AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA | DEPARTMENT OF EDUCATION AND SCIENCE

Scéim Mharcála — *Scrúduithe Ardteistiméireachta, 2002*

Matamaitic — *Gnáthleibhéal*

Marking Scheme — *Leaving Certificate Examination, 2002*

Mathematics — *Ordinary Level*

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2002

MATHEMATICS

ORDINARY LEVEL

PAPER 1

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3,......, S1, S2, S3,...., M1, M2, etc. Note that these lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), it is essential to note that
   - any correct relevant step in a part of a question merits *at least* the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,....etc.

4. The *same* error in the *same* section of a question is penalised *once* only.

5. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

6. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks only.

7. The phrase “and stops” means that no more work is shown by the candidate.

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

> Copper and zinc are mixed in the ratio 19 : 6.
> The amount of copper used is 133 kg.
> How many kilogrammes of zinc are used?

(a) **10 marks** **Att 3**

I:	II:		III:
$\frac{19}{25}$ths = 133 …(3m)			
$\frac{1}{25}$th = $\frac{133}{19}$ or 7	$\frac{133}{19}$ or 7	or $\frac{133 \times 6}{19}$	$\frac{1}{25}$th = $\frac{133}{19}$ or 7 …(7m)
Zn = 6(7) = 42	Zn = 6(7) = 42	= 42	=> total = 175 => Zn = 175 – 133 = 42 ..(10m)

IV:		
$\frac{19}{6}$ or 3 1/6	or $\frac{6}{19}$	…(3m)
133 ÷ 3 1/6	133 x (6/19) as in II(b)	..(7m)
= 42	= 42	..(10m)

* If decimals are used instead of fractions, work should be correct to at least 1 decimal place: otherwise S2 applies.
* Allow, without penalty: "19 = 133, 1 = 7 .: 6 = 42"
* Accept 42 without work for full marks.
* Incorrect answer without work, zero marks.

Blunders (-3)
B1 Takes 133 as the total, e.g 133 / 25 and continues.
B2 Inverted or incorrect fraction, and continues, e.g. 1/6 = 133 and continues.
B3 13/19 used.
B4 In effect, takes Zn : Cu as 19:6, e.g. in method I, 6/25 = 133 => 19/25 = 421.16.

Slips (-1)
S1 Numerical slips in multiplication or division.
S2 Rounding error. Once only.

Attempts (3 marks)
A1 Mentions 25 or 1/19 or 1/6 and stops.

Part (b) **20 marks (10, 10)** **Att (3, 3)**

> Four telephone calls cost €3.85, €7.45, €8.40 and €11.55.
> **(i)** John estimates the total cost of the four calls by ignoring the cent part in the cost of each call. Calculate the percentage error in his estimate.
> **(ii)** Anne estimates the total cost of the four calls by rounding the cost of each call to the nearest euro. Calculate the percentage error in her estimate.

(b) (i) **10 marks** **Att 3**

I: 3.85 + 7.45 + 8.40 + 11.55 = 31.25	II: Errors = .85 + .45 + .40 + .55 = 2.25 …(3m)
Est= 3+7+ 8 +11 = 29 => error = 2.25	3.85 + 7.45 + 8.40 + 11.55 = 31.25 …(7m)
% error = $\frac{2.25}{31.25}$ x 100 = 7.2 %	% error = $\frac{2.25}{31.25}$ x 100 = 7.2 % ..(10m)

* Correct answer without work: 10 marks.
* 'Total' found or 31.25, and 29, and stops: 4 marks.
* 29 ÷ 31.25 = 0.928 and stops: 4 marks.

Blunders (-3)
B1 Actual values not added at any stage.
B2 (2.25 / 29) x 100% = 7.758%, OR (29 / 31.25) x 100% = 92.8 and stops.
B3 Incorrect 'estimate', e.g. 3.8, 7.4, 8.4, 11.5; or 3.9, 7.5, 8.4, 11.6. Apply once.
OR: incorrect error (if not S1 below), e.g. error = .15 + .55 + .6 + .45. Apply Once.
B5 Failure to multiply by 100, e.g. answer = 0.072%, or % error = error/100.

Slips (-1)
S1 Totting error.
S2 (2.25 ÷ 31.25) x 100 = 7%. (Not a decimal blunder).

Attempts (3 marks)
A1 Any correct estimate, or John's total 'estimate' = 29 and stops.
A2 Any individual 'error' correct (e.g. 3.85 => error .85), or an average of individual errors.
A3 Any correct addition and stops, e.g. adds the errors (0.85 + 0.45 + …) and stops.

Worthless (0)
W1 Incorrect answers without work.

(b) (ii) **10 marks** **Att 3**

I: 4 + 7 + 8 + 12 = 31	II: Anne's est. costs 4, 7 , 8 and 12 …(3m)
error = 31.25 – 31 = 0.25	Differences = +.15 – .45 – .40 + .45 = – .25 *or* Differences = –.15 + .45 + .40 – .55 = .25 …(7m)
% error = $\frac{0.25}{31.25}$ x 100 = .8%	% error = $\frac{0.25}{31.25}$ x 100 = .8% ..(10m)

* Accept candidate's actual total from (i) without penalty.
* Correct answer without work: 10 marks.
* 'Total' found or 31, and 31.25, and stops: 4 marks.
* 31 ÷ 31.25 = 0.992 and stops: 4 marks.

Blunders (-3)

B1 Rounded values not added at any stage.
B2 (0.25 / 31) x 100% = 0.8 %
B3 (31 / 31.25) x 100% = 99.2 and stops.
B4 Incorrect 'estimate'. Apply once.
B5 Failure to multiply by 100, e.g. answer = 0.008%, or % error = error/100.

Attempts (3 marks)
A1 Any correct rounding ('estimate'), or addition of estimates.
A2 Any correct (new) addition.
A3 31.25, or candidate's actual total from (i), written down again in (ii).

Worthless (0)
W1 Incorrect answers without work.

Part (c) **20 marks (10, 5, 5)** **Att (3, 2, 2)**

A raffle to raise money for a charity is being held.
The first prize is €100, the second is €85, the third is €65 and the fourth is €50.
The cost of printing tickets is €42 for the first 500 tickets and €6 for each additional 100 tickets. The smallest number of tickets that can be printed is 500.
Tickets are being sold at €1.50 each.

(i) What is the minimum possible cost of holding the raffle?
(ii) If 500 tickets are printed, how many tickets must be sold in order to avoid a loss?
(iii) If 1000 tickets are printed and 65% of the tickets are sold, how much money will be raised for the charity?

(c)(i) **10 marks** **Att 3**

100 + 85 + 65 + 50 = 300 …(7m)

+ 42 = 342 ..(10m)

* Candidates needed to sift and select the relevant information, a lot of it for part (i).

Blunders (-3)
B1 300 and stops. (i.e. 7m).
B2 300 + incorrect amount, [e.g. 300 + 42(500), or 300(42) + 500]: 7 marks.
B3 Cost of prizes (300) not included, e.g. 'cost = 42', even without work, merits 7m.

Slips (-1)
S1 Totting error.

Attempts (3 marks)
A1 Any correct (but partial) addition.

(c)(ii) **5 marks** **Att 2**

I: $\frac{342}{1.5}$ … (2m) = 228 …(5m)	II: $\frac{342}{750}$ x 500 …(2m) = 228 …(5m) [Ignore the addition of 1 to 228]

* No penalty if [candidate's answer to (c)(i), e.g. 42 or 300] is used in place of €342.
* Accept 228 (or 229) without work.

Blunders (-3)
B1 Full costs, as in candidate's answer to (c)(i), not used – – if not already penalised.
Example: Answer (c)(i) = 342. Continues here with 300 ÷ 1.5 , or 42 ÷ 1.5.
B2 500 ÷ 1.5 = 333.33 (or 333 or 334).
B3 (500 tickets @ €1.50) *less* [costs from (c)(i)], e.g. 500 x 1.50 – 342 = 408.

Attempts (2 marks)
A1 Trial and error, if unsuccessful.
A2 500(1.5) = 750.

Worthless (0)
W1 Answer 30, or 30 + 42 = 72.

(c)(iii) **5 marks** **Att 2**

I: 65% of 1000 = 650 @ €1.50 each = €975	II: Cost = 342 + 5(6) = €372	…(2m)
Cost = 342 + 5(6) = €372	65% of 1000 = 650 @ €1.50 each = €975	
Profit = 975 – 372 = €603	Profit = 975 – 372 = €603	...(5m)

* Accept: cost = candidate's Answer (c)(i) + 5(6).
* Order of the first two steps is reversible along with the marks assigned.
* 65% of (1000 x €1.50) – €372 = €603 is the basis of the methods above, but 65% of [(1000 x €1.50) – €372] = 65% of [€1500 – €372] = 65% of €1128 = €733.20: B3.
* "Cost = 42 + 30 = 72": Att 2m.
* 65% of candidate's 'cost' calculated, e.g. 65% of 300, 342, 372, 42 or 72: Att 2m.

Blunders (-3)
B1 Cost = €342, or cost = €348, or cost = €342 + €42.
B2 Profit = 650 – 372 = €278.
B3 Incorrect handling of expenses, e.g. omitting the extra €30. See also note 1 above.
B4 Full costs (prizes + printing) incorrect, i.e. not equal to candidate's answer (c)(i) + €30.
B5 Omits the last line of either method.

Attempts (2 marks)
A1 Calculates 650 and/or 975 and/or 372 and/or 348, and stops.

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 6**

Part (a)	**10 marks**	**Att 3**

Solve for x

$$\frac{x-7}{2}=\frac{x+3}{6}.$$

(a)	**10 marks**	**Att 3**

I:	II:	III:	IV:
6(x–7) = 2(x + 3)	$\frac{6(x-7)}{2} = \frac{6(x+3)}{6}$	$\frac{3(x-7)}{6} = \frac{x+3}{6}$	½ x – 3½ = 1/6x + ½
6x – 42 = 2x + 6	3x – 21 = x + 3	3x – 21 = x + 3	½ x – 1/6x = 3½ + ½
4x = 48 => x = 12	2x = 24 => x = 12	2x = 24 => x = 12	1/3 x = 4 => x = 12

* Three steps in each method: 3m, 7m, 10m.
* Correct answer without work: if verified, award 10m; if not verified, Att 3.
* Trial/error substitutions: if correct answer found,10m; if not found, 0m.

Blunders (-3)

B1 (x /2) – 7 = (x/6) + 3 and continues. See A2.
B2 (6x – 42)/(2x + 6) = k, where k ≠ 1, and continues. See A1.
B3 2(x – 7) = 6(x+ 3) and continues. See W1.
B4 Brackets error, e.g. 6(x – 7) = 6x – 7 and/or 2(x + 3) = 2x + 3. Apply once if consistent.
B5 Transposition or sign error. Each time.
B6 Fraction error, e.g. ½ – 1/6 ≠ 1/3. Each time.

Slips (-1)

S1 Arithmetical, e.g. 6(x – 7) = 6x – 35.

Attempts (3 marks)

A1 6(x – 7) or 6x – 42 or 2(x + 3) or 2x + 6 mentioned.
A2 (x /2) – 7 = (x/6) + 3 and stops.

Worthless (0)

W1 2(x – 7) = 6(x + 3) and stops.

Part (b)	**20 marks (10, 10)**	**Att (3, 3)**

> **(i)** Show that $x+2$ is a factor of $2x^3+7x^2+x-10$.
>
> **(ii)** Hence, or otherwise, find the three roots of $2x^3+7x^2+x-10=0$.

(b) (i)	**10 marks**	**Att 3**

I:	II:
f(-2) ...(3m)	$x+2 \,\big)\, 2x^3+7x^2+x-10$ ...set up (3m), quotient $2x^2+3x-5$
$= 2(-2)^3+7(-2)^2+(-2)-10$ ...(7m)	$2x^3+4x^2$ ...(7m)
	$3x^2+x$
	$3x^2+6x$
	$-5x-10$
$=-16+28-2-10=0$ ...(10m)	$-5x-10$ ..(10m)

* Check (b)(ii) first -- if (b)(ii) done fully, then see Note 1 in (b)(ii).

* In method I above, candidate has 'x = –2 is a root', but does not do (b)(ii): award 10m for (b)(i) and Att 3m for (b)(ii).

* Candidate divides correctly as in method II above, but doesn't do (b)(ii): award 10 marks for (b)(i) and 4 marks for (b)(ii) -- see Method I of (b)(ii).

* Divides x + 2 correctly into f(x), concludes that x + 2 is a 'root': no penalty in this part.

* Final numbers (step 3 of method I) need not be added if they sum to zero. If the numbers don't sum to zero, they must be totted and a correct conclusion drawn: otherwise, apply B(–3) for an incorrect or no conclusion.

Blunders (-3 marks)

B1 Error in algebraic division, to a max. of 2 blunders. e.g. error in mult., division, addition, subtraction (e.g. signs), cancellations etc.

B2 Mathematical error in indices or brackets, e.g. $(-2)^3=-6$. Once per line if consistent.

B3 Incorrect root from factor, e.g. substitutes x = 2 and/or then divides by x – 2.

B4 Missing or incomplete step.

Slips (-1)

S1 Arithmetical slips in method I (but not sign errors).

Attempt (3 marks)

A1 First line of any correct method and stops.

A2 Tries to find f(2), e.g. $2(2)^3$, or f(0) found.

A3 Says x = – 2 is a root and stops.

Worthless (0)

W1 Substitutes (x + 2), i.e. $f(x+2)=2(x+2)^3+7(x+2)^2+(x+2)-10$ and continues.

(b) (ii) **10 marks** **Att 3**

Method I

$x = -2$ or $f(-2)$ anywhere in (b) ...(3m)

$$\begin{array}{r} 2x^2 + 3x - 5 \\ x+2 \,\big)\, 2x^3 + 7x^2 + x - 10 \\ \underline{2x^3 + 4x^2} \qquad\qquad \\ 3x^2 + x \qquad\quad \\ \underline{3x^2 + 6x} \qquad\quad \\ -5x - 10 \\ \underline{-5x - 10} \end{array}$$

$2x^2 + 3x - 5$ ...(4m)

$(x-1)(2x+5)$ ...(7m)

$x = 1,\ x = -5/2$ ...(10m)

Method II

A continuation of

Method II of (b)(i):

$2x^2 + 3x - 5$ ...(4m)

$(x - 1)(2x + 5)$ ...(7m)

$x = 1,\ x = -5/2, -2$ ...(10m)

Method III

$f(1) = 0$ or 1 is a root ...(3m)

$x - 1$ is a factor

$$\begin{array}{r} 2x^2 + 9x + 10 \\ x-1 \,\big)\, 2x^3 + 7x^2 + x - 10 \\ \underline{2x^3 - 2x^2} \qquad\qquad \\ 9x^2 + x \qquad\quad \\ \underline{9x^2 - 9x} \qquad\quad \\ 10x - 10 \\ \underline{10x - 10} \end{array}$$

$2x^2 + 9x + 10$ ...(4m)

$(x+2)(2x+5)$ ...(7m)

$x = -2,\ x = -5/2$, and 1 ..(10m)

Method IV:

$f(1) = 0$ or 1 is a root ...(3m)

$(x - 1)(x + 2) = x^2 + x - 2$ ...(7m)

$$\begin{array}{r} 2x + 5 \\ x^2 + x - 2 \,\big)\, 2x^3 + 7x^2 + x - 10 \\ \underline{2x^3 + 2x^2 - 4x} \qquad \\ 5x^2 + 5x - 10 \\ \underline{5x^2 + 5x - 10} \end{array}$$

$x = -5/2$, and -2 ...(10m)

Method V:

$f(1) = 0$ or 1 is a root ...(3m)

Divides $x - 1$ into $2x^3 + 7x^2 + x - 10$ to get $2x^2 + 9x + 10$ ...(4m)

Then divides $x + 2$ into $2x^2 + 9x + 10$ to get $2x + 5$ ...(7m)

$x = -5/2$, and -2. ...(10m)

Method VI: (Synthetic division)

...(3m)

$\boxed{-2}$	2	7	1	-10
		-4	-6	10
	2	3	-5	0

$\Rightarrow\ 2x^2 + 3x - 5$ ...(4m)

$(x - 1)(2x + 5)$ ...(7m)

$x = 1, -5/2, -2$ ...(10m)

Or, synthetic division by $x - 1$.

Method VII: $f(x) = (x + 2)(2x^2 + ax - 5)$ ...for setting up factors ...(3m)

mult. out $[2x^3 + (a+4)x^2 + (2a-5)x - 10]$; comparing coeff. $\Rightarrow a = 3 \Rightarrow 2x^2 + 3x - 5$ ...(4m)

and this quadratic solved to give $x = 1$ and $x = -5/2$. Also $x = -2$. ..(10m)

* If (b)(ii), as above, if given as an answer to both parts of (b), award 20m. But if '$x = -2$', 'root -2' or '$f(-2)$' is not mentioned anywhere in (b)(ii), then apply B(–3): award 10+7m.

* Draws graph of cubic and gets 3 roots from graph: allow 10+10 marks for roots 1,–2, –5/2. Mark candidate's work on slip and blunder out of 10 (Att 3) + 10 (Att 3).

Blunders (-3)

B1 Error in algebraic division, to a max. of 2 blunders (Mult., div., addn., subtr., signs, etc).

B2 Mathematical error in indices. Once per line if consistent.

B3 Incorrect factors. Apply once.

B4 Incorrect root or no root from factor(s), e.g. stops after finding factors. Apply once.

B5 Quadratic formula error (in formula, substitution or simplification). Max of 2 blunders.

B6 No indication that $x = -2$ is a root anywhere in (b). [States $x + 2$ a factor is insufficient].

Attempts (3 marks)

A1 Correct quadratic formula and stops. (If applied to cubic coefficients, still only Att marks).

A2 Effort to use Remainder Theorem, e.g. finds f(1) and stops.

A3 Differentiates the cubic. (Newton Raphson method relevant).

Part (c) **20 marks (10, 10)** **Att (3, 3)**

(i) Express b in terms of a and c where $\frac{8a-5b}{b}=c$.
(ii) Hence, or otherwise, evaluate b when $a=2^{\frac{5}{2}}$ and $c=3^3$.

(c)(i) **10 marks** **Att 3**

I: $8a - 5b = bc$ …(3m) cross–multiplies $8a = bc + 5b$ …(7m) b terms together $8a = b(c+5)$ $\frac{8a}{c+5} = b$ …(10m) divides	II: $\frac{8a}{b} - 5 = c$ …(3m) divides $\frac{8a}{b} = c + 5$ …(7m) transposes $\frac{8a}{c+5} = b$ …(10) finishes (swaps or cross–m.)

* A variation of method I: (8a – 5b)/b = (bc)/b => 8a – 5b = bc and continues as in I.
* A variation of method II: line 2 => b/(8a) = 1/(c + 5) => b = 8a / (c+5).
* b not fully isolated, e.g. b = (8a – 5b)/c: two blunders, award 4 marks.
* (8a – 5b)/b = c => 8a – 4b = c. Apply B1 twice. Could finish up with 4 marks.

Blunders (-3)
B1 Math. errors (multiplication, cross-mult., div., transposition, inversion, sign). Each time.
B2 Bracket error, e.g. b(c + 5) = bc + 5.
B3 Expressing – *b* in terms of *a* and *c*. [May occur in addition to note 3 above].
B4 Expressing *a* in terms of *b* and *c*.

Attempts (3 marks)
A1 Any unhelpful but correct alteration or transposition, e.g. (8a – 5b)/(b) – c = 0 and stops.
A2 8a – 5 = c.

Worthless (0)
W1 In one move, b = 8a – 5 + c, or b = (8a – 5b).c.
W2 Substitutes values at this stage.

(c)(ii) **10 marks** **Att 3**

I: $\frac{8.(2)^{5/2}}{3^3+5}$ …(3m) $= \frac{8.\sqrt{2^5}}{27+5}$ *or* $\frac{8.\sqrt{32}}{27+5}$ *or* $\frac{2^3.2^{5/2}}{27+5}$ $= \frac{8.4.\sqrt{2}}{32}$ | $= \frac{\sqrt{32}}{4}$ | $= \frac{2^{11/2}}{2^5}$ $= \sqrt{2}$ | $= \sqrt{2}$ | $= 2^{1/2}$	II: $\frac{8.(2)^{5/2}}{3^3+5}$ …(3m) $= \frac{8(5.656..)}{27+5}$ $= \frac{45.2548...}{32}$ $= 1.4$	III: $\frac{8.(2)^{5/2} - 5b}{b} = 3^3$ …(3m) $8.(2)^{5/2} - 5b = 27b$ $8.(2)^{5/2} = 32b$ $\frac{8.(2)^{5/2}}{32} = b$ $\frac{(2)^{5/2}}{4} = b \Rightarrow b = 2^{1/2}$

* Accept substitution into the candidate's answer from (c)(i).
* No steps after the first one (3m). Mark out of 10m using slips and blunders.

Blunders (-3)
B1 Error evaluating indices or square roots, e.g. $2^{5/2} = 2^5$ or 10/2 or 5. Apply each time.
B2 Transposition error. Apply each time.

Attempts (3 marks)
A1 $3^3 = 27$ or any correct relevant calculation, or a partial substitution.

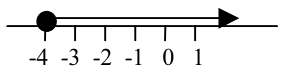
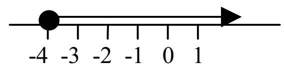
QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

Solve the inequality $5x + 1 \geq 4x - 3$ for $x \in \mathbf{R}$ and illustrate the solution set on a number line.

(a) **10 marks** **Att 3**

I: $5x - 4x \geq -3 - 1$ …(3m) $x \geq -4$ …(7m)  ..(10m)	II: $3 + 1 \geq 4x - 5x$ …(3m) $4 \geq -x$ $-4 \leq x$ or $x \geq -4$ …(7m)  ..(10m)

* Trial and error with substituted values: for x ≥ – 4, award 7m. May graph to get final 3m.
* x ≥ – 4 graphed correctly (with no other work): 10m.

Blunders (-3)
B1 Transposition or sign error. Apply once per step, e.g. 5x + 4x ≥ –3 + 1 is B1 once.
B2 Inequality sign error, e.g. fails to change inequality sign if required. (May rectify).
B3 Simplification error, e.g. – 3 –1 ≠ – 4, or 5x – 4x ≠ x.
B4 Graphing error, e.g. x ∉R, e.g. plots – 4, – 3, – 2, etc. on number line, or graphs x ≥ –3.
B5 Graph omitted or incorrect graphing of candidate’s result.

Misreadings (-1)
M1 Each misreading as long as it doesn’t cause the loss of x (as 5x + 1 ≥ 5x –3 would), e.g. 5x + 1 ≤ 4x – 3, or 5x + 1 > 4x – 3. [If 5x + 1 ≥ 5x –3 is continued, att 3 max.].

Slips (–1)
S1 Candidate’s graph excludes x = – 4, i.e. graphs x > – 4 only.

Attempts (3 marks)
A1 One step or part of a step correct.
A2 Any correct transposition, e.g. 5x + 1 – 4x ≥ – 3.
A3 5x + 1 = 4x – 3 solved *correctly,* x = – 4, and then stops.
A4 At least one value by trial/error substitution, tested correctly (e.g. x = 2 => 11 ≥ 5).
A5 Number line scaled (tick marks plus one number).

Worthless (0)
W1 5x + 1 = 4x – 3 and stops, or solved incorrectly.
W2 Incorrect answer without work.

Part (b)	**20 marks (15, 5)**	**Att (5, 2)**

(i) Solve for x and y

$$y = 10 - 2x$$

$$x^2 + y^2 = 25.$$

(ii) Hence, find the two possible values of $x^3 + y^3$.

(b) (i)	**15 marks**	**Att 5**

I :	II : $x = (10 - y)/2$
$x^2 + (10 - 2x)^2 = 25$ …(5m)	$\{(10 - y)/2\}^2 + y^2 = 25$ …(5m)
$x^2 + 100 - 40x + 4x^2 = 25$ …(6m)	$\{100 - 20y + y^2\}/4 + y^2 = 25$ …(6m)
	$100 - 20y + y^2 + 4y^2 = 100$
$5x^2 - 40x + 75 = 0$	$5y^2 - 20y = 0$
$x^2 - 8x + 15 = 0$	$y^2 - 4y = 0$
$(x - 5)(x - 3) = 0$ …(9m)	$y(y - 4) = 0$ …(9m)
$x = 5, \ x = 3$ ..(12m)	$y = 0, \ y = 4$ ..(12m)
$y = 0, \ y = 4.$ ..(15m)	$x = 5, \ x = 3$ ..(15m)

* If a graphical solution, or trial and error, yields the correct co-ordinates then they must be verified in both equations to earn full marks (15m); incomplete or no verification of correct co-ordinates merits the Att marks. Also, verification of *either* (3,4) or (5,0) in both equations merits Att marks. Incorrect co-ordinates by such methods merit 0 m.

* $x^2 + y^2 = 25 \Rightarrow x + y = 5$ and solves equations for *both* x and y values, award Att 5m. If no effort made to solve for 2nd variable, award 0m instead.

* Candidate finds values of first variable, substitutes into 2nd degree equation, finds correct and incorrect values and presents them all as solutions: no penalty (ignore excess answers).
However, if only the incorrect ones are found as solutions, apply B(-3).

Blunders (-3)

B1 Squaring error, e.g. $(10 - 2x)^2 = 100 - 4x^2$, or $100 + 4x^2$. Apply once.
B2 Algebraic error when totting/simplifying, etc. Example: $4x^2 - 40x + 75 = 0$.
Candidate may solve the quadratic generated without further penalty.
B3 Quadratic formula error (in formula, substit. or simplification). Each time to a max of 2.
B4 Incorrect factors. Apply once.
B5 Incorrect root(s) from candidate's factor(s). Apply once.
B6 One value for x when two available, or one value for y when two available. Apply once.
B7 Fails to find value(s) of the second variable. (B6 and B7 could both apply).
B8 Finds x but substitutes back for y.

Attempts (5 marks)

A1
$$\begin{array}{rcl} 4x^2 + y^2 &=& 100 \\ \underline{x^2 + y^2} &=& \underline{25} \\ 3x^2 &=& 75 \end{array}$$
As in note 1, if no effort made to solve for 2nd variables: 0 marks.
$\Rightarrow x^2 = 25 \Rightarrow x = +5, -5 \Rightarrow y = 0, 20$ resp. (2nd variables needed).
A2 Correct quadratic formula and stops.
A3 Effort to find the *second* variable, having found (not invented) the first. See W2.

Worthless (0)

W1 Incorrect values without work.
W2 Using $y = 10 - 2x$, $x = 0 \Rightarrow y = 10$; $y = 0 \Rightarrow x = 5$. ($x = 0$ invented, or assumed).

(b) (ii) **5 marks** **Att 2**

$5^3 + 0^3$	and	$3^3 + 4^3$	…(2m)
		$= 27 + 64$	
$= 125$		$= 91$	...(5m)

* Correct answers without work (based on candidate's values of x and y): full marks.
* One correct answer without work: Att 2 marks.
* No penalty for swapping x and y values , e.g. (0,5) used instead of (5,0).

Blunders (-3)
B1 Incorrect cubing, e.g. $5^3 = 15$, or $3^3 + 4^3 = 7^3$ or 12^3.
B2 Only one set of (x, y) evaluated, when two sets were found in (b)(i).
B3 Incorrect pairing, e.g. uses (x, x) instead of (x, y) values when cubing. Apply once, e.g. 5^3+3^3. Also, apply B3 once if two x values (no y values) found in (i), and used in (ii).

Misreading (-1)
M1 Error in reading values from (b)(i), if not B2 or B3 above.

Slips (–1)
S1 Error totting 27 and 64.

Attempts (2 marks)
A1 Any correct cubing of relevant numbers.
A2 Values substituted correctly (even partially).
A3 $x^3 + y^3$ factorised correctly, and stops. [Could continue and finish correctly for 5m].

Worthless (0)
W1 Incorrect answers without work.

Part (c)	20 marks (15, 5)	Att (5, 2)

> Let $f(x) = x^2 + ax + t$ where $a,\ t \in \mathbf{R}$.
> **(i)** Find the value of a, given that $f(-5) = f(-1)$.
> **(ii)** Given that there is only one value of x for which $f(x) = 0$, find the value of t.

(c) (i)	15 marks	Att 5

$f(-5) = 25 - 5a + t$	...(5m)	Interchangeable: One correct => 5m.
$f(-1) = 1 - a + t$	...(6m)	Both correct => 6m.
$1 - a + t = 25 - 5a + t$	…(9m)	
$4a = 24$	…(12m)	
$=> a = 6$	…(15m)	

* 25 – 5a + t = 0 and 1 – a + t = 0 may be solved as simultaneous equations to give a = 6.

Blunders (-3)
B1 Transposition error.
B2 Squaring error, e.g. $(-5)^2 = 2(-5)$ or $(-5)^2 = -(5)^2$.
B3 Calculates f(5) for f(–5) and/or f(1) for f(–1). Apply once.

Misreading (-1)
M1 Misreading which does not simplify the question, e.g. $x^3 + ax + t$.
Slips (–1)
S1 Numerical, e.g. multiplication error not involving signs.

Attempts (5 marks)
A1 Some correct substitution (even partial) of –5 or –1 into $x^2 + ax + t$.

Worthless (0)
W1 Answer without work, whether correct or not.

(c) (ii)	5 marks	Att 2

I:	II:	III:	IV:	
$x^2 + 6x + t$	$t = (6/2)^2$	$b^2 - 4ac = 0$ or $36 - 4t = 0$	$(x + 3)(x + 3)$	…(2m)
completing square.. $=> x^2 + 6x + 9$ $t = 9$	$t = 9$	$t = 9$	$t = 9$	…(5m)

* Value of *a* must be from (c)(i).
* Allow full marks for correct answer without work if consistent with answer for (c)(i).

Blunders (-3)
B1 Transposition error.
B2 Formula error, e.g. $b^2 - ac = 0$ or $b^2 + 4ac = 0$, or $t = a/2$, e.g. $x^2 + 6x => t = 3$.

Attempts (2 marks)
A1 Quadratic formula correct and stops.
A2 Value of *a* from (c)(i) substituted into $x^2 + ax + t$, and stops, e.g. $x^2 + 6x + t$ and stops.
A3 Effort to set up factors and stops, e.g. (x +)(x +).
A4 (c)(i) not answered (i.e. no *a* value found), but $(a/2)^2$ mentioned in (c)(ii), and stops.
A5 Invented value of *a* used correctly, other than *a* = 0 or *a* = 4.

Worthless (0)
W1 1 – 6 + t = 0 => t = 5; or, 25 – 5(6) + t = 0 => t = 5.
W2 Incorrect answer without work, but -- caution -- see note 2 above.

QUESTION 4

Part (a)	**10 marks**	**Att 3**
Part (b)	**25 marks**	**Att 9**
Part (c)	**15 marks**	**Att 5**

Part (a) **10 marks** **Att 3**

Given that $i^2 = -1$, simplify

$$2(3-i)+i(4+5i)$$

and write your answer in the form $x+yi$ where $x, y \in \mathbf{R}$.

(a) **10 marks** **Att 3**

I:	II: 2 (3 – i) = 6 – 2i …(3m) Interchaneable…one right (3m)
6 – 2i + 4i + 5i^2 …(7m)	i.(4 + 5i) = 4i + 5i^2 …(7m) both right (7m)
1 + 2i ..(10m)	adding => 1 + 2i ..(10m)

* i = -1 or i = 1 used: oversimplified, Att 3m is the maximum possible.

Blunders (-3)

B1 Bracket error, e.g. 2(3 – i) = 6 – i. Apply once if consistent.

B2 $i^2 \neq -1$, or mishandles $5i^2$, e.g. $5i^2 = 5(-1) = -4$.

B3 $-2i + 4i = -8i^2$.

B4 Adds real and imaginary parts, e.g. $4i + 5i^2 = 9i$.

Slips (-1)

S1 Numerical slip when adding real to real, or imaginary to imaginary. Each time.

Attempts (3 marks)

A1 Removes one bracket correctly and stops, e.g. 2(3 – i) = 6 – 2i and stops.

Part (b) **25 marks (10, 5, 10)** **Att (4, 2, 3)**

Let $z = 5 + 4i$.

(i) Plot z and $\bar{z}$ on an Argand diagram, where $\bar{z}$ is the complex conjugate of z.

(ii) Calculate $z\bar{z}$.

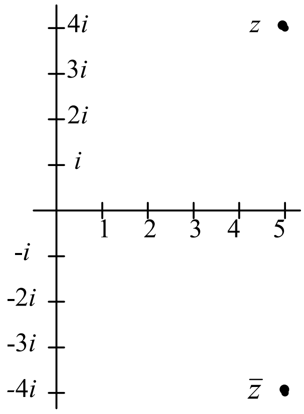
(iii) Express $\frac{z}{\bar{z}}$ in the form $u + vi$ where $u, v \in \mathbf{R}$.

(b) (i) **10 (5, 5) marks** **Att (2,2)**

Plot $z = 5 + 4i$ …(5m)

$\bar{z} = 5 - 4i$ …(2m)

Plot $\bar{z}$ …(5m)



* If the axes are reversed they must be identified, or B1 applies

* Unlabelled axes: assume horizontal axis real, e.g.(4, 5) plotted on unlabelled axes is B1.

* (4,5) & (– 4,5) plotted on unlabelled axes: award 2m for (i) and, if $\bar{z} = (5,-4)$, 5m for (ii). i.e. penalise reversal of axes *once* in (a).

* Axes drawn once (or twice), with one (or two) points plotted incorrectly: award 2 marks for z and 2 marks for $\bar{z}$.

* One unnamed point plotted: assume it is *z*.

* The same incorrect $\bar{z}$ will be penalised once only in (b), i.e. no penalty if reused later.

Blunders (-3)

B1 Incorrect plotting of z, e.g. plots 5 + 4i as 9i, or plots 5 + 4i as line linking (5,0) to (0,4).

B2 Incorrect calculation of $\bar{z}$, e.g. $\bar{z} = -5 + 4i$, or $\bar{z} = -5 - 4i$. Apply once in part (b).

B3 Incorrect plotting of candidate's $\bar{z}$.

Attempts (2 marks)

A1 A correct set of scaled axes (ticks sufficient). Apply once, unless drawn for each section.

A2 $\bar{z}$ incorrectly calculated, but result plotted correctly. If both incorrect, check Note 4 above.

(b) (ii) **5 marks** **Att 2**

I: $(5 + 4i)(5 - 4i) = 25 - 20i + 20i - 16i^2$ …(2m) $= 25 + 16 = 41$ …(5m)	II: $(x + yi)(x - yi) = x^2 + y^2$ …(2m) $= 25 + 16 = 41$ …(5m)

* No penalty for using the $\bar{z}$ formed in (b)(i).
* If conjugate is confused with modulus: max award is Att 2 for substitution (see A1) but, note: $|z|^2$ or $|\bar{z}|^2$ or $|z| . |\bar{z}|$ are correct methods of finding $\bar{z}z$.
* 41 without work: full marks (5m).

Blunders (-3)
B1 Incorrect $\bar{z}$, if not already penalised in (b)(i). See Note 1 above.
B2 Brackets error, e.g. 5(5 – 4i) = 25 – 4i. Apply once if consistent.
B3 $i^2 \neq -1$, or $-16i^2$ mishandled, e.g. $-16i^2 = -16(-1) = -17$.
B4 Adds real and imaginary parts, e.g. $20i - 16i^2 = 4i$.

Slips (-1)
S1 Numerical slips in adding real to real, or imaginary to imaginary.

Attempts (2 marks)
A1 States (in this part) that $\bar{z} = 5 - 4i$ & stops, or substitutes both of $z\bar{z}$ correctly & stops.
A2 Some relevant multiplication.
A3 A correct conjugate formula and stops, e.g. $\overline{a+bi} = a - bi$, and stops.

(b) (iii) **10 marks** **Att 3**

I: $\frac{(5+4i)}{(5-4i)} . \frac{(5+4i)}{(5+4i)}$ …(3m) $= \frac{25 + 20i + 20i + 16i^2}{25 + 20i - 20i - 16i^2}$ …(7m) for numerator $= \frac{9}{41} + \frac{40i}{41}$ ..(10m)	II: $\frac{z}{z} . \frac{z}{\bar{z}} = \frac{z^2}{z . \bar{z}}$ …(3m) $= \frac{25 + 20i + 20i + 16i^2}{41}$ …(7m) for num. $= \frac{9}{41} + \frac{40i}{41}$ ..(10m)

* First half of line 1 (either method) merits 3 marks.
* No penalty for using the $\bar{z}$ and/or $z . \bar{z}$ from previous parts of (b).
* Method III: (5 + 4i)/(5 – 4i) = u + vi, cross-multiplies, forms simultaneous equations, then solves them to get answer (9/41) + (40/41)i: mark on slip and blunder out of 10m.
* 41/(9 – 40i) is B1; 41 / (9 + 40i) is B4: award 7m for these or candidate's equivalents.
* $\frac{(5+4i)}{(5+4i)} . \frac{(5-4i)}{(5-4i)}$ multiplied out correctly to $\frac{41}{41}$ apply M(-1) + B(-3) and award 6m.
but if $\frac{5+4i}{5+4i} = 1$, with no other work: oversimplified, Att 3m.

Blunders (-3)
B1 Incorrect $\bar{z}$ used, if not penalised already in (b); e.g. $\bar{z} = 5 - 4i$ used $\Rightarrow$ Ans 41/(9–40i).
B2 Error multiplying out brackets. Apply once if consistent.
B3 $i^2 \neq -1$. Apply once.
B4 Inverts in the last step, e.g. 41/(9 + 40i); or forgets to multiply denom. by conjugate.
B5 Real and imaginary parts mixed up, e.g. when adding.
B6 Equates z.z and $\bar{z}$.z (instead of dividing them).

Slips (-1)
S1 Numerical slip when adding real to real, or imaginary to imaginary.
S2 (9 + 40i)/41 and stops, i.e. not in x + yi form.

Attempts (3 marks)
A1 States (in this part) that $\overline{z} = 5 - 4i$ and stops, or substitutes $z / \overline{z}$. correctly and stops.
A2 Any correct and relevant multiplication.

Part (c)	**15 marks (10, 5)**	**Att (3,2)**

> p and k are real numbers such that $p(2+i)+8-ki=5k-3-i$.
> **(i)** Find the value of p and the value of k.
> **(ii)** Investigate if $p+ki$ is a root of the equation $z^2-4z+13=0$.

(c) (i)	**10 marks**	**Att 3**

$2p + pi + 8 - ki = 5k - 3 - i$		…(3m)	
$2p + 8 = 5k - 3$; <u>or</u> $2p - 5k = -11$		…(3m)	still.
$p - k = -1$		…(4m)	Either equation (3m); both (4m).
$2p - 5k = -11$	$2p - 5k = -11$		
$\underline{2p - 2k = -2}$	$\underline{5p - 5k = -5}$		
$-3k = -9$	$-3p = -6$		
$k = 3$	$p = 2$	…(7m)	One value found (k or p)
$\therefore\ p - 3 = -1 \Rightarrow p = 2$	$2 - k = -1 \Rightarrow k = 3$	..(10m)	Second value found.

* Correct values for p and k, by trial/error substitution: allow full marks.
* Incorrect values for p and k, by trial/error substitution: 0 marks.
* Candidate may form equations as follows: $2p + 8 - 5k + 3 = - pi + ki - i$
$\Rightarrow 2p - 5k + 11 = i.(- p + k - 1)$, true if $0 = i.(0)$ $\therefore$ $2p - 5k = -11$ and $p - k = -1$.
* RHS = 5k – 3i could be a misreading (–1m) or a blunder (–3m) depending on the context.

Blunders (-3)
B1 Error multiplying out brackets.
B2 <u>Two</u> simultaneous linear equations not formed.
B3 Error equating real to real or imag.to imag., for simult. eqn(s). Apply once per eqn.
B4 Finds one value and stops.
B5 Sign error. Each time.

Slips (-1)
S1 Numerical error when adding real to real, or imaginary to imaginary.

Attempts (3 marks)
A1 A correct multiplying out of brackets, and stops.
A2 A correct transposition and stops.

(c) (ii) **5 marks** **Att 2**

I: $(2 + 3i)^2 - 4(2+3i) + 13$ …(2m) $4 + 12i + 9i^2 - 8 - 12i + 13$ $4 + 12i - 9 - 8 - 12i + 13 = 0$ …(5m)	II : $z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{4 \pm \sqrt{-36}}{2}$ ...(2m) $= (4 \pm 6i)/2 = 2 \pm 3i$ …(5m)
III: 2^{nd} root $= 2 - 3i \Rightarrow z^2 - (2 + 3i + 2 - 3i)z + (2+3i)(2-3i) = 0$ …(2m) $\Rightarrow z^2 - 4z + 13 = 0.$ …(5m)	
IV: 2^{nd} root $= 2 - 3i \Rightarrow [z - (2 + 3i)][z - (2 - 3i)] = 0$ *or* $[(z - 2) - 3i)][(z - 2) + 3i] = 0$ ...(2m) $\Rightarrow z^2 - 4z + 13 = 0.$ …(5m)	

* No penalty for dropping the – from the ± in formula (method II).

* Not necessary to say '= 0' in last line of Method I, if the numbers cancel out.

Blunders (-3)

B1 Squaring error, e.g. $(2 + 3i)^2 = 4 + 9i^2$.

B2 Error multiplying $-4(2 + 3i)$, or equivalent.

B3 $i^2 \neq -1$.

B4 Quadratic formula error. (Max 2 blunders)

B5 Only one root used in method III or IV.

B6 p and k substituted separately, i.e. $p^2 - 4k + 13 = 0$ examined. Apply once.
OR: any relevant number substituted for z (such as p, k or p + ki).

Attempts (2 marks)

A1 Correct substitution for z and/ or z^2 and stops.

A2 Quadratic formula correct and stops.

A3 Substitutes 'correctly' into incorrect relevant formula (II, III or IV above).

A4 Correct multiplying out of any brackets.

QUESTION 5

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 8**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks (5, 5)** **Att (2, 2)**

> Write down the next three terms in each of the following arithmetic sequences
>
> **(i)** – 10, – 8, – 6, ...
>
> **(ii)** 4.1, 4.7, 5.3, ...

(a) (i) **5 marks** **Att 2**

I:	II:
– 4, –2, 0 No work required, if listed in correct order.	$a = -10,\ d = -8 - (-10) = 2$ $t_4 = a + 3d = -10 + 3(2) = -4$ $t_5 = a + 4d = -10 + 4(2) = -2$ $t_6 = a + 5d = -10 + 5(2) = 0$

* Marking scheme the same as (a)(ii)

(a) (ii) **5 marks** **Att 2**

I:	II:
5.9, 6.5, 7.1 No work required, if listed in correct order.	$a = 4.1,\ d = 4.7 - 4.1 = 0.6$ $T_4 = a + 3d = 4.1 + 3(0.6) = 5.9$ $T_5 = a + 4d = 4.1 + 4(0.6) = 6.5$ $T_6 = a + 5d = 4.1 + 5(0.6) = 7.1$

* (a)(i) and (a)(ii) are separate questions. Slips and blunders apply separately to each.
* No penalty if d is called r but otherwise used correctly in an AP.
* In (i), allow incorrect d without penalty, if rectified in subsequent use (e.g. d = –2 but terms listed or calculated correctly).
* If candidate swaps correct *a* and correct *d*, (making both incorrect), apply B1 once.

Blunders (-3)
B1 Incorrect *d* used (but see notes 2 and 3 above), or incorrect *a* used.
B2 Subtraction of 2 or 0.6 instead of addition.
B3 Incorrect T_n of AP formula, e.g. $T_4 = -6(2)$ in (i) or $T_4 = 4.1(0.6)$ in (ii).
B4 Uses S_n formula instead of T_n, i.e. must substitute.
B5 Incorrect term without work. If all terms are incorrect, W1 applies. See also S2.
The order of listing matters, e.g. 0 – 2 , – 4 without work merits 2m (two wrong terms).

Slips (-1)
S1 Arithmetic error calculating *d*. (Once).
S2 First two terms correct and stops: award 4m (i.e. T_6 incorrect or missing).

Attempts (2 marks)
A1 *a* or *d* correct, or correct formula for *a* or *d*, e.g. $a = T_1$, or $d = T_2 - T_1$, or $d = T_n - T_{n-1}$.
A2 Correct T_n of AP formula, and stops, or $T_4 = -6 + d$ in (i) , or $T_4 = 5.3 + d$ in (ii).
A3 One extra term listed, e.g. – 4 or 5.9.

Worthless (0)
W1 All terms incorrect (which includes the order of terms).
W2 Any use of GPs or ratio unless A1 applies.

Part (b)	20 marks (5, 15)	Att (2, 6)

The sum of the first n terms of an arithmetic series is given by

$$S_n = \frac{3n}{2}(n+3).$$

(i) Calculate the first term of the series.

(ii) By calculating S_9 and S_{10}, find T_{10} (the tenth term of the series).

(b) (i)	5 marks	Att 2

$$\frac{3(1)(1+3)}{2} \quad \text{...(2m)}$$
$$= 6 \quad \text{...(5m)}$$

* Correct answer without work: full marks (5m). Allow $S_1 = 6$ for full marks.

* If S_n tidied up, allow use of 'new' incorrect Sn in other parts of (b), if not oversimplified.

Blunders (-3)

B1 $S_1 \neq T_1$ [but see Note 1]. Example: $S_1 = S_3 - S_2$.

B2 Incorrect n substituted, i.e. $n \neq 1$, or n retained after substitution.

B3 Substitutes & stops (at line 1), or error in calculations without work e.g. $(3/2)(1+3) \neq 6$.

B4 Error tidying up S_n before substitution.

Slips (-1)

S1 Arithmetical errors.

Attempts (2 marks)

A1 $T_1 = a$, and stops, or $T_n = a + (n-1)d$ isolated, and stops.

A2 Correct S_n of AP formula and stops, or substituted for n.

A3 Tries to tidy up the given S_n (with some work or merit) and stops.

Worthless (0)

W1 Incorrect answer without work

W2 Any use of GPs or ratios, unless A1 applies.

(b) (ii)	15 marks (5, 5, 5)	Att 6 (2, 2, 2)

$S_9 = \frac{3(9)(9+3)}{2}$ ...(2m)	$S_{10} = \frac{3(10)(10+3)}{2}$ ...(2m)	$T_{10} = 195 - 162$ ...(2m)
$= 162$ ...(5m)	$= 195$ ...(5m)	$= 33$ ...(5m)

* Treat as three separate 'questions', with the scheme applying separately to each.

* Accept correct answers (to each part) without work.

* Accept candidate's S_9 and S_{10} when used in T_{10}.

* $d = S_{10} - S_9 = 33$ used in $T_{10} = a + 9d$: ignore work after 33, and award full 5m.

Blunders (-3)

B1 Incorrect value of n substituted (could occur more than once).

B2 Mathematical error in simplification, e.g. 3(10)/2 + (13).

B3 $T_{10} \neq S_{10} - S_9$, e.g. $T_{10} = S_9 - S_{10}$, or $T_{10} = S_9 + S_{10}$.

B4 Incorrect d used in T_{10}, e.g. $d = S_2 - S_1$ (but not $d = S_{10} - S_9$, see note 4 above).

Slips (-1)

S1 Arithmetical errors, each time to a max of 3 in each 'question'.

Attempts (2 marks)

A1 Candidate's value of *a* stated, and stops. (Each time if repeated in each 'question').
A2 Line 1 and stops. Could apply to each 'question'.
A3 $T_{10} = a + 9d$ and stops, or $T_n = a + (n–1)d$ isolated, and stops.
A4 Some substitution.
A5 $S_9 = T_1 + T_2 + \ldots + T_9$, and stops, or $S_{10} = T_1 + T_2 + \ldots + T_{10}$. Each time.
A6 Correct S_n of AP formula and stops, or substituted for n. (A6 could apply in S_9 and S_{10}).

Worthless (0)

W1 Incorrect answer without work.
W2 Any use of GPs or ratios, unless A1 applies.
W3 Substitutes for n but still retains n, e.g. n become n+9 rather than 9.

Part (c) **20 marks (10, 10)** **Att (3, 3)**

The first three terms of a geometric sequence are

$$k - 3,\ 2k - 4,\ 4k - 3, \ldots\ldots\ldots.$$

where *k* is a real number.

(i) Find the value of *k*.

(ii) Hence, write down the value of each of the first four terms of the sequence.

(c) (i) **10 marks** **Att 3**

$$\frac{2k - 4}{k - 3} = \frac{4k - 3}{2k - 4} \quad \ldots(3m)$$

$$(k - 3)(4k - 3) = (2k - 4)(2k - 4) \quad \ldots(4m)$$

$$4k^2 - 12k - 3k + 9 = 4k^2 - 8k - 8k + 16 \quad \ldots(7m)$$

$$k = 7 \quad ..(10m)$$

* Correct answer obtained by trial and error substitutions: full marks if work is shown.
Correct answer without work merits attempt marks only.
Incorrect answers without work: worthless as in W1.

Blunders (-3)

B1 Incorrect fraction, e.g. (k–3)/(2k–4) on left in line 1 above. If both inverted, work is correct.
B2 Cross–multiplication error. Apply once if consistent. [While $4k^2 + 9 = 4k^2 + 16$ is B2 *once*, it would cause another B(–3). And both would apply].
B3 Brackets or multiplication error. Each time.
B4 Transposition or sign errors. Each time.

Slips (-1)

S1 Arithmetical error totting coefficients, to a maximum of 3.

Attempts (3 marks)

A1 $a = k - 3$ and stops.
A2 Correct T_n of GP formula.
A3 Correct S_n of GP formula.

A4 One correct fraction formed from given terms, and stops,
i.e. (2k – 4) / ((k – 3), or (4k – 3) / (2k – 4), and stops.
A5 $r = T_2 / T_1$ and stops.
A6 One correct transposition or correct multiplication.

Worthless (0)
W1 Incorrect answers without work.
W2 Incorrect formulae for Sn of GP, or Tn of GP, unless A1 applies.
W3 Any use of APs or common differences, unless A1 applies.

(c) (ii) **10 marks** **Att 3**

$T_1 = 7 - 3 = 4$	…(3m)	T_1
$T_2 = 2(7) - 4 = 10$ and $T_3 = 4(7) - 3 = 25$	…(7m)	T_2, T_3
$r = 10/4 = 2.5 \Rightarrow T_4 = 25.r = 25(2.5) = 62.5$ <u>*OR*</u> $T_4 = a.r^3 = 4(2.5)^3 = 62.5$	..(10m)	T_4

* Unless W1 applied to (c)(i), accept candidate's value of *k* from (c)(i), even if GP not generated.
* If candidate found more than one value for *k* in (c)(i), accept whichever value of k candidate uses in(c)(ii).
* Correct answer without work (all 4 terms): 10m. First 3 terms correct without work: 7m.

Blunders (-3)
B1 Each missing or incorrect term, e.g. T_4 incorrect or impossible to find.
B2 Incorrect value of r (using candidate's terms).
B3 Incorrect formula for Tn or T_4, e.g. $T_4 \neq ar^3$, or $T_4 \neq r.(T_3)$.
B4 Sign error.

Slips (-1)
S1 Arithmetic errors (to a max of 3).

Attempts (3 marks)
A1 One term correct, using candidate's k, if not an invented k. See note 1 above.
A2 A value for r found, using candidate's terms.
A3 $T_4 = r.(T_3)$, and stops.
A4 $a = k - 3$ and stops, or a = candidate's T_1 and stops.
A5 $T_n = ar^{n-1}$ stated, and stops.

Worthless (0)
W1 Incorrect answers without work.
W2 Uses an invented *k* value.

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks** **Att 3**

> Let $f(x) = \frac{1}{3}(x-8)$ for $x \in \mathbf{R}$.
>
> Evaluate $f(5)$.

(a) **10 marks** **Att 3**

f(5) = (1/3)(5 – 8) …(7m)		Accept if f(5) omitted or f(x) written.
= – 1 ..(10m)		

* 7 marks for first step, but Att mark is still 3m.
* No penalty for omission of brackets, provided the blunders below are avoided.
* Correct answer without work: full marks (10).

Blunders (-3)
B1 Incorrect substitution, e.g. x = 23 from solving ($^1/_3$)(x – 8) = 5. See W2.
B2 Sign error.
B3 Serious error in calculation, e.g. $^1/_3$)(5 – 8) = ($^1/_3$)(5) – 8 = – 6 $^1/_3$, or ($^1/_3$)(–3) = – 9.

Attempts (3 marks)
A1 f(5) = (1/3)(x – 8) and stops (i.e. some substitution).

Worthless (0)
W1 Incorrect answer without work.
W2 (1/3)(x – 8) = 5 solved (x = 23), and stops. No merit without substitution. See B1 above.

Part (b) **20 marks (10, 10)** **Att 6 (3, 3)**

> **(i)** Find $\frac{dy}{dx}$ where $y = (x-1)^7$ and evaluate your answer at $x = 2$.
>
> **(ii)** Find $\frac{dy}{dx}$ where $y = (x^3 - 3)(x^2 - 4)$ and simplify your answer.

(b) (i) **10 marks** **Att 3**

I:	II:
	u = x – 1 => du/dx = 1
	y = u^7 => dy/du = $7u^6$ …(3m)
dy/dx = $7(x – 1)^6$ …(7m)	dy/dx = (dy/du).(du/dx) = $7u^6$.1 <u>or</u> $7(x–1)^6$ …(7m)
x = 2 => $7(1)^6$ = 7 ..(10m)	x = 2 => $7(1)^6$ = 7 ..(10m)

III: $(x – 1)^7$ = $x^7 – 7x^6 + 21x^5 – 35x^4 + 35x^3 – 21x^2 + 7x – 1$	…(3m)
dy/dx = $7x^6 – 42x^5 + 105x^4 – 140x^3 + 105x^2 – 42x + 7$	…(7m)
x = 2 => 448 – 1344 + 1680 – 1120 + 420 – 84 + 7 = 7	..(10m)

* <u>Caution</u>: Method I is marked differently to usual scheme for derivatives. <u>See B1</u>.
* y = $(x – 1)^7$ => dy/dx = $7(2 – 1)^6$ and stops: s(-1) + B(-3) : 6 m;
if continued to get answer 7, award 9 m.
But y = $(x – 1)^7$ => dy/dx = $7(1)^6$ and stops: B(-3) + B(-3) : 4 m;
if continued to answer 7, award 7m.

Blunders (-3)

B1 In method I, *each* error in $7(x-1)^6$: check the 7, the $(x-1)$, and the power 6.
B2 In II and III, differentiation error. Once per term. (Two terms to check in Method II).
B3 In method II, error applying the chain rule formula. Apply once.
B4 In III, each omitted or incorrect term of expansion of $(x-1)^7$, to a max of 2 blunders.
B5 Mathematical error after substitution. Apply once, from last 3m.
B6 Incorrect or no substitution of $x = 2$ into dy/dx, e.g. $7(2)^6$.
B7 If not using method III, $dy/dx = 7x^6$.
(In III, $(x-1)^7 = x^7 - 1 \Rightarrow dy/dx = 7x^6$ is B4 + B4).

Slips (-1)

S1 Arithmetical errors, e.g. in multiplication or totting in method III.

Attempts (3 marks)

A1 Some element of $7(x-1)^6.1$ correct , even the multiplication by 1 written down.
A2 $u = x - 1$, or any other relevant statement in method II, and stops.
A3 Any correct term of the expansion of $(x-1)^7$and stops, e.g. $(x-1)^7 = x^7 - 1$ and stops.
A4 $y = (2-1)^7 = 1 \Rightarrow dy/dx = 0$.

Worthless (0)

W1 Substitutes $x = 2$ into f(x), no differentiation. But see A4.

(b) (ii) **10 marks** **Att 3**

I:	II:
$dy/dx = (x^3-3).2x + (x^2-4).3x^2$ …(7m)	$y = x^5 - 4x^3 - 3x^2 + 12$ …(3m)
$= 2x^4 - 6x + 3x^4 - 12x^2$ ..(10m)	$dy/dx = 5x^4 - 12x^2 - 6x$ ..(10m)
or $5x^4 - 12x^2 - 6x$	

* In method I, no penalty for omission of brackets as long as multiplication is implied.
* If u/v used, apply B2 twice (central sign, division by v^2). There may be other errors.

Blunders (-3)

B1 Differentiation error. Once per term. (Two terms to check in I, four terms in II).
B2 Error in u.v formula, e.g. central sign.
B3 Vice versa switching of derivatives, i.e. does u.(du/dx) + v.(dv/dx). Apply once.
B4 In I, incorrect terms after multiplying out brackets. Apply once.
B5 In II, each omitted or incorrect term in the expansion (line 1) to a max of 2 blunders.

Attempts (3 marks)

A1 Any correct derivative, e.g. $dy/dx = (3x^2).(2x)$.
A2 $u = x^3 - 3$ or $v = x^2 - 4$, or vice versa, and stops.
A3 Any correct multiplication (in method II).

Worthless (0)

W1 u.v or u/v rule (from Tables) and stops.

Part (c)	**20 marks (10, 10)**	**Att (3, 3)**

Let $f(x) = x^3 - ax + 7$ for all $x \in \mathbf{R}$ and for $a \in \mathbf{R}$.

(i) The slope of the tangent to the curve $y = f(x)$ at $x = 1$ is -9.
Find the value of a.

(ii) Hence, find the co-ordinates of the local maximum point and the local minimum point on the curve $y = f(x)$.

(c) (i)	**10 marks**	**Att 3**

I:

$f'(x) = 3x^2 - a$ …(3m)

$f'(1) = 3(1)^2 - a$ …(7m)

$\Rightarrow 3 - a = -9 \Rightarrow a = 12$ ..(10m)

* If no differentiation done or mentioned, see W1.

* Three steps: differentiating; substituting x = 1; then simplifying, equating to –9 and solving.
Candidate may take these steps in a different order.

* If $f'(-9) = 1$, penalise with <u>one blunder</u> <u>instead of</u> B2 + B3 below. But B4 may also occur.
i.e. one blunder for mixing up (or swapping) the –9 and 1. Then mark on.

Blunders (-3)

B1 Differentiation error. Each term. (Three terms to check).

B2 $f'(-9)$ found, or any incorrect substitution for x into $f'(x)$.

B3 $f'(x) = 1$ or $f'(x) = 0$, i.e. $f'(x)$ not equated to –9. See Note 3 above.

B4 Index or mult error tidying up the substituted $f'(x)$, e.g. $3(1)^2 - a = 9 - a$ in the final line.

B5 $f''(x)$ used for slope. [B2 will also apply].

Attempts (3 marks)

A1 Any of the <u>first two</u> terms correctly differentiated.

A2 dy/dx or $f'(x)$ mentioned or any reference to differentiation.

Worthless (0)

W1 No mention of differentiation, e.g. substitutes into f(x) and continues. But see A2.

(c) (ii) **10 marks** **Att 3**

I:	II:	III:
I: $3x^2 - 12 = 0$ …(3m) $x^2 - 4 = 0$ $x^2 = 4$ $x = 2$ and $x = -2$ …(7m) => $y = -9$ and $y = 23$ ..(10m)	II: $3x^2 - 12 = 0$ …(3m) $x^2 - 4 = 0$ $(x-2)(x+2) = 0$ $x = 2$ and $x = -2$ …(7m) => $y = -9$ and $y = 23$ ..(10m)	III: $3x^2 - 12 = 0$ …(3m) Uses quadratic formula to find $x = 2$ and $x = -2$ …(7m) => $y = -9$ and $y = 23$ ..(10m)

* Three steps: equating $f'(x)$ to zero, solving for x values, and corresponding y values.
* No need to distinguish between max and min points – not asked in question. So, ignore errors in candidate's work relating to 2nd derivative.
* In line 1, an implied use of '= 0' is acceptable when factors found, etc.
* Accept candidate's $f'(x)$ from (c)(i), but if $f'(x)$ is not a quadratic expression, then award Att 3 marks, even if continued.
* Tries to find max and min by drawing a graph: check the graph of candidate's f(x) for max and min (using candidate's *a* value) , allowing a tolerance of ±0.2 on correct coordinates:
if co-ordinates of both max and min correct (to within ±0.2), award 7 marks;
if co-ordinates of either max or min correct (to within ±0.2), award 4 marks.
if co-ordinates of neither max nor min correct (to within ±0.2), see A5.
* $x^2 = 4 \Rightarrow x = 2$ and stops: apply B6 + B7

Blunders (-3)

B1 $f'(x)$ not equated to zero.
B2 Incorrect factors. Apply once.
B3 Incorrect roots from factors. Apply once.
B4 Quadratic formula error (in formula, substitution or simplification). Max of 2 blunders.
B5 Errors finding y values (apart from slips), e.g. substitutes into $f'(x)$ rather than f(x). Once.
B6 Calculates only one x value.
B7 y co-ordinates not found. Apply once.

Slips(–1)

S1 Numerical, e.g. totting y values.

Attempts (3 marks)

A1 Substitutes for *a* in f(x), i.e. into $x^3 - ax + 7$; or first step in any method above.
A2 Statement that derivative = 0, e.g. '*max and min at dy/dx = 0*', and stops.
A3 Candidate's $f'(x) = 0$ and stops.
A4 Quadratic formula correct, and stops.
A5 Point or points found for graph of $f(x) = x^3 - 12x + 7$ or candidate's $f(x)$.

Worthless (0)

W1 Effort to solve $f(x) = 0$.
W2 Draws axes only or sketches without calculated points.

QUESTION 7

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 8**

Part (a) **10 marks** **Att 3**

Differentiate $7x^3 - 3x^2 + 9x$ with respect to x.

(a) **10 marks** **Att 3**

$21x^2 - 6x + 9$ …(10m)

* Correct answer without work or notation: full marks, 10m.
* If done from first principles, ignore errors in procedure – just mark the answer.
* Only one term correctly differentiated, award 4 marks.

Blunders (-3)
B1 Differentiation error. Once per term. (Three terms to check. See note 3 above).

Attempts (3 marks)
A1 Unsuccessful effort at first principles, e.g. $y + \Delta y$ on L.H.S., or x replaced by $x + \Delta x$ on R.H.S., 'limit' mentioned, $\Delta x \to 0$, f($x + h$), etc.

Worthlesss (0)
W1 No term differentiated correctly.
W2 Writes down the notation 'dy/dx' or '$f'(x)$' and stops.

Part (b) **20 marks (10, 10)** **Att (3, 3)**

(i) Differentiate $x^5 - 17 + \frac{1}{x^5}$ with respect to x.

(ii) Differentiate $\frac{2x}{x-1}$ with respect to x and simplify your answer.

(b) (i) **10 marks** **Att 3**

I:
$y = x^5 - 17 + x^{-5}$ …(3m)
$\frac{dy}{dx} = 5x^4 \quad -5x^{-6}$ …(10m)

II:
$5x^4 + \frac{x^5(0) - 1(5x^4)}{(x^5)^2}$ …(10m)

III:
$$y = \frac{x^{10} - 17x^5 + 1}{x^5}$$ …(3m)
$$\frac{dy}{dx} = \frac{x^5.[10x^9 - 85x^4] - [x^{10} - 17x^5 + 1].(5x^4)}{(x^5)^2}$$ …(10m)

* Correct answer without work or notation: full marks, 10m.
* No need to have 'y = …' and 'dy/dx = …'
* If done from first principles, ignore errors in procedure – just mark the answer.
* $5x^4 + 1/(5x^4)$ merits 7m (for two correct derivatives out of three).

Blunders (-3)

B1 Differentiation error. Once per term. (Three terms to check).

B2 Multiplies by x^5 before differentiating, i.e. $y = x^{10} - 17x^5 + 1$, or similar. Apply once. (Then continue marking as if that had been the question, applying B1 as required). However, if stops at $y = x^{10} - 17x^5 + 1$, award Att 3m.

Attempts (3 marks)

A1 Unsuccessful effort at first principles, e.g. $y + \Delta y$ on L.H.S., or $x + \Delta x$ on R.H.S., etc.

A2 Cancels x^5, then differentiates some term correctly, e.g. $y = 1 - 17 + 1$, or -15; => $dy/dx = 0$.

Worthless (0)

W1 No term differentiated correctly.

(b) (ii) **10 marks** **Att 3**

I: $\dfrac{(x-1).2 - 2x.(1)}{(x-1)^2}$ …(7m)	II: $y = 2x.(x-1)^{-1}$ …(3m) $dy/dx = 2x.[-1(x-1)^{-2}] + (x-1)^{-1}.2$ …(7m)
$= \dfrac{2x - 2 - 2x}{(x-1)^2}$ or $\dfrac{-2}{(x-1)^2}$ ..(10m)	$= \dfrac{-2x}{(x-1)^2} + \dfrac{2}{x-1}$ ..(10m)

* No penalty for omission of brackets if multiplication implied. (Decide by later work).

* If u.v used (even if u/v identified initially) apply B2 + B3, and others if necessary.

* No marks for writing down u/v or u.v formula from Tables, and stopping.

* $(x-1)^2$ need not have been multiplied out — ignore errors if it was.

* $y = 2x.(x-1)^1$: apply B5 + B3.

Blunders (-3)

B1 Differentiation error. Once per term. (Two terms to check in both methods).

B2 Central sign incorrect in formula.

B3 No $\div$ by v^2 in method I.

B4 Vice versa substitution for du/dx and dv/dx in u/v formula. Apply once.

B5 $y \neq 2x.(x-1)^{-1}$ in method II. Also apply B3 if power of $(x-1)$ is 1.

B6 Bracket error multiplying out numerator. Apply once if necessary. See Note 1.

Attempts (3 marks)

A1 $u = x^2$ and $v = x - 1$ and stops.

A2 Any correct derivative, du/dx or dv/dx, and stops; *e.g.* $dy/dx = 2x/1$ or $2/1$ or 2.

Part (c) **20 marks (5, 5, 5, 5)** **Att (2, 2, 2, 2)**

A marble rolls along the top of a table. It starts to move at $t = 0$ seconds.
The distance that it has travelled at t seconds is given by

$$s = 14t - t^2$$

where s is in centimetres.

(i) What distance has the marble travelled when $t = 2$ seconds?
(ii) What is the speed of the marble when $t = 5$ seconds?
(iii) When is the speed of the marble equal to zero?
(iv) What is the acceleration of the marble?

(c)(i) **5 marks** **Att 2**

$$t = 2 \Rightarrow s = 14(2) - 2^2 \quad \text{...(2m)}$$
$$= 24 \quad \text{...(5m)}$$

* If B1 and B2 both apply, then award 0 marks.

Blunders (-3)
B1 Incorrect t substituted, e.g. subs. solution of ds/dt = 0 (i.e. 7), or solution of s = 0 (i.e 0,14).
B2 Incorrect equation substituted, e.g. substitutes t =2 into ds/dt. See note 1 above.
B3 Mathematical error simplifying line 1, e.g. $14(2) - 2^2 = 16 - 4$, or $14 - 2$.

Slips (-1)
S1 Numerical error simplifying line 1, e.g. $14(2) - 2^2 = 26 - 4$; or $28 - 4 = 22$, etc.

Attempts (2 marks)
A1 Substitutes any incorrect value of t into $14t - t^2$.

Worthless (0)
W1 Solves $s = 14t - t^2 = 0$ or any other number, but does not substitute. See A1.

(c)(ii) **5 marks** **Att 2**

$$ds/dt = 14 - 2t \quad \text{...(2m)}$$
$$t = 5 \Rightarrow ds/dt = 14 - 2(5) = 4 \quad \text{...(5m)}$$

* No differentiation (to find *ds/dt*), or no reference to differentiation: award 0m. See A2.
* No penalty for notation, e.g. using *dy/dx* or $f'(x)$, instead of ***ds/dt***.

Blunders (-3)
B1 Differentiation error. Once per term. (Two terms to check).
B2 Incorrect t substituted in ds/dt, e.g. substitutes t = 0.
B3 Incorrect equation substituted, e.g. subst. t = 5 into $d^2s/dt^2 = -2$.
B4 Mathematical errors in calculation, e.g. $14 - 2(5) = 12(5) = 60$.

Attempts (2 marks)
A1 One term differentiated correctly and stops.
A2 *ds/dt* mentioned, e.g. speed = *ds/dt*, or mentions *dy/dx* or $f'(x)$, or differentiation implied.

Worthless (0)
W1 Substitutes $t = 5$ into the distance equation $s = 14t - t^2$. (No differentiation: no marks)
W2 Effort to use Speed = Distance ÷Time.
W3 $14t - t^2 = 5$. See Note 1.

(c)(iii) **5 marks** **Att 2**

I:	$14 - 2t = 0$ …(2m) $t = 7$ …(5m)
II: May graph <u>distance</u> against time and see from graph (or table) that max height (49) occurs when $t = 7$ sec => speed = 0 when $t = 7$ seconds …(5m) For reference: (0,0),(1,13),(2,24),(3,31),(4,40),(5,45),(6,48),(7,49),(8,48),(9,45),(10,40), …	
III: May graph speed against time and see from graph (or table) that speed = 0 at $t = 7$ …(5m) For reference: (0,14),(1,12),(2,10,(3,8),(4,6),(5,4),(6,2),(<u>7,0</u>)…..	

* Correct answer without work: full marks, 5m.
* Accept candidate's use of *ds/dt* from (ii), provided it was a derivative.
* Substitutes t = 0 into 14 – 2t: Att 2 (as in A1 below).

Blunders (-3)
B1 Solves *ds/dt* = an incorrect number, i.e. not 0, e.g. $14 - 2t = 5$.
B2 Transposition error.

Attempts (2 marks)
A1 *ds/dt* mentioned again, or *ds/dt* = 14 – 2t found again
A2 '*ds/dt* = 0'& stops; or $14 - 2t = 0$, stops; or candidate's derivative in (c)(ii) = 0 & stops.
A3 Effort to tabulate, graph or use trial/error, e.g. 2 points correctly calculated or plotted, or 2 correct values found using a trial/error approach.

Worthless (0 marks)
W1 *ds/dt* not mentioned, found or used, e.g. subs into distance equation.$s = 14t - t^2$. See A1.

(c)(iv) **5 marks** **Att 2**

I d^2s/dt^2 …(2m) $= -2$ …(5m)	II $a = (v - u)/t$ correctly substituted for speed, e.g. $(4 - 14)/5$ or $(0 - 4)/2$ …(2m) $= -2$ …(5m)

* Correct answer without work: full (5) marks.
* $v = u + f.t$ isolated: 0m (in Tables); but $v = u + a.t$ and stops (i.e. subst. for f): Att 2m

Blunders (-3)
B1 Differentiation error. Once per term. Two terms to check.

Attempts (2 marks)
A1 Mentions d^2s/dt^2, dv/dt, d^2v/dt^2 or similar, i.e. 2nd derivative.
A2 Finds ds/dt = 14 – 2t *for the first time*, and stops.
A3 a = $(v - u)/t$ and stops.

Worthless (0 marks)
W1 2nd derivative not found or mentioned, unless Method II used.

QUESTION 8

Part (i)	**10 marks**	**Att 3**
Part (ii)	**5 marks**	**Att 2**
Part (iii)	**10 marks**	**Att 3**
Part (iv)	**10 marks**	**Att 3**
Part (v)	**10 marks**	**Att 3**
Part (vi)	**5 marks**	**Att 2**

Part (i) **10 marks** **Att 3**

Let $f(x) = \dfrac{1}{x+2}$.

(i) Find $f(-6)$, $f(-3)$, $f(-1)$, $f(0)$ and $f(2)$.

(i) **10 marks** **Att 3**

$f(-6) = \dfrac{1}{-6+2}$	$f(-3) = \dfrac{1}{-3+2}$	$f(-1) = \dfrac{1}{-1+2}$	$f(0) = \dfrac{1}{0+2}$	$f(2) = \dfrac{1}{2+2}$	(1m) H/M
					+
$= \dfrac{1}{-4}\ or\ \dfrac{-1}{4}$	$= \dfrac{1}{-1}\ or\ -1$	$=1$	$= \dfrac{1}{2}$	$= \dfrac{1}{4}$	(1m) H/M

* Each unsimplified fraction merits 1 mark if correct and 0 marks if incorrect. Hit/Miss. Each such fraction (*correct or not*) merits 1m if it is then simplified correctly. Hit/Miss.
* If a candidate earns only 1 or 2 marks in total, replace with the overall Att.3 marks.
* Blunders do not apply in (i).
* Accept the use of decimals.
* Correct answer without work: full marks (2m) each time.
* f(–6) = (–1/6) + 2 = 1.83 merits 0m+1m; f(–3) = (–1/3)+2 = 1.66 merits 0m+1m; etc. 5m possible if continued. But f(–6) = 1.83 without work: 0 m (out of the 2m). Note that f(-1) could be correct, meriting 2m, but the minimum positive mark is Att 3m.

Part (ii) **5 marks** **Att 2**

For what real value of x is $f(x)$ not defined?

(ii) **5 marks** **Att 2**

x + 2 = 0 …(2m)
x = –2 …(5m)

* Allow $^1/_{(x+2)}$ = 0 => x + 2 = 0 = > x = – 2 without penalty.

Attempts (2 marks)
A1 x + 2 = 0 and stops, or $^1/_{(x+2)}$ = 0 (for recognition of '= 0') and stops.
A2 1/0 and stops.

Worthless (0)
W1 x = 0 and stops.

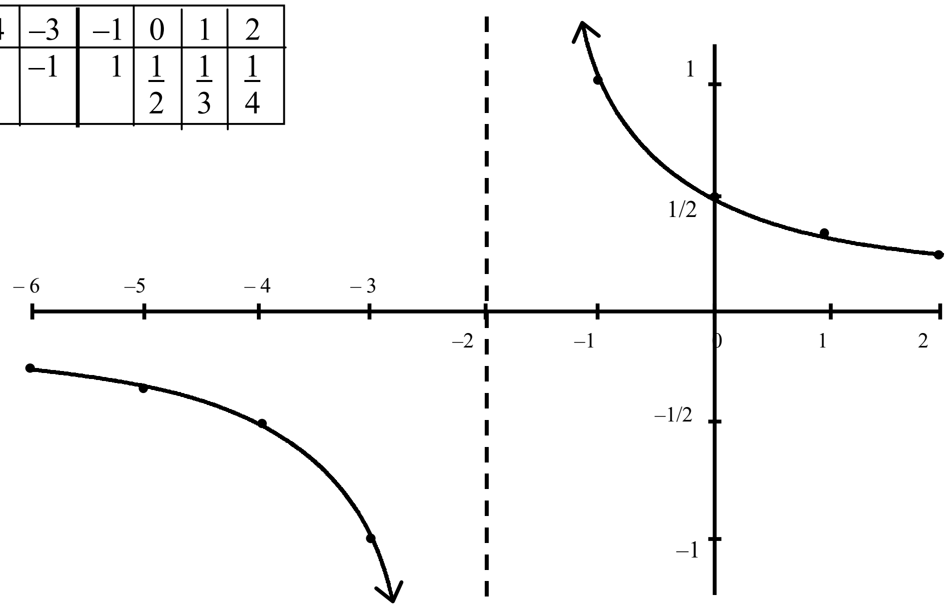
Part (iii) **10 marks** **Att 3**

> Draw the graph of $f(x) = \dfrac{1}{x+2}$ for $-6 \le x \le 2$.

(iii) **10 marks** **Att 3**

x	–6	–5	–4	–3	–1	0	1	2
f(x)	$\frac{-1}{4}$	$\frac{-1}{3}$	$\frac{-1}{2}$	–1	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$

I: **Using Table:**



II: **Sketching in relation to asymptotes (Table not used):**

<u>Award 3m</u> <u>(Hit/Miss)</u> for correct asymptote (drawn/implied, not intersected by curve).
<u>Award 4m</u> <u>(Hit/Miss)</u> for one side of the curve (approx. shape).
<u>Award 3m</u> <u>(Hit/Miss)</u> for the other side of the curve (approx. shape).
} First check if the 'better' side merits 4m or 0m.

* Graph may be drawn from a table or in relation to the asymptote x = – 2.
* If new table is drawn up, mark accordingly: <u>do not refer back</u> to the earlier values in (i). However, candidates may import values from (i) to the table or direct to the graph.
* Be lenient in regard to approximate scaling of axes. (Ignore incorrect scale if possible).
* Asymptote x = – 2 need not be drawn: an implied vertical asymptote (or visible gap) is fine.
* If left and right branches of the curve are joined, apply B1. e.g. (–3,–1) joined to (–1,1). If curve cuts horizontal asymptote *elsewhere* (e.g. left or right extremity), apply B1 again.

Table Method:

Blunders (-3)

B1 Asymptote error. See notes 4 and 5 above.
B2 0, ¼, $^1/_3$, ½ and 1 equally scaled on y axis (str. line graphs), or similar poor scaling.
B3 If f(x) = 1/(x) + 2 re–calculated and graphed in (iii). Other error may also occur.
B4 Points plotted but not joined. (Max available 7m).

Slips (-1)

S1 Check the points on the left and on the right of the vertical asymptote (x = – 2) and penalise <u>each</u> incorrect point, subject to a max penalty of –3m on each side.
In this case, an incorrect point means one which is wrongly calculated and/or plotted.
Allow candidate to leave out a few points if shape of graph is not affected.
S2 $^1/_3$, ¼ incorrectly scaled (or – $^1/_3$, –¼) if causing a blip in the graph. Apply once.

Attempts (3 marks) (for either method):
A1 One point correctly calculated and/or plotted, e.g. point correct in Table but no graph.
A2 Draws axes and stops
A3 x = – 2 an asymptote and stops, or states that x axis is an asymptote and stops.

Part (iv) **10 marks** **Att 3**

Find $f'(x)$, the derivative of $f(x)$.

(iv) **10 marks** **Att 3**

I:	II:
$f'(x) = \dfrac{(x+2).0 - (1)(1)}{(x+2)^2}$ ..(10m)	$f(x) = (x+2)^{-1}$ …(3m) $f'(x) = -1.(x+2)^{-2}$ ..(10m)

* No penalty for omission of brackets as long as multiplication is implied.
* 1/(x + 2) is clearly identified by candidate as u/v, but uv formula is used: apply B2 and B3. Other penalties may/may not arise.

Blunders (-3)
B1 Differentiation error, once per term. Method I has two terms to check, II has one term.
B2 Central sign error in u/v formula.
B3 No (or incorrect) division by v^2 (whether v^2 mentioned in formula or not).
B4 Vice versa switching of derivatives, i.e. does $[v.dv/dx) - u.(du/dx)]/v^2$.
B5 $f(x) \neq (x+2)^{-1}$ if using method II. But if f(x) taken as $(x+2)^1$ or x +2, see A3 and W2.

Attempts (3 marks)
A1 Any correct derivative and stops, e.g. $f'(x) = 0/1$, or 0.
A2 $f(x) = (x+2)^{-1}$ and stops.
A3 f(x) = x + 2 and differentiates this correctly. (Oversimplified).

Worthless (0)
W1 No differentiation done in method I, or step 1 missing (and not subsumed) in method II.
W2 f(x) = x + 2 and stops.

Part (v) **10 marks** **Att 3**

Find the two values of x at which the slope of the tangent to the graph is $-\frac{1}{9}$.

(v) **10 marks** **Att 3**

$-1.(x+2)^{-2} = \dfrac{-1}{9}$ or $\dfrac{-1}{(x+2)^2} = \dfrac{-1}{9}$ …(3m)

$(x+2)^2 = 9$ …(4m)

	$x^2 + 4x + 4 = 9$	$x^2 + 4x + 4 = 9$	…still 4m
$x + 2 = \pm 3$ ..7m	$x^2 + 4x - 5 = 0$	$x^2 + 4x - 5 = 0$	…(7m)
$x = \pm 3 - 2$	$(x-1)(x+5) = 0$	$x = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(-5)}}{2}$	
x = 1, x = –5	x = 1, x = – 5	x = 1, x = – 5	..(10m)

* Allow the candidate to use $f'(x)$ from previous part; however, if this oversimplifies the question then the max mark attainable is Att 3.
* Candidate may cross–multiply in line 1 to get: $-(x+2)^2 = -9 \Rightarrow (x+2)^2 = 9$.
* $x^2 + 4x + 4 = 9 \Rightarrow x^2 + 4x = 5 \Rightarrow x(x + 4)= 5 \Rightarrow x = 5, x + 4 = 5 \Rightarrow$ answers x = 5 and 1: apply 2 blunders.

Blunders (-3)

B1 Index or inversion error.
B2 Only one root extracted, e.g. $(x + 2)^2 = 9 \Rightarrow x + 2 = 3 \Rightarrow x = 1$.
B3 Transposition or sign error.
B4 Incorrect factors. Apply once.
B5 Incorrect roots from factors. Apply once.
B6 Quadratic formula error (in formula, substitution or simplification). (Only one applies).

Attempts (3 marks)

A1 Candidate's answer to (iv) equated to –1/9, and stops.
A2 Tidies up in (v) the $f'(x)$ obtained in (iv).
A3 dy/dx or $f'(x) = -1/9$ and stops; 'dy/dx or $f'(x)$ = slope of tangent' and stops.
A4 Quadratic formula correct, and stops.

Worthless (0)

W1 $x = \pm 1/3$.
W2 $1/(x + 2) = -1/9$ and stops.

Part (vi)	**5 marks**	**Att 2**

Show that there is no tangent to the graph of f that is parallel to the x-axis.

(vi)	**5 marks**	**Att 2**

I: $\dfrac{-1}{(x+2)^2} = 0$ ...(2m) $\Rightarrow -1 = 0$ impossible ...(5m)	II: $(x + 2)^2$ has to be positive. …(2m) $\Rightarrow \dfrac{-1}{(x+2)^2}$ cannot be zero. …(5m)

* Allow candidate's $f'(x)$ from (iv) unless it oversimplifies the question. Oversimplified questions merits Att 2 at most.
* "No tangent is possible … because the curve continually approaches the x axis": 5 marks; or "…because the curve is never parallel to the axis": 5 marks; or similar reasons relating to the curve: 5 marks. See also A4 below. If A4 + a reason such as above: 5 marks.

Blunders (-3)

B1 Cross–multiplication error, e.g. '$-1 = (x + 2)^2$, which is impossible'

Attempts (2 marks)

A1 $-1 = (x + 2)^2$ and stops, or continues.
A2 Parallel lines have equal slopes.
A3 Slope of the x axis is 0.
A4 Candidate draws some tangents to the curve showing them to be not parallel to x axis, or, draws some lines parallel to the axis showing them cutting the curve.

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2002

MATHEMATICS

ORDINARY LEVEL

PAPER 2

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3,......, S1, S2, S3,...., M1, M2, etc. Note that these lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), it is essential to note that
   - any correct relevant step in a part of a question merits *at least* the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,....etc.

4. The *same* error in the *same* section of a question is penalised *once* only.

5. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

6. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks only.

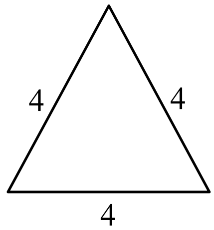
7. The phrase “and stops” means that no more work is shown by the candidate.

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 8**

Part (a) **10 marks** **Att 3**

Each side of an equilateral triangle measures 4 units.
Calculate the area of the triangle, giving your answer in surd form.



Note: Area of a triangle = $\frac{1}{2}ab\sin C$.

(a) **10 marks** **Att 3**

α: $\theta = 60^o$ Area = ½(4)(4)sin60^o or ½(4)(4)($\frac{\sqrt{3}}{2}$) $= 4\sqrt{3}$	β: $h^2 + 2^2 = (4)^2$ $h = \sqrt{12}$ Area = ½(4)($\sqrt{12}$) $= 2\sqrt{12}$	γ: $s = {}^{(4+4+4)}/_2 = 6$ Area $= \sqrt{6(6-4)(6-4)(6-4)}$ or $\sqrt{6(2)(2)(2)}$ $= \sqrt{48}$	3m 4m 7m 10m

Blunders (-3)

B1 Mathematical errors, e.g. indices, fractions.
B2 Incorrect substitution or no substitution, e.g. $\theta \neq 60^o$, e.g. ½ (4)(4) sin90^o = 8.
[S3 also applies here, i.e. ½ (4)(4) sin90^o = 8 merits 6 marks.]
B3 Confuses radians with degrees.
B4 Incorrect function read.
B5 Incorrect use of Pythagoras.
B6 Incorrect relevant formula. e.g. ½ |ab|sinC=½(4)sin60^o $= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$ or *ab*sinC → $8\sqrt{3}$.
B7 Incomplete calculations, e.g. 8 sin60^o with work and stops or avoids reading sinC
B2 + B7 for ½ (4)(4)sinC or continued to 8 sinC and stops.

Slips (-1)

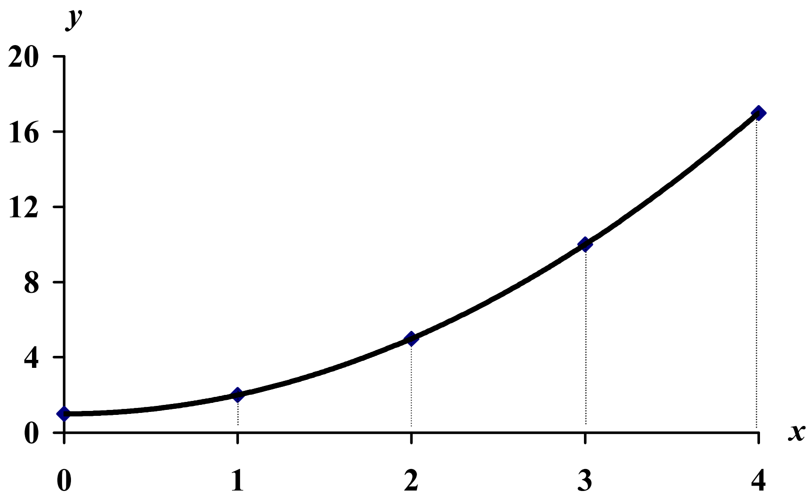
S1 Each numerical slip to a maximum of 3.
S2 Obvious slip in reading tables or calculator.
S3 Answer not in surd form.

Attempts (3 marks)

A1 Some correct substitution in reasonable formula, e.g. ½(4)(4) or ½(4)(4) = 8 or ½(4)(4)(1).
A2 Some work on formula from tables, e.g. Area = ½ ah = ½(4)h and stops.
A3 Relevant formula not transcribed from the tables.
A4 Some relevant work, e.g. states Theorem of Pythagoras.
A5 Correct answer without work.
A6 ½ × 4 × 4 × ${}^4/_4$ = 8.
A7 ½ × 4 × arbitrary value, even if completed.

Part (b) **20 marks (10, 10)** **Att 6 (3, 3)**

The diagram shows the curve $y = x^2 + 1$ in the domain $0 \le x \le 4$.



(i) Copy the following table. Then, complete it using the equation of the curve:

x	0	1	2	3	4
y					

(ii) Hence, use Simpson's Rule to estimate the area between the curve and the x-axis.

(b)(i) **10 marks** **Att 3**

α :

x	0	1	2	3	4
y	1	2	5	10	17

β :

x	0	1	2	3	4	
x^2	0	1	4	9	16	7marks
1	1	1	1	1	1	
y	1	2	5	10	17	10m

* Method α: Values of y without showing work: Award 5 × 2 (hit or miss) subject to Att 3.

Blunders (-3)

B1 Mathematical error unless an obvious slip, e.g.1 treated as $1x$ **or** x^2 incorrect **more** than once (once each row).

B2 Omits the row with 1 in method β.

B3 Includes the x row in the totals for y in method β.

B4 $y = (x + 1)^2$ and continues correctly. [Note: $y = x+1$ and continues, merits Att3, at most.]

Slips (-1)

S1 Each arithmetical slip to a maximum of 3, e.g. x^2 incorrect **once** but correct 4 times.

Attempts (3 marks)

A1 One correct y and stops or work with x^2 row not included.

A2 Copies table and stops.

A3 Some correct relevant work, e.g. $3^2 = 9$ and stops.

(b)(ii) **10 marks** **Att 3**

α: Area $= \frac{1}{3}\{F + L + 2(\text{odds}) + 4(\text{evens})\}$	3m	β: $\frac{1}{3}\{F + L + TOFE\}$	3m
$= \frac{1}{3}\{1 + 17 + 2(5) + 4(2 + 10)\}$	7m	F/L O E	
$= \frac{1}{3}\{18 + 10 + 4(12)\}$		1 5 2	
$= \frac{1}{3}\{28 + 48\}$		17 10	
$= \frac{1}{3}\{76\}$		$\frac{1}{3}\{ \quad \times 2 \quad \times 4\}$	7m
$= \frac{76}{3}$	10m	$= \frac{1}{3}\{18 + 10 + 4(12)\}$	
		$= \frac{1}{3}\{28 + 48\}$	
		$= \frac{1}{3}\{76\}$	
		$= \frac{76}{3}$	10m

* Accept candidate's y values from table.
* Marks for part (i) may be awarded for work done in part (ii).
* Allow $\frac{h}{3} = \{1^{st} + \text{last} + TOFE\}$ and penalise in calculations if formula not used correctly.

Blunders (-3)
B1 Incorrect $h/3$ (once).
B2 Incorrect F and/or L or extra terms with F and L (once).
B3 Incorrect TOFE (once).
B4 E or O omitted (once).
B5 Mathematical blunder, e.g. distribution error. Penalise once.
B6 Correct answer using integration.
B7 Uses x or x^2 values as heights **or** his/her inconsistent heights explicitly written.

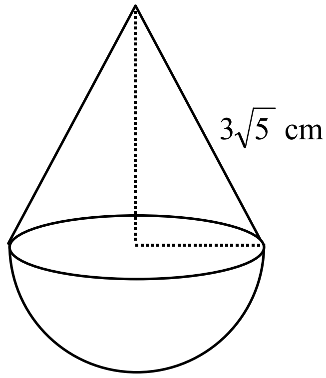
Slips (-1)
S1 Each arithmetical slip to a maximum of 3.

Attempts (3 marks)
A1 Identifies F and/or L or odds or evens and stops.
A2 Statement of Simpson's Rule, i.e. $\frac{h}{3}\{F + L + TOFE\}$.
A3 E **and** O omitted.
A4 Some correct relevant calculation only.
A5 Completes one rectangle or area of one rectangle.
A6 Completes all rectangles, but no calculations.
A7 Completes all rectangles and adds areas.
A8 Correct or consistent answer without showing work.
A9 Some correct step, e.g. some correct step in integrating.

Worthless (0 marks)
W1 Formula taken from tables and stops.
W2 Incorrect answer without work.

Part (c) **20 marks (5, 5, 5, 5)** **Att 8 (2, 2, 2, 2)**

A solid is in the shape of a hemisphere surmounted by a cone, as in the diagram.



(i) The volume of the hemisphere is 18π cm^3.
Find the radius of the hemisphere.

(ii) The slant height of the cone is $3\sqrt{5}$ cm.
Show that the vertical height of the cone is 6 cm.

(iii) Show that the volume of the cone equals the volume of the hemisphere.

(iv) This solid is melted down and recast in the shape of a solid cylinder. The height of the cylinder is 9 cm. Calculate its radius.

(c)(i) **5 marks** **Att 2**

Volume of hemisphere = 18π	$\Rightarrow \frac{2}{3}\pi r^3 = 18\pi$	
	$\Rightarrow r^3 = 27$	2m
	$\Rightarrow r = 3$ or $\sqrt[3]{27}$	5m

* No penalty if tables read as $^4/_8\pi r^3$.
* Substitution for π only does not merit marks.
* Must use volume of hemisphere to merit full marks.

Blunders (-3)
B1 Incorrect relevant formula, e.g. omits the ½ and deals with $^4/_3\pi r^3$, or uses curved surface area of hemisphere: $2\pi r^2$.
B2 Incorrect substitution.
B3 Mathematical error.
B4 Transposition error.

Slips (-1)
S1 Each arithmetical slip to a maximum of 3.

Attempts (2 marks)
A1 Correct answer without work.
A2 Some correct step and stops, e.g. Vol. of hemisphere = 18π, or 18π added to diagram.

(c)(ii) **5 marks** **Att 2**

α: $h^2 + r^2 = l^2 \Rightarrow h^2 + 3^2 = (3\sqrt{5})^2 \Rightarrow h^2 = 45 - 9 \Rightarrow h^2 = 36$	$\Rightarrow h = 6$ or $\sqrt{36}$
β: $h^2 + r^2 = 36 + 9 = 45 \Rightarrow l = \sqrt{45}$	$= 3\sqrt{5}$

* Allow candidate to use his value of *r* from part (i).
* No conclusion is required.
* Must use $3\sqrt{5}$ to merit full marks.

Blunders (-3)
B1 Incorrect use of Pythagoras.
B2 Mathematical error, e.g. $h^2 = 36 \Longrightarrow h = 18$ or $(3\sqrt{5})^2 = 15$.
B3 Transposing error.

Slips (-1) max.3

S1 Each arithmetical slip to a maximum of 3.

Misreadings (-1)

M1 Reads $3\sqrt{5}$ as $\sqrt[3]{5}$ and continues correctly.

Attempts (2 marks)

A1 Some correct substitution or diagram with dimension (other than $3\sqrt{5}$) shown and stops.

A2 Some relevant step, e.g. states Pythagoras or 3^2 and stops.

A3 Some correct trigonometrical statement, e.g. $\cos A = {}^{3}/_{3\sqrt{5}}$ and stops.

(c)(iii) **5 marks** **Att 2**

α: Volume of cone = $^1/_3\pi r^2h$ = $^1/_3\pi(3)^2(6)$	β: $^1/_3\pi r^2h = 18\pi$	2m
$= 18\pi$	$^1/_3 \times \pi \times 3^2 \times 6 = 18\pi$	5m

* Accept candidate's *r* and *h* from previous sections.

* No conclusion required.

Blunders (-3)

B1 Incorrect relevant cone formula, e.g. $^1/_3\pi rh$, $^1/_3 r^2h$, πr^2h, πrl.

B2 Incorrect substitution.

B3 Mathematical error.

B4 Correctly fills in formula and stops.

Attempts (2 marks)

A1 Some relevant step, e.g. a correct substitution, or diagram with dimension shown, and stops.

A2 Correct answer without work.

(c)(iv) **5 marks** **Att 2**

Volume of cylinder = Volume of solid *or* $\pi r^2h = 18\pi + 18\pi$ *or* $\pi r^2(9) = 36\pi \Rightarrow r^2 = 4$	2m
$r = 2$ or $\sqrt{4}$	5m

Blunders (-3)

B1 Incorrect relevant cylinder formula, e.g. $2\pi rh$, $^1/_3\pi r^2h$, πrh, r^2h and continues.

B2 Incorrect substitution.

B3 Mathematical error.

B4 Transposing error.

Attempts (2 marks)

A1 Some relevant step, e.g. indicates addition of volumes.

A2 A correct substitution or diagram with correct dimension shown.

A3 Correct answer without work.

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**40 marks**	**Att 14**

Part (a)	**10 marks**	**Att 3**

Find the co-ordinates of the point of intersection of the line $4x + y = 5$ and the line $3x - 2y = 12$.

(a)	**10 marks**	**Att 3**

α method	β method	Marks
α: 4x + y = 5 ⇒ 8x + 2y = 10	β: 4x + y = 5 ⇒ y = - 4x + 5	3 m
3x - 2y = 12 <u>3x - 2y = 12</u>	3x - 2y = 12 ⇒ 3x -2(- 4x + 5) = 12	
11x = 22	11x = 22	
x = 2	x = 2	7 m
y = -3	y = -3	10 m

* Accept correct point verified in **both** equations for 10 marks.

* Award at least 7 marks if candidate uses value of (calculated) first variable correctly to find second variable.

Blunders (-3)

B1 Fails to find the second variable.

B2 Mathematical error, e.g. (-2)(-4) = -8 *or* $8x + 3x = 11x^2$.

B3 Transposing error, e.g. $11x = 22 \Rightarrow x = {}^{11}/_{22}$ *or* 22–11

or 11x–10 = 12 ⇒ 11x = 12–10.

B4 Fails to multiply **both** sides in α method (once).

Slips (-1)

S1 Each numerical slip to a maximum of 3.

Attempts (3 marks)

A1 Some relevant step, e.g. 8*x* in α method and stops.

A2 Verifies any correct point in **one** equation.

A3 Tests an incorrect point in one or both equations **or** finds one or two points on either line.

A4 Graphical solution, written or indicated.
[Must be (2, -3) or consistent with a misreading. Subject to A3.]

A5 Correct answer without work.

Worthless (0 marks)

W1 Incorrect answer without work or inconsistent with correct answer from a graph.

Part (b)	**40 marks (10, 5, 10, 5, 5, 5)**	**Att 14 (3, 2, 3, 2, 2, 2)**

The line L has equation $4x - 5y = -40$.
$a(0,8)$ and $b(-10,0)$ are two points.

(i) Verify that a and b lie on L.

(ii) What is the slope of L?

(iii) The line K is perpendicular to L and it contains b. Find the equation of K.

(iv) K intersects the y-axis at the point c. Find the co-ordinates of c.

(v) d is another point such that $abcd$ is a rectangle. Calculate the area of $abcd$.

(vi) Find the co-ordinates of d.

(b) (i)		**10 marks**	**Att 3**
α: L:	$4x - 5y = -40$		β:
a(0, 8):	$4(0)$	3 m	Slope ab = $\frac{(8-0)}{(0+10)}$ or $\frac{4}{5}$
	$-5(8) = -40$	7 m	Eq^n ab is $y - 8 = \frac{4}{5}(x - 0)$
b(-10, 0):	$4(-10) - 5(0) = -40$	10 m	Eq^n ab is $4x - 5y = -40$

* Accept $y_1 - y = m(x_1 - x)$ or $y_2 - y_1 = m(x_2 - x_1)$ as equation of a line.
* 7 marks for one point.
* Finds midpoint of [ab], tests it correctly and stops merits 7 marks.
* Conclusion is not required, even if there is an error.
* Attempt marks for diagrams:
A diagram with *a* and/or *b* plotted once……………….Award Att mark for (i) **or** (ii).
[Do not award Att mark for **both** (i) **and** (ii) unless plotted twice.]
A line perpendicular to *ab*……………………………..Award Att mark for (iii).
Identifies where this line cuts y axis as point *c*………..Award Att mark for (iv) **or** (v).
[Do not award Att mark for **both** (iv) **and** (v) unless indicated separately.]
Completes rectangle *abcd*……………………………..Award Att mark for (vi).

Blunders (-3)
B1 Mixes up x and y entries (once).
B2 Mathematical error, e.g. -5(8) = 40 **or** -5(8) = ± 13.
B3 Transposing error, e.g. 4x + 5y = - 40 **or** 4x - 32 = 5x in β method.
B4 Incorrectly treats points as (x_1, x_2) and (y_1, y_2).
B5 Incorrect relevant formula for slope, e.g. $\frac{x_2 - x_1}{y_2 - y_1}$ *or* $\frac{y_2 + y_1}{x_2 + x_1}$ [error in **both** signs].
B6 Incorrect relevant formula for line, e.g. $x - x_1 = m(y - y_1)$ **or** $y + y_1 = m(x + x_1)$ [**both** signs].
B7 Uses perpendicular slope for equation of line in β method.

Slips (-1)
S1 Each numerical slip to a maximum of 3, e.g. 4 × 0 = 4.
S2 Error in **one** sign in slope or line formula, e.g. $\frac{y_2 - y_1}{x_2 + x_1}$ or $y + y_1 = m(x - x_1)$.
S3 Error in **one** sign when substituting into $y_2 - y_1$ or $x_2 - x_1$ or similar, if formula is written.

Attempts (3 marks)

A1 Some relevant step, e.g. some effort at substituting.

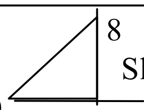
A2 Correct relevant formula, e.g. $\dfrac{y_2 - y_1}{x_2 - x_1}$ or $y - y_1 = m(x - x_1)$ or $y = mx \pm c$.

A3 Formula with $x_2 - x_1$ and/or $y_2 - y_1$ with some correct substitution.

A4 Point a and/or b plotted reasonably well.

A5 Writes "*if a point is on a line, it must satisfy its equation*" or similar.

(b) (ii) **5 marks** **Att 2**

α: L: $4x - 5y = -40$	β: Pts (0, 8),(-10, 0)	γ:  8	
$\Rightarrow -5y = -4x - 40$	Slope = $^{(0-8)}/_{(-10-0)}$	Slope = $^{\text{vertical}}/_{\text{horizontal}}$	2 m
$y = {}^4/_5 x + 8 \Rightarrow$ Slope = $^4/_5$	Slope = $^{-8}/_{-10}$ or $^4/_5$	-10 = $^8/_{10}$ or $^4/_5$	5 m

* Accept correct answer without work.
* Accept correct slope in part (i) for 5 marks in part (ii).

Blunders (-3)

B1 Error in transposing, e.g. $-5y = 4x$ or $y = {}^5/_4$.

B2 Answer given as $^5/_4$ or $^{-4}/_5$ without work.

B3 Incorrect relevant formula, e.g. $\dfrac{x_2 - x_1}{y_2 - y_1}$ *or* $\dfrac{y_2 + y_1}{x_2 + x_1}$ *or* $\dfrac{y_2 - y_1}{x_1 - x_2}$ or $^a/_b$ and continues.

B4 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2).

B5 Error in more than one sign when substituting.

B6 Switches x and y when substituting.

B7 Mathematical error, e.g. $0 - -10 = -10$.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

S2 Error in **one** sign in slope formula, e.g. $\dfrac{y_2 - y_1}{x_2 + x_1}$.

[Note: Accept $y = mx - c$, if used correctly.]

S3 **One** incorrect substitution or sign when substituting into $y_2 - y_1$ or $x_2 - x_1$ or similar, if formula is written, e.g. $\dfrac{y_2 - y_1}{x_2 - x_1} = {}^{(0+8)}/_{(-10-0)}$.

S4 Slope = $^{-a}/_b = {}^{-4}/_5$.

Attempts (2 marks)

A1 Some relevant step, e.g. any correct transposition, e.g. $4x = 5y$ and stops.

A2 Correct relevant formula, e.g. $^{\text{vertical}}/_{\text{horizontal}}$ *or* $^{\text{opposite}}/_{\text{adjacent}}$ *or* $\tan\theta$ *or* $\dfrac{y_2 - y_1}{x_2 - x_1}$

or $y - y_1 = m(x - x_1)$ *or* $y = mx \pm c$ and stops.

A3 Formula with $x_2 - x_1$ and/or $y_2 - y_1$ and some correct substitution.

A4 Point *a* and/or *b* plotted reasonably well **for this part**.

(b) (iii) **10 marks** **Att 3**

α: Slope of L is $^4/_5$	β: L is $4x - 5y = -40$, Slope $^4/_5$	3m

Slope of K is $-\,^{5}/_{4}$ Eq^n K: $y-0 = -\,^{5}/_{4}(x - -10)$ *or* $5x + 4y = -50$	K is $5x + 4y = c$ or $(-10, 0) \to 5(-10) + 4(0) = c$ $c = -50$ or K is $5x + 4y = -50$	7m 10m
γ: Slope of L is $^{4}/_{5}$		3m
Slope K is $-\,^{5}/_{4}$ or K is $y = -\,^{5}/_{4}x + c$ or $(-10, 0) \to 0 = -\,^{5}/_{4}(-10) + c$		7m
$c = -12\frac{1}{2}$ or K is $y = -\,^{5}/_{4}x - 12\frac{1}{2}$		10m

* Step 2 presupposes step 1.
* Errors in simplifying equation of K to be penalised in later part, if used.
* Answers **without work**:

$y - 0 = -\,^{5}/_{4}(x + 10)$ or any correct variation — Award full marks.

$y - 0 = {}^{4}/_{5}(x + 10)$ **or** $5x + 4y = c,\ c \neq -50$
or $y = -\,^{5}/_{4}x + c,\ c \neq -12\frac{1}{2}$ — Award 7 marks.

$y - 0 = m(x + 10)$, m not relevant — Award 4 marks.

$5x - 4y + c = 0$, (i.e. equation of line with neither slope nor point correct): — Award Att 3 marks.

Blunders (-3)

B1 $m_1.m_2 \neq -1$.
B2 Incorrect slope of ***L*** (with or without work) and continues correctly using that slope.
B3 Incorrect relevant formula for slope, e.g. $\frac{x_2 - x_1}{y_2 - y_1}$ *or* $\frac{y_2 + y_1}{x_2 + x_1}$ *or* $\frac{y_2 - y_1}{x_1 - x_2}$.
B4 Incorrect relevant formula for a line, e.g. $x - x_1 = m(y - y_1)$ **or** $y + y_1 = m(x + x_1)$.
B5 Switches x and y, e.g. $y - -10 = -\,^{5}/_{4}(x - 0)$ [once].
B6 Transposing error, e.g. $12\frac{1}{2} + c = 0 \Rightarrow c = 12\frac{1}{2}$.
B7 Uses a point other than *a* or *b*.

Slips (-1)

S1 **One** incorrect sign in formula.
S2 One incorrect sign in substitution, if formula is written.

Misreadings (-1)

M1 Uses point *a* instead of point *b*. [Other penalties may apply.]

Attempts (3 marks)

A1 Correct relevant formula, e.g. $\frac{y_2 - y_1}{x_2 - x_1}$ or $y - y_1 = m(x - x_1)$ or $y = mx \pm c$ and stops.
A2 Some relevant step, e.g. $m_1.m_2 = -1$ and stops.
A3 Slope of L = = $^{4}/_{5}$ or $^{-a}/_{b}$ and stops.
A4 Draws a pair of ⊥ lines with b as point of intersection.
A5 Formula with $x_2 - x_1$ and/or $y_2 - y_1$ and some correct substitution.

(b) (iv)	**5 marks**	**Att 2**

α: $x = 0$ or $y - 0 = -\frac{5}{4}(0 + 10)$ $y = -12\frac{1}{2}$	β: $x = 0$ or $y = -\frac{5}{4}(0) - 12\frac{1}{2}$ $y = -12\frac{1}{2}$

* Accept correct answer without work or $y = mx + c$ **and** $y = {}^{-5}/_4\,x - 12{\cdot}5$.

Blunders (-3)

B1 Mathematical error, e.g. $y - 0 = -\frac{5}{4}(x + 10) \Rightarrow y = -5x - 50$.
B2 Takes $y = 0$.
B3 Transposing error.

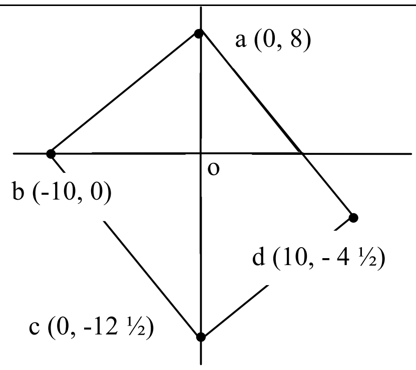
Slips (-1)

S1 Each numerical slip to a maximum of 3, e.g. $-\frac{5}{4}(0) = -\frac{5}{4}$.

Attempts (2 marks)

A1 Some relevant step, e.g. $x = 0$ or $y = 0$ and stops.
A2 Plots K and indicates c on the y axis reasonably well.
A3 Simplifies K to form $y = mx + c$, even if done in previous part, e.g. $y = {}^{-5}/_4\,x - 12{\cdot}5$ and stops.

(b) (v)	**5 marks**	**Att 2**

α: $(0, 8) \rightarrow (0, 0)$ $a(0, 8), b(-10, 0), c(0, -12\frac{1}{2})$ $a'(0, 0), b'(-10, -8), c'(0, -20\frac{1}{2})$ Area $\Delta abc = \frac{1}{2}\lvert x_1.y_2 - x_2.y_1\rvert$ $= \frac{1}{2}\lvert(-10)(-20\frac{1}{2}) - (0)(-8)\rvert$ $= 102\frac{1}{2}$ Area rectangle = 205	β: Area $\Delta abc = \frac{1}{2}\lvert ac\rvert.\lvert ob\rvert$ $= \frac{1}{2}(20\frac{1}{2})(10)$ $= 102\frac{1}{2}$ Rectangle = 205	
δ: $\frac{1}{2}\left\lvert\begin{array}{cc} 0 & 8 \\ -10 & 0 \\ 0 & -12\frac{1}{2} \\ 0 & 8 \end{array}\right\rvert$ $= \frac{1}{2}\lvert 0 + 125 + 0 + 80 - 0 - 0\rvert$ $= 102\frac{1}{2}$ $\Rightarrow$Rectangle = 205	γ: Area $\Delta abc = \Delta abo + \Delta bco$ $= \frac{1}{2}(10)(8) + \frac{1}{2}(10)(12\frac{1}{2})$ $= 40 + 62\frac{1}{2} = 102\frac{1}{2}$ Rectangle = 205	

ε: $\frac{1}{2}\lvert\{x_1.y_2 + x_2.y_3 + x_3.y_1\} - \{y_1.x_2 + y_2.x_3 + y_3.x_1\}\rvert$ $= \frac{1}{2}\lvert\{(0)(0)+(-10)(-12\frac{1}{2})+(0)(8)\} - \{(8)(-10)+(0)(0)+(-12\frac{1}{2})(0)\}\rvert$ $= \frac{1}{2}\lvert 0 + 125 + 0 + 80 - 0 - 0\rvert \Rightarrow$ Area $\Delta abc = 102\frac{1}{2}$ $\Rightarrow$Rectangle = 205	λ: Rectangle = $\lvert ab\rvert.\lvert bc\rvert$ $= \sqrt{100+64}\sqrt{100+165.25}$ $= \sqrt{164}\sqrt{256.25}$ $= \sqrt{42025}$ or 205

* No penalty for Area = - 205.
* Accept candidate's point c from (iv) and allow $\lvert ab\rvert \times \lvert bc\rvert$.
* Image triangle after $\overrightarrow{bo}$ is $a'(10, 8), b'(0,0), c'(10,-12\frac{1}{2})$;
after $\overrightarrow{co}$ is $a'(0,20\frac{1}{2}), b'(-10,-12\frac{1}{2}), c'(0,0)$.
* Part (vi) may be done before part (v) and point $d(10, -4\frac{1}{2})$ used to find area of rectangle.

Blunders (-3)

B1 Incorrect rel. formula and cont., e.g. $\frac{1}{2}|x_1.y_2 + x_2.y_1|$ *or* $\frac{1}{2}|x_1.y_2.x_2.y_1|$ *or* central sign.

B2 Two or more non-central signs incorrect in formula, e.g. $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$.

B3 Two or more incorrect substitutions.

B4 Mathematical error, e.g. ½(8)(10) = 20 or (-10)(-20½) = - 205.

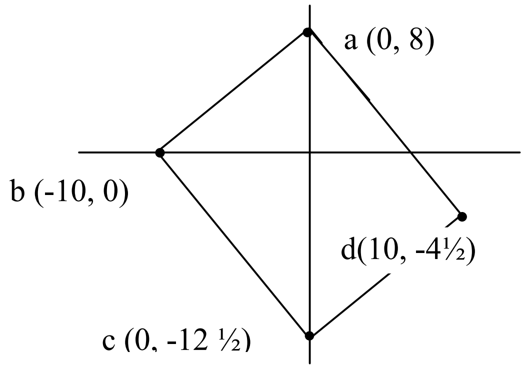
B5 Fails to find area of 2nd triangle in γ method.

B6 Fails to double area of triangle, i.e. leaves answer as 102½.

B7 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2).

B8 Two or more ordinates incorrect after translation in α method, e.g. $\overrightarrow{oa}$ instead of $\overrightarrow{ao}$.

B9 No translation where translation is required.



Slips (-1)

S1 Each numerical slip to a maximum of 3.

S2 **One** sign incorrect in formula for δ or ε method.

S3 **One** sign error in substitution, if formula is written.

S4 **One** ordinate incorrect after translation in α method.

S5 Rounding off of decimal to give, say, 204·8 as an answer.

Attempts (2 marks)

A1 Correct answer without work.

A2 Correct relevant formula (not transcribed from tables) and stops.

A3 Points a and/or b and/or c plotted reasonably well **for this part**.

A4 $\frac{1}{2}|x_1.y_2 + x_2.y_1|$ or $\frac{1}{2}|x_1.y_2.x_2.y_1|$ or similar with one correct substitution and stops.

A5 Formula with $x_2 - x_1$ and/or $y_2 - y_1$ and some correct substitution.

A6 Some relevant work, e.g. writes "*Area of rectangle is twice area of triangle*" or similar.

(b) (vi) **5 marks** **Att 2**

α: $\overrightarrow{ba}$ $\Rightarrow$x up 10, y up 8 $\Rightarrow$ c(0 -12½) $\to$ d(10, - 4½)	β: $\Rightarrow$ x up 10, y down 12½ $\Rightarrow$ a(0, 8) $\to$ d(10, - 4½)	γ: Eq^n ad is 5x + 4y = 32 Eq^n cd is 8x - 10y = 125 {d} = ad $\cap$ cd = (10, - 4½)

* Accept correct answer without work. * Accept candidate's point c from previous part.

Blunders (-3)

B1 Uses wrong translation, e.g. $\overrightarrow{ab}$ instead of $\overrightarrow{ba}$ or deals with ▱ abdc $\to$ d(-10, -20·5).

B2 Two incorrect ordinates, having used correct translation (either stated or indicated).

B3 Incorrect relevant formula for slope or equation of line in γ method.

B4 Finds only one ordinate.

Slips (-1)

S1 **One** correct and one incorrect ordinate after a correct translation.

Attempts (2 marks)

A1 Rectangle abcd plotted reasonably well **for this part**.

A2 Writes $\overrightarrow{ba}$ or $\overrightarrow{bc}$ or indicates on a diagram.

A3 Gives answer as (10, 0) or only one ordinate correct or consistent without work.

A4 Correct relevant formula and stops.

A5 Formula with $x_2 - x_1$ and/or $y_2 - y_1$ and some correct substitution.

A6 Some correct step, e.g. *up 10.*

QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 6**

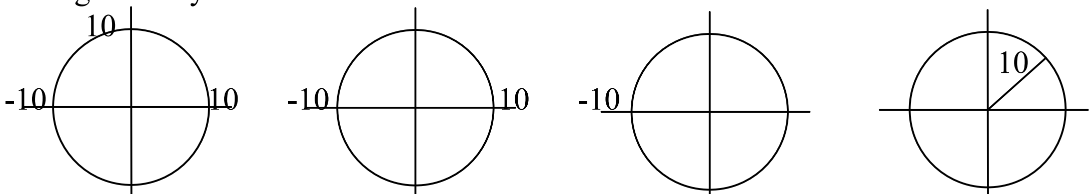
Part (a)	**10 marks**	**Att 3**

Write down the co-ordinates of any three points that lie on the circle with equation $x^2 + y^2 = 100$.

(a)	**10 marks**	**Att 3**

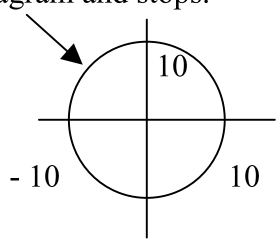
Any three of (0, 10), (0, −10), (10, 0), (−10, 0), (6, 8), ….,(1, $\sqrt{99}$), etc

* Award 3 marks for **one** correct, 7 marks for **two** correct, 10 marks for **three** correct.
* Award marks for correct points. Do not penalise for interspersed incorrect points.
* Accept correct points without work *or* with the co-ordinates written on a diagram.
* Accept $0^2 + 10^2 = 100$, $(-10)^2 + 0^2 = 100$, $10^2 + 0^2=100$. Do **not** penalise subsequent errors.
[Award 3 marks for one such, 7 marks for two 10 marks for three.]
* Diagram only:



7 marks   4 marks   3 marks   3 marks

Blunders (-3)
B1 Only two correct points.
B2 $x^2 = 2x$ or $\sqrt{x} = ½ x$. [Apply penalty once in this part.]
B3 Error in transposing (once).
B4 One of the ordinates incorrect in a point if not an obvious slip with work.
B5 Draws axes and circle with correct centre with r =10 written on a diagram and stops.



Slips (-1)
S1 Each numerical slip to a maximum of 3.

Attempts (3 marks)
A1 One correct point only.
A2 Correct relevant formula, e.g. $x^2 + y^2 = r^2$.
A3 Some relevant work towards substituting a value for x or y, e.g. writes 3^2.
A4 A relevant step, e.g. r = 10 *or* $r^2 = 100$ *or* centre (0, 0) *or* draws axes & circle & stops.

Part (b)	**20 marks (10, 10)**	**Att 6 (3, 3)**

The circle *C* has equation $(x-2)^2 + (y+1)^2 = 8$.

(i) Find the co-ordinates of the two points at which *C* cuts the *y*-axis.

(ii) Find the equation of the tangent to *C* at the point (4, 1).

(b) (i) **10 marks** **Att 3**

α	β	γ	
α: $x = 0 \Rightarrow (0-2)^2 + (y+1)^2 = 8$	β: $x^2 - 4x + 4 + y^2 + 2y + 1 = 8$	γ: $(0-2)^2 + (y+1)^2 = 8$	3m
$4 + y^2 + 2y + 1 = 8$	$x = 0 \Rightarrow \quad 4 + y^2 + 2y + 1 = 8$	$(y+1)^2 = 4$	4m
$y = {}^{(-2 \pm \sqrt{16})}/_2$ or $y = 1$	$y = {}^{(-2 \pm \sqrt{16})}/_2$ or $y = 1$	$y + 1 = \pm 2$ or $y = 1$	7m
$y = -3$	$y = -3$	$y = -3$	10m

Blunders (-3)

B1 Mathematical error, e.g. $(-2)(-2) = -4$ or $-2x - 2x = +4x$ or $(y+1)^2 = y^2 + 1$.

B2 Incorrect factors or incorrect relevant formula,

$$\text{e.g. } \frac{-b \pm \sqrt{b^2 + 4ac}}{2a} \text{ or } \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2} \text{ or } \sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}\ .$$

B3 Only one value for y (either value).

B4 Transposing error (once for each step and once for each type).

B5 $y = 0$ instead of $x = 0 \Rightarrow x = {}^{(4 \pm \sqrt{28})}/_2$ for 4 marks $\Rightarrow x = 4{\cdot}6$ and $-0{\cdot}6$ for 7 marks.

Slips (-1)

S1 Each numerical slip to a maximum of three, e.g. $-4(0) = -4$.

S2 One non-central sign wrong in distance or circle formula, e.g. $\sqrt{(x_2 - x_1)^2 + (y_2 + y_1)^2}$

Attempts (3 marks)

A1 Some relevant step, e.g. states $x = 0$ or writes x^2 and stops.

A2 States $r^2 = 8$ and stops or $r = \sqrt{8}$ and stops or centre $(0, 0) \Rightarrow (0, \sqrt{8})$.

A3 States centre of the circle is $(2, -1)$ and stops.

A4 Correct relevant formula & stops, e.g. $(x-h)^2 + (y-k)^2 = r^2$ *or* $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

A5 Correct answer without work or from graph (if both not verified).

(b) (ii) **10 marks** **Att 3**

α	β	γ	
α: Centre $(2, -1)$	β: $2(x-2) + 2(y+1)^{dy}/_{dx} = 0$	γ: Centre $(2, -1) = (-g, -f)$	3m
Slope $(2, -1)$ to $(4, 1) = \frac{1+1}{4-2}$	$\frac{dy}{dx} = \frac{-x+2}{y+1}$	$(x_1, y_1) = (4, 1)$, $g^2 + f^2 - c = 8$	4m
Tangent slope $= -1$	At $(4, 1)$ slope $= {}^{(-4+2)}/_{(1+1)}$ *or* -1	$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$	7m
Equation: $y - 1 = -1(x - 4)$ or $x + y = 5$	Equation: $y - 1 = -1(x - 4)$ or $x + y = 5$	$4x + 1y - 2(x+4) + 1(y+1) - 3 = 0$ or $x + y = 5$	10m

δ: $(2, -1) \to (0, 0) \Rightarrow (4, 1) \to (2, 2)$
$xx_1 + yy_1 = r^2 \Rightarrow 2x + 2y = 8$
$(0, 0) \to (2, -1) \Rightarrow 2x + 2y = 8 \to x + y = 5$

* Candidate may not lose more than 3 marks for incorrect slope of radius, if used correctly.
* Allow candidate to use other valid points to find the slope of the relevant radius.
* Case: $xx_1 + yy_1 = r^2$
  $4x + 1y = 17$ or 8…….. Award 4 marks.

Blunders (-3)

B1 Incorrect relevant formula, e.g. $\frac{x_2 - x_1}{y_2 - y_1}$ *or* $\frac{y_2 + y_1}{x_2 + x_1}$.

[No further penalty if continued correctly]

B2 x and y switched in substitution.

B3 Error in more than one sign when substituting.

B4 Transposing error, e.g. in method β.

B5 Error in getting slope of tangent, i.e. leaves as 1 and not –1.
B6 Mathematical error, e.g. 4 –2 = –2
B7 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2) [once].
B8 Incorrect centre of circle, e.g. (–2, 1) both signs incorrect.
B9 Incorrect point used.

Slips (-1)
S1 Each numerical slip to a maximum of three.
S2 **One** non-central sign incorrect, e.g. (2, 1) or $\frac{y_2 - y_1}{x_2 + x_1}$ or $y - y_1 = m(x + x_1)$ or sign error in substituting.

Attempts (3 marks)
A1 Writes a relevant formula and stops, e.g. $y - y_1 = m(x - x_1)$ or $m_1 \times m_2 = -1$.
A2 Some relevant step, e.g. centre of circle is (2, - 1) or (2, 1) or proves (4, 1)$\in$C and stops.
A3 States that radius of circle is $\sqrt{8}$ and stops.
A4 Correct answer without work.
A5 $x_2 - x_1$ or $y_2 - y_1$ with some correct substitution.

Worthless (0 marks)
W1 Equation of circle instead of line (if candidate does not merit marks for other work).

Part (c) **20 marks (10, 10)** **Att 6 (3, 3)**

$a(-5,1)$, $b(3,7)$ and $c(9,-1)$ are three points.

(i) Show that the triangle *abc* is right-angled.

(ii) Hence, find the centre of the circle that passes through *a, b* and *c* and write down the equation of the circle.

(c)(i) **10 marks** **Att 3**

α:	Slope = $(y_2-y_1)/(x_2-x_1)$ Slope ab = $^{(7-1)}/_{(3+5)} = {}^6/_8$ Slope bc = $^{(-1-7)}/_{(9-3)} = {}^{-8}/_6$ $^6/_8 \times {}^{-8}/_6 = -1$	β: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\|ab\| = \sqrt{100}$ $\|bc\| = \sqrt{100}$, $\|ac\| = \sqrt{200}$ $\|ab\|^2 + \|bc\|^2 = \|ac\|^2$	3m 6m 9m 10m
γ:	$(y-y_1)/(x-x_1) = (y_2-y_1)/(x_2-x_1)$ Eq^n ab: $3x - 4y + 19 = 0$ Eq^n bc: $4x + 3y - 33 = 0$ $^3/_4 \times {}^{-4}/_3 = -1$	δ: $\vec{ab} = \vec{b} - \vec{a}$ $\vec{ab} = 8\vec{i} + 6\vec{j}$ $\vec{bc} = 6\vec{i} - 8\vec{j}$ $\vec{ab}.\vec{bc} = 0$	3m 6m 9m 10m

* Accept slope ab = ¾ , slope bc = $^{-4}/_3$, $¾ \times {}^{-4}/_3 = -1$ for 10 marks.

Blunders (-3)

B1 Incorrectly treats the couples as (x_1, x_2) and (y_1, y_2) or x and y switched when substituting (once).

B2 Incorrect relevant formula $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$ *or* $(y_2 + y_1)/(x_2 + x_1)$.

B3 Incorrect use of √ , e.g. |ac| = 200.

B4 Mathematical errors, e.g. $8^2 = 16$ or $(-8)^2 = -64$ etc.

B5 Two or more incorrect substitutions or error in more than one sign when substituting.

B6 Deals only with two lines that are not perpendicular.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

S2 **One** incorrect sign in $(x_2 - x_1)$ **or** $(y_2 - y_1)$.

S3 **One** incorrect substitution or one incorrect sign when substituting if formula is written.

S4 Last line omitted in any of four methods shown.

Attempts (3 marks)

A1 Relevant step, e.g. states $m_1 \times m_2 = -1$ and stops.

A2 States Pythagoras' theorem.

A3 Writes any correct relevant formula and stops (including area of triangle).

A4 Plots points and stops.

A5 sin/cos/tan defined.

(c)(ii) **10 marks** **Att 3**

α: Midpt. of [ac] = $\left(\frac{-5+9}{2}, \frac{1-1}{2}\right)$ = (2, 0)	β: $x^2 + y^2 + 2gx + 2fy + c = 0$	γ: $x^2 + y^2 + 2gx + 2fy + c = 0$	3m
	Centre (2, 0), g = -2, f = 0	*a*: 25 + 1 - 10g + 2f + c = 0	
		b: 9 + 49 + 6g + 14f + c = 0	
		c: 81 + 1 + 18g - 2f + c = 0	
|radius| = $\sqrt{(2+5)^2 + (0-1)^2} = \sqrt{50}$	$x^2 + y^2 - 4x + 0 + c = 0$	g = -2, f = 0, centre (2, 0)	7m
	b(3, 7): 9 + 49 - 12 + c = 0	[c = - 46]	
Eq.: $(x-2)^2 + (y-0)^2 = (\sqrt{50})^2$ or 50	c = - 46		10m

* γ method is not *hence* and merits 7 marks.
* Case: Uses (c)(ii) to prove (c)(i), e.g. γ method ⇒ centre (2, 0)
  Centre (2, 0) is midpoint of [ac].
  ⇒ |∠abc| = 90° Merits 7 + 10 marks.
* Accept $(h - x)^2 + (k - y)^2 = r^2$ and continues.
* Accept: centre (2, 0) **and** $(h - 2)^2 + (k - 0)^2 = 50$ for 10 marks.
* Cases without work:

Centre (2, 0) written and: $(x-2)^2 + y^2 = 50$…………………………. award 10 marks

$(x-2)^2 + y^2 = r^2$, $r^2 \neq 50$…………………. award 7 marks

$(x-h)^2 + (y-k)^2 = 50$, $(h, k) \neq (2, 0)$…………… award 7 marks

$(x-h)^2 + (y-k)^2 = r^2$, $r^2 \neq 50$, $(h, k) \neq (2, 0)$….. award 4 marks

$(x-h)^2 - (y-k)^2 = r^2$, $r^2 \neq 50$, $(h, k) \neq (2, 0)$……. award 3 marks.

Centre ≠ (2, 0) or not written and $(x-2)^2 + y^2 = 50$……………………. award 7 marks

Centre (h, k) ≠ (2, 0) and

$(x-h)^2 + (y-k)^2 = r^2$, r^2 consistent,……………. award 7 marks

$(x-h)^2 + (y-k)^2 = r^2$, r^2 not consistent,…………. award 4 marks

$(x-h)^2 + (y-k)^2 = r^2$, (h, k) not consistent,…….. award 4 marks

$(x-h)^2 + (y-k)^2 = r^2$, neither r^2 nor (h, k) consistent: award 3 marks

Blunders (-3)

B1 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2), *or* x and y switched when substituting, (apply once).
B2 Incorrect relevant formula, e.g. $[(x_1 - x_2)/2, (y_1 - y_2)/2]$ or $(x - h)^2 - (y - k)^2 = r^2$.
B3 Confuses radius with diameter.
B4 Incorrectly leaves out √ , e.g. r = 50.
B5 Two or more incorrect substitutions.
B6 Transposing error.
B7 Mathematical error e.g. $(1)^2 = 2$ or $(-7)^2 = -49$.
B8 Incorrect centre, e.g. midpoint of [ab] = (- 1, 4) and continues correctly.

Slips (-1)

S1 Each numerical slip to a maximum of 3.
S2 **One** incorrect non-central sign in formula, e.g. $(x - h)^2 + (y + k)^2 = r^2$.
S3 **One** incorrect substitution.

Attempts (3 marks)

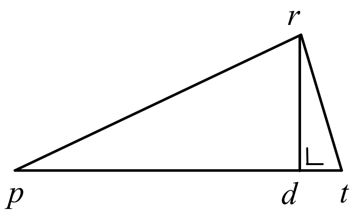
A1 Any correct relevant formula and stops,
e.g. $(x-h)^2 + (y-k)^2 = r^2$, $x^2 + y^2 + 2gx + 2fy + c = 0$
or midpoint or line (mediator) formula.
A2 Draws a circle containing the three points **for this part**.
A3 States radius is the distance from the centre to the point a, b, or c.
A4 States the centre is halfway between the points a and c.
A5 Mentions that an angle in a semi-circle is 90^{o}.
A6 Some relevant step.

QUESTION 4

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 8**

Part (a) **10 marks** **Att 3**

The area of the triangle *rpt* is 30 cm^2.
rd is perpendicular to *pt*.
Given that $|pt| = 12$ cm, calculate $|rd|$.



(a) **10 marks** **Att 3**

½ |pt|.|rd| = Area Δrpt	3 m
½ (12).|rd| = 30	7 m (all subs.)
|rd| = 5	10 m

* Accept correct answer without work..

Blunders (-3)
B1 Omits ½ in formula.
B2 Error in substitution.
B3 Mathematical error in calculations, e.g. ½ (12) = 24 or error in transposing.

Slips (-1)
S1 Each obvious numerical slip to a maximum of 3.

Attempts (3 marks)
A1 A correct formula for the area of a triangle not lifted from the tables, e.g. ½ base × height.
A2 Some correct substitution in relevant formula, e.g. ½(12).|pr| sin|∠rpt|.
A3 ½(12)(30), even if continued to 180.
A4 Some correct work, e.g. adds something to diagram.

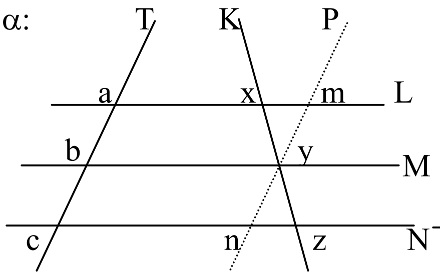
Worthless (0 marks)
W1 Transcribes formula from tables and stops.

Part (b) **20 marks** **Att 7**

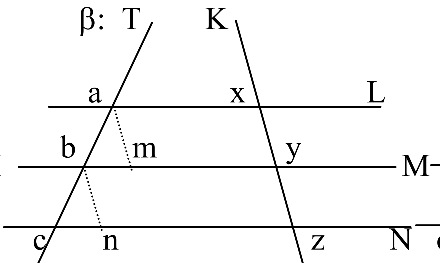
> Prove that if three parallel lines make intercepts of equal length on a transversal, then they will also make intercepts of equal length on any other transversal.

(b) **20 marks** **Att 7**

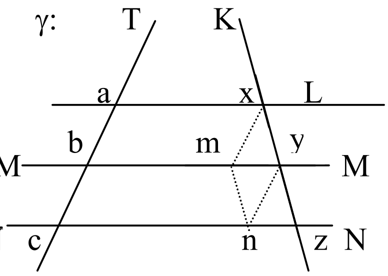
α:



β:



γ:



Given |ab| = |bc|

To Prove |xy| = |yz|

α	β	γ	
Construction Through y draw P//T. Let P intersect L and N at m and n	Cons Draw am//xy, bn//yz. m∈M, n∈N	Cons Draw xm//T, yn//T. m∈M, n∈N	5
Proof |ab| = |my|, |bc| = |yn| ==> |my = |yn|	Proof |∠bam| = |∠cbn|, |∠abm| = |∠bcn|	⇒ |xm| = |ab| = |bc| = |yn|, xm // yn	5, 2
		⇒ xmny is a parallelogram	
|∠xym| = |∠nyz|, |∠mxy| = |∠nzy|	|ab| = |bc| Δabm ≡ Δbcn	⇒ |mn| = |xy|	5, 2
Δmxy and ynz are congruent.	⇒ |am| = |bn|	and |mn| = |yz|	5, 2
⇒ |xy| = |yz|	|am| = |xy|, |bn| = |yz|		

* If a candidate's work is not worthless, he/she must be awarded at least 7 marks.
* Candidate is not required to write |xy| = |yz| at the end of the proof.
* Award 0 marks for a step that is omitted and not implied.
* Accept steps clearly indicated on diagram subject to B2.

δ: Based on diagram in γ
Cons Complete ▱ abmx and ▱ xmny.
Proof |yn| = |xm| = |ab| = |bc| => cn // by => n∈N (because bcny is a ▱)
⇒ |mn| = |xy|
and mnzy is a ▱
⇒ |mn| = |yz|

Blunders (-3)

B1 Incorrect step or part of step omitted.
B2 Steps written in an illogical order (once).

Attempts (7 marks)

A1 Construction and stops.
A2 A relevant theorem stated or proved, e.g. opposite sides of a parallelogram.
A3 Reasonable diagram without construction.
A4 States or illustrates a special case, e.g. with T⊥N.
A5 No construction merits ⇒ attempt marks, at most.
A6 Some relevant step.

Worthless (0 marks)

W1 An irrelevant theorem.

Part (c) **20 marks (5, 5, 5, 5)** **Att 8 (2, 2, 2, 2)**

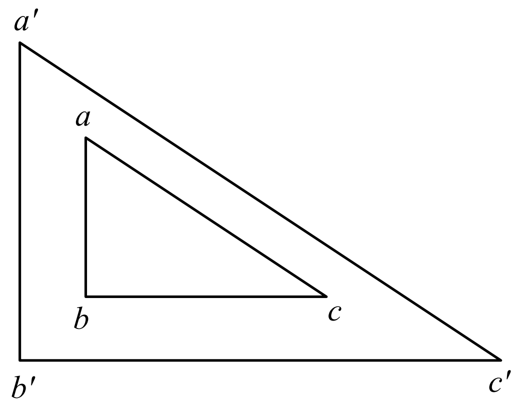
The triangle $a'b'c'$ is the image of the triangle abc under an enlargement.

(i) Find, by measurement, the scale factor of the enlargement.

(ii) Copy the diagram and show how to find the centre of the enlargement.

(iii) Units are chosen so that $|bc| = 8$ units.
How many of these units is $|b'c'|$?

(iv) Find the area of triangle abc, given that the area of $a'b'c'$ is 84 square units.



(c)(i) **5 marks** **Att 2**

Scale factor (k) = 2

* Accept 1·5 ≤ k ≤ 2·5 without work for 5 marks.
* Accept k = 2 used in later parts for 5 marks in part (i), if not already awarded.
* Accept words, e.g. *double*.
* Accept answer without work or accept scale factor = 2 used in later parts for 5 marks in (i).

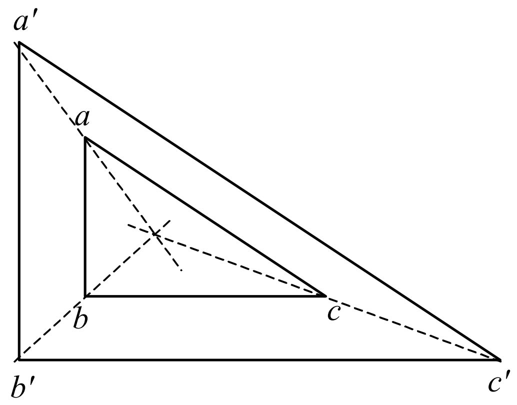
Blunders (-3)
B1 k > 2·5 or 1 ≤ k < 1·5 with or without work.
B2 Confuses images, e.g. $k = {}^{|b'c'|}/_{|ab|}$.

Slips (-1)
S1 Each obvious slip with work to a maximum of 3.

Misreadings (-1)
M1 k = ½ with or without work.
M2 A reciprocal of an acceptable k with work.

Attempts (2 marks)
A1 $k = {}^{|a'b'|}/_{|ab|}$ or ${}^{|ab|}/_{|a'b'|}$ and stops.
A2 Length of side given:
|ab| = 21 mm, |a'b'| = 42, |ac| = 38, |a'c'| = 76, |bc| = 32, |b'c'| = 64 ±5.
[or 0·8 inches, 1·6, 1·5, 3·0, 1·25, 2·5]
A3 Some relevant step, e.g. $k^2 = {}^{\Delta a'b'c'}/_{\Delta abc}$ and stops.

(c)(ii) **5 marks** **Att 2**



* Accept two lines, say, a'a and b'b intersecting and ignore subsequent errors.
* Do not insist on three construction lines being concurrent.
* Accept line segment [b'o] with b the midpoint of [b'o] or |b'b| = |bo| or indicated.

Blunders (-3)
B1 Only one relevant construction line, e.g. b'b.

Slips (-1)
S1 Two or three relevant construction lines but not intersecting.

Attempts (2 marks)
A1 Any relevant step, e.g. draws one or two triangles and stops.
A2 Mention of construction step, e.g. *intersect* or *parallel* or *join b'b.*

(c)(iii) **5 marks** **Att 2**

α: $\|b'c'\| = 8 \times 2$ or 16	β: (Unitary method) 32 mm $\rightarrow$ 8 units => 1mm $\rightarrow$ $^{8}/_{32}$ units 64 mm $\rightarrow$ ($^{8}/_{32}$) $\times$ 64 or 16

* Accept answer which is correct or consistent with candidate's k from (i) without work.

Blunders (-3)
B1 $8 \div k$ or $k \div 8$.
B2 $k^2 \times 8$.

Slips (-1)
S1 Each obvious slip to a maximum of 3.

Attempts (2 marks)
A1 $8 \times r$ or $8 \div r$, where r is an arbitrary inconsistent number.
A2 8 + k or 8 - k or 8+ r or 8 - r, where r is an arbitrary inconsistent number.
A3 Any relevant step, e.g. 32 = 8.

(c)(iv) **5 marks** **Att 2**

α: Area of Δabc = Area Δa'b'c' ÷ k^2 = 84 ÷ 4 = 21:	δ: Δa'b'c' = 84 ⇒ ½|b'c'|.h' = 84
β: Δabc: Δa'b'c' = 21^2 :42^2 ⇒ Δabc = 84 ÷ 4 = 21	½(16)h' = 84 ⇒ h' = 10·5
γ: ½|a'b'|.|b'c'|. sin|∠a'b'c'| = 84 ⇒ ½(10·5)(16)sin|∠a'b'c'| = 84 sin|∠a'b'c'| = 1⇒ Δabc = ½(5·25)(8)(1) = 21	h = 10·5 ÷ 2 = 5·25 Δabc = ½(8)(5·25) = 21

* Accept candidate's k from previous work and |b'c'| from part (iii).
* Accept correct or consistent answer without work.

Blunders (-3)

B1 84 ÷ k or 84 × k or 84 × k^2, even if candidate continues.
B2 k ÷ 84 or k^2 ÷ 84, even if candidate continues.
B3 Transposing error.

Slips (-1)

S1 Each obvious slip to a maximum of 3.
S2 Stops at 84 ÷ 4.

Attempts (2 marks)

A1 84 × r^2 or 84 ÷ r^2 or 84 ± r^2, where r is an arbitrary inconsistent number.
A2 84 × r or 84 ÷ r or 84 ± r, where r is an arbitrary inconsistent number.
A3 Finds k^2 and stops.
A4 Some relevant step in any method and stops.
A5 Area formula from tables with some correct substitution.
A6 Relevant formula, not copied from the tables, e.g. ½ base × h.
A7 ½ × 8 × 5·25 and stops.

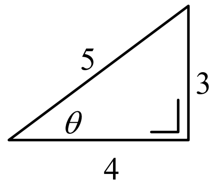
QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks** **Att 3**

Use the information given in the diagram to show that

$$\sin\theta + \cos\theta > \tan\theta.$$



(a) **10 marks** **Att 3**

α: $\sin\theta = {}^3/_5$ or 0·6 — 3 marks	β: $\theta = \sin^{-1}(0{\cdot}6) = 36{\cdot}8698\ldots^o$ or 0·6435…rads. — 3 m
$\cos\theta = {}^4/_5$ or 0·8 — 6 m	$\cos\theta = 0{\cdot}8$ — 6 m
$\tan\theta = {}^3/_4$ or 0·75 — 9 m	$\tan\theta = 0{\cdot}75$ — 9 m
$\sin\theta + \cos\theta = {}^7/_5$ or 1·4 — 10 m	$\sin\theta + \cos\theta = 1{\cdot}4$ — 10 m
or $\sin\theta + \cos\theta > \tan\theta$	or $\sin\theta + \cos\theta > \tan\theta$

* Accept ${}^3/_5 + {}^4/_5 > {}^3/_4$ or ${}^3/_5\,\theta + {}^4/_5\,\theta > {}^3/_4\,\theta$ without work.

* Accept any valid rearrangement of the inequality, e.g. 0·6 > 0·75 - 0·8.

Blunders (-3)

B1 Incorrect or inverted ratio, e.g. $\sin\theta = {}^5/_3$ (once for same error repeated).
B2 Each missing ratio, subject to attempt mark.
B3 Misplaced decimal, e.g. $\sin\theta = 6$ (if correct value is not written).
B4 Mathematical error, e.g. $\sin\theta + \cos\theta = {}^7/_{10}$ (if correct values are not written).

Slips (-1)

S1 Each obvious slip to a maximum of 3, e.g. $\sin^{-1}(0{\cdot}6) = 36{\cdot}6898\ldots^o$
S2 No conclusion having written the three ratios.

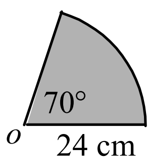
Attempts (3 marks)

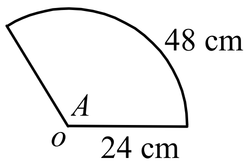
A1 Any correct trigonometrical definition and stops, e.g. $\sin = {}^{opp}/_{hyp}$ or correct mnemonic.
A2 States Pythagoras' theorem.
A3 Uses an arbitrary angle, e.g. $\sin45^o + \cos45^o > \tan 45^o$ and/or values read.
A4 Some relevant step.

Part (b) **20 marks (10, 10)** **Att 6 (3, 3)**

A circle has radius 24 cm and centre *o*.

(i) Calculate the area of a sector which has 70° at *o*.
Take $\pi = \dfrac{22}{7}$.





(ii) An arc of length 48 cm subtends an angle *A* at *o*.
Calculate *A*, correct to the nearest degree.

(b) (i) **10 marks** **Att 3**

α: Area of sector $= {}^{70}/_{360} \times \pi r^2$	β: $70^\circ = {}^{70\pi}/_{180}$ radians [Area of sector $= \frac{1}{2} r^2\theta$]	3m
Area of sector $= {}^{70}/_{360} \times {}^{22}/_{7} \times 24 \times 24$	Area of sector $= \frac{1}{2} \times 24 \times 24 \times {}^{70\pi}/_{180}$	7m
$= 352$	$= 352$	10m

* Accept $\pi = 3{\cdot}14 \rightarrow 351{\cdot}68$ or E.C. $\rightarrow 351{\cdot}8583\ldots$ with work shown.
* Cases: $\frac{1}{2} \times 24^2 \times 70 = 20160$……………………………Award 7 marks [B3].
$\frac{1}{2}\pi r^2\theta = \frac{1}{2} \times {}^{22}/_{7} \times 24^2 \times 70 = 63\ 360$……………Award 4 marks [B4 and B3].
$\frac{1}{2} \times 24^2 \times 70\pi = 63\ 360$…………………………..Award 4 marks [B4 and B3].

Blunders (-3)
B1 Mathematical error when dealing with fractions (once).
B2 Treats r^2 as $2r$, i.e. $24^2 = 48$.
B3 Converts to radians incorrectly in method β.
B4 Uses an incorrect relevant formula, e.g. ½ ab sinC (once).
B5 Calculates length of the arc for part (i)→arc = 29·3215...[Note: See γ method (b) (ii)]
B6 Misplaced decimal point (once).

Slips (-1)
S1 Each numerical slip to a maximum of 3.
S2 Answer in terms of $\pi \rightarrow 112\pi$ with work shown.
S3 Early rounding of decimals yielding an answer $<351{\cdot}5$ or $\geq 352{\cdot}5$, e.g. $\pi = 3{\cdot}1 \rightarrow 347{\cdot}2$.

Attempts (3 marks)
A1 Acceptable answer without work, e.g. $351{\cdot}5 \leq$ answer $< 352{\cdot}5$.
A2 Any mention of ${}^{70}/_{360}$ or ${}^{70}/_{180}$ or 24^2.
A3 Some correct substitution into a relevant formula and stops.
A4 A relevant step.

(b) (ii) **10 marks** **Att 3**

α: Circumference $= 2\pi r = 2 \times \pi \times 24$	β: Arc $= r\theta \Rightarrow 48 = 24(A)$	γ: From (i) $70^\circ \rightarrow {}^{24.70.\pi}/_{180}$ cm	3m
$\frac{A}{360} \times 2 \times \pi \times 24 = 48$ or $A = \dfrac{360 \times 48}{2 \times \pi \times 24}$ deg.	$A = 2$ rads or $\dfrac{180 \times 2}{\pi}$ deg.	$\Rightarrow A = \dfrac{70 \times 180 \times 48}{24 \times 70 \times \pi}$ deg	7m
$\Rightarrow A = {}^{360}/_{\pi}$ or $114{\cdot}\dot{5}\dot{4}$ or $114{\cdot}649$ or $114{\cdot}591\ldots$			9m
$A = 115^\circ$			10m

* Note: $\pi = {}^{22}/_{7} \rightarrow 114{\cdot}\dot{5}\dot{4}$, $\pi = 3{\cdot}14 \rightarrow 114{\cdot}649\ldots$, π from E.C. $\rightarrow 114{\cdot}591\ldots$

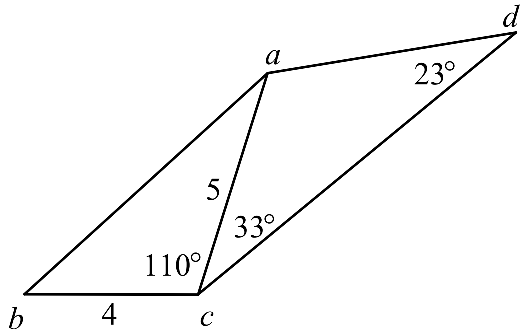
Blunders (-3)
B1 Error in relevant formula or substitution.
B2 Confuses radians and degrees or π radians $\neq 180^\circ$.
B3 Mathematical error in fractions.
B4 Works from *area of sector = 48* instead of *length of arc = 48* $\rightarrow 9{\cdot}\dot{5}\dot{4} = 10^\circ$ with work.

Slips (-1)
S1 Each numerical slip to a maximum of 3.
S2 Fails to round off correctly (once).

Attempts (3 marks)
A1 Acceptable answer without work, i.e. $114{\cdot}5 \leq$ answer $< 115{\cdot}5$.
A2 A relevant step and stops, e.g. ${}^{48}/_{24}$.
A3 Some substitution into a relevant formula and stops.
A4 π radians $= 180^\circ$.

Part (c) **20 marks (10, 10)** **Att 6 (3, 3)**

In the quadrilateral *abcd*, $|ac| = 5$ units, $|bc| = 4$ units, $|\angle bca| = 110°$, $|\angle acd| = 33°$ and $|\angle cda| = 23°$.



(i) Calculate $|ab|$, correct to two decimal places.

(ii) Calculate $|cd|$, correct to two decimal places.

(c)(i) **10 marks** **Att 3**

α:

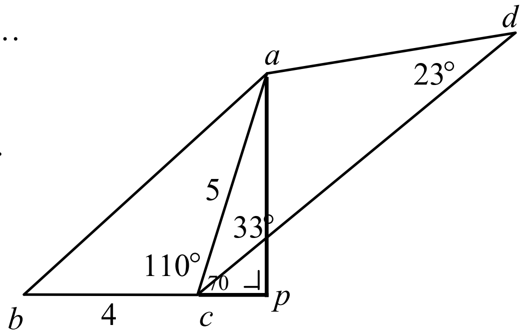
$\|ab\|^2 = \|bc\|^2 + \|ac\|^2 - 2\,\|bc\|.\|ac\|.\cos\|\angle acb\|$	3 marks
$= \|bc\|^2 + \|ac\|^2 - 2\,(4)(5).\cos\|\angle acb\|$	4 m (incomplete substitution)
$\|ab\|^2 = 4^2 + 5^2 - 2(4)(5)\cos 110^o$	7 m (complete substitution)
$\|ab\|^2 = 41 - 40\cos 110^o$	
$\|ab\|^2 = 41 + 13{\cdot}6808\ldots = 54{\cdot}6808\ldots$	
$\|ab\| = \sqrt{54 \cdot 6808...}$ or $7{\cdot}3946\ldots$	9 m
$\|ab\| = 7{\cdot}39$	10 m

β:

$^{|cp|}/_5 = \cos 70^o \Rightarrow |cp| = 5\cos 70^o$ or $1{\cdot}7101\ldots$

$|bp| = 5{\cdot}7101\ldots$

$^{|ap|}/_5 = \sin 70^o \Rightarrow |ap| = 5\sin 70^o$ or $4{\cdot}6984\ldots$



$|ab|^2 = |ap|^2 + |bp|^2$

$= 22{\cdot}0755\ldots + 32{\cdot}6052\ldots = 54{\cdot}6808\ldots$

$|ab| = \sqrt{54 \cdot 6808...}$ or $7{\cdot}3946\ldots$

$|ab| = 7{\cdot}39$

Blunders (-3)

B1 Error in cosine formula (once).
B2 Incorrect substitution and continues (once).
B3 Misplaced decimal point (once).
B4 Incorrect function read, e.g. $\sin 110^o = 0{\cdot}9396\ldots$ instead of $\cos 110^o$ (once in this part).
B5 Incorrectly uses radian mode (once in each part of (c)).
B6 Treats figure as parallelogram and continues, e.g. $^{|ab|}/_{\sin 110} = {}^{5}/_{\sin 37} \rightarrow 7{\cdot}8071 \approx 7{\cdot}81$.
B7 $41 - 40\cos 110^o = 1\cos 110^o$.
B8 $|ab|^2 = 41 - 40\cos 110^o = 41 - 13{\cdot}6808\ldots = 27{\cdot}319\ldots \Rightarrow |ab| \simeq 5{\cdot}23$.

Slips (-1)

S1 Each numerical slip to a maximum of 3.
S2 Fails to round off correctly.
S3 Slip in reading tables/calculator.

Attempts (3 marks)

A1 7·39 or 7·39something without work.

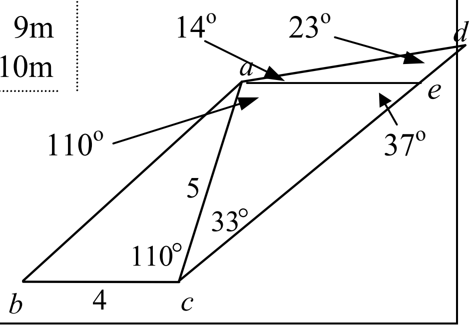
A2 Some but not all substitution into sine formula and stops.
A3 Mention of 5^2 and/or 4^2 and/or $\cos 110^o$ and stops.
A4 $|ab|^2 = 4^2 + 5^2 = 41 \Rightarrow |ab| = \sqrt{41} \simeq 6{\cdot}40$.
A5 Some use of sin/cos/tan.
A6 Some relevant step, e.g. correct relevant formula or $180^o - 110^o = 70^o$ and stops.

Worthless (0 marks)
W1 Incorrect answer without work subject to A1.
W2 Formula taken from tables and stops.

(c)(ii) **10 marks** **Att 3**

α:
$|\angle cad| = 124^o$ 3m
$^{|cd|}/_{\sin 124} = {}^5/_{\sin 23}$ or $^{|cd|}/_{0{\cdot}8290} = {}^5/_{0{\cdot}3907}$ or $|cd| = {}^{5\sin 124}/_{\sin 23}$ 7m
$|cd| = 10{\cdot}6091$ 9m
$|cd| \simeq 10{\cdot}61$ 10m

β: Draw ae // bc 3m
$^{|ce|}/_{\sin 110} = {}^5/_{\sin 37}$ or $|ce| = 7{\cdot}8071$ 4m
$^{|ae|}/_{\sin 33} = {}^5/_{\sin 37}$ or $|ae| = 4{\cdot}5250$ 4m
$^{|ed|}/_{\sin 14} = {}^{4{\cdot}5250}/_{\sin 23}$ or $|ed| = 2{\cdot}8017$ 7m
$|cd| = |ce| + |ed| = 10{\cdot}6088$ 9m
$\simeq 10{\cdot}61$ 10m



Blunders (-3)
B1 Error in sine rule and continues (once).
B2 Incorrect substitution and continues.
B3 Misplaced decimal point (once).
B4 Incorrect function read, e.g. $\cos 23^o$ instead of $\sin 23^o$ (once in this part).
B5 Incorrectly uses radian mode (once).
B6 Uses ab//dc or bc//ad or isosceles triangle and continues with sine or cosine rule.
B7 Error in calculating $|\angle cad|$, if not an obvious slip.
B8 Calculates |ad| instead of |cd| correctly, with work, if task is not oversimplified.

Slips (-1)
S1 Each numerical slip to a maximum of 3.
S2 Fails to round off correctly.

Attempts (3 marks)
A1 10·61 or 10·60something without work.
A2 Sin 23^o or similar and stops.
A3 $|\angle cad| = 124^o$.
A4 Some correct use of sin/cos/tan.
A5 Some relevant step.
A6 Oversimplified work, e.g. Δacd is isosceles => |ad| = 5 or abcd is a ▱ $\Rightarrow |cd| = 7{\cdot}39$.

Worthless (0 marks)
W1 Incorrect answer without work subject to A1.
W2 Formula taken from tables and stops.

QUESTION 6

Part (a)	10 marks	Att 4
Part (b)	20 marks	Att 8
Part (c)	20 marks	Att 8

Part (a)	10 marks (5, 5)	Att 4 (2, 2)

There are eight questions on an examination paper.

(i) In how many different ways can a candidate select six questions?

(ii) In how many different ways can a candidate select six questions if one particular question must always be selected?

(a) (i)	5 marks	Att 2

α	β	γ	δ	ε	λ	θ
$\binom{8}{6}$	$\frac{8.7.6.5.4.3}{6.5.4.3.2.1}$	28	$\frac{20160}{720}$	$\frac{8!}{6!.2!}$	$\frac{{}^8P_6}{6!}$	List
$\binom{8}{2}$	$\frac{8.7}{2.1}$		$\frac{56}{2}$		$\frac{{}^8P_2}{2!}$	

(a) (ii)	5 marks	Att 2

α	β	γ	δ	ε	λ	θ
$1.\binom{7}{5}$	$\frac{7.6.5.4.3}{5.4.3.2.1}$	21	$\frac{2520}{120}$	$\frac{7!}{5!.2!}$	$\frac{{}^7P_5}{5!}$	List
$\binom{7}{2}$	$\frac{7.6}{2.1}$		$\frac{42}{2}$		$\frac{{}^7P_2}{2!}$	

* Accept correct answer without work.
* No penalty for $\left(\frac{8}{6}\right)$ in (i) but $\frac{8}{6}$ is B2 + B3, i.e. award Att 2.
* If sections of (a) are not identified and it is not obvious which section is being attempted treat each section in order.
* No penalty for omitting 1 in (ii) α method.
* Apply penalties to each part, even for repeated errors.
* Multiplication, if it is required, must be indicated or implied.
* Accept candidate to use his/her answer from part (i) in part (ii), e.g. (i) ${}^6C_2$, (ii) ${}^5C_2$ merits 2 + 5 marks **or** (i) ${}^{10}C_6$, (ii) ${}^9C_5$ merits 2 + 5 marks.

Blunders (-3)

B1 Inverted, e.g. ${}^6C_8$.

B2 "Top" section incorrect, e.g. ${}^9C_6$ or ${}^9C_2$ or ${}^{(8.7.6.5.4.3.2.1)}/_{(6.5.4.3.2.1)}$.

B3 "Bottom" section incorrect, e.g. ${}^8C_3$ or ${}^8P_6$ or ${}^7P_5$.

B4 Addition instead of multiplication or necessary multiplication not indicated, e.g. [8] [7]

Slips (-1)

S1 Each selection omitted in List method, subject to attempt.

Attempts (2 marks each part)

A1 One relevant step and stops, e.g. 6! in (i) or 5! in (ii).

A2 Any relevant integer written, i.e.1 to 8 in (i) or1 to 7 in (ii) or any integer from our solutions.

A3 Writes any combination, permutation or factorial symbol & stops, e.g. ! or C or ( ) or L

Part (b) **20 marks (5, 5, 5, 5)** **Att 8 (2, 2, 2, 2)**

A meeting is attended by 23 men and 21 women.
Of the men, 14 are married and the others are single.
Of the women, 8 are married and the others are single.

(i) A person is picked at random. What is the probability that the person is a woman?
(ii) A person is picked at random. What is the probability that the person is married?
(iii) A man is picked at random. What is the probability that he is married?
(iv) A woman is picked at random. What is the probability that she is single?

(b)(i) **5 marks** **Att 2**

α	β	γ	δ
$^{21}/_{44}$ or $^{8}/_{44}+{}^{13}/_{44}$	0·47 or 0·48	$1-{}^{23}/_{44}$	Sample Space and continues

(b)(ii) **5 marks** **Att 2**

α	β	γ	δ
$^{22}/_{44}$ or ½ or equivalent or $^{14}/_{44}+{}^{8}/_{44}$	0·5	$1-{}^{22}/_{44}$	Sample Space and continues

(b)(iii) **5 marks** **Att 2**

α	β	γ	δ
$^{14}/_{23}$	0·60 or 0·61	$1-{}^{9}/_{23}$	Sample Space and continues

(b)(iv) **5 marks** **Att 2**

α	β	γ	δ (Sample Space)
$^{13}/_{21}$	0·61 or 0·62	$1-{}^{8}/_{21}$	mw1 mw2 mw3 mw4 mw5 mw6 mw7 mw8 sw1 sw2 sw3 sw4 sw5 sw6 sw7 sw8 sw9 sw10 sw11 sw12 sw13

married woman ← mw; single woman ← sw

and continues to $^{13}/_{21}$

* Accept correct or consistent answers without work.
* Accept equivalent values, e.g. 21:44 or $^{42}/_{88}$ for (i).
* Accept decimals, rounded off or truncated.
* If sections of (b) are not identified and it is not obvious which section is being attempted, treat each section in order.

	Men	Women	Total
Married	14	8	22 (ii)
Single	9 (iii)	13 (i) (iv)	22
Total	23	21	44 (i) (ii)

* Do not penalise same error in sample space for (i) and (ii), e.g. $^{21}/_{46}$ for (i) and $^{22}/_{46}$ for (ii).
* Once a fraction is written, ignore subsequent errors.
* Penalise same error in each part, but allow candidate to correctly use an incorrect answer from one part in a subsequent part.

Blunders (-3)
B1 Inverted fraction (penalise once in each part, if relevant).
B2 Each incorrect #Event (without work and if not consistent with previous value).
B3 Each incorrect #Sample space (without work and if not consistent with previous value).

Slips (-1)
S1 Each obvious numerical slip to a maximum of 3.

Attempts (2 marks each part)
A1 One correct step, e.g. any incorrect ratio (in the form $^{a}/_{b}$ or a:b) without work.

A2 Relevant integer written down with or without work and with or without other integers. (i) 8,13, 21, 23, 44. (ii) 1, 2, 8,14, 22, 44 (iii) 9, 14, 23. (iv) 8, 13, 21. } or any number written in relevant solution.

A3 Award 1 × Att 2 for each part where an incomplete (obvious) sample space has been used, e.g. the boxed numbers in the table merit 4 × Att 2 for the indicated parts.

Part (c) **20 marks (5, 5, 5, 5)** **Att 8 (2, 2, 2, 2)**

The digits 0, 1, 2, 3, 4, 5 are used to form four-digit codes. A code cannot begin with 0 and no digit is repeated in any code.

(i) Write down the largest possible four-digit code.
(ii) Write down the smallest possible four-digit code.
(iii) How many four-digit codes can be formed?
(iv) How many of the four-digit codes are greater than 4000?

(c)(i) **5 marks** **Att 2**

5432

(c)(ii) **5 marks** **Att 2**

1023

* If sections of (c) are not identified and it is not obvious which section is being attempted, treat each section in order.
* Accept correct or consistent answers without work.
* Accept 5, 4, 3, 2 or any symbol between the digits.
* Four-digit code not beginning with 0 and no digit repeated:
(c)(i) Largest code……………………………Award 5 marks
Not largest: Award 3 marks if 1st digit is *5*, plus 1 mark if others are in descending order.
If 1st digit is not *5*, award Att 2 marks.
(c)(ii) Smallest code…………………………Award 5 marks.
Not smallest: Award 3 marks if 1st digit is *1*, plus 1 mark if others are in ascending order.
If 1st digit is not *1*, award Att 2 marks.

Misreadings (-1)

M1 Each **obvious** misreading of an available digit to a maximum of 3 from **total** of 10 marks.

Attempts (2 marks each part) [subject to M1]

A1 Any four-digit code containing digits other than those available (each part).
A2 Any four-digit code containing only available digits but beginning with 0 (each part).
A3 Any four-digit code containing only available digits but with digit(s) repeating (each part) subject to the award of 3 marks above.
A4 Any non-four-digit number with digits in correct order, e.g. 54321 and 012345.

Worthless (0 marks)

W1 Other non-four-digit codes without work or 0,1,2,3,4,5 (in the question paper).

(c)(iii)		**5 marks**		**Att 2**
α	β	γ	δ	
5.5.4.3	300	$5.{}^5P_3$	List	

(c)(iv)			**5 marks**			**Att 2**
α	β	γ	δ	ε	λ	θ
2.5.4.3	120	$2.{}^5P_3$	List	5.4.3 + 5.4.3	$^2/_5$ Ans(iii)	Ans(iii) - 180

* Accept correct or consistent answers or any expression that will lead to correct answers without work.

* Multiplication must be clearly indicated. [See B1.]

* Penalise repeated errors in each section but allow the candidate to use the (incorrect) result of part (iii) in part (iv), e.g. 10 for (iii) and $^2/_5(10)$ or 4 for (iv) merits 0 marks for (iii) and 5 marks for part (iv), if without work.

Blunders (-3)

B1 Multiplication not clearly indicated, e.g. [5] [5] [4] [3] (each part).
B2 Each incorrect or omitted "box".
B3 Addition instead of multiplication and vice versa.
B4 (iv) = k(iii), k ≠ $^2/_5$, with work.
B5 Combination instead of permutation.

Slips (-1)

S1 Each entry omitted in List method, subject to attempt.

Attempts (2 marks each part)

A1 One correct step.
A2 One or more boxes drawn.
A3 At least one four-digit code listed each part where relevant.
A4 Writes any permutation or factorial or combination and stops.
A5 Correct relevant fraction, e.g. $^2/_5$.
A6 Relevant integer written down, (iii) 3, 4 or 5.
(iv) 2, 3, 4, 5 or 180.
A7 If an attempt mark in part (iii) oversimplifies a solution to part (iv), then the candidate may be awarded, at most, the attempt mark for part (iv), e.g. using the list method.
A8 5432 - 4000 for part (iv), even if completed to 1432.

QUESTION 7

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a)	**10 marks**	**Att 3**

Calculate the mean of the following numbers:

1, 0, 1, 5, 2, 3, 9.

(a)	**10 marks**	**Att 3**

α: 1+0+1+5+2+3+9 or 7 $\Rightarrow \dfrac{1+0+1+5+2+3+9}{7} = {}^{21}/_{7}$ or 3

β: ${}^{1}/_{7} + {}^{0}/_{7} + {}^{1}/_{7} + {}^{5}/_{7} + {}^{2}/_{7} + {}^{3}/_{7} + {}^{9}/_{7} = {}^{21}/_{7}$ or 3

γ: (Assumed mean) $\dfrac{(1-2)+(0-2)+(1-2)+(5-2)+(2-2)+(3-2)+(9-2)}{7} + 2 = {}^{7}/_{7}+2$ or 3

δ: $0{\cdot}142 + 0{\cdot}000 + 0{\cdot}142 + 0{\cdot}710 + 0{\cdot}284 + 0{\cdot}426 + 1{\cdot}278 = 2{\cdot}982 \simeq 3$

ε: Sum of numbers is 21 $\Rightarrow {}^{21}/_{7}$ or 3

* Accept 3 without work.
* Method δ: Leaves answer as 2·982 with work…Award 9 marks.
* Method α: No penalty if 0 is omitted in numerator.

Blunders (-3)

B1 Multiplies top numbers instead of adds $\rightarrow {}^{270}/_{7}$ or ${}^{0}/_{7} = 38{\cdot}57$ or 0 with work.

B2 Incorrect divisor (except 1), e.g. ${}^{(1+1+5+2+3+9)}/_{6}$.

B3 Omits a number (other than 0 in addition), e.g. ${}^{1+0+1+5+2+3)}/_{7}$ or ${}^{1+0+1+5+2+3)}/_{6}$.

B4 Uses numbers <u>also</u> as frequencies, i.e. ${}^{(1\times1+0\times0+1\times1+5\times5+2\times2+3\times3+9\times9)}/_{21} = {}^{121}/_{21}$.

Slips (-1)

S1 Each arithmetical slip to a maximum of 3.

Misreadings (-1)

M1 Each obvious misreading of a number from the question, if it does not simplify the task.

Attempts (3 marks)

A1 Partial addition and stops.

A2 Indicates knowledge of mean, e.g. ${}^{\Sigma x}/_{n}$.

A3 States "*Median is 2*" or "*mode is 1*".

Worthless (0 marks)

W1 Incorrect answer without work.

W2 *Median is 5* and stops.

Part (b) **20 marks (5, 10, 5)** **Att 7 (2, 3, 2)**

The following cumulative frequency table refers to the ages of 70 guests at a wedding:

Age (in years)	< 20	< 40	< 60	< 90
Number of guests	6	23	44	70

(i) Copy and complete the following frequency table:

Age (in years)	0 – 20	20 – 40	40 – 60	60 – 90
Number of guests				

[Note: 20 – 40 means 20 years old or more but less than 40 etc.]

(ii) Using mid-interval values, calculate the mean age of the guests.

(iii) What is the greatest number of guests who could have been over 65 years of age?

(b) (i) **5 marks** **Att 2**

Age (in years)	0 – 20	20 – 40	40 – 60	60 – 90
Number of guests	6	17	21	26

* Award 2 marks for 1 correct or consistent and 1 mark for each of the others correct or consistent, e.g. 6, 17, 27, 43 merits 3 marks, i.e. 2 + 1.

Blunders (-3)

B1 Adds instead of subtracts, i.e.

0 - 20	20 - 40	40 - 60	60 - 90
6	29	67	114

. .

B2 Cumulative cumulative table, i.e

0 - 20	20 - 40	40 - 60	60 - 90
6	29	73	143

.

Slips (-1)

S1 Each arithmetical slip to a maximum of 3.

S2 Each frequency omitted, subject to attempt.

Attempts (2 marks)

A1 One correct frequency and stops.

A2 Copies either given table **for this part** and stops.

(b) (ii) **10 marks** **Att 3**

Mid-intervals 10, 30, 50, 75 3m

Mean = $\dfrac{(10\times 6)+(30\times 17)+(50\times 21)+(75\times 26)}{6+17+21+26}$ or $\dfrac{60+510+1050+1950}{70}$ 7m

$^{3570}/_{70}$ or 51 10m

* Accept $^{3570}/_{70}$ or 51 or consistent mean without work.

* $^{70}/_{3570}$ without work is one blunder, B5. Otherwise incorrect (inconsistent) answer without work merits 0 marks, subject to attempt.

* Accept work consistent with candidate's frequency table in part (i).

Blunders (-3)

B1 Mid-intervals not used, with work. [Lower limits → 39·1…, Upper limits → 62·8…, Uses interval widths $^{(6\times 20\,+\,17\times 20\,+\,21\times 20\,+\,26\times 30)}/_{70}$ →23·7…].

B2 Multiplies instead of adds in denominator with work. [$^{3570}/_{55692}$ = 0·0641…]

B3 Adds instead of multiplies in numerator with work. [$^{235}/_{70}$ = 3·357…]

B4 Incorrect (inconsistent) denominator, e.g. $^{3570}/_{4}$ or no denominator (3570) and stops.

B5 $^{70}/_{3570}$ or 0·196… with work.
B6 No frequencies, i.e. $^{(10+30+50+75)}/_{70} = {}^{165}/_{70} = 2{\cdot}357\ldots$ with work.
B7 Omits an interval, each time subject to attempt.
B2 + B3 $^{235}/_{55692} = 0{\cdot}00421\ldots$ with work.
B1 + B6 $^{(20+20+20+30)}/_{70} = {}^{90}/_{70} = 1{\cdot}2857\ldots$ with work.
B4 + B6 $^{(10+30+50+75)}/_{4} = {}^{165}/_{4} = 41{\cdot}25$ with work.

Slips (-1)
S1 Each arithmetical slip to a maximum of 3.
S2 Each incorrect mid-interval to a maximum of 3.

Attempts (3 marks)
A1 Mean = $^{\Sigma fx}/_{\Sigma f}$ and stops.
A2 One correct mid-interval and stops.
A3 A relevant multiplication and stops.
A4 Some correct work, e.g. Σf.

(b) (iii) **5 marks** **Att 2**

Greatest number of guests over 65 years = 70 - 44 or 26

* Accept answer consistent with candidate's frequency table.
* Accept answer without work.

Blunders (-3)
B1 44 and stops, i.e. those under 65 years, subject to candidate's frequency table.
B2 70 + 44 and stops or 70 + 44 = 114.

Misreadings(-1)
M1 Obvious misreading from table with work, e.g. *44 under 65 years of age* and stops.

Attempts (2 marks)
A1 70 - k, where k is not relevant.
A2 $^{26}/_{2}$ or 13 without work.
A3 $26 \times {}^{25}/_{30}$.
A4 Any incorrect grouping of frequencies with work.
A5 Draws a (cumulative) frequency curve reasonably well, using either table of part (b), whether the number of guests over 65 years of age is found or not.

Worthless (0 marks)
W1 Any incorrect number other than those given above as meriting marks without work.

Part (c) **20 marks (5, 5, 10)** **Att 7 (2, 2, 3)**

The grouped frequency table below refers to the marks obtained by 85 students in a test:

Marks	0 – 40	40 – 55	55 – 70	70 – 100
Number of students	16	18	27	24

[Note: 40 – 55 means 40 marks or more but less than 55 etc.]

(i) What percentage of students obtained 55 marks or higher?

(ii) Name the interval in which the median lies.

(iii) Draw an accurate histogram to represent the data.

(c)(i) **5 marks** **Att 2**

$$\frac{(27+24)\times 100}{85} = {}^{5100}/_{85}\ \%\text{ or }60\%$$

* Accept $^{5100}/_{85}$ or 60 without work.

Blunders (-3)

B1 $^{(27 \times 24 \times 100)}/_{85} = {}^{64800}/_{85}$ or 762·352… with work.

B2 $^{(27 + 24 + \text{a number})}/_{85}$ and continues, e.g. $^{(27 + 24 + 18)}/_{85} = {}^{6900}/_{85}$ or 81·176… with work.

B3 Omits the 100 or divides by 100. [$^{(27 + 24)}/_{85} = {}^{51}/_{85}$ or $^{3}/_{5}$ with work **or** $^{51}/_{8500}$ or 0·006]

B4 Omits the 27 or 24, i.e. $^{(24 \times 100)}/_{85} = {}^{2400}/_{85}$ or 28·235… with work **or** $^{2700}/_{85}$ = 31·76…

B5 (27 + 24) × $^{85}/_{100}$ and continues.

Slips (-1)

S1 Each arithmetical slip to a maximum of 3.

Misreadings (-1)

M1 ($^{(16+18)}/_{85}$)×100 = $^{3400}/_{85}$ or 40% got <u>less than</u> 55 marks and stops. Apply penalties.

Attempts (2 marks)

A1 Any correct step, e.g. 27 + 24 and stops or indicates ÷ 85 or indicates × 100.

A2 Any other incorrect grouping of frequencies with some work.

Worthles (0 marks)

W1 Any other incorrect answer without work, e.g. 40%.

(c)(ii) **5 marks** **Att 2**

55 - 70

* Accept *3rd interval* or *2nd last interval* or *27* or similar.

* Accept interval implied, e.g. $^{(77 - 55) \times 9}/_{27}$.

Misreadings (-1)

M1 Finds median from ogive (without naming the interval), i.e. $\simeq$ 59·7 with work.

Attempts (2 marks)

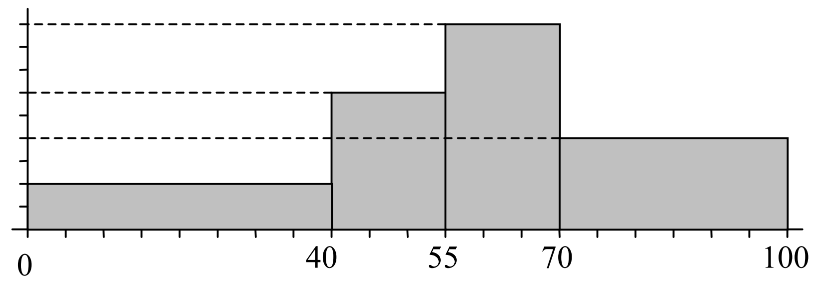
A1 Makes out the cumulative frequency table and/or draws the ogive and stops.

A2 Some correct step if not worth more than attempt, e.g. mentions 43rd or 42·5 or middle student or middle or mid-interval without work.

Worthless (0 marks)

W1 Incorrect interval without work.

(c)(iii) **10 marks** **Att 3**



* Accept any heights in the ratio 2:6:9:4 to the eye, i.e. (without work), e.g. 6:18:27:12.
* Rectangle(s) drawn with correct bases and
  - I: no two having heights in correct ratio…………....Award 3 marks,
  - II: only (any) two having heights in correct ratio.…..Award 4 marks,
  - III: only (any) three having heights in correct ratio.…Award 7 marks.
* Correct histogram possible without labels.
* If three rectangles are correct, candidate must be awarded at least 7 marks.
* Accept axes interchanged.

Blunders (-3)

B1 Each incorrect rectangle or rectangle omitted, subject to attempt.
B2 Inconsistent scale (once). [Do **not** penalise same rectangle twice.]
B3 Draws a frequency polygon or cumulative chart instead of a histogram subject to slips and blunders thereafter.
B4 Each incorrect height, subject to B2.

Slips (-1)

S1 Each obvious numerical slip to a maximum of 3.
S2 Spaces between rectangles. [Penalise once.]

Attempts (3 marks)

A1 Draws axes and stops (even, without labels or scale).
A2 Some relevant work at calculating a height or makes out a table of heights but no histogram.

QUESTION 8

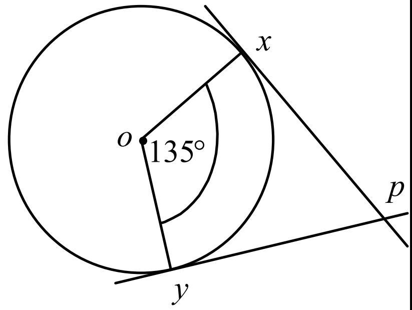
Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks (5, 5)** **Att 4 (2, 2)**

x and y are two points on a circle with centre o.
px and py are tangents to the circle, as shown.

(i) Write down $|\angle pxo|$.

(ii) Given that $|\angle xoy| = 135°$, find $|\angle ypx|$.



(a) (i) **5 marks** **Att 2**

$|\angle pxo| = 90^o$ **or** $|\angle pxo|$ is a right-angle **or** ox $\perp$ xp **or** correctly indicated on diagram

Attempts (2 marks)

A1 $|\angle pxo| = |\angle pyo|$.

A2 $|\angle o| + |\angle x| + |\angle p| + |\angle y| = 360^o$ **for this part**.

A3 Some relevant step, e.g. joins x to y or mention of 22·5.

A4 Any correct mathematical statement about the diagram (unless it merits more marks), e.g. $|\angle ypx| = 45^o$. [Note: $|\angle ypx| = 45^o$ **in (i)** merits 2 marks for (i) and 5 marks for (ii).]

A5 Copies diagram and adds anything relevant, e.g. joins p to o or mentions 135÷2 or 67·5.

(a) (ii) **5 marks** **Att 2**

α:	$|\angle ypx| = [360^o - (90^o + 90^o + 135^o)]$	$= 45^o$	.........4 angles sum to 360^o
β:	$|\angle ypx| = 180^o - 135^o$	$= 45^o$	.........2 right-angles excluded
γ:	$|\angle ypx| = [180^o - (67{\cdot}5^o + 67{\cdot}5^o)]$	$= 45^o$	.........Dealing with Δxyp
δ:	$|\angle ypx| = 2[180^o - (67{\cdot}5^o + 90^o)]$	$= 45^o$	.........Dealing with Δxop

* Accept correct answer without work. * Accept $|\angle ypx| = 45^o$ written in part (i) or diagram.

Blunders (-3)

B1 Sum of 4 angles of quadrilateral $\neq 360^o$ or sum of 3 angles of triangle $\neq 180^o$ when used.

B2 Adds instead of subtracts and vice versa.

B3 Omits an angle in calculations with work, e.g. forgets one $67{\cdot}5^o$ in γ method.

Slips (-1)

S1 Each obvious numerical slip to a maximum of 3.

Attempts (2 marks)

A1 Any relevant step and stops, e.g. mentions 180^o or 360^o.

A2 Relevant construction **for this part** & stops, or addition to diagram relevant only to (ii).
[Same construction does not merit 2 × Att(2) unless repeated.]

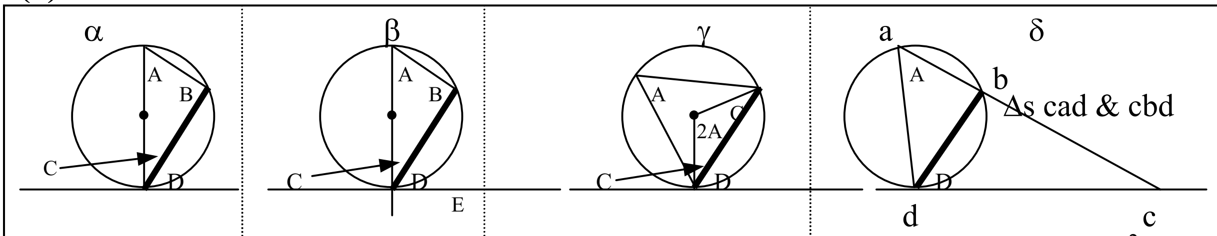
A3 States a relevant theorem and stops.

A4 $|ox| = |xp| \rightarrow |\angle ypx| = 135^o$.

Part (b) **20 marks** **Att 7**

Prove that an angle between a tangent ak and a chord $[ab]$ of a circle has degree-measure equal to that of any angle in the alternate segment..

(b) **20 marks** **Att 7**



α	β	γ	δ	
				5m
$D + C = 90^o$	$E + D = A + B$	$D + C = 90^o$	$\|ca\|.\|cb\| = \|cd\|^2$	5,2
$A + C = 90^o$	$E = B \quad (90^o)$	$2A+2C = 180^o$ or $A+C = 90^o$	$^{\|cb\|}/_{\|cd\|} = {}^{\|cd\|}/_{\|ca\|}$ =>similar Δs	5,2
D = A = (any angle in that segment)				5,2

* If candidate merits more than 0 marks, he/she must get at least 7 marks.

* Accept steps marked clearly on diagram subject to B2.

* In methods α and β the statement "*A = any angle in that segment*" or similar, stated or indicated, is required.

Blunders (-3)

B1 Incorrect step or part of step.

B2 Steps in an illogical order (once).

Attempts (7 marks)

A1 Outline diagram and stops, e.g. circle with tangent **or** chord.

A2 Only one step or part of a step and stops.

A3 Special case, e.g. measured from diagram.

A4 Relevant statement, e.g. *angles in the same segment of a circle are equal in measure*.

A5 Proves *angle at centre of circle* theorem or any of that family of theorems.

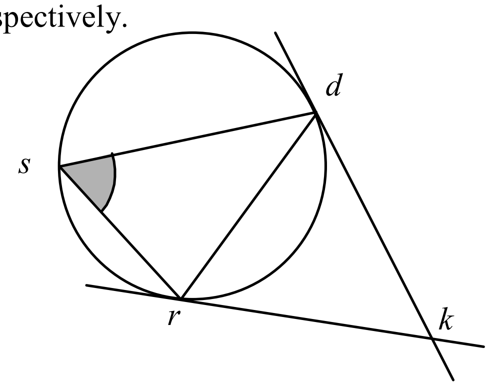
Part (c) **20 marks (5, 10, 5)** **Att 8 (2, 3, 2)**

The lines kd and kr are tangents to a circle at d and r respectively.
s is a point on the circle as shown.

(i) Name two angles in the diagram equal in measure to $\angle dsr$.

(ii) Find $|\angle rkd|$, given that $|\angle dsr| = 65°$.

(iii) Is $|dk| = |rk|$? Give a reason for your answer.

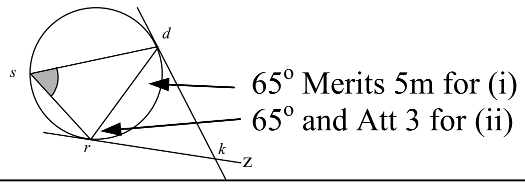


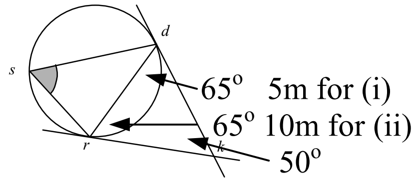
(c)(i) **5 marks** **Att 2**

∠krd	and	∠kdr	**or**	Clearly indicated on the diagram

* Award 4 marks for one correct angle and 5 marks for both correct angles.
* If a candidate has named one correct angle, he/she must be awarded at least 4 marks.
* If a correct angle clearly marked on the diagram, do not penalise incorrect order of points.

Case:





Attempts (2 marks)

A1 Names any pair of angles which are equal in measure, e.g. |∠srz| = |∠sdr|.
A2 Makes a relevant statement, e.g. *angle between a tangent and a chord etc.*
A3 A relevant step, e.g. draws one or two angles in the same segment of the circle as ∠dsr.

Worthless (0 marks)

W1: Incorrect answer without work, subject to attempt.

(c)(ii) **10 marks** **Att 3**

180°	$- (65^\circ + 65^\circ)$	$= 50^\circ$

* Accept correct answer without work.
* Accept candidate's values (if any) for |∠krd| and |∠kdr| from part (c)(i).

Blunders (-3)

B1 Uses $180^\circ - 65^\circ = 115^\circ$.

Slips (-1)

S1 Each obvious numerical slip to a maximum of 3.

Attempts (3 marks)

A1 Any relevant step, statement or addition to diagram, e.g. 180° or $65^\circ+65^\circ$ **for this part**.

(c)(iii) **5 marks** **Att 2**

Yes	Because *Isosceles Δ* **or** *Tangents from same point are equal in measure* **or** …..

* Accept any fair reason. * Accept answer consistent with previous work.

Blunders (-3)

B1 An obviously nonsensical reason.
B2 *Yes* without any reason.

Slips (-1)

S1 A correct relevant reason but did not state that |dk| = |rk| or did not write *yes* or similar.

Attempts (2 marks)

A1 *No* with a plausible reason.
A2 Any correct step or mathematical statement about the diagram **for this part**.
A3 Effort at constructing a tangent to a circle from an external point, e.g. joins centre to k.

QUESTION 9

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 8**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks (5, 5)** **Att 4 (2, 2)**

Let $\vec{p} = -\vec{i} + 2\vec{j}$ and $\vec{w} = 3\vec{i} - 4\vec{j}$.

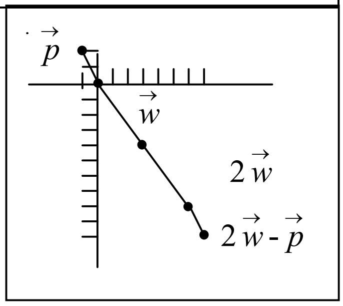
Express, in terms of $\vec{i}$ and $\vec{j}$,

(i) $2\vec{w}$

(ii) $2\vec{w} - \vec{p}$.

(a) (i) **5 marks** **Att 2**

$2\vec{w} \;=\; 2(3\vec{i} - 4\vec{j}) \;=\; 6\vec{i} - 8\vec{j}$



* Accept correct answer without work.

Blunders (-3)

B1 Distributive law error, e.g. $6\vec{i} - 4\vec{j}$ or $3\vec{i} - 8\vec{j}$.

B2 Error with signs, if not an obvious slip, e.g. $6\vec{i} + 8\vec{j}$.

B3 Mixes up $\vec{i}$ s and $\vec{j}$ s.

Slips (-1)

S1 Each numerical slip to a maximum of 3, e.g. $2 \times 3 = 5$. [Not to be confused with B1.]

S2 Correct vector plotted accurately but not expressed in $\vec{i}$ s and $\vec{j}$ s as in the diagram.

Misreadings (-1)

M1 An obvious misreading, e.g. $2\vec{w} = 2(3\vec{i} + 4\vec{j}) = 6\vec{i} + 8\vec{j}$. [S2 and M1 may apply.]

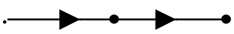
M2 Finds $2\vec{p} = -2\vec{i} + 4\vec{j}$ instead of $2\vec{w}$. [S2 and M2 may apply.] [Task is not simplified.]

Attempts (2 marks)

A1 Stops at $2(3\vec{i} - 4\vec{j})$ or $3\vec{i} - 4\vec{j} + 3\vec{i} - 4\vec{j}$ **or** $2\vec{w} = \vec{w}^2 = 9\vec{i}^2 - 24\vec{i}.\vec{j} + 16\vec{j}^2$ or 25.

A2 $6\vec{i}$ **or** $-8\vec{j}$ without work.

A3 Arbitrary vector drawn and doubled (to the eye), e.g.



A4 A relevant step, e.g. plots w correctly.

A5 $|w| = 5$. [Note: 5 only merits 0 marks.]

(a) (ii) **5 marks** **Att 2**

$2\vec{w} - \vec{p} = 2(3\vec{i} - 4\vec{j}) - (-\vec{i} + 2\vec{j}) = 6\vec{i} - 8\vec{j} + \vec{i} - 2\vec{j} \qquad = 7\vec{i} - 10\vec{j}$

* Accept correct answer without work.

* Accept candidate's $2\vec{w}$ from part (i).

Blunders (-3)

B1 Distributive law error, e.g. $-(-\vec{i} + 2\vec{j}) = \vec{i} + 2\vec{j}$.

B2 Error with signs, if not an obvious slip, e.g. $-8\vec{j} - 2\vec{j} = 10\vec{j}$.

B3 Mixes up $\vec{i}$ s and $\vec{j}$ s.

Slips (-1)

S1 Each numerical slip to a maximum of 3, e.g. $-8\vec{j} - 2\vec{j} = -11\vec{j}$.

S2 Correct vector plotted accurately but not expressed in $\vec{i}$ s and $\vec{j}$ s as in the diagram.

Misreadings (-1)

M1 Obvious misreading that does not simplify the task, e.g. writes $\vec{p} - 2\vec{w} = -7\vec{i} + 10\vec{j}$.

Attempts (2 marks)

A1 Stops at $2(3\vec{i} - 4\vec{j}) - (-\vec{i} + 2\vec{j})$ or $6\vec{i} - 8\vec{j} + \vec{i} - 2\vec{j}$.

A2 $7\vec{i}$ **or** $-10\vec{j}$ without work.

A3 $2\vec{w} + \vec{p}$ found correctly. [Task is simplified.]

A4 p or w plotted correctly **for this part**.

A5 Diagram showing correct triangle or parallelogram law.
[Unmarked triangle is worthless.]

Part (b) **20 marks (5, 5, 5, 5)** **Att 8 (2, 2, 2, 2)**

abcd is a parallelogram. The diagonals intersect at the point *m*.

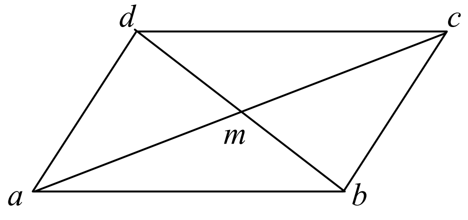
Express each of the following as a single vector

(i) $\vec{ab} + \vec{bm}$

(ii) $\vec{ab} + \vec{ad}$

(iii) $\vec{ac} - \vec{ab}$

(iv) $\frac{1}{2}\vec{ac} + \frac{1}{2}\vec{db}$.

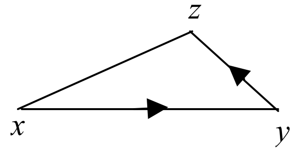


(b)(i) **5 marks** **Att 2**

α: $\vec{ab}+\vec{bm}=\vec{am}$ or $\vec{mc}$ or $\frac{1}{2}\vec{ac}$ or $-\vec{ma}$	β: $\vec{ab}+\vec{bm}=\vec{md}+\vec{dc}=\vec{am}$ or $\vec{mc}$ or ½ $\vec{ac}$
γ: $\vec{ab}+\vec{bm}=\vec{b}-\vec{a}+\vec{m}-\vec{b}=-\vec{a}+\vec{m}=\vec{am}$	δ: $\vec{ab}+\vec{bm}=\vec{ao}+\vec{ob}+\vec{bo}+\vec{om}=\vec{ao}+\vec{om}=\vec{am}$

* Accept any correct answer without work.

ε: $\vec{ab}+\vec{bm}=\vec{xz}$



Blunders (-3)

B1 Incorrect distance, e.g. $\vec{ab}+\vec{bm}=\vec{ac}$.

B2 Incorrect direction, e.g. $\vec{ab}+\vec{bm}=\vec{ma}$.

B3 Incorrect use of origin, e.g. $\vec{ab}=\vec{b}$, unless $\vec{a}$ is stated as the origin.

B4 Error in triangle law, e.g. $\vec{ab}=\vec{a}+\vec{b}$.

B5 Failure to reduce to a single vector, e.g. stops at $\vec{m}-\vec{a}$ in γ method.

Attempts (2 marks)

A1 Any relevant knowledge, e.g. triangle law stated or indicated or $\vec{am}=\vec{mc}$.

A2 Adds to diagram, e.g. draws an arrow or indicates an arbitrary origin.

A3 $\vec{ab}=\vec{b}$ or $\vec{ab}=-\vec{a}$ or answer given as $\vec{m}$ without work and stops.

(b)(ii) **5 marks** **Att 2**

α: $\vec{ab}+\vec{ad}=\vec{ac}$ [parallelogram law] or $-\vec{ca}$	γ: $\vec{ab}+\vec{bc}=\vec{b}-\vec{a}+\vec{c}-\vec{b}=-\vec{a}+\vec{c}=\vec{ac}$
β: $\vec{ab}+\vec{ad}=\vec{ab}+\vec{bc}=\vec{ac}$ [triangle law]	δ: $\vec{ab}+\vec{bc}=\vec{ao}+\vec{ob}+\vec{bo}+\vec{oc}=\vec{ao}+\vec{oc}=\vec{ac}$

* Accept correct answer without work.

Blunders (-3)

B1 Incorrect distance, e.g. $\vec{ab}+\vec{ad}=\vec{am}$.

B2 Incorrect direction, e.g. $\vec{ab}+\vec{ad}=\vec{ca}$.

B3 Incorrect use of origin, e.g. $\vec{ab}=\vec{b}$.

B4 Error in triangle law, e.g. $\vec{ab}=\vec{a}+\vec{b}$ or parallelogram law, e.g. $\vec{ab}+\vec{ad}=\vec{bd}$.

B5 Failure to reduce to a single vector, e.g. stops at $\vec{c}-\vec{a}$ in γ method.

Slips (-1)

S1 Each obvious slip to a maximum of 3.

Misreadings (-1)

M1 Obvious misreading that does not simplify the task, e.g. writes question as $\vec{ab}+\vec{da}$.

Attempts (2 marks)

A1 Any relevant knowledge, e.g. triangle or par. law, stated or indicated, or $\vec{bc}=\vec{ad}$.

A2 Adds to diagram, e.g. draws an arrow or indicates an arbitrary origin **for this part**.

A3 $\vec{ab}=\vec{b}$ or $\vec{ab}=-\vec{a}$ or answer given as $\vec{c}$ without work **for this part** and stops.

(b)(iii) **5 marks** **Att 2**

α: $\overrightarrow{ac} - \overrightarrow{ab} = \overrightarrow{ad}$ or $\overrightarrow{bc}$ or $-\overrightarrow{da}$	γ: $\overrightarrow{ac} - \overrightarrow{ab} = \vec{c} - \vec{a} - (\vec{b} - \vec{a}) = \vec{c} - \vec{a} - \vec{b} + \vec{a}) = \vec{c} - \vec{b} = \overrightarrow{bc}$
β: $\overrightarrow{ac} - \overrightarrow{ab} = \overrightarrow{ac} - \overrightarrow{dc} = \overrightarrow{ad}$ $= \overrightarrow{ao} + \overrightarrow{oc} - \overrightarrow{ao} - \overrightarrow{ob} = \overrightarrow{oc} - \overrightarrow{ob} = \overrightarrow{bc}$	δ: $\overrightarrow{ac} - \overrightarrow{ab} = \overrightarrow{ao} + \overrightarrow{oc} - (\overrightarrow{ao} + \overrightarrow{ob})$

* Accept correct answer without work

Blunders (-3)

B1 Incorrect distance.

B2 Incorrect direction, e.g. $\overrightarrow{ac} - \overrightarrow{ab} = \overrightarrow{da}$ **or** $-\overrightarrow{ab} = \overrightarrow{ab}$ or $\overrightarrow{dc}$.

B3 Incorrect use of origin, e.g.. $\overrightarrow{ac} = \vec{c}$.

B4 Error in triangle law, e.g. $\overrightarrow{ab} = \vec{a} + \vec{b}$ or parallelogram law.

B5 Failure to reduce to a single vector, e.g. stops at $\vec{c} - \vec{b}$ in γ method.

B6 Error in signs when removing brackets in method γ or δ.

Slips (-1)

S1 Each obvious slip to a maximum of 3 or obvious misreading.

Attempts (2 marks)

A1 Any relevant knowledge, e.g. triangle or parallelogram law **or** $\overrightarrow{ad} = \overrightarrow{bc}$ **or** $-\overrightarrow{ab} = \overrightarrow{ba}$.

A2 Adds to diagram, e.g. draws an arrow or indicates an arbitrary origin **for this part**.

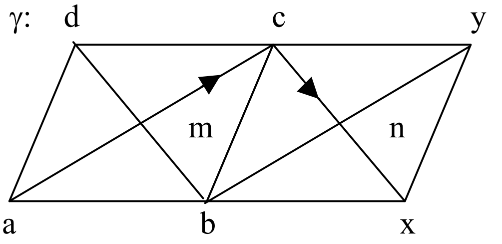
A3 $\overrightarrow{ac} = -\vec{a}$ or $\overrightarrow{ac} = \vec{c}$ or answer given as $\vec{d}$ without work **for this part** and stops.

(b)(iv) **5 marks** **Att 2**

α: $\frac{1}{2}\overrightarrow{ac} + \frac{1}{2}\overrightarrow{db} = \overrightarrow{am} + \overrightarrow{mb} = \overrightarrow{ab}$ or $\overrightarrow{dc}$ or $-\overrightarrow{ba}$	β: $\frac{1}{2}\overrightarrow{ac} + \frac{1}{2}\overrightarrow{db} .= \overrightarrow{mc} + \overrightarrow{dm} = \overrightarrow{dc}$ or $\overrightarrow{ab}$

* Accept correct answer without work.

γ:



$\frac{1}{2}\overrightarrow{ac} + \frac{1}{2}\overrightarrow{db} .= \overrightarrow{mc} + \overrightarrow{cn} = \overrightarrow{mn}$ or $\overrightarrow{bx}$

Blunders (-3)

B1 Incorrect distance, e.g. deals incorrectly with ½ .

B2 Incorrect direction, e.g. $\overrightarrow{am} + \overrightarrow{mb} = \overrightarrow{ba}$.

B3 Incorrect use of origin, e.g.. $\overrightarrow{ac} = \vec{c}$.

B4 Error in triangle law or parallelogram law.

B5 Fails to reduce to single vector, e.g. stops at $\vec{b} - \vec{a}$.

Slips (-1)

S1 Each obvious slip to a maximum of 3 or a misreading which does not simplify the task.

Attempts (2 marks)

A1 Any relevant knowledge, e.g. triangle or par. law **or** $\overrightarrow{am} = \overrightarrow{mc}$ **or** $\overrightarrow{am} = ½\ \overrightarrow{ac}$.

A2 Adds to diagram, e.g. draws an arrow or indicates an arbitrary origin **for this part**.

A3 $\overrightarrow{ac} = -\vec{a}$ or $\overrightarrow{ac} = \vec{c}$ or answer given as $\vec{b}$ without work **for this part** and stops.

Part (c) **20 marks (10, 10)** **Att 6 (3, 3)**

Let $\vec{x} = 3\vec{i} + 4\vec{j}$ and $\vec{y} = 5\vec{i} + 12\vec{j}$.

(i) Show that $\left|\vec{x}\right| + \left|\vec{y}\right| > \left|\vec{x} + \vec{y}\right|$.

(ii) Write down $\vec{x}^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$ and hence, show that $\left|\vec{x}\right|^2 + \left|\vec{x}^{\perp}\right|^2 = \left|\vec{x} - \vec{x}^{\perp}\right|^2$.

(c)(i) **10 marks** **Att 3**

α:	β:	γ: Pythagoras	
$\lvert\vec{x}\rvert = \sqrt{a^2+b^2}$	$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$		
$= \sqrt{9+16} = \sqrt{25}$ or 5	$= \sqrt{(3-0)^2+(4-0)^2} = \sqrt{25}$ or 5	$= \sqrt{25}$ or 5	3m
$\lvert\vec{y}\rvert = \sqrt{25+144} = \sqrt{169}$ or 13	$\sqrt{(5-0)^2+(12-0)^2} = \sqrt{169}$ or 13	$= \sqrt{169}$ or 13	7m
$\lvert\vec{x}+\vec{y}\rvert = \lvert 8\vec{i}+16\vec{j}\rvert = \sqrt{320}$ or 17·8	$\sqrt{(8-0)^2+(16-0)^2} = \sqrt{320}$ or 17.8	$= \sqrt{320}$ / 17·8	9m
	Conclusion		10m

* Accept each measure without work.

* Accept the 3 vectors plotted and *measure of 2 sides of Δ are together greater than 3[rd]* .

Blunders (-3)

B1 Incorrect relevant formula,
e.g. $\sqrt{(x_2-x_1)^2-(y_2-y_1)^2}$ or $\sqrt{(x_2+x_1)^2+(y_2+y_1)^2}$ or $\sqrt{a^2-b^2}$.

B2 Mathematical blunder, e.g. $8^2 = 16$.

B3 Leaves measure as $\sqrt{5^2+12^2}$ (once).

B4 $\vec{i}^{\,2}$ or $\vec{j}^{\,2} \neq 1$ (once in this part).

Slips (-1)

S1 Numerical slips to a maximum of 3, e.g. 5 + 3 = 9.

S2 **One non-central** sign in dist. formula or its substitution, e.g. $\sqrt{(x_2-x_1)^2+(y_2+y_1)^2}$.

Attempts (3 marks)

A1 Correct relevant formula or one with $x_2–x_1$ or $y_2–y_1$ with a correct substitution & stops.

A2 Some relevant step, e.g. $3^2 = 9$ or 3 + 5 = 8 or $\sqrt{9\vec{i}+16\vec{j}}$.

A3 Plots one or more relevant vectors reasonably well.

(c)(ii) **10 marks** **Att 3**

$\vec{x}^{\perp} = -4\vec{i} + 3\vec{j}$	3 marks
$\lvert\vec{x}\rvert^2 + \lvert\vec{x}^{\perp}\rvert^2 = [(3)^2 + (4)^2] + [(-4)^2 + (3)^2] = [9 + 16] + [16+9] = 25 + 25$ or 50	7 LHS **or** RHS
$\lvert\vec{x} - \vec{x}^{\perp}\rvert^2 = \lvert(3\vec{i} + 4\vec{j}) - (-4\vec{i} + 3\vec{j})\rvert^2 = \lvert 3\vec{i} + 4\vec{j} + 4\vec{i} - 3\vec{j}\rvert^2$	
$= \lvert 7\vec{i} + \vec{j}\rvert^2 = 7^2 + 1^2 = 49 + 1$ or 50	10 LHS **and** RHS

* Candidate may use methods outlined in (c) (i).
* Conclusion is not required.

Blunders (-3)

B1 Blunder in finding $\vec{x}^{\perp}$.
B2 Incorrect relevant formula,
e.g. $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$ or $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$ or $\sqrt{a^2 - b^2}$.
B3 Mathematical blunder, e.g. $7^2 = 14$.
B4 Leaves measure as $\sqrt{7^2 + 1^2}$ **or** $\lvert\vec{x}\rvert = \sqrt{25}$, $\lvert\vec{x}^{\perp}\rvert = \sqrt{25}$, $\lvert\vec{x} - \vec{x}^{\perp}\rvert = \sqrt{50}$ (once).

B5 Error removing brackets, e.g. $(3\vec{i} + 4\vec{j}) - (-4\vec{i} + 3\vec{j}) = 3\vec{i} + 4\vec{j} - 4\vec{i} - 3\vec{j}$.

B6 $\vec{i}^{\,2}$ or $\vec{j}^{\,2} \neq 1$ (once in this part).

Slips (-1)

S1 Numerical slips to a maximum of 3.
S2 **One non-central** sign in dist. formula or its substitution, e.g. $\sqrt{(x_2 - x_1)^2 + (y_2 + y_1)^2}$.

Misreadings (-1)

M1 Obvious misreading which does not simplify the task, e.g. deals with $\vec{y}^{\perp}$ correctly.

Attempts (3 marks)

A1 Correct relevant formula or one with $x_2 - x_1$ or $y_2 - y_1$ with a correct substitution and stops.
A2 Some relevant step, e.g. $3^2 = 9$ or $\sqrt{9\vec{i} + 16\vec{j}}$ or $\lvert\vec{x}\rvert^2 = 25$ and stops.
A3 Image of $\vec{x}$ under symmetry in origin or axis, e.g. $3\vec{i} - 4\vec{j}$ or coeffs. switched and stops.
A4 Plots one or more relevant vectors reasonably well **for this part**.

QUESTION 10

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 6**

Part (a) (i)&(ii)	**10 marks**	**Att 3**

(i) Calculate each of the following correct to three decimal places

$$\left(\frac{1}{2}\right)^6, \left(\frac{1}{2}\right)^7, \left(\frac{1}{2}\right)^8, \left(\frac{1}{2}\right)^9.$$

(ii) Hence, write down $\lim_{n\to\infty}\left(\frac{1}{2}\right)^n$.

(a) (i) & (ii)	**10 marks**	**Att 3**

(a) (i)

α: $(^1/_2)^6 = {^1/_2} \times {^1/_2} \times {^1/_2} \times {^1/_2} \times {^1/_2} \times {^1/_2} = {^1/_{64}} = 0{\cdot}015625 \simeq 0{\cdot}016$
β: $= 0{\cdot}5 \times 0{\cdot}5 \times 0{\cdot}5 \times 0{\cdot}5 \times 0{\cdot}5 \times 0{\cdot}5 = 0{\cdot}015625 \simeq 0{\cdot}016$
γ: $= {^1/_{64}} = 0{\cdot}015625 \simeq 0{\cdot}016$ } 3 marks

$(^1/_2)^7 =$ similar $= 0{\cdot}0078125 \simeq 0{\cdot}008$
δ: $½(^1/_{64}) = {^1/_{128}} = 0{\cdot}0078125 \simeq 0{\cdot}008$
ε: $½(0{\cdot}015625 = 0{\cdot}0078125 \simeq 0{\cdot}008$
λ: $½(0{\cdot}016) \simeq 0{\cdot}008$ } 5 marks

$(^1/_2)^8$ $\quad {^1/_{256}} = 0{\cdot}00390625 \simeq 0{\cdot}004$ } 6 marks

$(^1/_2)^9$ $\quad {^1/_{512}} = 0{\cdot}001953125 \simeq 0{\cdot}002$ } 7 marks

(a) (ii) $\lim_{n\to\infty}\left(\frac{1}{2}\right)^n = \lim_{n\to\infty}\frac{1}{2^n} = 0$ — 10 marks

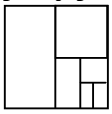
* Accept answers without work.
* Award 3 marks any part of (i) correct or consistent, a further 2 marks for a 2nd part, a further 1 mark for each of a 3rd and/or 4th part correct or consistent, subject to **one** blunder.
* Award 3 marks for part (ii) correct, hit or miss.

Blunders (-3)
B1 Leaves in fraction form (once).
B2 Error in use of indices, e.g. $(^1/_2)^6 = {^1/_{12}}$ or ${^6/_2}$ or ${^1/_{32}}$ **or** $(^1/_2)^6 = (^1/_2)^3 \times (^1/_2)^2$ [once].
B3 Error in use of indices going from one part to another, e.g. $(^1/_2)^7 = (^1/_2)^6 + (^1/_2)^1$ [once and subject to 2nd * above].

Slips (-1)
S1 Each numerical slip to a maximum of 3.
S2 Fails to round off or rounds off incorrectly or too early, if it affects answer (once).

Attempts (3 marks)
A1 Some correct manipulation or power of $^1/_2$, e.g. $^1/_4$ and stops.
A2 Some relevant knowledge of indices, $x^6 \times x = x^7$, or some correct term in expansion of $(1+x)^6$, e.g. 6x or $(1+x)^6 = 1$. [1 without work merits 0 marks.]

A3 Any expression (arbitrary or otherwise) divided by 2, e.g. 0·5 or $^x/_2$ where x ≠ 1.
* If candidate got 0 marks for part (i), he/she may merit the Att (3) from (ii) [**Not both**]
A4 Any term of ($^1/_2)^n$ for n > 9, e.g. 0·002 ÷ 2 (= 0·001).
A5 $1^n = 1$ or $^1/_\infty$ and stops
A6 $(^1/_2)^n = \frac{1}{2^n}$ or $(^1/_2)^n = {^1/_2} \times {^1/_2} \times {^1/_2} \times {^1/_2} \times \ldots$ or 0·5 × 0·5 × 0·5 × 0·5 ×…
A7 $S_\infty = {^a/_{(1-r)}}$, even if continued.
A8 Some relevant step, e.g. *getting smaller* or diagram.



Part (b) **20 marks (10, 5, 5)** **Att 7 (3, 2, 2)**

The first term of a geometric series is 3. The second term of the series is 12. **(i)** Write down the common ratio. **(ii)** What is the fifth term of the series? **(iii)** Calculate the sum of the first nine terms of the series.

(b)(i) **10 marks** **Att 3**

Common ratio is $^{12}/_3$ or 12 ÷ 3 or 4

* Accept correct answer without work.
* If 10 marks are not awarded for (b)(i), award the marks if 4 is subsequently used or implied in (b)(ii) or (iii).

Blunders (-3)
B1 $^3/_{12}$ (or $^1/_4$), even without work or unless it is not an obvious slip.
B2 $3r = 12$ and stops or continues with an error in transposition, e.g. $3r = 12 \Rightarrow r = 9$.

Slips (-1)
S1 Each obvious slip to a maximum of 3.

Attempts (3 marks)
A1 $\frac{T_2}{T_1}$ *or* $\frac{T_{n+1}}{T_n}$ and stops.
A2 Some relevant step, e.g. *ar* and stops or ar^k for any *k*.

Worthless (0 marks)
W1 $r = 12 - 3 = 9$ only or $3 + r = 12 \Rightarrow r = 9$ or 9 without work.

(b)(ii) **5 marks** **Att 2**

α:	3, 12, 48, 192, 768
β:	$T_5 = ar^4 = 3 \times 4^4 = 3 \times 256$ or 768

* Accept correct answer without work. * Accept candidate's r from (i).

Blunders (-3)
B1 Mathematical blunder, e.g. $3(4)^4 = 12^4 = 20736$ or $3 \times 4^4 = 3 \times 16$.
B2 Incorrect relevant formula, e.g. ar^5 or $a + r^4$ or S_5 or inconsistent substitution and continues.
B3 Leaves answer as 3×4^4.

Slips (-1)
S1 Each obvious numerical slip or misreading to a maximum of 3.

Misreadings(-1)
M1 An **obvious** misreading, e.g. $T_4 = 3(4)^3 = 192$ and stops.

Attempts (2 marks)
A1 $T_n = ar^{n-1}$ or $T_1.r^{n-1}$ written **for this part** or $T_5 = ar^4$ and stops.
A2 Some relevant step, e.g. 48, i.e. T_3 **or** 4^2 or 4^3 or 4^4 written and stops.

Worthless (0 marks)
W1 A.P. formula or $T_1 + 4r$, even if completed, subject to attempt.

(b)(iii) **5 marks** **Att 2**

α:	$S_n = \dfrac{a(1-r^n)}{1-r} \Rightarrow S_9 = \dfrac{3(1-4^9)}{1-4}$ = - 1(1 - 262 144)	= 262 144 - 1 or 262 143
β:	3 + 12 + 48 + 192 + 768 + 3 072 + 12 288 + 49 152 + 196 608	= 262 143

* Accept candidate's *r* from above.

Blunders (-3)
B1 Incorrect relevant formula, e.g. $\dfrac{a(1+r^n)}{1+r}$ or $\dfrac{a(1-r)^n}{1-r}$ or $\dfrac{1-r^n}{1-r}$ **or** ar^{n-1} and continues.
B2 Mathematical error, e.g. $4^9 = 36$ or $-1(-262\ 143) = -262\ 143$.
B3 Incorrect and inconsistent T_1 or *r*.
B4 Omits a term in method β or omits last step in method α or β.

Slips (-1)
S1 Each numerical slip to a maximum of 3.
S2 **One** sign incorrect in formula.

Attempts (2 marks)
A1 Correct formula for S_n or S_9 and stops.
A2 Correct new term (except for T_9 as in B1), 48 **for this part**, and stops.
A3 Adds some terms correctly, i.e. extra terms written correctly.
A4 Correct or consistent answer without work.

Worthless (0 marks)
W1 Uses formula for arithmetic series.
W2 Writes S_9 and stops, i.e. expression "S_9".

Part (c) **20 marks (10, 10)** **Att 6 (3, 3)**

(i)	€100 is invested at 10% compound interest per annum. Show that the value of the investment is less than €1000 after 24 years and more than €1000 after 25 years.
(ii)	The sum to infinity of a geometric series is 2. The common ratio and the first term of the series are equal. Find the common ratio.

(c)(i) **10 marks** **Att 3**

$A = P[1 + {}^{r}/_{100}]^{n}$ or $P[{}^{(100+r)}/_{100}]^{n}$ 3m

α: 24 years: $A = 100[1 + {}^{10}/_{100}]^{24} = 100[1\cdot1]^{24} = 100[9\cdot8497\ldots] =$ € 984·97 7m

25 years: $A = 100[1 + {}^{10}/_{100}]^{25} = 100[1\cdot1]^{25} = 100[10\cdot8347\ldots] =$ € 1083·47 10m

or 25 years: A = 984·9732…+ 98·497… = € 1083·47 **or** A = 984·9732…× 1·1 = € 1083·47

[3m] [5m] [6m]

β: **1.** 110, **2.** 121, **3.** 133·1, **4.** 146·41, **5.** 161·051, **6.** 177·156, **7.** 194·872, **8.** 214·359, **9.** 235·795, **10.** 259·375, **11.** 285·313, **12.** 313·844, **13.** 345·228, **14.** 379·751, **15.** 417·726, **16.** 459·499, **17.** 505·449, **18.** 555·994, **19.** 611·593, **20.** 672·752, **21.** 740·027, **22.** 814·030, **23.** 895·433,

After 24 years € 984·976 [7m] **After 25 years** € 1083·474 [10m]

γ: $€1000 = 100[1 + {}^{10}/_{100}]^{n} \Rightarrow 10 = [1\cdot1]^{n} \Rightarrow n = {}^{\log 10}/_{\log 1\cdot1} = {}^{1}/_{0\cdot041..} = 24\cdot158\ldots$ years.

δ: 24 years: $A = 100[1 + {}^{10}/_{100}]^{24} = 100[1\cdot1]^{24}$

$\Rightarrow \log A = 2 + 24\log 1\cdot1 = 2 + 0\cdot993.. \Rightarrow A =$ €984·97

25 years $A = 100[1 + {}^{10}/_{100}]^{25} = 100[1\cdot1]^{25}$

$\Rightarrow \log A = 2 + 25\log 1\cdot1 = 2 + 1\cdot034.. \Rightarrow A =$ €1083·47

ε: $100(1\cdot1), 100(1\cdot1)^2, 100(1\cdot1)^3\ldots$ or T_{24} of G.P. $= ar^{23} = 100(1\cdot1)(1\cdot1)^{23} =$ €984·97

$T_{25} = 100(1\cdot1)(1\cdot1)^{24} =$ €1083·47

* Conclusion not required.
* Accept € 984·97… **and** €1083·47… without work.
* In β method, accept rounding off or truncating to 1 or more decimal places, but see B5.

Blunders (-3)

B1 Incorrect relevant formula for compound interest or geometric sequence, e.g. $P[1 - {}^{r}/_{100}]^{n}$ or treats as investment of €100 at beginning of **each** year (once).
B2 Each incorrect substitution in compound interest or geometric sequence formula unless it is an **obvious** slip.
B3 Mathematical blunder, e.g. $100[1\cdot1]^{24} = [110]^{24}$ or incorrect use of logs.
B4 Misplaced decimal point (once).
B5 Rounds off to the nearest euro each year in method β (once).
B6 Transposing error, e.g. in method γ.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

Attempts (3 marks)

A1 Calculates interest or amount for 1 year.
A2 Correct relevant formula & stops or incorrect relevant formula with a correct substitution.

A3 Some relevant step, e.g. 1·1 or $^1/_{10}$ or an obvious 10% of some value.
A4 Simple interest used (with work) or states simple interest formula. [€340 and €350.]
A5 Answer (with €983, €984 or €985 and/or €1082, €1083 or €1084) or (€984·97 **or** €1083·47) without work. [Remember, (€984·97 **and** €1083·47 merits 10m.]

(c)(ii) **10 marks** **Att 3**

	[3m]	[7m]	[10m]
α: $S_\infty = {}^a/_{(1-r)} \Rightarrow {}^a/_{(1-r)} = 2 \Rightarrow {}^r/_{(1-r)} = 2$		$\Rightarrow r = 2(1-r)$	$\Rightarrow r = {}^2/_3$ or $a = {}^2/_3$
β: $a + ar + ar^2 + ar^3 + \ldots = 2$			
$\div a \Rightarrow 1 + a + ar + ar^2 + \ldots = {}^2/_a$		Subtracting $\Rightarrow -1 = 2 - {}^2/_a$	$\Rightarrow a = {}^2/_3$ or $r = {}^2/_3$
γ: $\lim\limits_{n\to\infty} S_n = \lim\limits_{n\to\infty} \dfrac{a(1-r^n)}{1-r} = 2 \Rightarrow \lim\limits_{n\to\infty}(r - r^{n+1}) = 2(1-r)$		$\Rightarrow r - 0 = 2(1-r)$	$\Rightarrow r = {}^2/_3$ or $r = {}^2/_3$

* Accept decimals.

Blunders (-3)
B1 Incorrect relevant formula, e.g. $^a/_{(1+r)}$.
B2 Transposing error, e.g. $1 - r = 2r$ or $r = 2 - 2r \Rightarrow r - 2r = 2$ or $3r = 2 \Rightarrow r = -1$.
B3 Mathematical error, e.g. $2(1 - r) = 2 - r$ or deals with signs incorrectly.
B4 Leaves answer at $3r = 2$.

Slips (-1)
S1 Each obvious numerical slip to a maximum of 3.

Attempts (3 marks)
A1 Correct relevant formula, e.g. $^a/_{(1-r)}$ or $S_n = \dfrac{a(1-r^n)}{1-r}$ and stops.
A2 A relevant step, e.g. $a + ar + ar^2 + \ldots$ or $a + a^2 + a^3 + \ldots$ or $r = {}^{T2}/_{T1}$.
A3 Correct answer **or** 0·6 or better without work.

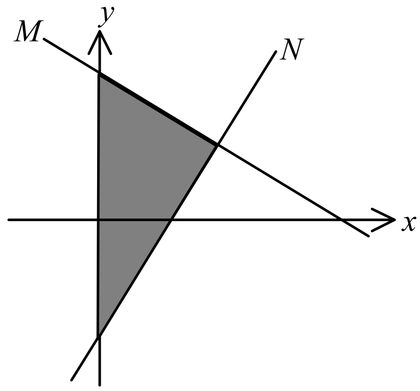
QUESTION 11

Part (a)	**15 marks**	**Att 6**
Part (b)	**35 marks**	**Att 14**

Part (a) **15 marks** **Att 6**

The equation of the line M is $2x + y = 10$.
The equation of the line N is $4x - y = 8$.

Write down the three inequalities that define the shaded region in the diagram.



(a) **15 marks (5, 5, 5)** **Att 6**

M: $2x + y \le 10$	5 marks, Att 2
N: $4x - y \le 8$	5 marks, Att 2
$x \ge 0$	5 marks, Att 2

* Accept correct answers without work.
* Accept answers even if the " = " sign is omitted.
* It is possible to award 2 marks (only) for part (a).

Case 1:	$2x+y = 10$	
	$0+0 < 10$	5 m
Case 2:	$2x+y = 10$	
	$0 + 0 > 10$	2 m

Blunders (-3)
B1 Incorrect inequality symbol (each time).
B2 Mathematical error when testing a point (e.g. sign error).
B3 Incorrect or no conclusion, e.g. case 2.

Slips (-1)
S1 Numerical slip, e.g. $2 \times 0 = 2$ to a maximum of 3.

Attempts (2 marks for each inequality)
A1 Tests (0,0) or any other point and stops (each inequality).
A2 Finds or plots one or more correct points on given line(s) (each line).
* NOTE: (5,0) **and** (2,0) merits 2×Att 2, but (3, 4) merits 2×Att 2.[It is on both lines.]
A3 Some correct step in solving simultaneous equations (once).
A4 $y \ge 0$ or $y \le 0$ instead of $x \ge 0$ (once).
A5 $x = 0$ and stops (once).
A6 $M \le 10$ merits 1 × Att 2, also $N \le 8$ merits 1 × Att 2, each without work.
A7 Some relevant step, e.g. adds something relevant, such as a half-plane arrow, to the diagram.

Worthless (0 marks)
W1 $M \le 0$ and/or $N \le 0$ without work.
W2 Copies diagram and adds nothing to it.

Part (b)	35 marks (20, 10, 5)	Att 14 (8, 4, 2)

A new ship is being designed. It can have two types of cabin accommodation for passengers — type A cabins and type B cabins.

Each type A cabin accommodates 6 passsengers and each type B cabin accommodates 3 passengers. The maximum number of passengers that the ship can accommodate is 330.

Each type A cabin occupies 50 m^2 of floor space. Each type B cabin occupies 10 m^2 of floor space. The total amount of floor space occupied by cabins cannot exceed 2300 m^2.

(i) Taking x to represent the number of type A cabins and y to represent the number of type B cabins, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The income on each voyage from renting the cabins to passengers is €600 for each type A cabin and €180 for each type B cabin. How many of each type of cabin should the ship have so as to maximise income, assuming that all cabins are rented?

(iii) What is the maximum possible income on each voyage from renting the cabins?

(b) (i) Inequalities	10 marks (5, 5)	Att 4 (2, 2)

Passengers $6x + 3y \le 330$
Space $50x + 10y \le 2300$

	A	B	Max.
Passengers	6x	3y	330
Space	50x	10y	2300

* Accept correct multiples/fractions of inequalities or different letters.
* Do not penalise here for incorrect or no inequality sign. Penalise in graph if used.
* Case: 6 3 330 / 50 10 2300 } Award 10 m, but penalise in graph if linkup is incorrect.

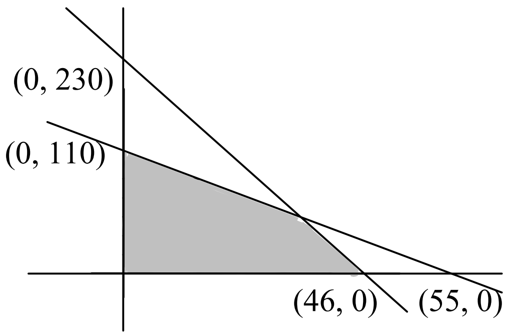
Blunders (-3)

B1 Mixes up x's and y's (once if consistent error).
B2 Confuses rows and columns, e.g. $6x + 50y \le 330$ (once if consistent).
B3 Misplaced decimal point, e.g. $5x + y \le 23$.

Attempts (2 marks for each inequality)

A1 Incomplete relevant data in table and stops (each inequality).
A2 Any other correct inequality, e.g. $x \ge 0$, $y \ge 0$, (each time).
A3 Some variable ≤ 330 or ≤ 2300 (each time).
A4 6x and/or 3y and stops (1 × Att 2).
A5 50x and/or 10y and stops (1 × Att 2).

(b) (i) Graph **10 marks (5, 5)** **Att 4 (2, 2).**



* Each half-plane merits 5 marks, attempt 2 marks.
* Points or scale required.
* Half-planes required but no penalty for not indicating intersection if half-planes are indicated. If half-planes are indicated correctly, do not penalise for incorrect shading.
* Accept correct shading of intersection for half-planes but candidates may shade out areas that are not required and leave intersection blank
* Correct shading over-rules arrows.
* Two lines drawn and intersecting area not indicated apply only one of the following
  Case 1: Two sets of arrows in expected direction………………..………. (full marks)
  Case 2: Two sets of arrows in unexpected direction ………….…..………….(full marks)
  Case 3: Two sets of arrows, inconsistent……..….(1×-3) (one incorrect, one correct)
  Case 4: One set of arrows……………………..(1×-3) (one correct, one line without)
  Case 5: No arrows…………………………………………(2×-3) (two lines without)

Blunders (– 3)
B1 No half-plane indicated (each time).
B2 Blunder in plotting a line or calculations (each line).
B3 Incorrect shading (once), e.g. one or both of small triangles shaded.

Attempts (2 marks each half-plane)
A1 Some relevant work towards a point on a line, i.e. 2 marks for each line attempted.
A2 Draws axes or axes and one line ($1 \times$ attempt 2).
A3 Draws axes and two lines reasonably accurately (award Att 2 + Att 2).

(b) (ii) Intersection of lines **5 marks** **Att 2**

$6x + 3y = 330$	or $2x + y = 110$	$x = 40$
$50x + 10y = 2300$	$5x + y = 230$	$y = 30$

* Accept candidate's own equations from previous parts.
* If x is calculated accept consistent value for y without further work and vice versa.

Blunders (- 3)
B1 Fails to multiply/divide both sides of equation(s) correctly when eliminating variable.
B2 Sign error.
B3 x **or** y value only.
B4 Transposing error.

Slips (- 1)
S1 Each arithmetical slip to a maximum of 3.

Attempts (2 marks)

A1 Correct or consistent answer without work or from a graph.
[Should get same values from graph as if they had been found algebraically.]

A2 Any relevant step towards solving equations.

Worthless (0 marks)

W1 Incorrect answer without work and inconsistent with graph.

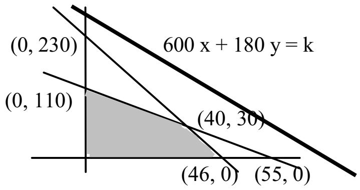
(b) (ii) Income **5 marks** **Att 2**

Income $600x + 180y$

α:	Vertices	600x	+	180y	Income
Step 1	Vertices	600x	+	180y	Income
Step 2	(0, 110)	0	+	19 800	19 800
Step 3	(40, 30)	24 000	+	5 400	29 400
Step 4	(46, 0)	27 600	+	0	27 600

Step 5 40 of A and 30 of B

β:



(40, 30) ⇒ 24 000 + 5 400 ⇒29,400
40 of A and 30 of B

* Accept point of intersection from previous part.
* Information does not have to be in table form.
* Award 1 mark for each consistent step, subject to the attempt mark.
* Accept only vertices consistent with previously accepted work, not arbitrary ones.
If (0, 230) is tested **and** result is used to give max. income, apply -1. Otherwise, ignore.
Accept correct vertices or vertices from candidate's indicated area on non-simplified graph.
* Accept any correct multiple or fraction of 600x + 180y in this part of (b) (ii).
* If no marks have been awarded for intersection of lines and this point is written here, award att. 2 marks for the previous part and also reward it in this part if the step is correct.
* Answer must be explicit, eg. award 4 marks if step 3 is indicated but step 5 not written.
* Testing only (40, 30) to get €29 400 merits Att 2 for this part of (ii), even if candidate writes *40 of A and 30 of B.* [Method α]

Slips (-1)

S1 Each arithmetic slip to a maximum of 3.

S2 Each step of the solution omitted, subject to the attempt mark. [Step 1 may be implied.]

Attempts (2 marks)

A1 Any relevant work involving x or y and/or 600, 180 or similar.

A2 Any attempt at substituting coordinates into some expression.

A3 States 40 of A and/or 30 of B with no other work.

(b) (iii) **5 marks** **Att 2**

Maximum income € 29 400

* 5 steps in table of part (ii) merit 5 marks for part (ii) and 4 marks for part (iii).
* Answer must be explicitly written to merit full marks.
* Accept maximum income consistent with candidate's work with at least 3 non-zero vertices.
* Testing only (40, 30) to get €29 400 [α] and writing *Max = €29 400* merits Att 2 for (iii).
* Cases: At least 3 vertices in table or similar in part b(ii) and
  maximum income specified…………………………..Award 5 marks in b(iii)
  maximum row indicated but income not specified.….Award 4 marks in b(iii)
  incorrect maximum given…………………………....Award A1 subject to 1st *
  maximum neither indicated nor specified……………Award A2.

Less than 3 vertices in table or similar in b(ii) and at least one income calculated…..
Award A2 in b(iii).

Attempts (2 marks)

A1 Some value from the table, other than candidate's max income, as answer, subject to *.
A2 Correct answer without work. Table in (ii) may be regarded as work for (iii).
A3 Multiple or fraction of 600x + 180y leading to incorrect answer in (iii).