3x+5y+11 \geq 0$.

$3(18)+5(-15)+11\downarrow = 54-75+11\downarrow = -10\downarrow \leq 0$. The point does not satisfy the inequality.

[*3 marks*] [*4 marks*] [*7 marks*] [*10 marks*]

Blunders (-3)

B1 Switches x and y in substituting.

B2 Conclusion incorrect or not stated clearly.

Slips (-1)

S1 One incorrect substitution in the inequality.

S2 Numerical slips to a maximum of 3.

Attempts (3 marks)

A1 The co-ordinates of the point written with x_1 and y_1 identified.

A2 Attempt at a graphical answer.

A3 Tests an arbitrary point in the inequality.

A4 Answer "no" without work or justification.

(a) (ii) **5 marks** **Att 2**

$x+2y+4 \leq 0$

* Accept correct inequality without work and $<$ for $\leq$ or $>$ for $\geq$.

Blunders (-3)

B1 Incorrect half-plane i.e. $x+2y+4 \geq 0$.

Attempts (2 marks)

A1 Substitutes any point and stops.

A2 Incorrect or no conclusion e.g. $x+2y+4=0 \Rightarrow 0+2(0)+4=0$.

A3 Mathematical error in testing a point (e.g. sign error).

A4 Answer given is $K \leq 0$.

A5 Gives both inequalities without clearly selecting either as the answer.

Worthless (0 marks)

W1 Writes equation of K or draws the given diagram and stops.

W2 Any inequality involving an axis e.g. $x \geq 0$ or $y \leq 0$.

Part (b) **35 (10, 5, 5, 5, 5, 5) marks** **Att 3, 2, 2, 2, 2, 2**

A small restaurant offers two set lunch menus each day: a fish menu and a meat menu.
The fish menu costs €12 to prepare and the meat menu costs €18 to prepare.
The total preparation costs must not exceed €720.
The restaurant can cater for at most 50 people each lunchtime.

(i) Taking x as the number of fish menus ordered and y as the number of meat menus ordered, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The price of a fish menu is €25 and the price of a meat menu is €30. How many of each type would need to be ordered each day to maximise income?

(iii) Show that the maximum income does not give the maximum profit.

(b) (i) Inequalities **15 (10, 5) marks** **Att (3, 2)**

Cost: $12x + 18y \le 720$ or $2x + 3y \le 120$
People: $x + y \le 50$

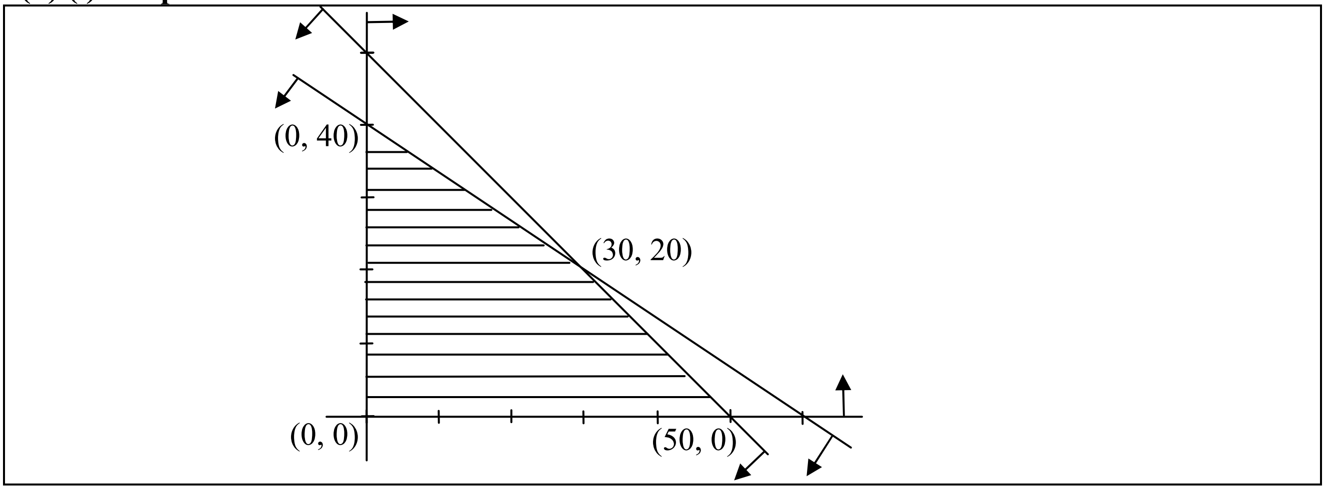
	Fish x	Meat y	Maximum
Cost People	12 1	18 1	720 50

- * Accept correct multiples or fractions of inequalities or the use of different letters.
- * Award 10 marks for one correct inequality, 5 marks for the second correct inequality in any order.
- * Apply a slip (-1), once, to these marks if no inequality sign or the incorrect inequality sign is written.

Blunders (-3)
B1 Mixes up x's and y's (once if consistent error).
B2 Confuses rows and columns in table, e.g. $12x + y \le 720$ (once if consistent).

Attempts (3 marks and 2 marks - each inequality)
A1 Incomplete relevant data in table and stops e.g. x or $12x$ or $\le$ 720 (each inequality).
A2 Any other correct inequality, e.g. $x \ge 0$, $y \ge 0$, (each time).

(b) (i) Graph **5 marks** **Att 2**



* Points or scales required.

* Correct shading over-rules arrows or correct arrows overrule shading.

Award marks as follows:

5 marks: Both half-planes illustrated either correctly or consistently.

2 marks: Some relevant work e.g. one half-plane graphed or both lines graphed with no half-planes.

0 marks: Worthless work.

Blunders (-3)

B1 Blunder in plotting a line or calculations.

B2 Incorrect shading e.g. one or both of the small triangles shaded.

Attempts (2 marks)

A1 Some relevant work towards a point on a line.

A2 Draws scaled axes or axes and one line.

(b) (ii) Intersection of lines **5 marks** **Att 2**

$$\begin{array}{r} 2x+3y=120 \\ \underline{2x+2y=100} \\ y\ =20 \end{array} \quad \Rightarrow x=30$$

* Accept candidate's own equations from previous sections.

* If solving incorrect equations, the point found may be outside the feasible set – award marks for correct work and accept in later sections.

Blunders (-3)

B1 Fails to multiply / divide both sides of equation(s) correctly when eliminating variable.

B2 x or y value only found.

Slips (-1)

S1 Numerical slips to a maximum of 3.

Attempts (2 marks)

A1 Correct or consistent answer without work or from a graph.

[Should get the *exact same* values from graph as if they had been found algebraically.]

A2 Any relevant step towards solving equations.

Worthless (0 marks)

W1 Incorrect answer without work and inconsistent with graph.

(b) (ii) Income **5 marks** **Att 2**

Step 1	Vertices	$25x + 30y$	Income
Step 2	(50, 0)	1250 + 0	1250
Step 3	(30, 20)	750 + 600	1350
Step 4	(0, 40)	0 + 1200	1200

Step 5 30 fish and 20 meat to maximise income.

* Information does not have to be in table form.

* Accept any correct multiple or fraction of $25x + 30y$ here.

* Accept work on a feasible set of points formed by axes and one line without further penalty.

* Accept only vertices consistent with previously accepted work, not arbitrary ones. If (60, 0) or (0, 50) is tested and result is used to give maximum income, award zero for step 5.

* If no marks have been awarded for intersection of lines and this point is written here award Att 2 for the previous work and also reward it here if the step is correct.

* Step 5 must be explicitly written to gain the final mark. Otherwise (-1).

* Testing only (30, 20) to get 1350 merits Att 2 even if the candidate writes 30 fish and 20 meat i.e. no comparison means the attempt mark at most.

Award marks as follows:

5 marks: Answer is fully correct or consistent.
4 marks: The maximum value is identified but step 5 not stated.
2 marks: Some relevant work.
0 marks: Worthless work.

Attempts (2 marks)

A1 Any relevant work involving x or y and / or 25, 30 or similar.
A2 Any attempt at substituting co-ordinates into some relevant expression.

Worthless (0 marks)

W1 Writing €25 or €30 without further work.

(b) (iii) Profit **5 marks** **Att 2**

	Vertices	$13x + 12y$	Profit
Profit at	(50, 0)	650 + 0	650
Profit at	(30, 20)	390 + 240	630

Conclusion: Maximum profit is not at $x = 30$ and $y = 20$.

* Accept candidate's vertices and income from previous section.

Award marks as follows:

5 marks: Answer is fully correct or consistent, with conclusion.
2 marks: Some relevant work.
0 marks: Worthless work.

Attempts (2 marks)

A1 Works with the expression $12x + 18y$ and fails to complete.

MARCANNA BREISE AS UCHT FREAGAIRT TRÍ GHAEILGE

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. 198 marc × 5% = 9·9 ⇒ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle [300 – bunmharc] × 15%, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0