# Coimisiún na Scrúduithe Stáit
# State Examinations Commission

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## LEAVING  CERTIFICATE  EXAMINATION, 2006

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# MATHEMATICS − ORDINARY  LEVEL

## PAPER 1  (300 marks)

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## THURSDAY, 8 JUNE − MORNING, 9:30 to 12:00

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Attempt **SIX QUESTIONS**  (50 marks each).

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**WARNING:  Marks will be lost if all necessary work is not clearly shown.**

**Answers should include the appropriate units of measurement, where relevant.**

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**1.** **(a)** €320 is $\dfrac{4}{9}$ of a prize fund.   Find the total prize fund.

**(b)** Aoife pays a fixed monthly charge of €15 for her mobile phone. This charge includes 100 free text messages and 50 minutes free call time each month.  Further call time costs 28 cent per minute and additional text messages cost 11 cent each.
In one month Aoife sends 140 text messages and her call time is 2 hours.

    **(i)** Find the total cost of her fixed charge, text messages and call time.

    **(ii)** VAT is added to this cost at the rate of 21%.
Find the amount paid, including VAT.

**(c)** The standard rate of income tax is 20% and the higher rate is 42%.
Colm has weekly tax credits of €50 and a standard-rate cut-off point of €240.
Until recently, Colm had a gross weekly income of €900.

    **(i)** Calculate the tax Colm paid each week.

    **(ii)** After getting a pay rise, Colm's weekly after-tax income increased by €20.30.
Calculate the increase in Colm's gross weekly income.

**2.** **(a)** Simplify   $3(2x+4) - 5(x+1)$.

**(b)** Let $f(x) = 2x^3 + ax^2 + bx + 14$.

    **(i)** Express $f(2)$ in terms of $a$ and $b$.

    **(ii)** If $f(2) = 0$ and $f(-1) = 0$, find the value of $a$ and the value of $b$.

**(c)** **(i)** Find the smallest natural number $k$ such that
$$2x + 4(x+3) + 7(2x+4) \; < \; 20(x+k).$$

    **(ii)** The lengths of the sides of a triangle are
$$4\sqrt{x}, \quad (x-4) \quad \text{and} \quad (x+4), \qquad \text{where} \; x > 4.$$

    Prove that the triangle is right-angled.

**3.** **(a)** Find the value of $\dfrac{ab-c}{2}$ when $a=3$, $b=\dfrac{2}{3}$ and $c=1$.

**(b)** Solve for $x$ and $y$
$$x-2y=10$$
$$x^2+y^2=20.$$

**(c)** Solve for $x$
$$x = \frac{3+2x}{x-2}, \quad x \neq 2$$

and give your solutions in the form $a \pm \sqrt{b}$, where $a, b \in \mathbf{N}$.

Write one of your solutions correct to two decimal places. Using this value, show that the difference between the values of the left hand side and the right hand side of the given equation is less than 0.1.

**4.** **(a)** Let $u=3-6i$ where $i^2=-1$.
Calculate $|u+2i|$.

**(b)** **(i)** Solve $z^2-4z+29=0$.
Write your answers in the form $x+yi$ where $x, y \in \mathbf{R}$.

**(ii)** Write in its simplest form $i\left(i^4+i^5+i^6\right)$.

**(c)** **(i)** Express $\dfrac{3-2i}{1-4i}$ in the form $x+yi$.

**(ii)** Hence, or otherwise, find the values of the real numbers $p$ and $q$ such that
$$p+2qi = \frac{17(3-2i)}{1-4i}.$$

**5.** **(a)** The first term of an arithmetic sequence is 17 and the common difference is $-8$.
Find, in terms of $n$, an expression for $T_n$, the $n$th term.

**(b)** The $n$th term of a geometric series is
$$T_n = 4\left(\frac{1}{2}\right)^n.$$

**(i)** Find $a$, the first term.

**(ii)** Find $r$, the common ratio.

**(iii)** Write $4 - S_{10}$ in the form $\dfrac{1}{2^k}$, $k \in \mathbf{N}$, where $S_{10}$ is the sum of the first ten terms.
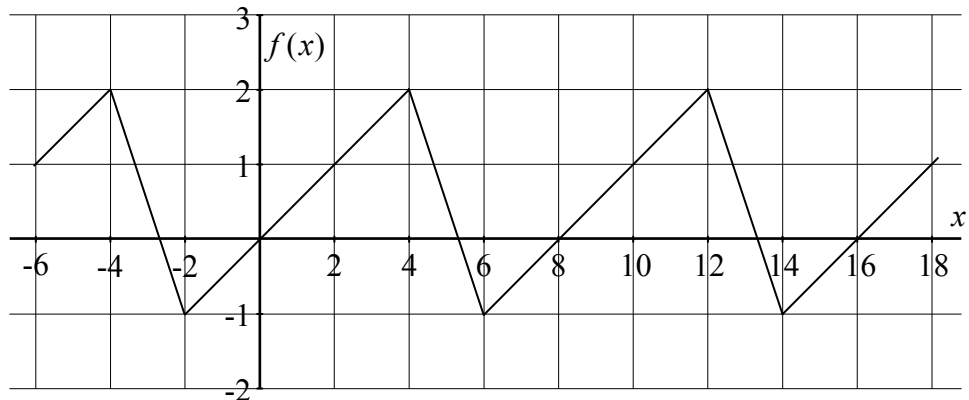
**(c)** The first three terms of an arithmetic sequence are
$$h + 3, \quad 5h - 2, \quad 6h - 13$$
where $h$ is a real number.

**(i)** Find the value of $h$.

**(ii)** Hence, write down the value of each of the first three terms.

**(iii)** Find the value of the eleventh term.

**6. (a)** $f : x \rightarrow f(x)$ is a periodic function defined for $x \in \mathbf{R}$.
The period is as indicated in the diagram.



   **(i)** Write down the period and the range of the function.

   **(ii)** Find $f(44)$.

**(b)** The temperature, $C$, in degrees Celsius, of a liquid in an insulated container is related to time $t$, in hours, by
$$C = 86 - 6t.$$

   **(i)** Draw the straight line graph of this relation, putting $t$ on the horizontal axis, for $0 \leq t \leq 8$.

   **(ii)** Use your graph to estimate the temperature when $t = 5.5$ hours.

   **(iii)** Use your graph to estimate the time it takes for the temperature to fall from 80 degrees to 60 degrees.

**(c)** Let $f(x) = 3 + 8x - 2x^2$, $x \in \mathbf{R}$.

   **(i)** Find the co-ordinates of the point at which the curve $y = f(x)$ cuts the $y$-axis.

   **(ii)** Find the value of $x$ for which $f(x)$ is a maximum.

   **(iii)** For what range of values of $x$ is $f'(x) > 4$?

**7.** **(a)** Differentiate $5x^3 - 4x + 7$ with respect to $x$.

**(b)** **(i)** Differentiate $\dfrac{x^2 - 1}{x + 1}$ with respect to $x$ and write your answer in its simplest form.

**(ii)** Given that $y = (5 - x^2)^3$, find $\dfrac{dy}{dx}$ when $x = 2$.

**(c)** A missile is fired straight up in the air. The height, $h$ metres, of the missile above the firing position is given by

$$h = t(200 - 5t)$$

where $t$ is the time in seconds from the instant the missile was fired.

**(i)** Find the speed of the missile after 10 seconds.

**(ii)** Find the acceleration of the missile.

**(iii)** One second before reaching its greatest possible height, the missile strikes a target. Find the height of the target.

**8.** **(a)** Let $g(x) = \dfrac{3}{x + 1}$, $x \in \mathbf{R}$, $x \neq -1$.

Evaluate $g(0.5) - g(-0.5)$.

**(b)** Let $h(x) = x^2 + 2x - 1$, $x \in \mathbf{R}$.

**(i)** Simplify $h(x - 5)$.

**(ii)** Find the value of $x$ for which $h(x - 5) = h(x) - 5$.

**(c)** Let $f(x) = \dfrac{1}{x - 2}$, $x \in \mathbf{R}$, $x \neq 2$.

**(i)** Find $f'(x)$, the derivative of $f(x)$.

**(ii)** Find the values of $x$ for which $f'(x) = -1$.

**(iii)** Find the co-ordinates of the two points on the curve $y = f(x)$ at which the slope of the tangent is $-1$.

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