Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2004

MATHEMATICS – ORDINARY LEVEL

PAPER 1 (300 marks)

THURSDAY, 10 JUNE – MORNING, 9:30 to 12:00

Attempt **SIX QUESTIONS** (50 marks each).

WARNING: Marks will be lost if all necessary work is not clearly shown.

1. **(a)** There are 240 eggs in a box.
2.5% of the eggs are broken.
Find the number of eggs that are broken.

(b) The standard rate of income tax is 20% and the higher rate is 42%.
Orla has a gross income of €58 000 for the year and a standard-rate cut-off point of €35 000.

(i) Calculate the amount of tax due at the standard rate.

(ii) Calculate the total amount of gross tax due.

(iii) Orla has tax credits of €3400 for the year.
After tax is paid, what is Orla's income for the year?

(c) A faulty petrol pump actually delivers 1.02 litres of petrol for every 1 litre that the pump registers. During one day the pump registers 2650 litres.

(i) What was the actual volume of petrol delivered?

(ii) Customers paid 85 cent for every litre of petrol registered.
Find the total amount paid for the petrol.

(iii) If the pump had registered the correct volume delivered, how much more would have been paid?

2. **(a)** Find the value of $3(2p - q)$ when $p = -4$ and $q = 5$.

(b) **(i)** Solve $2x^2 - 7x + 3 = 0$.

(ii) Show that $x - 2$ is a factor of $x^3 - 3x^2 - x + 6$.

(c) **(i)** Evaluate $8^{\frac{1}{3}}$.

(ii) Express $4^{\frac{1}{4}}$ in the form 2^k, $k \in \mathbf{Q}$.

(iii) Solve for x the equation

$$\left(8^{\frac{1}{3}}\right)\left(4^{\frac{1}{4}}\right) = 2^{5-x}.$$

3. **(a)** Solve for x

$$2x = 3(5 - x).$$

(b) Solve for x and y

$$x + y = 1$$
$$x^2 + y^2 = 13.$$

(c) p is a positive number and f is the function $f(x) = (2x + p)(x - p)$, $x \in \mathbf{R}$.

(i) Given that $f(2) = 0$, find the value of p.

(ii) Hence, find the range of values of x for which $f(x) < 0$.

4. **(a)** Given that $i^2 = -1$, simplify

$$4(2 - i) + i(3 + 5i)$$

and write your answer in the form $x + yi$, where $x, y \in \mathbf{R}$.

(b) **(i)** Let $w = 1 - 2i$.
Plot w and $\overline{w}$ on an Argand diagram, where $\overline{w}$ is the complex conjugate of w.

(ii) Solve $z^2 - 10z + 26 = 0$.
Write your answers in the form $a + bi$, where $a, b \in \mathbf{R}$.

(c) Let $z_1 = 5 + 12i$ and $z_2 = 2 - 3i$.

(i) Find the value of the real number k such that $|z_1| = k|z_2|$.

(ii) p and q are real numbers such that

$$\frac{z_1}{z_2} = p(q + i).$$

Find the value of p and the value of q.

5. **(a)** The first term of an arithmetic sequence is 40 and the common difference is –5.
Write down the first five terms of the sequence.

(b) The *n*th term of an arithmetic series is given by

$$T_n = 1 + 5n.$$

(i) The first term is a and the common difference is d.
Find the value of a and the value of d.

(ii) Find the value of n for which $T_n = 156$.

(iii) Find S_{12}, the sum of the first 12 terms.

(c) The first term of a geometric series is 1 and the common ratio is −4.

(i) Write down the first three terms of the series.

(ii) Find S_6, the sum of the first 6 terms.

(iii) Show that $16S_4 - 3 = S_6$, where S_4 is the sum of the first 4 terms.

6. **(a)** Let $g(x) = 1 - kx$.
Given that $g(-3) = 13$, find the value of k.

(b) Let $f(x) = x^3 - 3x^2 + 1, \quad x \in \mathbf{R}$.

(i) Find $f(-1)$ and $f(3)$.

(ii) Find $f'(x)$, the derivative of $f(x)$.

(iii) Find the co-ordinates of the local maximum point and of the local minimum point of the curve $y = f(x)$.

(iv) Draw the graph of the function f in the domain $-1 \le x \le 3$.

Use your graph to:

(v) estimate the range of values of x for which $f(x) < 0$ and $x > 0$

(vi) estimate the range of values of x for which $f'(x) < 0$.

7. **(a)** Differentiate with respect to x:

(i) $2x^5$

(ii) $4(3-x^2)$.

(b) **(i)** Differentiate $(x^2-4)(x^2+3x)$ with respect to x.

(ii) Given that $y=(x^2-2x-3)^3$, show that $\frac{dy}{dx}=0$ when $x=1$.

(c) A jet is moving along an airport runway. At the instant it passes a marker it begins to accelerate for take-off. From the time the jet passes the marker, its distance from the marker is given by

$$s=2t^2+3t,$$

where s is in metres and t is in seconds.

(i) Find the speed of the jet at the instant it passes the marker ($t=0$).

(ii) The jet has to reach a speed of 83 metres per second to take off. After how many seconds will the jet reach this speed?

(iii) How far is the jet from the marker at that time?

(iv) Find the acceleration of the jet.

8. **(a)** Let $g(x)=3x-7$.

(i) Find $g(7)$.

(ii) Find the value of k for which $g(7)=k[g(0)]$.

(b) Differentiate x^2+3x with respect to x from first principles.

(c) Let $f(x)=\frac{1}{x+3}$, $x\in\mathbf{R}$, $x\neq -3$.

(i) Find $f'(x)$, the derivative of $f(x)$.

(ii) There are two points on the curve $y=f(x)$ at which the slope of the tangent is -1. Find the co-ordinates of these two points.

(iii) Show that no tangent to the curve $y=f(x)$ has a slope of 1.

Blank Page