Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2008

MATHEMATICS – ORDINARY LEVEL

PAPER 2 (300 marks)
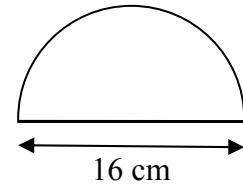
MONDAY, 9 JUNE – MORNING, 9:30 to 12:00

Attempt **FIVE** questions from **Section A** and **ONE** question from **Section B**.
Each question carries 50 marks.

WARNING: Marks will be lost if all necessary work is not clearly shown.

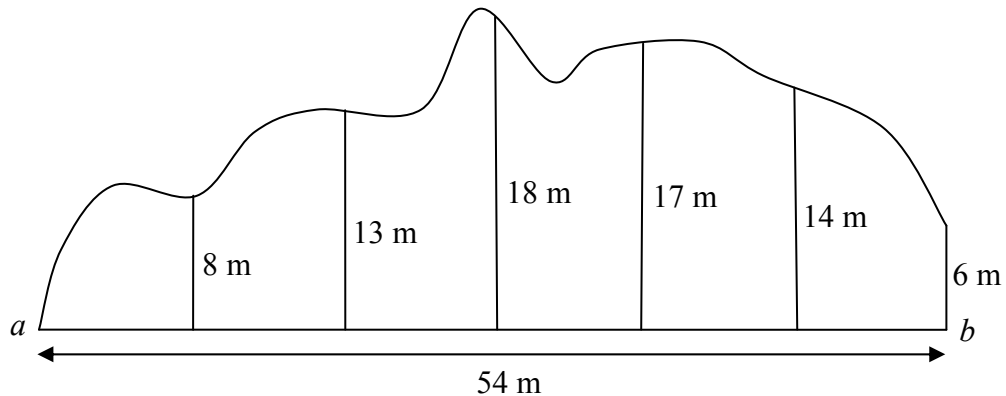Answers should include the appropriate units of measurement, where relevant.

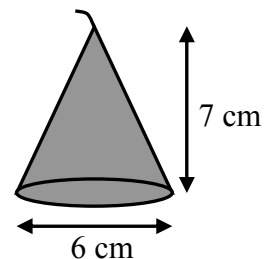1. **(a)** The semicircular shape shown in the diagram has diameter 16 cm.

   (i) Find the length of the perimeter of the shape, correct to the nearest centimetre.

   (ii) Find the area of the shape, correct to the nearest square centimetre.

   (b) The sketch shows a piece of land which borders the side of a straight road $[ab]$.
   The length of $[ab]$ is 54 m.
   At equal intervals along $[ab]$, perpendicular measurements are made to the boundary, as shown on the sketch.

   (i) Use Simpson's Rule to estimate the area of the piece of land.

   (ii) The land is valued at €480 000 per hectare. Find the value of the piece of land.
   Note: 1 hectare $= 10\ 000\ \text{m}^2$.

   (c) A wax candle is in the shape of a right circular cone.
   The height of the candle is 7 cm and the diameter of the base is 6 cm.

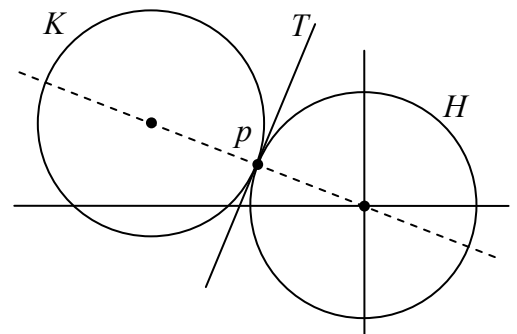   **(i)** Find the volume of the wax candle, correct to the nearest cm^3.

   (ii) A rectangular block of wax measuring 25 cm by 12 cm by 12 cm is melted down and used to make a number of these candles.

   Find the maximum number of candles that can be made from the block of wax if 4% of the wax is lost in the process.

2. **(a)** Find the area of the triangle with vertices $(0, 0)$, $(8, 6)$ and $(-2, 4)$.

(b) L is the line $y - 6 = -2(x + 1)$.

    (i) Write down the slope of L.

    (ii) Verify that $p(1, 2)$ is a point on L.

    (iii) L intersects the y-axis at t. Find the co-ordinates of t.

    (iv) Show the line L on a co-ordinate diagram.

(c) $o(0, 0)$, $a(5, 2)$, $b(1, 7)$ and $c(-4, 5)$ are the vertices of a parallelogram.

    (i) Verify that the diagonals $[ob]$ and $[ac]$ bisect each other.

    (ii) Find the equation of ob.

3. **(a)** A circle has equation $x^2 + y^2 = 16$.

    (i) Show the circle on a co-ordinate diagram.

    (ii) Mark the four points at which the circle intersects the axes and label them with their co-ordinates.
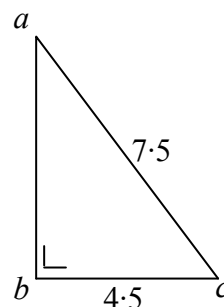
(b) The diagram shows two circles H and K, of equal radius.
The circles touch at the point $p(-2, 1)$.
The circle H has centre $(0, 0)$.

    (i) Find the equation of H.

    (ii) Find the equation of K.

    (iii) T is a tangent to the circles at p.
    Find the equation of T.

(c) The circle S has equation $(x - 3)^2 + (y + 2)^2 = 40$.
S intersects the x-axis at the point a and at the point b.

    (i) Find the co-ordinates of a and the co-ordinates of b.

    (ii) Show that $|ab|$ is less than the diameter of S.

    (iii) Find the equation of the circle with $[ab]$ as diameter.

4. **(a)** In the triangle abc,
$|\angle abc| = 90°$, $|bc| = 4{\cdot}5$ and $|ac| = 7{\cdot}5$.

Find $|ab|$.

(b) Prove that the opposite sides of a parallelogram have equal lengths.
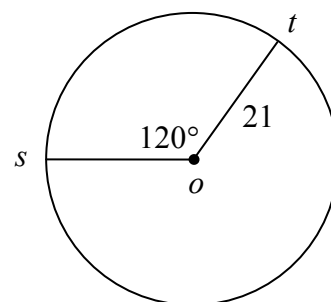
(c) **(i)** Construct an equilateral triangle pqr of side 8 cm.

(ii) Construct the image of the triangle pqr under the enlargement of scale factor $0{\cdot}75$ and centre q.

(iii) Given that the area of the triangle pqr is $16\sqrt{3}$ cm^2, find the area of the image triangle in the form $k\sqrt{3}$ cm^2.

5. **(a)** A circle has centre o and radius 21 cm.
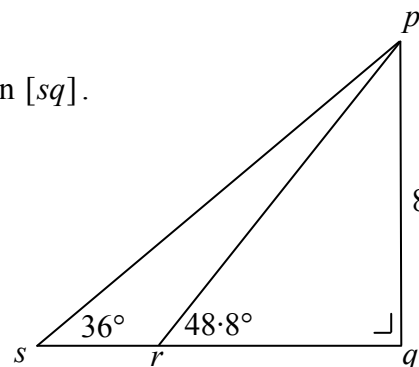s and t are two points on the circle and $|\angle sot| = 120°$.

Find the length of the shorter arc st,
correct to the nearest centimetre.

(b) In the right-angled triangle psq, p is joined to a point r on $[sq]$.
$|pq| = 8$ cm, $|\angle prq| = 48{\cdot}8°$ and $|\angle psq| = 36°$.

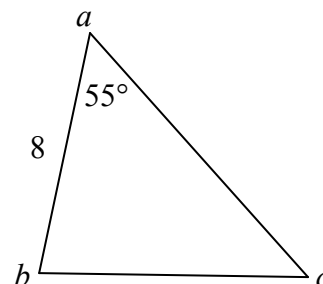**(i)** Find $|pr|$, correct to one decimal place.

(ii) Find $|sr|$, correct to the nearest centimetre.

(c) The area of the triangle abc is 33 cm^2.
$|ab| = 8$ cm and $|\angle cab| = 55°$.

(i) Find $|bc|$, correct to one decimal place.

(ii) Find $|\angle abc|$, correct to the nearest degree.

6. **(a)** Evaluate $5! + 6!$

(b) One shelf of a school library has 70 books. The books are on poetry and on drama and are either hardback or paperback.
The following table shows the number of each type.

	Hardback	Paperback
Poetry	23	17
Drama	14	16

A student selects one book at random from the shelf.

Find the probability that the book selected is

(i) a paperback poetry book

(ii) a hardback book

(iii) a poetry book

(iv) not a paperback drama book.

(c) There are 6 junior-cycle students and 5 senior-cycle students on the student council in a particular school.
A committee of 4 students is to be selected from the students on the council.

In how many different ways can the committee be selected if

(i) there are no restrictions

(ii) a particular student must be on the committee

(iii) the committee must consist of 2 junior-cycle students and 2 senior-cycle students.
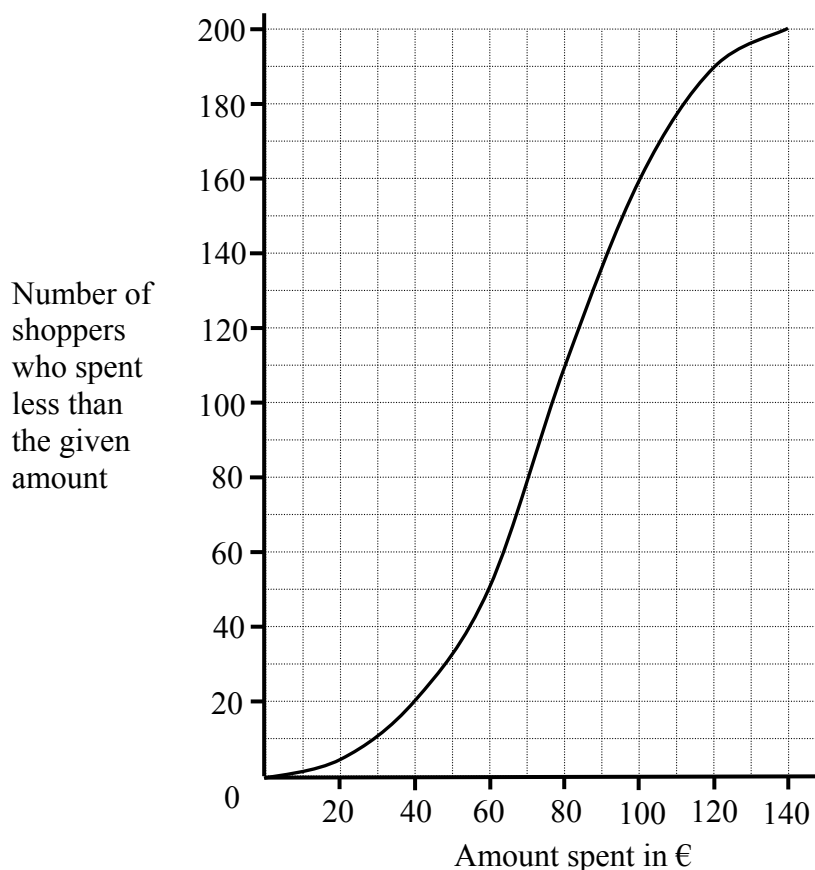
The committee of 4 students is chosen at random.

(iv) Find the probability that all 4 students are junior-cycle students.

7. **(a)** The ages of the members of a sports centre were analysed.   The results were:

Age	15 - 25	25 - 35	35 - 45	45 - 55	55 - 75
Number of members	40	100	60	80	120

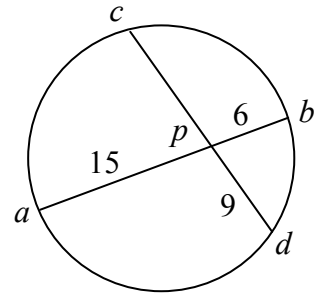[Note: 25 - 35 means 25 years old or more but less than 35, etc.]

(i) Draw a histogram to represent the data.

(ii) By taking the data at the mid-interval values, calculate the mean age per member.

(iii) What is the greatest possible number of members who could have been over 60 years of age?

(b) The amount of money spent by shoppers in a supermarket over a particular time period was recorded. The results are represented by the ogive below:

Number of shoppers who spent less than the given amount

Amount spent in €

(i) Estimate the median amount spent.

(ii) Estimate the interquartile range.

(iii) Estimate the number of shoppers who spent between €40 and €100.

(iv) Given that the mean amount spent was €80 per shopper, estimate the percentage of shoppers who spent more than the mean amount.
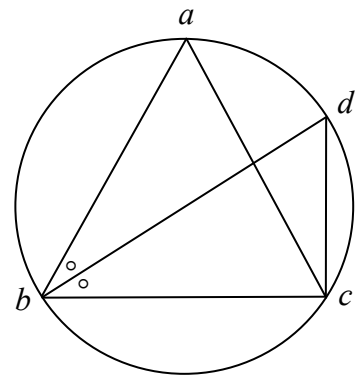
8. **(a)** The chords $[ab]$ and $[cd]$ of a circle
intersect at a point p inside the circle.
$|ap| = 15$, $|pb| = 6$ and $|pd| = 9$.

Find $|cp|$.

(b) Prove that the degree-measure of an angle subtended at the centre of a circle by a chord
is equal to twice the degree-measure of any angle subtended by the chord at a point of
the arc of the circle which is on the same side of the chordal line as is the centre.

(c) The points a, b, c and d lie on a circle.

$|ab| = |bc| = |ac|$ and $[bd]$ bisects $\angle abc$.

(i) Find $|\angle cab|$.

(ii) Find $|\angle cdb|$.

(iii) Find $|\angle bcd|$.

(iv) Is $[bd]$ a diameter of the circle?
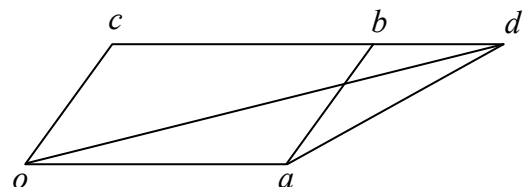Give a reason for your answer.

9. **(a)** Let $\vec{v} = 2\vec{i} + 3\vec{j}$ and $\vec{w} = \vec{i} - 4\vec{j}$.

(i) Express $\vec{v} + 2\vec{w}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Express $\vec{vw}$ in terms of $\vec{i}$ and $\vec{j}$.

(b) Let $\vec{m} = 4\vec{i} + 3\vec{j}$ and $\vec{n} = 15\vec{i} - 8\vec{j}$.

(i) Find $\vec{m} \cdot \vec{n}$, the dot product of $\vec{m}$ and $\vec{n}$.

(ii) Calculate $|\vec{m}|$ and $|\vec{n}|$.

(iii) Find the measure of the angle between $\vec{m}$ and $\vec{n}$, correct to the nearest degree.

(c) $oabc$ is a parallelogram. $[cb]$ is produced to d such that $|bd| = \frac{1}{2}|cb|$.

(i) Express $\vec{cd}$ in terms of $\vec{a}$.

(ii) Express $\vec{d}$ in terms of $\vec{a}$ and $\vec{c}$.

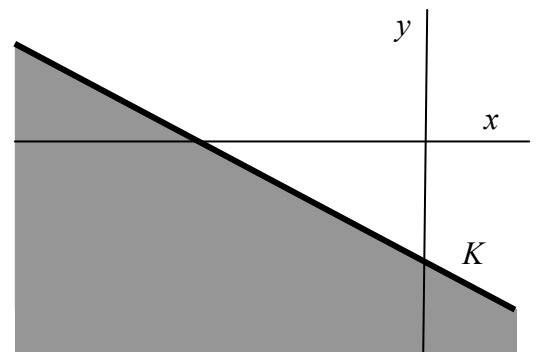**(iii)** Express $\vec{ad}$ in terms of $\vec{a}$ and $\vec{c}$.

10. **(a)** **(i)** Write out the first 3 terms in the expansion of $(1-x)^6$ in ascending powers of x.

(ii) Calculate the value of the third term when $x = 0 \cdot 1$.

(b) **(i)** Find the sum to infinity of the geometric series $\dfrac{7}{10} + \dfrac{7}{100} + \dfrac{7}{1000} + \ldots$

(ii) Hence, express the recurring decimal $1 \cdot 777\ldots$ in the form $\dfrac{a}{b}$, where $a, b \in \mathbf{N}$.

(c) **(i)** Tom gave a donation of €200 to a charity in 2004.
Tom agreed to increase his donation by €10 each year for the next 9 years.
Use the relevant series formula to find the total amount Tom will have donated to the charity after the 10 years.

(ii) Kate also gave a donation of €200 to the charity in 2004.
She agreed to increase her donation by a fixed amount each year for the next 9 years. After the 10 years Kate will have donated €3125.
By how much is Kate increasing her donation each year?

11. **(a)** **(i)** Does the point $(18, -15)$ satisfy the inequality $3x + 5y + 11 \geq 0$?
Justify your answer.

(ii) The equation of the line K is
$x + 2y + 4 = 0$.

Write down the inequality which defines
the shaded half-plane in the diagram.

(b) A small restaurant offers two set lunch menus each day: a fish menu and a meat menu.
The fish menu costs €12 to prepare and the meat menu costs €18 to prepare.
The total preparation costs must not exceed €720.
The restaurant can cater for at most 50 people each lunchtime.

(i) Taking x as the number of fish menus ordered and y as the number of meat menus ordered, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The price of a fish menu is €25 and the price of a meat menu is €30. How many of each type would need to be ordered each day to maximise income?

(iii) Show that the maximum income does not give the maximum profit.

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