Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2005

MATHEMATICS – ORDINARY LEVEL

PAPER 1 (300 marks)

THURSDAY, 9 JUNE – MORNING, 9:30 to 12:00

Attempt **SIX QUESTIONS** (50 marks each).

WARNING: **Marks will be lost if all necessary work is not clearly shown.**

Answers should include the appropriate units of measurement, where relevant.

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1. **(a)** Express 35 cm as a fraction of 1 m. Give your answer in its simplest form.

(b) **(i)** The approximation 50×80 was used for the calculation 51×79.
Find the percentage error, correct to one decimal place.

(ii) Express the ratio $\dfrac{1}{2} : \dfrac{1}{3} : \dfrac{1}{4}$ as a ratio of natural numbers.

Divide 325 in the ratio $\dfrac{1}{2} : \dfrac{1}{3} : \dfrac{1}{4}$.

(c) At the start of the year 2000 the population of a particular town was P.
During the year 2000, the population of the town increased by 10%.

(i) Express, in terms of P, the population of the town at the end of the year 2000.

(ii) During the year 2001 the population of the town increased by 4%.
During the year 2002 the population increased by 2%.
Find the total percentage increase in the population of the town over the three years.

(iii) The actual increase in the population was 8344. Find the value of P.

2. **(a)** Find the value of $x^2 - 5xy$ when $x = 3$ and $y = -2$.

(b) Solve for x and y
$$x + 3 = 2y$$
$$xy - 7y + 8 = 0.$$

(c) **(i)** Write $\sqrt{x} + \dfrac{1}{\sqrt{x}}$ as a single fraction.

(ii) Hence, or otherwise, simplify $\left(\dfrac{2\sqrt{x}}{1 + x}\right)\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)$.

(iii) Solve for x
$$\left(\dfrac{2\sqrt{x}}{1 + x}\right)\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right) = x - 3.$$

3. **(a)** Given that $ax + b = c$, express x in terms of a, b and c, where $a \neq 0$.

(b) **(i)** Find A, the solution set of $3x - 2 \leq 4$, $x \in \mathbf{Z}$.

(ii) Find B, the solution set of $\dfrac{1 - 3x}{2} < 5$, $x \in \mathbf{Z}$.

(iii) List the elements of $A \cap B$.

(c) Let $f(x) = 2x^3 - 3x^2 - 11x + 6$.

(i) Verify that $f(3) = 0$.

(ii) Solve the equation
$$2x^3 - 3x^2 - 11x + 6 = 0.$$

4. **(a)** Let $u = 4 - 2i$, where $i^2 = -1$.

Plot

(i) u

(ii) $u - 4$

on an Argand diagram.

(b) Let $w = 1 + 3i$.

(i) Express $\dfrac{2}{w}$ in the form $x + yi$, where $x, y \in \mathbf{R}$.

(ii) Investigate whether $|iw + w| = |iw| + |w|$.

(c) Let $z = 1 - 2i$.

(i) Write down $\bar{z}$, the complex conjugate of z.

(ii) Find the real numbers k and t such that
$$kz + t\bar{z} = 2z^2.$$

5. **(a)** The first term of an arithmetic sequence is 9 and the second term is 13.

     (i)   Find the common difference.

     (ii)   Find the third term.

(b) The sum of the first n terms of an arithmetic series is given by

$$S_n = n^2 + n.$$

     (i)   Find a, the first term.

     (ii)   Find S_2, the sum of the first two terms.

     (iii) Find d, the common difference.

     (iv) Write down the first five terms of the series.

(c) In a geometric sequence of positive terms, the third term is $\dfrac{1}{4}$ and the fifth term is $\dfrac{1}{16}$.

     (i)   Find r, the common ratio.

     (ii)   Find a, the first term.

     (iii) How many terms of the sequence are greater than 0.01?

6. **(a)** Let $g(x) = \dfrac{x+5}{2}, \quad x \in \mathbf{R}.$

     Find $g(0) + g(2)$.

(b) Differentiate $3x - x^2$ with respect to x from first principles.

(c) Let $f(x) = x^2 + px + 10, \quad x \in \mathbf{R}, \quad$ where $p \in \mathbf{Z}.$

     (i)   Find $f'(x)$, the derivative of $f(x)$.

     (ii)   The minimum value of $f(x)$ is at $x = 3$. Find the value of p.

     (iii) Find the equation of the tangent to $f(x)$ at the point $(0, 10)$.

7. **(a)** Differentiate $9 + 3x - 5x^2$ with respect to x.

(b) **(i)** Differentiate $(3x^2 - 2)(x^2 + 4)$ with respect to x.

(ii) Given that $y = \dfrac{x^2}{x-1}$, find $\dfrac{dy}{dx}$ when $x = 3$.

(c) A car begins to slow down at p in order to stop at a red traffic light at q.

p q

The distance of the car from p, after t seconds, is given by

$$s = 12t - \frac{3}{2}t^2$$

where s is in metres.

(i) Find the speed of the car as it passes p.

(ii) Find the time taken to stop.

(iii) The car stops exactly at q. Find the distance from p to q.

8. Let $f(x) = \dfrac{1}{x-1}$, $x \in \mathbf{R}$, $x \neq 1$.

(i) Find $f(-3)$, $f(-1.5)$, $f(0.5)$, $f(1.5)$, $f(5)$.

(ii) Draw the graph of the function f from $x = -3$ to $x = 5$.

(iii) On the same diagram, draw the graph of the function

$$g(x) = x + 1$$

in the domain $-2 \leq x \leq 2$, $x \in \mathbf{R}$.

(iv) Use your graphs to estimate the values of x for which $f(x) = g(x)$.

(v) Find, using algebra, the values of x for which $f(x) = g(x)$.

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