# Coimisiún na Scrúduithe Stáit
# State Examinations Commission

## LEAVING CERTIFICATE EXAMINATION, 2007

## MATHEMATICS – ORDINARY LEVEL

**PAPER 2 ( 300 marks )**

**MONDAY, 11 JUNE – MORNING, 9:30 to 12:00**

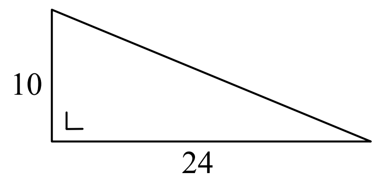
Attempt **FIVE** questions from **Section A** and **ONE** question from **Section B**.
Each question carries 50 marks.

**WARNING: Marks will be lost if all necessary work is not clearly shown.**

**Answers should include the appropriate units of measurement, where relevant.**

**SECTION A**
**Attempt FIVE questions from this section.**

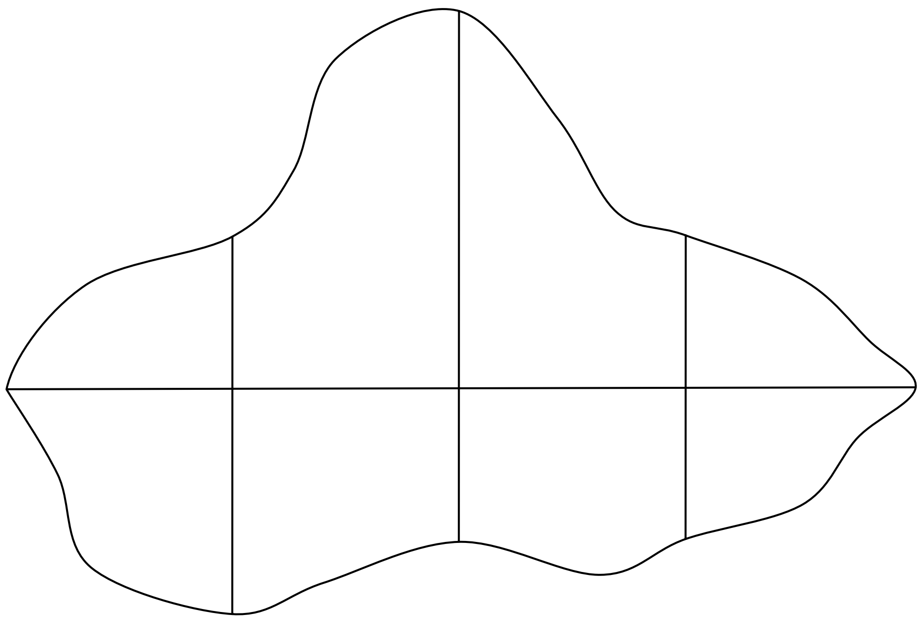
**1.** **(a)** The right-angled triangle shown in the diagram has sides of length 10 cm and 24 cm.




**(i)** Find the length of the third side.

**(ii)** Find the length of the perimeter of the triangle.

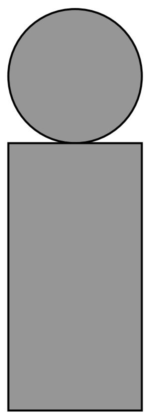
**(b)** In order to estimate the area of the irregular shape below, a horizontal line is drawn across the widest part of the shape and three offsets (perpendicular lines) are drawn at equal intervals along this line.



**(i)** Measure the horizontal line and the offsets, in centimetres.
Make a rough sketch of the shape in your answerbook and record the measurements on it.

**(ii)** Use Simpson's Rule with these measurements to estimate the area of the shape.

**(c)** A team trophy for the winners of a football match is in the shape of a sphere supported on a cylindrical base, as shown. The diameter of the sphere and of the cylinder is 21 cm.




**(i)** Find the volume of the sphere, in terms of $\pi$.

**(ii)** The volume of the trophy is $6174\pi$ cm$^3$.
Find the height of the cylinder.

**2.** **(a)** Find the co-ordinates of the mid-point of the line segment joining the points $(2, -3)$ and $(6, 9)$.

**(b)** The line $L$ intersects the $x$-axis at $(-4, 0)$ and the $y$-axis at $(0, 6)$.

**(i)** Find the slope of $L$.

**(ii)** Find the equation of $L$.

The line $K$ passes through (0, 0) and is perpendicular to $L$.

**(iii)** Show the lines $L$ and $K$ on a co-ordinate diagram.

**(iv)** Find the equation of $K$.

**(c)** $a(-4, 3)$, $b(6, -1)$ and $c(2, 7)$ are three points.

**(i)** Find the area of the triangle $abc$.

**(ii)** $abcd$ is a parallelogram in which $[ac]$ is a diagonal.
Find the co-ordinates of the point $d$.

**3.** **(a)** The circle $C$ has centre (0, 0) and radius 4.

**(i)** Write down the equation of $C$.

**(ii)** Verify that the point (3, 2) lies inside the circle $C$.

**(b)** The line $x - 3y = 0$ intersects the circle $x^2 + y^2 = 10$ at the points $a$ and $b$.

**(i)** Find the co-ordinates of $a$ and the co-ordinates of $b$.

**(ii)** Show that $[ab]$ is a diameter of the circle.

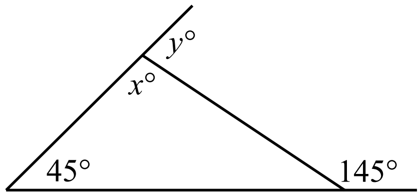
**(c)** The circle $K$ has equation $(x-5)^2 + (y+1)^2 = 34$.

**(i)** Write down the radius of $K$ and the co-ordinates of the centre of $K$.

**(ii)** Verify that the point $(10, -4)$ is on the circle.

**(iii)** $T$ is a tangent to the circle at the point $(10, -4)$.
$S$ is another tangent to the circle and $S$ is parallel to $T$.
Find the co-ordinates of the point at which $S$ is a tangent to the circle.

**4.** **(a)** In the diagram, two sides of the triangle are produced.



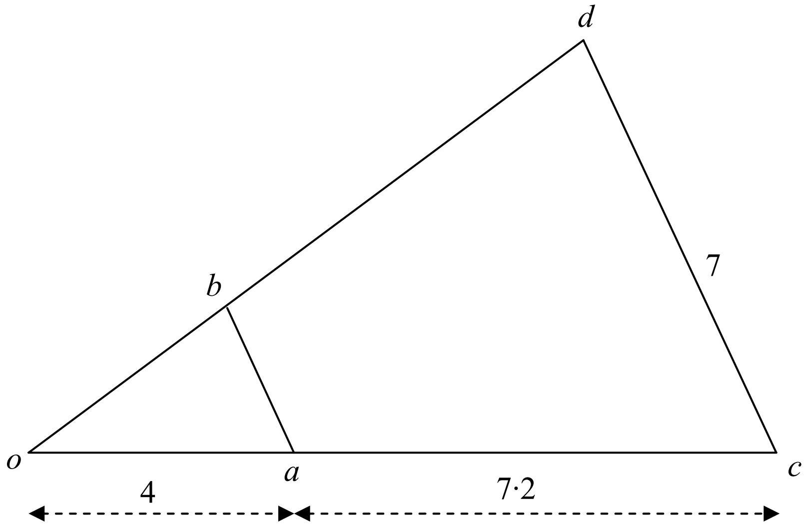

**(i)** Find $x$.

**(ii)** Find $y$.

**(b)** Prove that the products of the lengths of the sides of a triangle by the corresponding altitudes are equal.

**(c)** The triangle *ocd* is the image of the triangle *oab* under an enlargement with centre *o*.

$|oa| = 4$, $|ac| = 7{\cdot}2$ and $|cd| = 7$.



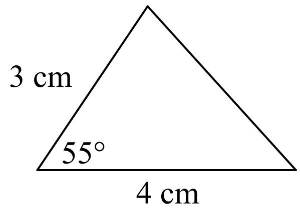

**(i)** Find the scale factor of the enlargement.

**(ii)** Find $|ab|$.

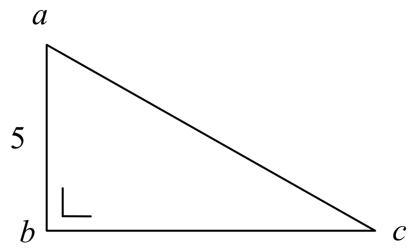
**(iii)** The area of the triangle *oab* is 4·5 square units.
Find the area of the triangle *ocd*.

**5.** **(a)** Calculate the area of the triangle shown.

Give your answer correct to one decimal place.




**(b)** In the right-angled triangle *abc*, $|ab| = 5$ cm.
The area of the triangle is $15\ \text{cm}^2$.

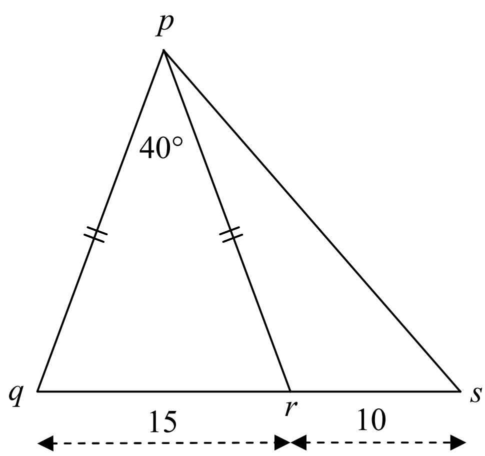



**(i)** Find $|bc|$.

**(ii)** Find $|\angle cab|$, correct to the nearest degree.

**(iii)** Find $|\angle bca|$, correct to the nearest degree.

**(c)** In the triangle *pqr*,
$|pq| = |pr|$, $|qr| = 15$ cm and $|\angle rpq| = 40°$.




**(i)** Find $|pr|$, correct to the nearest centimetre.

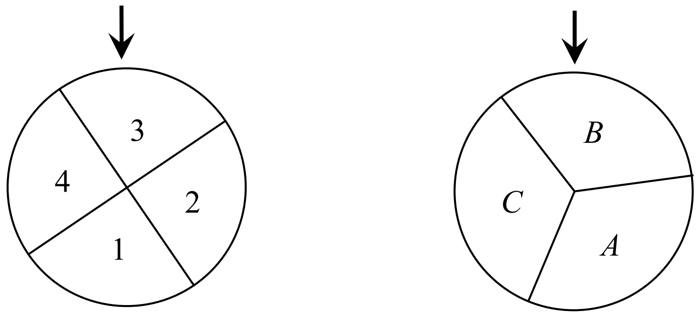
**(ii)** *s* is a point on *qr* such that $|rs| = 10$ cm.
Find $|ps|$, correct to the nearest centimetre.

**6.** **(a)** One letter is chosen at random from the letters of the word EUCLID.

**(i)** Find the probability that the letter chosen is D.

**(ii)** Find the probability that the letter chosen is a vowel.

**(b)** The diagram shows two wheels.
The first wheel is divided into four equal segments numbered 1, 2, 3 and 4.
The second wheel is divided into three equal segments labelled $A$, $B$ and $C$.




A game consists of spinning the two wheels and noting the segments that stop at the arrows. For example, the outcome shown is (3, $B$).

**(i)** Write down all the possible outcomes.

**(ii)** What is the probability that the outcome is (2, $C$)?

**(iii)** What is the probability that the outcome is an odd number with the letter $A$?

**(iv)** What is the probability that the outcome includes the letter $C$?

**(c)** **(i)** How many different three-digit numbers can be formed from the digits 2, 3, 4, 5, 6, if each of the digits can be used only once in each number?

**(ii)** How many of the numbers are less than 400?

**(iii)** How many of the numbers are divisible by 5?

**(iv)** How many of the numbers are less than 400 and divisible by 5?

7. **(a)** Find the median of the numbers

5, 11, 3, 16, 8.

**(b)** The table below shows the time, in minutes, that customers were waiting to be served in a restaurant.

| Time (minutes) | < 5 | < 10 | < 15 | < 20 | < 25 |
|---|---|---|---|---|---|
| Number of customers | 5 | 20 | 70 | 110 | 120 |

**(i)** Draw the cumulative frequency curve (ogive).

**(ii)** Use your curve to estimate the median waiting time.

**(iii)** Use your curve to estimate the interquartile range.

**(c)** The age of each person living in one street was recorded during a census.
The information is summarised in the following table:

| Age (in years) | 0 - 20 | 20 - 30 | 30 - 50 | 50 - 80 |
|---|---|---|---|---|
| Number of people | 16 | 12 | 32 | 12 |

**(i)** How many people were living in the street?

**(ii)** Using mid-interval values, calculate the mean age.

**(iii)** What is the greatest number of people who could have been aged under 40 years?

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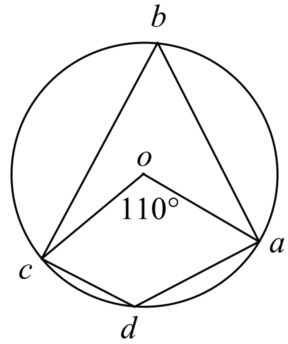
**SECTION B**
**Attempt ONE question from this section.**

**8.** **(a)** The points $a$, $b$, $c$ and $d$ lie on a circle, centre $o$.
$|\angle aoc| = 110°$.



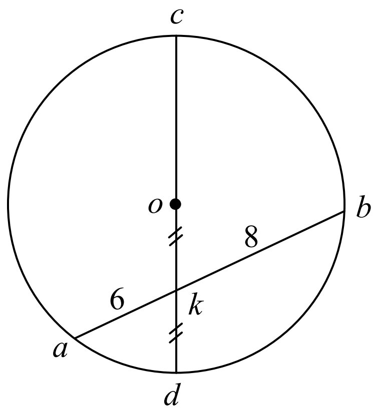

**(i)** Find $|\angle abc|$.

**(ii)** Find $|\angle cda|$.

**(b)** Prove that if $[ab]$ and $[cd]$ are chords of a circle and the lines $ab$ and $cd$ meet at the point $k$ inside the circle, then $|ak|.|kb| = |ck|.|kd|$.

**(c)** $[ab]$ and $[cd]$ are chords of the circle, centre $o$.
$[ab]$ bisects $[od]$ at the point $k$.
$|ak| = 6$ and $|kb| = 8$.

Find the length of the radius of the circle.

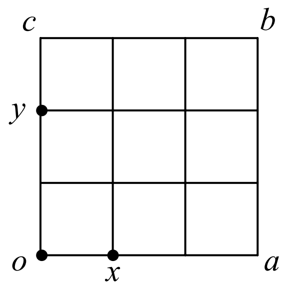



**9.** **(a)** $oabc$ is a square divided into nine equal squares.
$o$ is the origin and $x$ and $y$ are the points shown.

Copy the diagram and on it show

**(i)** the point $r$ such that $\vec{r} = \vec{x} + \vec{y}$

**(ii)** the point $s$ such that $\vec{s} = 2\vec{x} + \vec{y}$.




**(b)** Let $\vec{p} = 2\vec{i} - \vec{j}$ and $\vec{q} = -3\vec{i} + 4\vec{j}$.

**(i)** Find $|\vec{p}|$.

**(ii)** Express $5\vec{p} - \vec{q}$ in terms of $\vec{i}$ and $\vec{j}$.

**(iii)** Express $\overrightarrow{pq}$ in terms of $\vec{i}$ and $\vec{j}$.

**(iv)** Calculate $\vec{p}\,.\,\vec{q}$, the dot product of $\vec{p}$ and $\vec{q}$.

**(c)** Let $\vec{u} = 2\vec{i} + 5\vec{j}$ and $\vec{v} = 8\vec{i} + 10\vec{j}$.

**(i)** Find the scalars $h$ and $k$ such that $\vec{u} + h\vec{v} = k\vec{i}$.

**(ii)** Using your values for $h$ and $k$, verify that $\vec{u}^{\perp} + h\vec{v}^{\perp} = k\vec{i}^{\perp}$.

**10.** **(a)** Find the sum to infinity of the geometric series

$$2 + \frac{2}{5} + \frac{2}{25} + \ldots$$

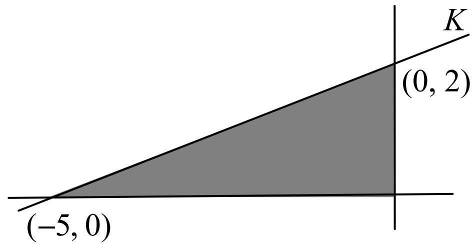
**(b)** **(i)** Expand $(1+2x)^3$ fully.

**(ii)** Given that $(1+2x)^3 + (1-2x)^3 = 2(a+bx^2)$, for all $x$,
find the value of $a$ and the value of $b$.

**(c)** **(i)** €2000 is invested at 4% per annum compound interest.
Find the value of the investment at the end of six years, correct to the nearest euro.

**(ii)** An investment account earns 4% per annum compound interest.
At the beginning of each year for six consecutive years €2000 is invested in the account.
Using the formula for the sum of the first $n$ terms of a geometric series, find the total value of the investment account at the end of the six years, correct to the nearest euro.

**11.** **(a)** The line $K$ cuts the $x$-axis at $(-5, 0)$ and the $y$-axis at $(0, 2)$.




**(i)** Find the equation of $K$.

**(ii)** Write down the three inequalities that together define the region enclosed by $K$, the $x$-axis and the $y$-axis.

**(b)** A developer is planning a holiday complex of cottages and apartments.
Each cottage will accommodate 3 adults and 5 children and each apartment will accommodate 2 adults and 2 children.
The other facilities in the complex are designed for a maximum of 60 adults and a maximum of 80 children.

**(i)** Taking $x$ as the number of cottages and $y$ as the number of apartments, write down two inequalities in $x$ and $y$ and illustrate these on graph paper.

**(ii)** If the rental income per night will be €65 for a cottage and €40 for an apartment, how many of each should the developer include in the complex to maximise potential rental income?

**(iii)** If the construction costs are €200 000 for a cottage and €120 000 for an apartment, how many of each should the developer include in the complex to minimise construction costs?